From Network Microeconomics to Network Infrastructure Emergence

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***Abstract* – This paper suggests that evolutionary models of network infrastructure in a market economy can be derived from the underlying selfish behavior of users and providers of network services in the same way as non-equilibrium thermodynamics is derived from the underlying statistical physics of interacting particles. This approach may be useful for overcoming restrictions of existing models failing to account for the effect of the details of user/provider selfish behavior on the infrastructure evolutionary path. Network security considerations may be a part of this user/provider behavior. Our main assumption is that "almost perfect competition" keeps the system close to the "social optimum".**

Keywords—network, economics, infrastructure, emergence.

I. INTRODUCTION

In a market economy network evolution is driven by technological constraints and self-interests of the participants. Each user generates demand in attempt to maximize its net utility, which is the difference between the utility of obtaining service and the service price while each service provider attempts to maximize its profit, which is the difference between its revenue and expenses. We model a network by a capacitated graph with links/nodes owned by different providers. The key feature of our model is that providers not only can expand/reduce network capacity within the infrastructure represented by the network graph, but also have the option to modify the network infrastructure by adding/eliminating network links. While capacity expansion within a fixed infrastructure can be done at the marginal cost of the resources, growing network infrastructure by adding links/nodes requires initial and typically significant investment, e.g., for putting fiber underground.

This initial investment typically destroys convexity and causes a multiplicity of Nash equilibrium infrastructures in the natural game-theoretic formalization of the selfish user/provider behavior, making interpretation of this game-theoretic model difficult. This difficulty has led to using game-theoretic models of selfish user/provider behavior only for fixed network infrastructure when the corresponding game typically has a unique Nash equilibrium in demand, pricing, investments, and capacities. Separate modeling of infrastructure growth typically relies on phenomenological random graph models [1]-[3].

However, accounting for the effect of user/provider economic incentives on the network infrastructure evolution may be essential. For example, the result of selfish investment in the network security [4] can be affected if network users and providers have an option of disconnecting from providers who create or propagating security risk. Another example is selfish investment in the bandwidth within existing infrastructure. The inefficiency of the competitive equilibrium in the bandwidth pricings and offerings [5] may be a result of inefficient network infrastructure, which prevents provider competition. In both examples provider ability to modify infrastructure may significantly affect the emerging network infrastructure.

This paper suggests that infrastructure evolutionary models can be derived from the underlying game-theoretic model of selfish user/provider behavior in the same way as non-equilibrium thermodynamics is derived from the underlying statistical physics of interacting particles. We assume time scale separation between "fast" convergence to user/provider equilibrium for a given network infrastructure and "slow" infrastructure evolution modeled by a Markov process. We consider the situation of almost perfect competition in which a large number of providers compete for demand generated by the same users [6]. Based on results from economics [6], as well as results for some particular networks [7]-[9], we conjecture that selfish user/provider behavior maximizes the aggregate user utility subject to provider profitability.

Exploiting similarities between utility maximization and entropy maximization in a closed physical system with fixed energy [10], we propose to model the infrastructure evolution by a time-reversible Markov process. The proposed model includes a possibility of selfish investment in network security and is open to various generalizations. The paper is organized as follows. Section II discusses user/provider competitive and socially optimal equilibria. Section III proposes network infrastructure models consistent with the underlying selfish user/provider interactions. Finally, Section IV briefly summarizes and outlines directions of future research.

II. NETWORK ECONOMIC MODEL

This Section describes economics driving user/provider strategic interactions. Subsection A describes the network model. Subsection B describes the bandwidth supply/demand model.

A. Network

We model the network by a directed capacitated graph with set of nodes N and set of links

$L=\{L_{nk}: n\in N, k\in N\setminus n\}$ where links $l\in L_{nk}$ directly connect node n to node k. The network infrastructure is determined by network graph $g=(N,L)$ and ownership of network elements. Let $s_{nk}=\dim\{L_{nk}\}$ be the number of links directly connecting node n to node k, and C_{nk} be the aggregate capacity of these links.

Limited link capacities impose constraints on the feasible end-to-end throughputs $x_{(n,k)}$ from nodes $n\in N$ to nodes $k\in N\setminus n$

$$x_{(n,k)} = \sum_{r\subset R_{nk}} x_r \, , \tag{1}$$

$$Y_{ij} = \sum_{r:(i,j)\in r\subset R_{nk}} x_r \le C_{ij} \, , \tag{2}$$

where Y_{nk} is the aggregate rate through links $l\in L_{nk}$, vector $x=(x_r, r\in R)$ characterizes end-to-end flow rates on all feasible routes $R=\bigcup_{n,k} R_{nk}$, and the set of feasible routes from node $n\in N$ to node $k\in N\setminus n$ is R_{nk}.

For simplicity of exposition we assume that all links $l\in L_{nk}$ directly connecting node n to node k are identical with respect to their capacities: $c_l = C_{nk}/s_{nk}$ and carried load: $y_l = Y_{nk}/s_{nk}$. In our economic model these assumptions are justifiable when the cost of providing bandwidth c, is the same $\phi_{nk}(c)$, for all links $l\in L_{nk}$. In this case bandwidth providers are inclined to charge the same price for bandwidth p_{nk} on all links $l\in L_{nk}$ and users are indifferent to using links $l\in L_{nk}$ [8]-[9]. Generalization to a case when the cost of providing bandwidth on different links $l\in L_{nk}$ may be different is straightforward.

We assume that bandwidth costs $\phi_{nk}(c)$ are non-decreasing in $c\ge 0$. Communication bandwidth costs $\phi_{nk}(c);\ n\in N, k\in N\setminus n$ are concave in $c\ge 0$, e.g.,

$$\phi_{nk}(c) = a_{nk} + b_{nk}c,\ \ n\in N, k\in N\setminus n \tag{3}$$

where $a_{nk}\ge 0$ characterizes the fixed cost, e.g., cost of putting the fiber underground and $b_{nk}\ge 0$ characterizes the marginal capacity cost.

Our model can be easily generalized to incorporate constraints imposed by limited node processing capacity C_{nn}, $n\in N$. It is natural to assume that costs of the processing bandwidths $\phi_{nn}(C), n\in N$ are convex in $C\ge 0$ due to technological constraints. For example, considered in [11] is the case of hard processing bandwidth constraints $C_{nn}\le C^*$ corresponding to processing cost $\phi_{nn}(C)=0$ if $C\le C^*$, and $\phi_{nn}(C)=\infty$ if $C>C^*$. In this paper we uniformly refer to network nodes and links as network elements.

B. Bandwidth Users and Providers

We model the effect of investment in the network security by assuming that traffic of rate x traversing network element l faces security risk $xh_l(q)$, where $q=(q_l, l\in L)$ is the vector of levels of investment in the security of network elements l is. We assume functions $h_l(q)$ to be decreasing in all vector q components due to positive externalities, i.e., improving security of a network element not only improves this element security; but also is beneficial for the security of the entire network [4].

Assuming that the owner of a network element l charges users price p_l for a unit of this element's bandwidth, the profit generated by network element l is

$$f_l = (p_l - q_l)\sum_{r:l\in r} x_r - \phi_{ll}(c_l) \tag{4}$$

Generally, there are S service providers, with service provider $s\in S$ owning network elements $l\in M_s$. Each network element is owned by some provider: $\bigcup_{s\in S} M_s \equiv L$. Each provider $s\in S$ attempts to maximize its aggregate profit

$$F_s = \sum_{l\in M_s} f_l \tag{5}$$

We assume that each user is uniquely identified by the origin-destination pair $(n,k), n\in N, k\in N\setminus n$. Let $u_{nk}(x)$ be user (n,k)'s utility of obtaining end-to-end bandwidth x from node n to node k. Utility functions $u_{nk}(x)$ are assumed to be monotonously increasing and concave in $x\ge 0$ at least in the case of file transfers. For example, weighted (α,w)- fair bandwidth allocation [12] is based on user utilities

$$u(x|\alpha,w) = \begin{cases} w x^{1-\alpha}/(1-\alpha) & if \quad \alpha\ne 1 \\ w\ln x & if \quad \alpha = 1 \end{cases} \tag{6}$$

where $\alpha, w>0$ are some parameters. In a particular case of proportional fairness, when $\alpha=1$, parameters $w=w_{nk}$ can be interpreted as user (n,k)'s willingness to pay for end-to-end bandwidth x from node n to node k.

User (n,k) net utility is its gross utility of having end-to-end bandwidth minus expenses and expected losses due to security risks:

$$U_{nk} = u_{nk}\left(\sum_{r\in R_{nk}} x_r\right) - \sum_{r\in R_{nk}} x_r \sum_{l\in r}\left[p_l + \eta_{nk} h_l(q)\right] \tag{7}$$

where parameter $\eta_{nk} \geq 0$ quantifies user (n,k)'s sensitivity to the security risk. Each user (n,k) attempts to maximize its net utility (7) over the vector of flow rates $(x_r, r \in R_{st})$, given security risks $h = (h_l, l \in L)$ and prices $p = (p_l, l \in L)$.

III. USER/PROVIDER EQUILIBRIA

This Section introduces notions of user/provider equilibria. Subsection A describes user/provider best responses. Subsection B discusses multiplicity of the competitive (Nash) equilibria of the user/provider game. Subsection C conjectures that under perfect competition these competitive equilibria approach the social optimum.

A. *User/Provider Best Responses*

It is easy to see that user net utility (7) maximization results in minimum cost routing where the adjusted cost of a route is the sum of the adjusted costs of network elements comprising the route

$$d_r(\eta) = \sum_{l \in r} d_l(\eta) \quad (8)$$

and network element l adjusted cost is

$$d_l(\eta) = p_l - \eta h_l(q) \quad (9)$$

Assuming sufficient capacity, the entire user (n,k) aggregate demand

$$x^*_{(nk)} = \arg\max_{x \geq 0} \left[u_{nk}(x) - d^*_{nk} x\right] \quad (10)$$

is sent over minimum cost routes $R^*_{nk} = \{r : d_r = d^*_{nk}, r \in R_{nk}\}$, where

$$d^*_{nk} = \min_{r \in R_{nk}} d_r(\eta_{nk}) \quad (11)$$

Users are indifferent between feasible routes of minimum cost. If there is a single minimum cost route: $R^*_{nk} = r^*_{nk}$, the entire user (n,k) demand is carried on this route:

$$x^*_r = \begin{cases} x^*_{nk} & if \quad r = r^*_{nk} \\ 0 & otherwise \end{cases} \quad (12)$$

In the particular case of user utilities (6): $u_{nk}(x) = u(x|\alpha_{nk}, w_{nk})$, user (n,k) aggregate demand is

$$x^*_{(nk)} = \left(w_{nk} / d^*_{nk}\right)^{1/\alpha_{st}} \quad (13)$$

Further we assume that users always generate the best response demand, and thus element l generates profit

$$f^*_l(p,q) = p_l \sum_{r:l \in r} x^*_r(p,q) - q_l - \phi_l\left[\sum_{r:l \in r} x^*_r(p,q)\right] \quad (14)$$

Expression (14) implies that the optimal provider responses ensure tightness of the capacity constraints (2). Indeed, providers have incentive to eliminate any spare capacity to reduce expenses while in a case of insufficient capacity the provider can increase its revenue by raising the price.

B. *Competitive Equilibria*

Consider a non-cooperative game of providers $s \in S$ attempting to maximize their profits

$$F^*_s(p,q) = \sum_{l \in M_s} f^*_l(p,q) \quad (15)$$

over pricing p_l, and investment in security q_l of their assets $l \in M_s$, where $f^*_l(p,q)$ are given by (14). Typically, this game has multiple Nash equilibria corresponding to different network infrastructures defined by a combination of network topology and ownership of network elements. These Nash equilibria are associated with attractors of the following evolutionary/learning provider adjustments [13]:

$$\dot{q}_l = \partial F^*_s(q,p)/\partial q_l, \quad \dot{p}_l = \partial F^*_s(q,p)/\partial p_l \quad l \in M_s \quad (16)$$

Note that since best user response (8)-(12) $x = x^*(q,p)$ is discontinuous in a case of equal minimum cost multipath routing, the differential equations (16) may have discontinuous right-hand sides. It can be shown that in this case system (16) should be understood as a differential inclusion [14] describing sliding modes along the discontinuity hyperplanes.

Bertrand's model of user/provider equilibria [9], [15] assumes that in "fast" time users/providers achieve equilibrium over user demands $x = (x_r, r \in R)$, bandwidth pricing $p = (p_l)$ and investment in security $q = (q_l)$ subject to the capacity constraints (2) for fixed capacities $c = (c_l)$. On a "slow" time scale bandwidth pricing and investment in security become functions of the network element capacities: $p = p^*(c)$ and $q_l = q^*(c)$. Nash equilibrium capacities $c = c^*$ are associated with attractors of the following bandwidth adjustments [9]:

$$\dot{c}_l = \partial F^*_s[q^*(c), p^*(c)]/\partial c_l \quad l \in M_s, \ s \in S \quad (17)$$

Figure 1 demonstrates multiplicity of provider equilibria achieved by adjustment process (17) on an example of two providers competing for the same demand [9].

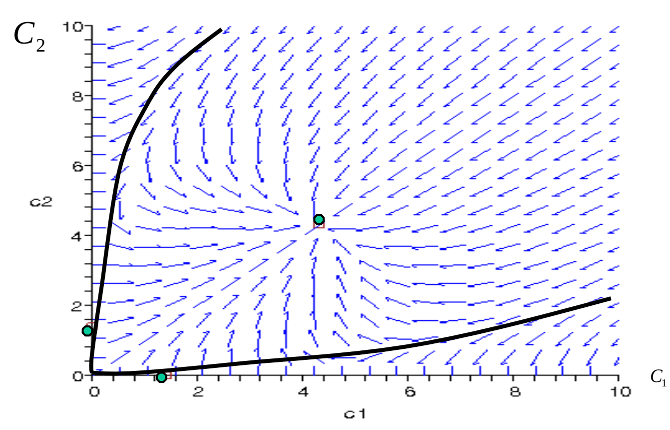

Fig. 1. Capacity adjustments by two competing providers.

Three equilibria are possible: one equilibrium with both providers supplying bandwidth and two equilibria with only one provider supplying bandwidth and another provider driven out of business.

C. Perfect Competition: Social Optimum

In competitive equilibrium providers ensure profitability by charging users a competitive price which is higher than the marginal bandwidth price. It is known from economics [6] that as the number of providers competing for the same demand increases, the competitive prices drop approaching the corresponding marginal prices. This increase in competition squeezes the provider profit margins but increases user net utilities as shown in Figures 2 and 3 respectively for N parallel links competing for demand generated by the same users [8]-[9].

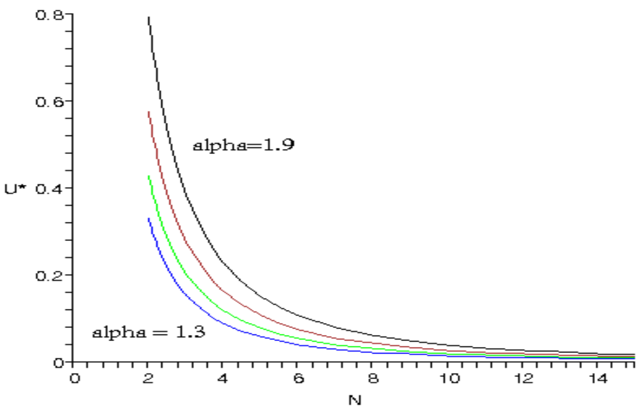

Fig. 2. Provider profit vs. number of competing providers N

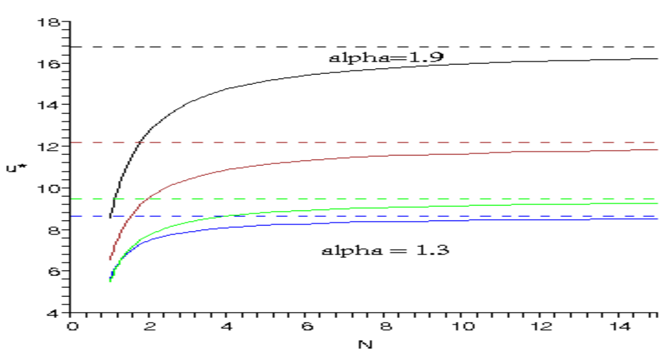

Fig. 3. User net utility vs. number of competing providers N

One may expect that in the limit of perfect competition with a large number of providers competing for the same users the competitive equilibria will approach a social optima which maximizes the aggregate user utility

$$\max_{(x,p,q)} U_\Sigma(x,p,q) \tag{18}$$

$$U_\Sigma(x,p,q) = \sum_{(n,k)} U_{nk}(x,p,q) \tag{19}$$

subject to provider profitability

$$F_M = \sum_{l\in M_s}\left[(p_l - q_l)\sum_{r:l\in r} x_r - \phi_l\left(\sum_{r:l\in r} x_r\right)\right] \geq 0 \tag{20}$$

Note that for bandwidth cost function (3) constraints (20) take the following form:

$$\sum_{l\in M_s}\left[a_l + (b_l + q_l - p_l)\sum_{r:l\in r} x_r\right] \leq 0 \tag{21}$$

It is easy to see that due to the assumed concavity of the communication bandwidth costs $\phi_{nk}(c)$; $n\in N, k\in N\setminus n$, at a social optimum (18)-(20) no more than one link $l = l_{nk}$ connects node $n\in N$ to node $k\in N\setminus n$. This observation allows us to assume in (18)-(20) that at most one link $l = l_{nk}$ connects a node $n\in N$ to a node $k\in N\setminus n$. However, it poses a fundamental question since the social optimization (18)-(20) views system performance from the user perspective, assuming that providers only recover their expenses without making any profit, and this may occur only close to perfect competition with a large number of providers competing for demand generated by the same users [6]. The rest of this Subsection qualitatively demonstrates that low barriers to entering the market are necessary for this situation to occur and for the optimization problem (18)-(20) to approximate the competitive equilibria.

Consider N providers competing for the same demand generated by a user with utility function $u(x) = w\log x$ disregarding network security $(\eta = 0)$. Each provider charges the user a competitive price $p_N x$ for bandwidth x, while the provider's own expense for supplying this bandwidth is $a + bx$. Since the user's individual optimization results in demand $x = w/p_N$ equally divided among all N providers, the provider profitability condition $p_N\, x/N > a + b\,x/N$ takes the form $N < (w/a)(1 - b/p_N)$. Since $p_N \uparrow b$ as $N\uparrow\infty$ (see [9]), the profitability condition imposes an upper bound on the number of competing providers $N < N^*$. Optimization problem (18)-(20) implies a large number of providers N^* driving provider profit down to zero, and it is easy to see that in our simple case this can occur only if the barrier for a provider to enter the market a is much less than user willingness to pay w: $a << w$. It can be shown that this qualitative conclusion holds for general topology networks.

IV. Infrastructure Models

Given set of network nodes N, we characterize network infrastructure by a binary vector $\delta = (\delta_l : l = (n,k), n\in N, k\in N\setminus n)$, where $\delta_{nk} = 1$ if directed link $l = l_{nk}$ from node n to node k exists and $\delta_{nk} = 0$ otherwise. This Section proposes equilibrium and evolutionary models for $\delta = \delta(t)$ under assumptions that "almost perfect" competition keeps the network infrastructure close to socially optimal. The proposed models are based on the underlying economic model of user/provider interactions. Subsection A introduces infrastructure social utility $W^*(\delta)$, which is the result of maximization of the social welfare over bandwidth demand/supply for a given vector δ. Subsection B suggests an entropy-based equilibrium model for vector δ with $W^*(\delta)$ playing the role of the negative energy. Subsection C suggests an approximation of infrastructure evolution by a time-reversible Markov process $\delta = \delta(t)$ used in physics to describe evolution of interacting spins.

A. Infrastructure Utility

Optimization problem (18)-(20) is non-convex due to the assumed concavity of the communication bandwidth cost functions $\phi_{nk}(c);\ n \in N, k \in N \setminus n$. It is known that this non-convexity leads to multiple local maxima of the aggregate utility (19). However, given the vector $\delta(x) = (\delta_l(x))$,

$$\delta_l(x) = \begin{cases} 1 & if \quad \sum_{r:l \in r} x_r > 0 \\ 0 & otherwise \end{cases}, \qquad (22)$$

describing network infrastructure, the aggregate utility maximization (18)-(20) typically becomes a convex optimization problem. The corresponding maximal aggregate utility is a function of vector $\delta(x) = \delta$: $U_\Sigma^*(\delta)$ assuming that infrastructure $\delta(x) = \delta$ can support provider profitability (20). It is natural to interpret $U_\Sigma^*(\delta)$ as the social utility of infrastructure δ. Of course actual evaluation of $U_\Sigma^*(\delta)$ is computationally infeasible, and in the rest of this subsection we briefly discuss some approximations.

Consider the Lagrangian of the optimization problem (18)-(20)

$$W = U_\Sigma - \gamma_M \sum_{l \in M_s} \left[(p_l - q_l) \sum_{r:l \in r} x_r - \phi_l \left(\sum_{r:l \in r} x_r \right) \right] \qquad (23)$$

where $\gamma_M = \gamma_M(p,q)$ are the Lagrange multipliers solving the corresponding dual problem. Let $W^*(\delta)$ be the maximum of function (23) over (p,q,x) for a given vector δ. Due to non-convexity, solutions to the primal (18)-(20) and the corresponding dual problems may differ, and thus $U_\Sigma^*(\delta) - W^*(\delta) \neq const$ (utilities are defined up to additive constant). Nevertheless, the function $W^*(\delta)$ can be viewed as an approximation of the infrastructure utility. The computational advantage of the function $W^*(\delta)$ over $U_\Sigma^*(\delta)$ is that to evaluate $W^*(\delta)$ one has to solve only one optimization problem rather than solve an optimization problem for each vector δ to evaluate function $U_\Sigma^*(\delta)$.

In a case of low barriers to entering the market, further simplification is possible since (23) can be approximated by the difference between the aggregate user utility (18) and aggregate provider expenses:

$$W(x,q) = U_\Sigma(x,q) - \sum_l \left[q_l + \phi_l \left(\sum_{r:l \in r} x_r \right) \right] \qquad (24)$$

In a case of bandwidth cost function (3) social welfare (24) takes the following form:

$$W(\delta,q,x) = -\sum_{l:\delta_l=1} \left[a_l + (b_l + q_l) \sum_{r:l \in r} x_r \right] + \sum_{(n,k)} \left[u_{nk} \left(\sum_{r \in R_{nk}(\delta)} x_r \right) - \eta_{nk} \sum_{r \in R_{nk}(\delta)} x_r \sum_{l \in r} h_l(q) \right] \qquad (25)$$

where the set of available routes from node $n \in N$ to node $k \in N \setminus n$ conditioned on infrastructure δ is

$$R_{nk}(\delta) = R_{nk} \bigcap \left\{ r : \prod_{l \in r} \delta_l = 1 \right\}. \qquad (26)$$

Define the social utility of infrastructure δ to be the maximum value of social welfare (20) achievable for this infrastructure:

$$W^*(\delta) = \max_{q,x} W(\delta,q,x) \qquad (27)$$

Infrastructure utility (25) can be evaluated effectively and even explicitly under realistic assumptions. Due to limited space we only note that in a case of network security insensitive users $(\eta_{nk} = 0)$ with utilities $u_{nk}(x) = w_{nk} \log x$ the infrastructure utility (25) is

$$W^*(\delta) = \sum_{(n,k)} \log \left(w_{nk} \Big/ \min_{r \in R_{nk}(\delta)} \sum_{l \in r} b_l \right) - \\ - \sum_{l:\delta_l=1} \left[a_l + b_l \sum_{(n,k):l \in r_{nk}^*(\delta)} \left(w_{nk} \Big/ \min_{r \in R_{nk}(\delta)} \sum_{l \in r} b_l \right) \right] \qquad (28)$$

where user (n,k) optimal route is

$$r_{nk}^*(\delta) = \arg \min_{r \in R_{nk}(\delta)} \sum_{l \in r} b_l \qquad (29)$$

B. Equilibrium Infrastructure

We model unavoidable uncertainties present in large networks by assuming that vector δ is a binary random variable with some probability distribution $\Omega(\delta)$ and entropy

$$H = -\sum_\delta \Omega(\delta) \log[\Omega(\delta)] \qquad (30)$$

Social optimization either maximizes the infrastructure expected social utility

$$\overline{W}^* = \sum_\delta W^*(\delta) \Omega(\delta) \qquad (31)$$

subject to low bound on entropy (30) or, equivalently, maximizes entropy (30) subject to low bound on the expected infrastructure utility (31).

Solution to these dual optimization problems is

$$\Omega^*(\delta) = Z^{-1} e^{\beta W^*(\delta)} \quad (32)$$

where the parameter $\beta > 0$ is determined by the corresponding optimization constraint and Z is the normalization constant. As uncertainty decreases: $H \to 0$ or equivalently $\beta \to \infty$, the distribution (32) recovers the optimal network infrastructure. Similar to approach [2] we define an equilibrium graph distribution by the entropy maximization procedure. The difference is that in our case the graph energy $E(\delta) = -W^*(\delta)$ is derived from user/provider microeconomics. This opens a possibility of exploring the effect of microeconomics, including network security considerations, on the network infrastructure emergence.

C. Infrastructure Emergence

We assume that competitive pressures may result in elimination of some network elements $l : (\delta_l = 1) \to (\delta_l = 0)$ due to un-profitability and/or adding some network elements $l : (\delta_l = 0) \to (\delta_l = 1)$ due to potential profit. Let vector δ^l be the result of flipping component $\delta_l : \delta_l \to 1 - \delta^l$ in vector δ, i.e., infrastructure described by vector δ^l contains (does not contain) link l if the infrastructure described by vector δ does not contain (contains) link l. Otherwise vectors δ^l and δ are identical.

Viewing vector $\delta = (\delta_l)$ as a system of interacting spins with energy $E(\delta) = -W^*(\delta)$, we model infrastructure evolution by a Markov process $\delta(t) = (\delta_l(t))$, with continuous time $t \in [0, \infty)$ and transitions $\delta \to \delta^l$ with rates $\mu(l, \delta) = \exp[\Delta_l W^*(\delta)]$, where the infrastructure utility change due to this transition is $\Delta_l W^*(\delta) = W^*(\delta^l) - W^*(\delta)$ [16]. Process $\delta(t)$ is time reversible and satisfies conditions of detailed balance. This Markov evolutionary model is consistent with the entropy-based equilibrium model (32) since the process $\delta(t)$ distribution converges to distribution (32) as $t \to \infty$. Moreover, process $\delta(t)$'s relative entropy $\sum_\delta \Omega(t,\delta) \log[\Omega(t,\delta)/\Omega^*(\delta)]$, where $\Omega(t,\delta) = \Pr\{\delta(t) = \delta\}$, monotonously decreases with time.

Process $\delta(t)$, which describes infrastructure drift towards infrastructure utility increase perturbed by random fluctuations, can provide a foundation for phenomenological models of network growth. Connecting a new node n to the network can be viewed as creating link l from node n to some already connected to the network node $k \in N$: $(\delta_l = 0) \to (\delta_l = 1)$. The corresponding change in the infrastructure utility $\Delta_l W^*(\delta)$ weights the benefit of connectivity to a specific node against cost of this connection. It can be shown that, after some simplifying assumptions, network growth based on infrastructure utility gain $\Delta_l W^*(\delta)$ takes the form of "heuristically optimized" [3].

V. Conclusion and Future Research

Assuming that the network infrastructure is represented by the network graph and ownership, this paper suggests that network infrastructure modeling should be based on the network microeconomics. The paper argues that methodologies used in transitions from statistical physics of a large number of interacting particles to thermodynamics of a small number of macro-variables may prove applicable to the networking domain. The proposed approach assumes that "almost perfect competition" keeps the system close to social optimum, which maximizes the aggregate user utility subject to provider profitability. Future research will be concentrated on demonstrating the practical validity of the proposed approach and extending this approach to competitive user/provider equilibria far from perfect competition.

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