Second SIM Metrology School

Time and Frequency Metrology: Fundamental concepts in Time and Frequency metrology

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The Time

- Every human group in history has been concerned with the importance of time:
  - Duration of the day – night
  - The seasons
  - The lunar and solar cycle
  - The birthday!
- A request for the time is the most asked question in the world
  - What time is it?

The Second

- The second is one of the 7 fundamental units of the SI and is denoted with the letter ***s***

- The second can be realized with an uncertainty of less than 10^{-15}

- Since 1970 the second has been defined by the atomic frequency of the Cesium 133 atom:

"The duration of 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom"

The Second

- Multiples and fractions of the second
  - 1 year = 365,2422 days
  - 1 day = 24 hours
  - 1 hour = 60 minutes
  - 1 minute = 60 second

  Based in a Sexagesimal system (base 60)

  - millisecond (ms) = 10^{-3} s
  - microsecond (μs) = 10^{-6} s
  - nanosecond (ns) = 10^{-9} s
  - picosecond (ps) = 10^{-12} s
  - femtosecond (fs) = 10^{-15} s

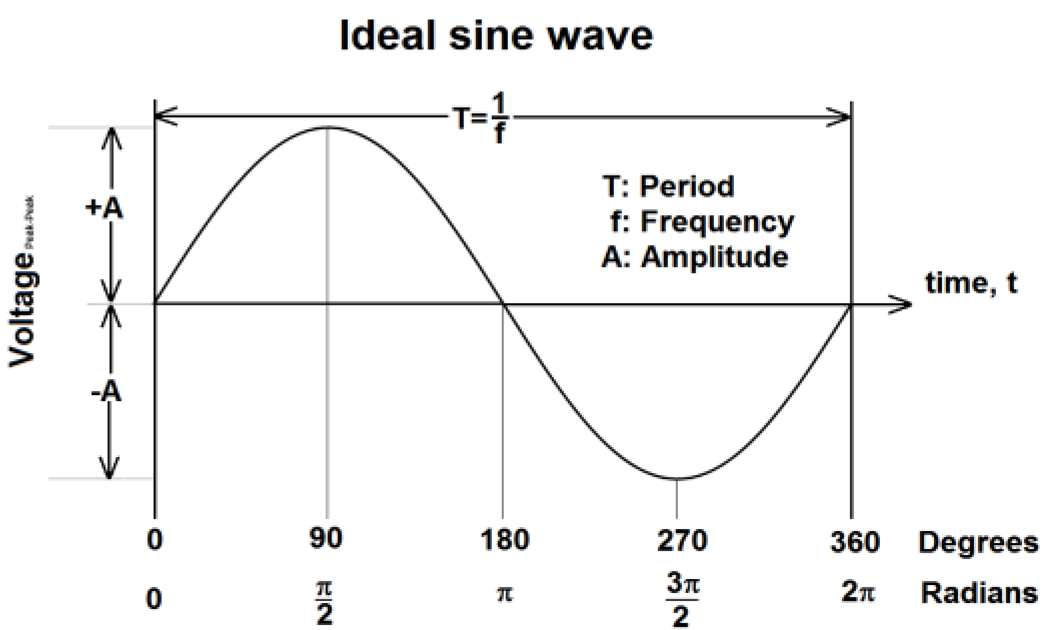
  Based in the Decimal system (base 10)

The Hertz

- A derived unit from the second
- Represent the rate of events per second
- The SI units is s^{-1} but is best know by Hz
- Multiples of the Hz:
  - hertz (Hz) = 1 event or cycle per second
  - kilohertz (kHz) = 10^3 Hz
  - megahertz (MHz) = 10^6 Hz
  - gigahertz (GHz) = 10^9 Hz

Relation between time and frequency

- Time and frequency are related. We can obtain the frequency of the event if we count his occurrences in a time interval (in our case in a second)
- The period of a signal is the time between events



$$T = \frac{1}{f}$$

Relation between time and frequency

- This relation give us the frequency offset (frequency deviation, fractional frequency, relative frequency error, etc.) and the time (phase) change with respect to a reference.

$$f_{off} = \frac{f_{measured} - f_{nominal}}{f_{nominal}} = \frac{\Delta f}{f}$$

One of the most important relationships between time and frequency changes

$$\frac{\Delta f}{f} = -\frac{\Delta t}{T}$$

Relationship between time and frequency

Frequency difference
$f_{MEASURED} - f_{REFERENCE}$

Time interval difference
$t_2 - t_1$

$$\frac{\Delta f}{f} = -\frac{\Delta t}{T}$$

Frequency Reference
$f_{REFERENCE}$

Time interval
(observation time)

- If the frequency offset is positive, the time difference is negative (your clock runs slower that the reference)
- If the frequency offset is negative, the time difference is positive (your clock runs faster that the reference)

Relationship between time and frequency

- If you had a microsecond phase change in one day in the time difference of a oscillator:

$$\frac{\Delta f}{f} = -\frac{(-0{,}0000001)}{86400}$$

- The frequency offset is 1,16E-11 Hz/Hz

- If you had a frequency offset of 5E-07 Hz/Hz, and you want to know the time difference in a day:

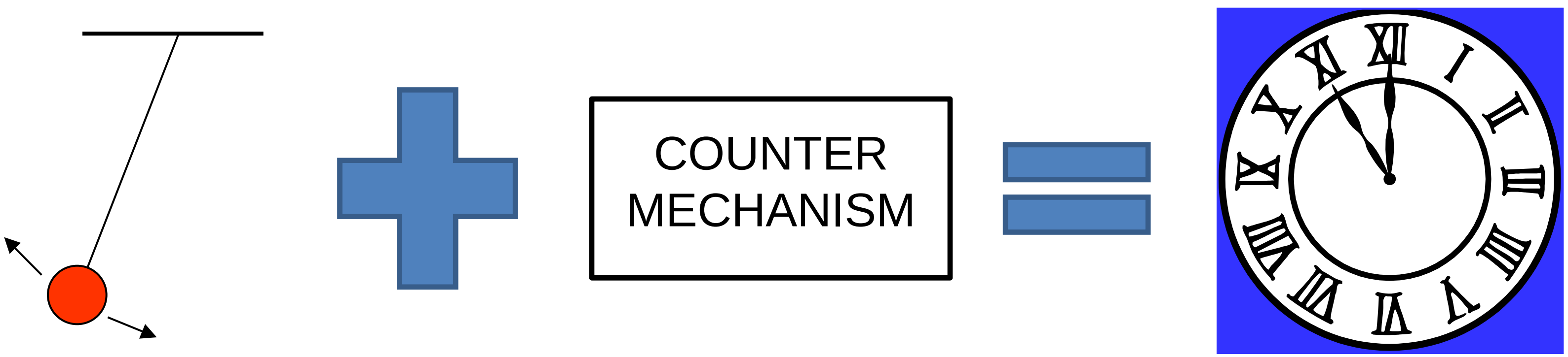
$$\Delta t = -\left(5 \times 10^{-07}\right) \cdot (86400)$$

- The delay is 43 milliseconds per day

Clocks and oscillators

- The oscillators are devices that produce a periodic signal with a repeatable rate
- This periodic signals need a stable frequency
- A clock is only a oscillator and a counter
- The quality of the clock is proportional to the quality of the oscillator

Clocks and oscillators



OSCILLATOR	COUNTER MECHANISM
Earth rotation	Sundial
Pendulum Swing	Clock Gears and Hands
Quartz Crystal Vibration	Electronic Counter
Cesium Atomic Vibration	Microwave Counter

Clocks and oscillators

- For a system with an ideal frequency of 5 Hz (period is 200 ms) that counts 5 events then:

$$T = 200\text{ ms} + 200\text{ ms} + 200\text{ ms} + 200\text{ ms} + 200\text{ ms}$$

$$T = 1000\text{ ms}$$

$$T = 1\text{ s}$$

- We assume that one second has passed

Clocks and oscillators

- But if the system frequency of 5 Hz is not ideal and have a deviation of Δt:

$$T = (200\text{ ms} + \Delta t) + (200\text{ ms} + \Delta t) + (200\text{ ms} + \Delta t) + (200\text{ ms} + \Delta t) + (200\text{ ms} + \Delta t)$$

$$T = 1000\text{ ms} + 5\,\Delta t$$

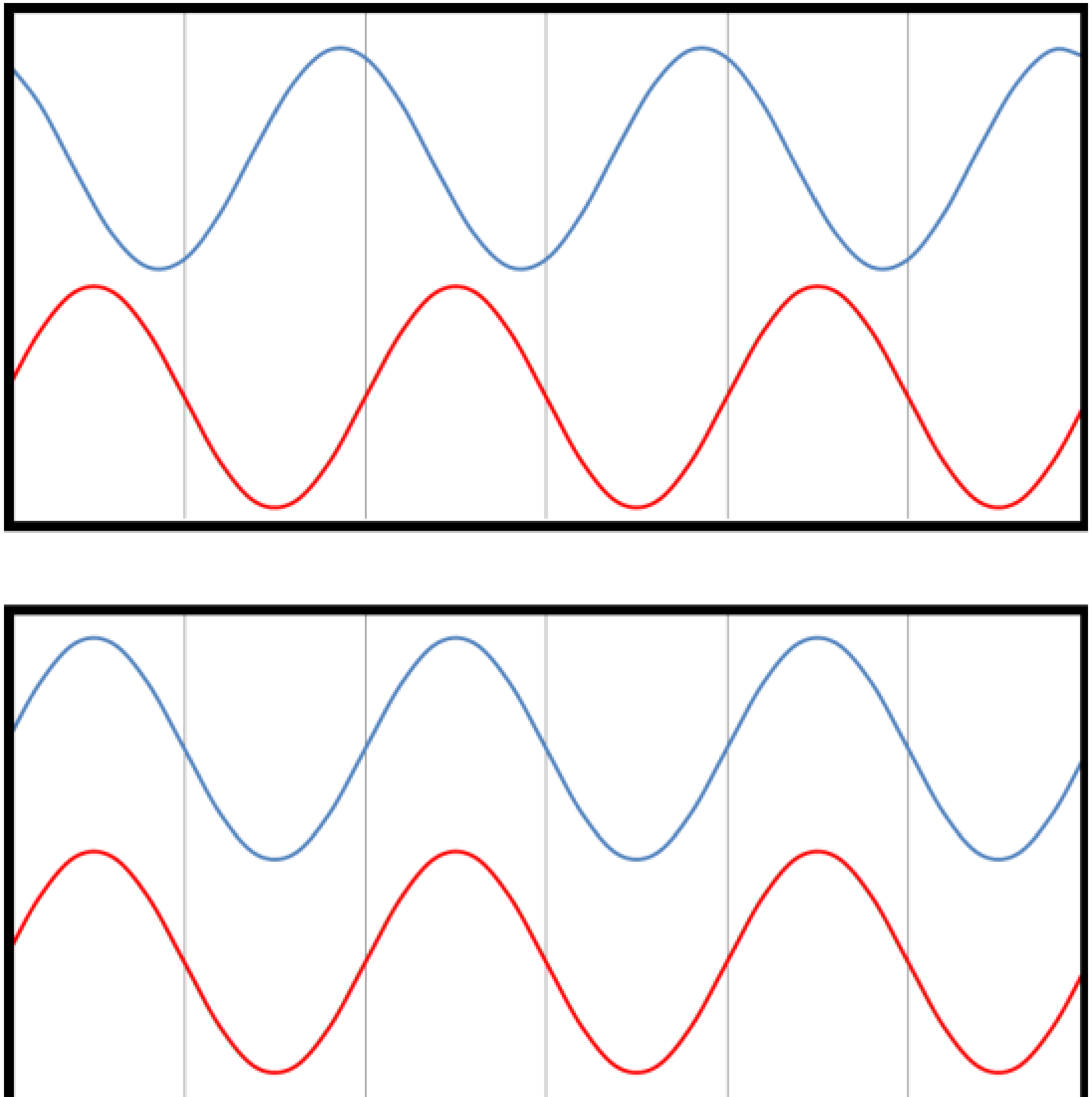
$$T = 1\text{ s} + 5\,\Delta t$$

- We can’t ensure that only one second has passed

Synchronization and Syntonization

- *Synchronization* is the process of setting two or more *clocks* to the same time
  - Clocks, time interval reference, telecommunication systems, digital systems, etc.
- *Syntonization* is the process of setting two or more *oscillators* to the same frequency
  - Disciplined oscillators (like GPSDO), radio and microwave links, etc.

Synchronization

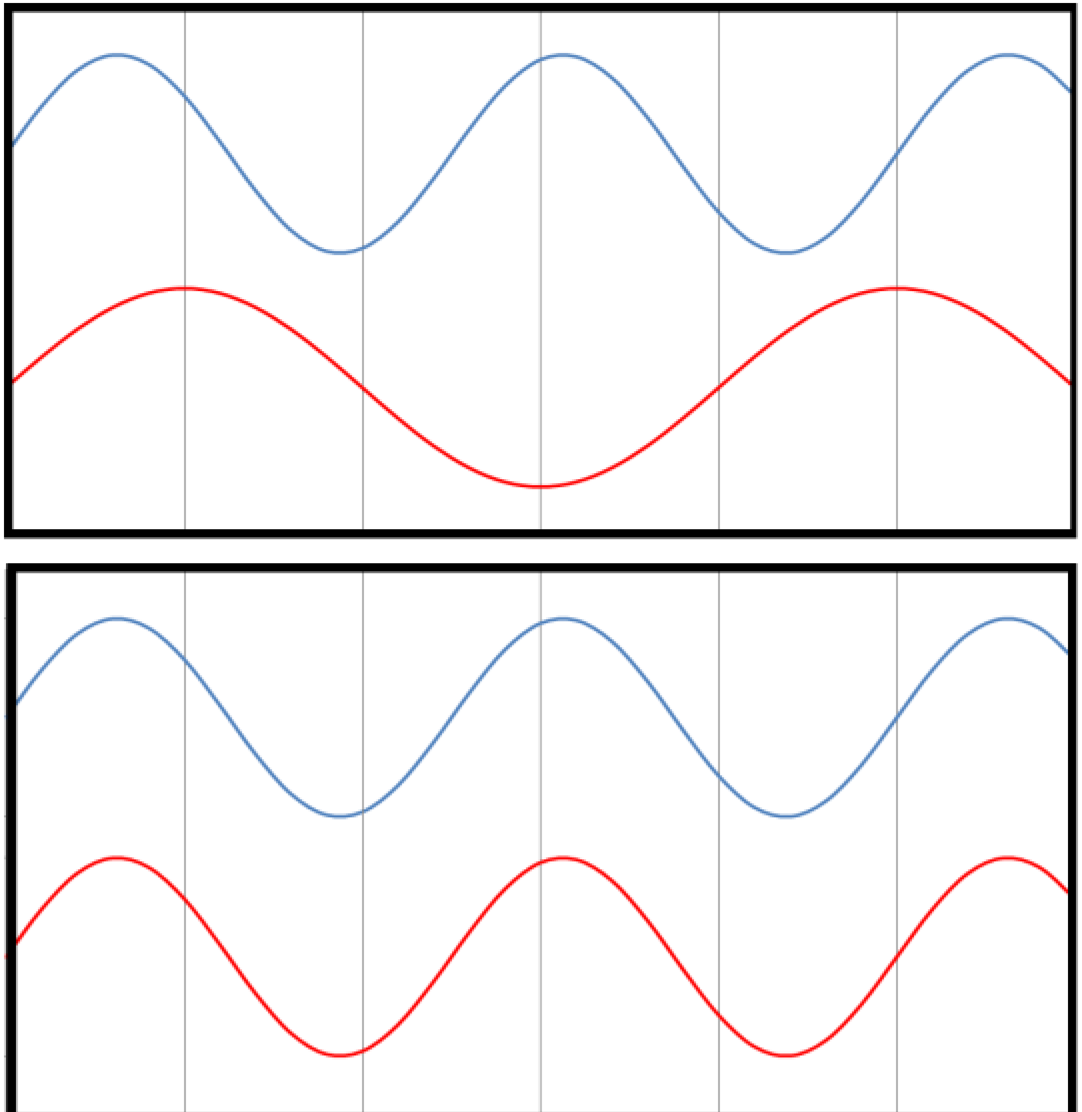


$$A \cdot \sin(2\pi \cdot f_A \cdot t + \varphi_A)$$

$$B \cdot \sin(2\pi \cdot f_B \cdot t + \varphi_B)$$

- For the same sampling time (t) and f_A=f_B
  - If $\varphi_A = \varphi_B$: Signals are synchronized
  - If $\varphi_A \neq \varphi_B$: Signals aren't synchronized

Syntonization



$$A \cdot \sin(2\pi \cdot f_A \cdot t + \varphi_A)$$
$$B \cdot \sin(2\pi \cdot f_B \cdot t + \varphi_B)$$

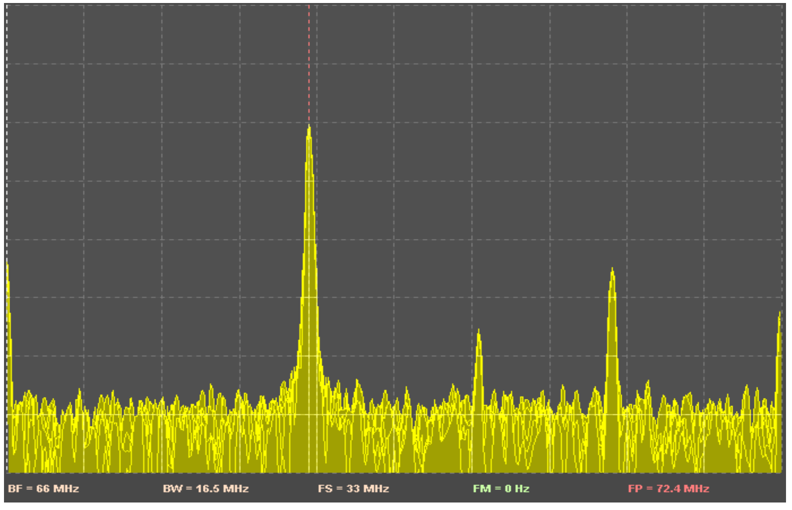
- For the same sampling time (t)
  - If $f_A = f_B$: Signals are syntonized
  - If $f_A \neq f_B$: Signals aren't syntonized
- Is not necessary that $\varphi_A = \varphi_B$

Frequency Domain and Time Domain

- In the mathematical language we have two form to analyze the systems
- The time domain indicates that the mathematical operations are done in time terms
- The frequency domain indicates that the mathematical operations are done in frequency terms
- We can estimate time uncertainty measuring frequency or estimate frequency uncertainty measuring time intervals

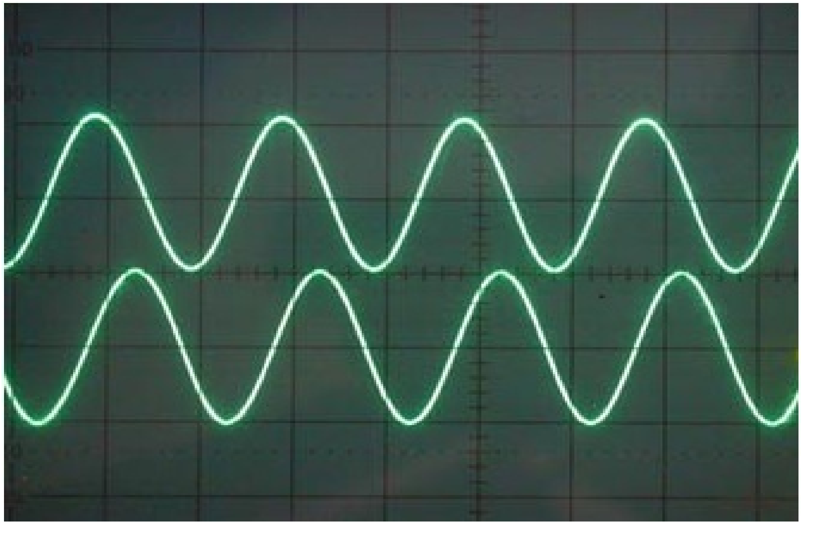
Frequency Domain

- Voltage and power is measured in function of the frequency
- Spectrum analyzers are typically used
- Separate the signal in his components and represent they with his power (using FFT algorithms)
- A perfect signal is like a line with zero bandwidth



Time Domain

- Voltage and power is measured in function of time
- Time interval counters and oscilloscopes are typically used
- Signals are generally sine or square
- A perfect signal had no frequency uncertainty or instability (using Allan Variance statistics)



Typical calibration reference oscillators

- In Time and Frequency metrology, the stability of an oscillator is its most important characteristic

- The typical reference oscillators are:
  – Quartz oscillators
  – Rubidium oscillators
  – Cesium oscillators
  – GPS Disciplined Oscillators

Quartz Oscillators

- The most common and least expensive of the stable oscillators
- Operating frequencies depend of the geometry and the piezoelectric capabilities
- The frequency changes due to temperature variations and mechanical vibrations
- Good short term stability. But in the long term the performance gets worse due to drift and aging

Rubidium Oscillators

- The most accessible of the atomic oscillators
- Is a good reference for a calibration laboratory
- Its long term stability is better than quartz oscillators

- Sensitive to temperature fluctuations and aging
- Is a secondary frequency reference standard (based on unperturbed ground-state hyperfine transition of Rubidium 87 atoms)

Cesium Oscillators

- The time and frequency primary reference
- Cesium is used to define the base unit of time (the second)
- Good short term stability and ~~a~~ excellent long term stability

- Unfortunately does not last as long as Quartz and Rubidium Oscillators
- Very expensive and requires some maintenance

GPSDO

- They are local oscillators disciplined (periodically corrected) by signals from the GPS satellite network

- The short term stability depends on the local oscillator (quartz, rubidium, etc.) and the long term stability depend on the GPS signals

Comparative table

Parameter	Quartz OCXO	Rubidium	Cesium	GPSDO
Life expectancy	Depend of the electronic (more than 10 years)	10 to 25 years	5 to 10 years	Depend of the electronic (more than 10 years)
Frequency accuracy after 30 minute warm-up	No guaranteed accuracy, must be set on frequency	5×10^{-10}	1×10^{-12}	1×10^{-12}
Stability at 1 s	1×10^{-12}	1×10^{-11}	1×10^{-11}	Parts in 10^{10}
Stability at 1 day	1×10^{-10}	2×10^{-12}	1×10^{-13}	2×10^{-13}
Stability at 1 month	Parts in 10^{9}	5×10^{-11}	Parts in 10^{14}	Parts in 10^{15}
Noise floor	≤ 10 s	≤ 1 000 s	Several days to 30 days	The noise is always down because of the frequency correction to agree with UTC
Aging	1×10^{-10} / day	5×10^{-11}/month	None, by definition	None, frequency is corrected by satellites
Cost (USD, in USA)	\$200 to \$2000	\$1000 to \$8000	\$35000 to \$75000	\$1000 to \$15000

Traceability

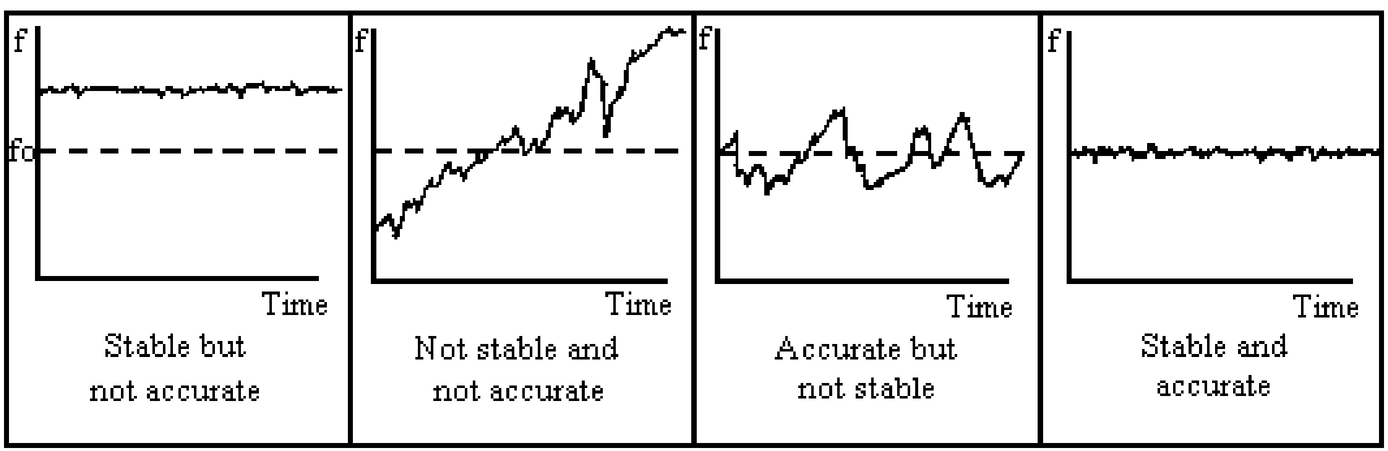
- The primary source of traceability is the key comparison that defines UTC (CCTF-k001.UTC)
- The monthly Circular T is the document used to know the difference of your local time, UTC(k), and the UTC
- To participate in this comparison you need (as minimal) a commercial Cesium clocks or a laboratory developed clock equal to or more stable than the commercial cesium clocks

Traceability

- The SIM have the SIMT, a regional comparison that help to bring traceability to all the time and frequency laboratories in America
- The participant laboratories can have GPSDO, Quartz, Rubidium, Cesium and laboratory developed oscillators.
- The SIMT is linked to the UTC through NRC (Canada), NIST (United States), CENAM (Mexico), CENAMEP (Panama), ONRJ (Brazil) and INTI (Argentina)

Oscillator Stability

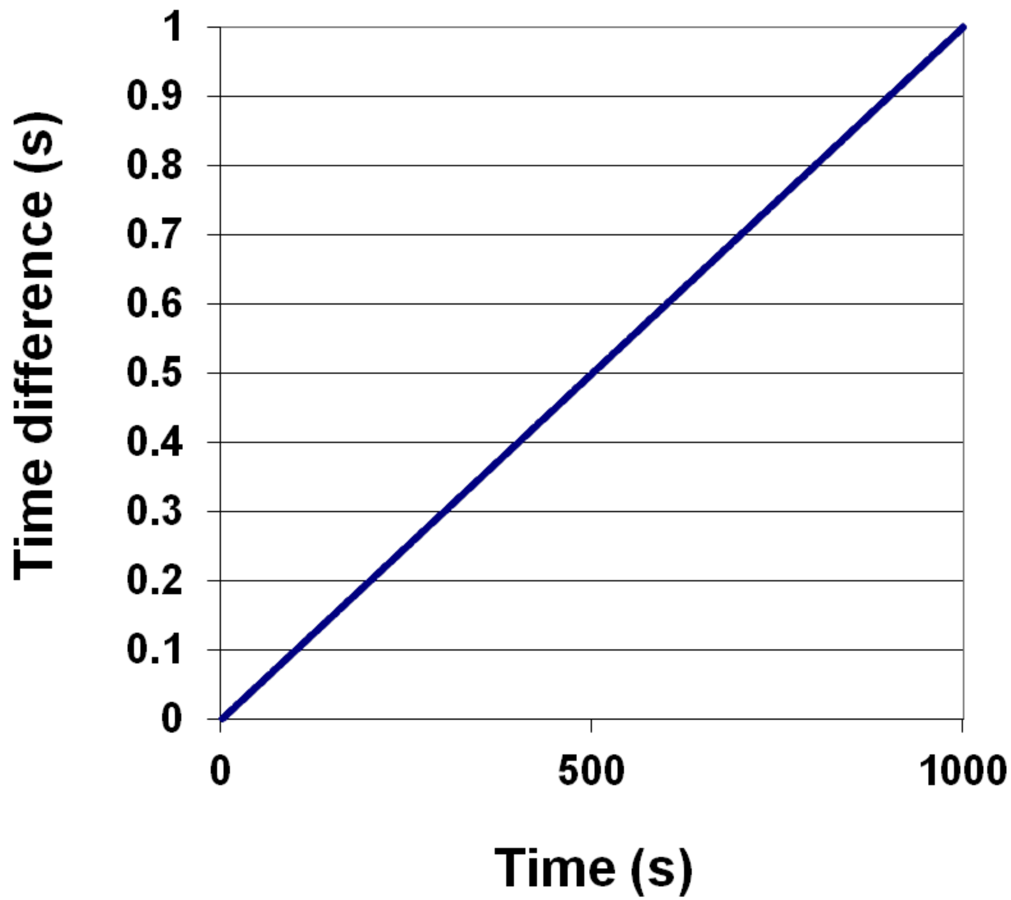
- Stability shows how the output frequency changes over time
- Accuracy shows how close the output frequency is to the nominal frequency
- The analyzed signals keep changing because of the technology and components used in the device



Oscillator Stability

- If the difference keeps growing, the mean keeps changing and the Variance of the data grows

- For this reason the classical Variance don't work in the oscillators analysis



Allan Deviation

- It’s a statistic used to estimate frequency stability
- It was named after David W. Allan, a physicist who retired from NIST in 1993
- He first published the statistic in 1966
- The variances is called Allan Variance (AVAR, σ_y^2) and the deviation is called Allan Deviation (ADEV, σ_y)

Allan Deviation

- The ADEV is computed by taking the differences between successive pairs of data points to remove the trend or slope contributed by the frequency offset.
- This is necessary because by removing the trend, we can make the data converge to a mean.
- ADEV is only useful for computing stability, and not for accuracy.

Allan Deviation

$$\sigma_x(\tau) = \sqrt{\frac{1}{2(N-2m)\tau^2}\sum_{i=1}^{N-2m}\left(x_{i+2m} - 2x_{i+m} + x_i\right)^2}$$

$$\sigma_y(\tau) = \sqrt{\frac{1}{2\tau(N-1)}\sum_{i=1}^{N-1}\left(y_{i+1} - y_i\right)^2}$$

σ_x Allan Deviation of time

σ_y Allan Deviation of frequency

N Number of frequency samples

τ Observation time

y_i *ith* fractional frequency

x_i *ith* time difference

m average factor

Allan Deviation

$$\sigma_y(\tau) = \sqrt{\frac{1}{2\tau(N-1)}\sum_{i=1}^{N-1}\left(y_{i+1} - y_i\right)^2}$$

Frequency (Hz)	Frequency Offset (Hz)
10 000 000,3	3,00E-08
9 999 999,4	-6,00E-08
9 999 999,3	-7,00E-08
10 000 000,2	2,00E-08
10 000 001,1	1,10E-07
9 999 999,3	-7,00E-08
10 000 000,5	5,00E-08
10 000 000,3	3,00E-08
9 999 999,4	-6,00E-08
10 000 000,8	8,00E-08

Standard frequency: 10 000 000 Hz

Measurement interval (tau): 1 s

$$\sigma_y(1) = \sqrt{\frac{1}{2 \cdot 1 \cdot (10-1)}\left(5{,}5167 \times 10^{-15}\right)}$$

$$\sigma_y(1) = 7{,}43 \times 10^{-08}$$

Allan Deviation

$$\sigma_x(\tau) = \sqrt{\frac{1}{2(N-2m)\tau^2}\sum_{i=1}^{N-2m}\left(x_{i+2m} - 2x_{i+m} + x_i\right)^2}$$

Time difference (s)
0,0E+00
1,1E-09
2,1E-09
3,2E-09
4,1E-09
5,4E-09
6,5E-09
7,9E-09
8,9E-09
1,0E-08

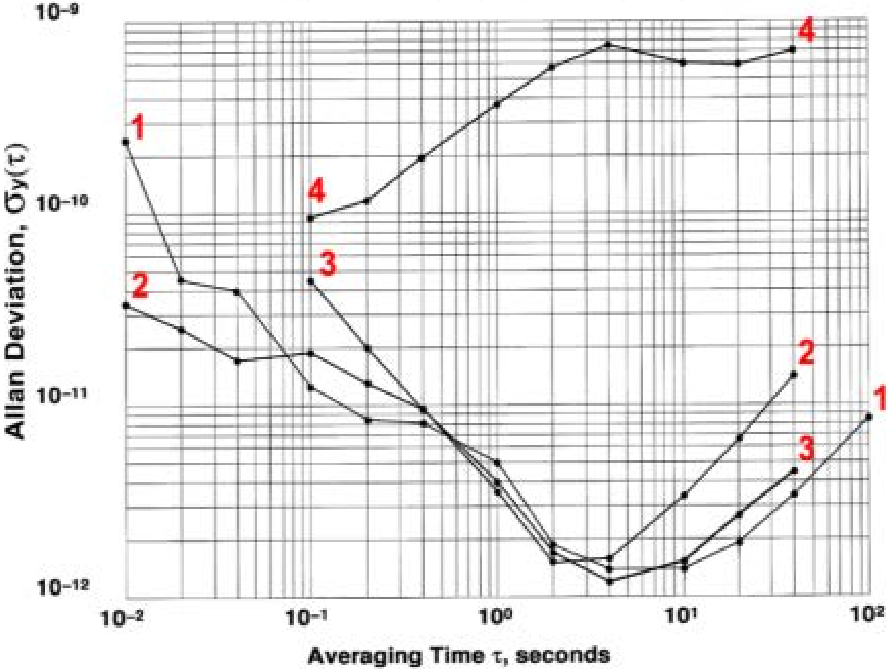
Measurement interval (tau): 10 s

Average factor = 1

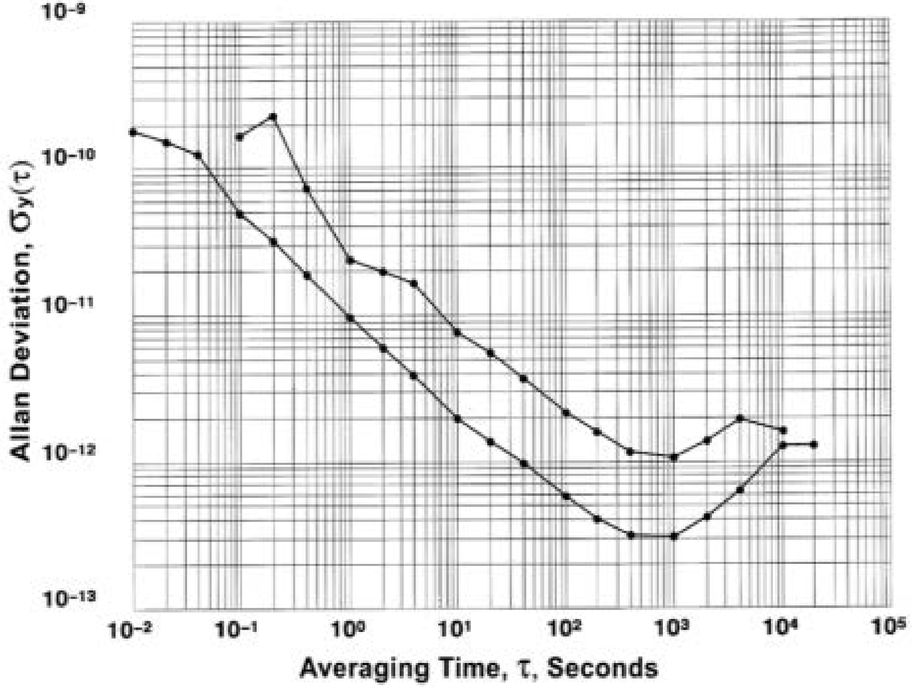
$$\sigma_x(10) = \sqrt{\frac{1}{2\cdot(8-2)\cdot 10^2}\left(3{,}25\times 10^{-22}\right)}$$

$$\sigma_x(10) = 1{,}80\times 10^{-11}$$

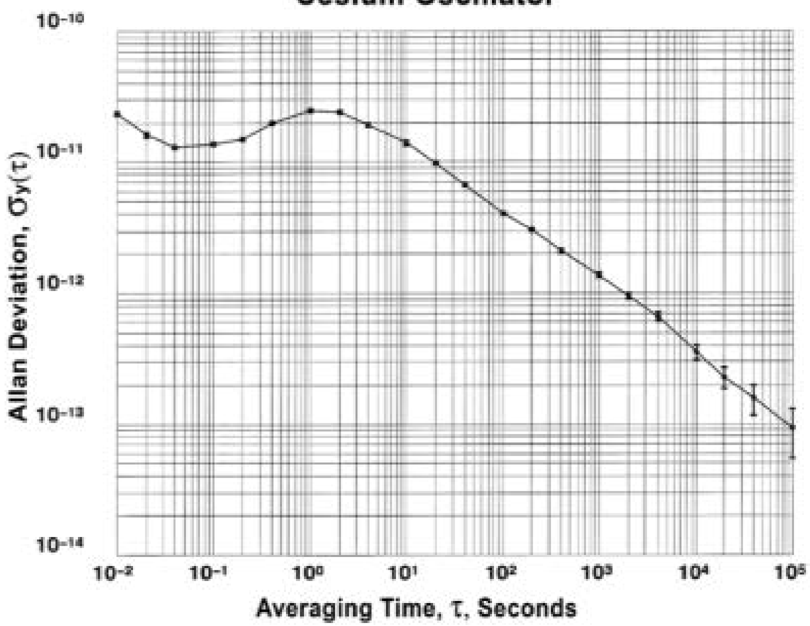
Comparison of Quartz Oscillators



Comparison of Rubidium Oscillators

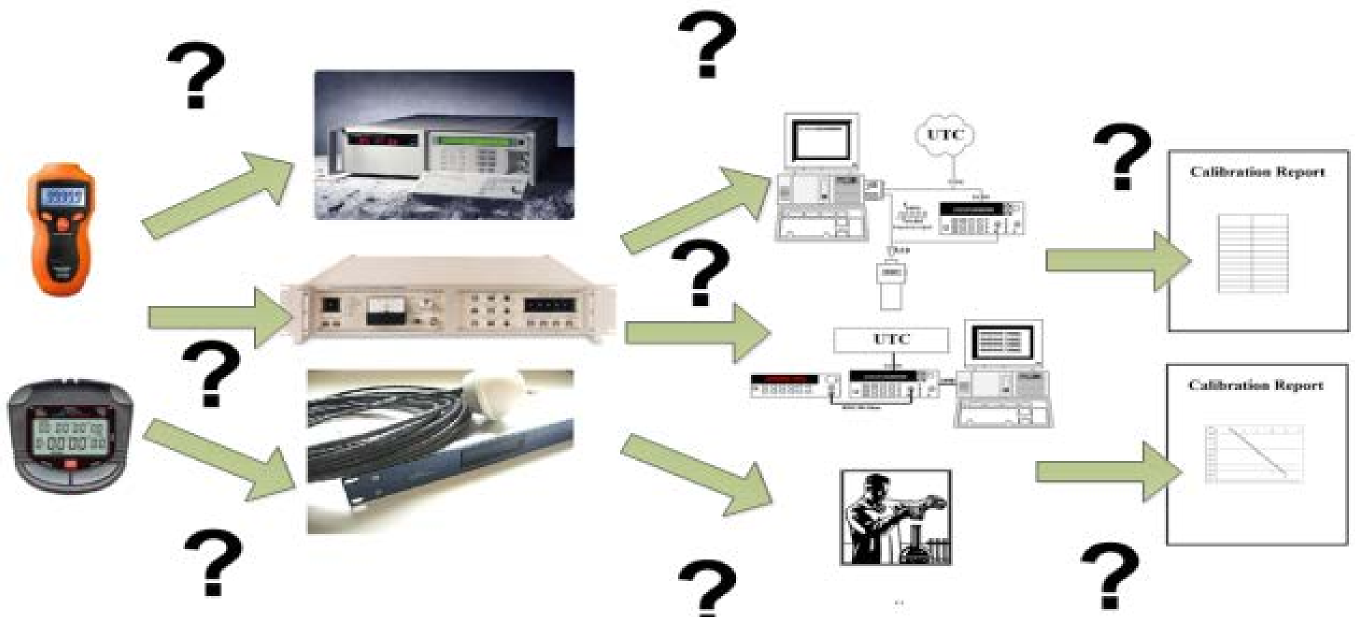


Cesium Oscillator



Calibration

- The calibration is formed by 4 parts:
  - The device under test
  - The calibration reference
  - The calibration method
  - The calibration results



Device Under Test

- The DUT can be any device that generates or measures a oscillating signal
- Can be a mechanic signal, electronic signal (digital or analog) or another origin signal (biological or physical)
- Some nationals laws and international recommendations require traceability (the National Time, taximeters, parking meter, etc.) to a time standard

Device Under Test

- References like clocks and oscillators (mechanical, quartz, atomic, etc.) can be DUTs
- Frequency meters, oscilloscopes, signal generators, spectrum analyzers, acoustic, acceleration and vibration source devices can be DUTs
- Some devices can be calibrated in the time and frequency metrology domain (like tachometers or strobe tuner) after some transformations

The Calibration Reference

- The reference is the source of traceability and can be a good quartz, atomic clock or GPSDO
- Some references are a “two-in-one” standard because they have a time output (1 PPS signal) and a frequency output (5 MHz or 10 MHz signal)
- Under the ISO 17025: is required that the reference be traceable to UTC through a comparison or a calibration

The Calibration Reference

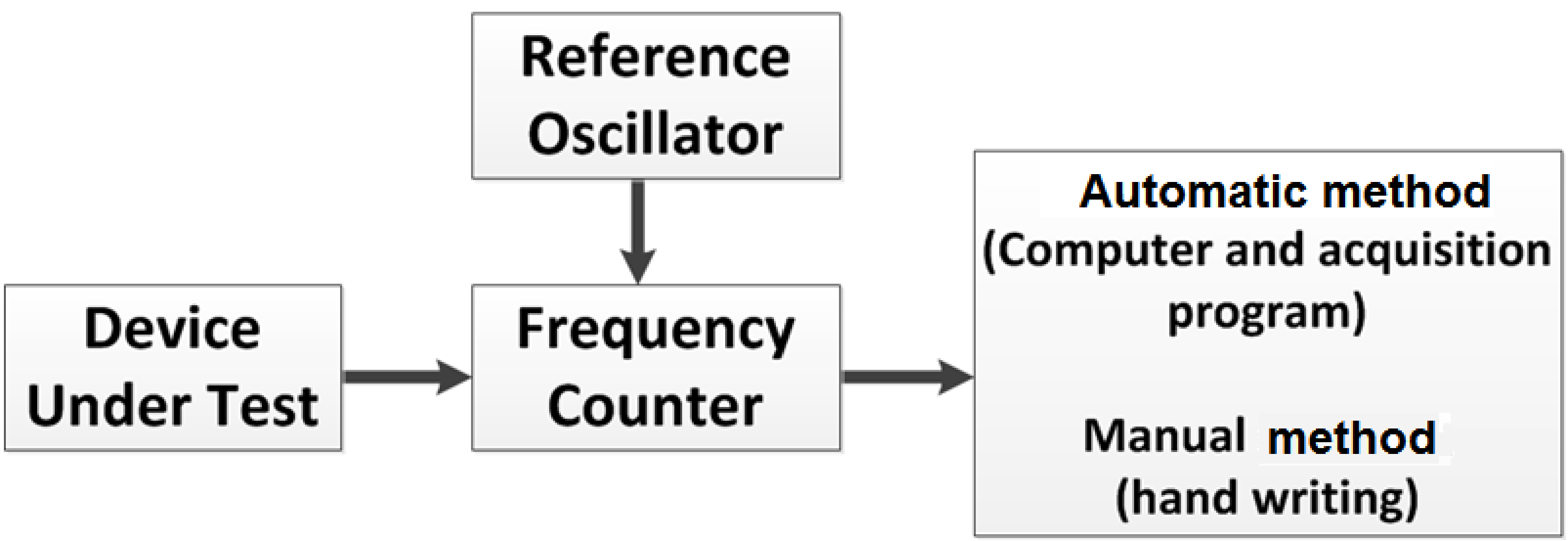
For this DUT (oscillator based in)	Recommended reference
Quartz	Rubidium Cesium GPSDO ***National Standard***
Rubidium	Cesium GPSDO ***National Standard***
Cesium	GPSDO, allow at least 1 week or maybe longer, for the calibration ***National Standard***
GPSDO	Normally not calibrated unless a ***National Standard*** is used.

Calibration Method

- The method is important to ensure the quality of your results and the repeatability of the experiment
- The calibration method depend directly on the DUT, the reference system and how you acquire the data
- The best ratio between the uncertainty of the calibration and your reference is 10:1 but the minimal ratio to calibrate a DUT is 3:1 (in some cases 4:1)

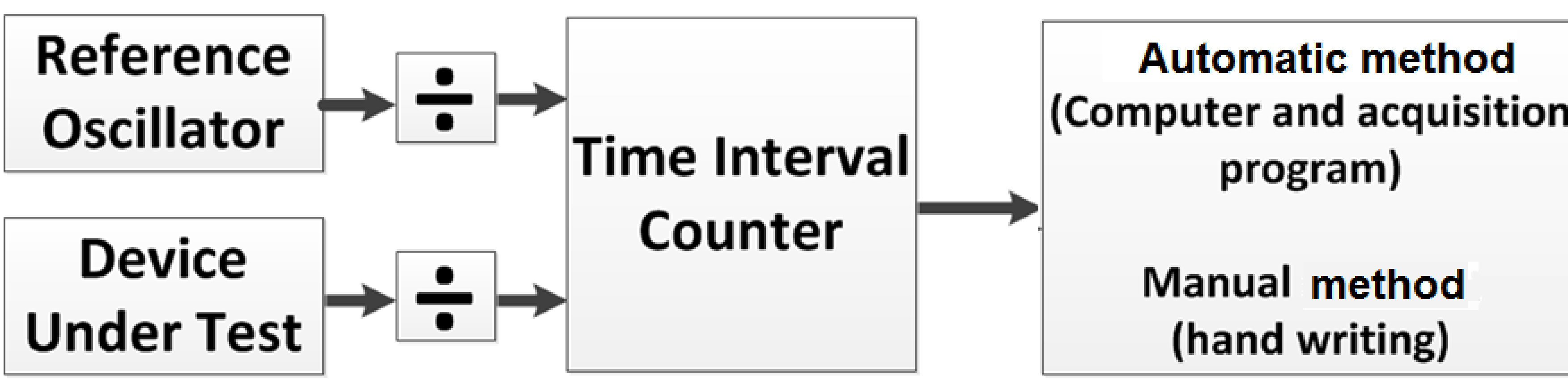
Calibration Method

- Using a frequency counter you can acquire directly the frequency
- You need only the reference oscillator, frequency counter, cables, adapters and a PC (if you can automate the frequency counter)



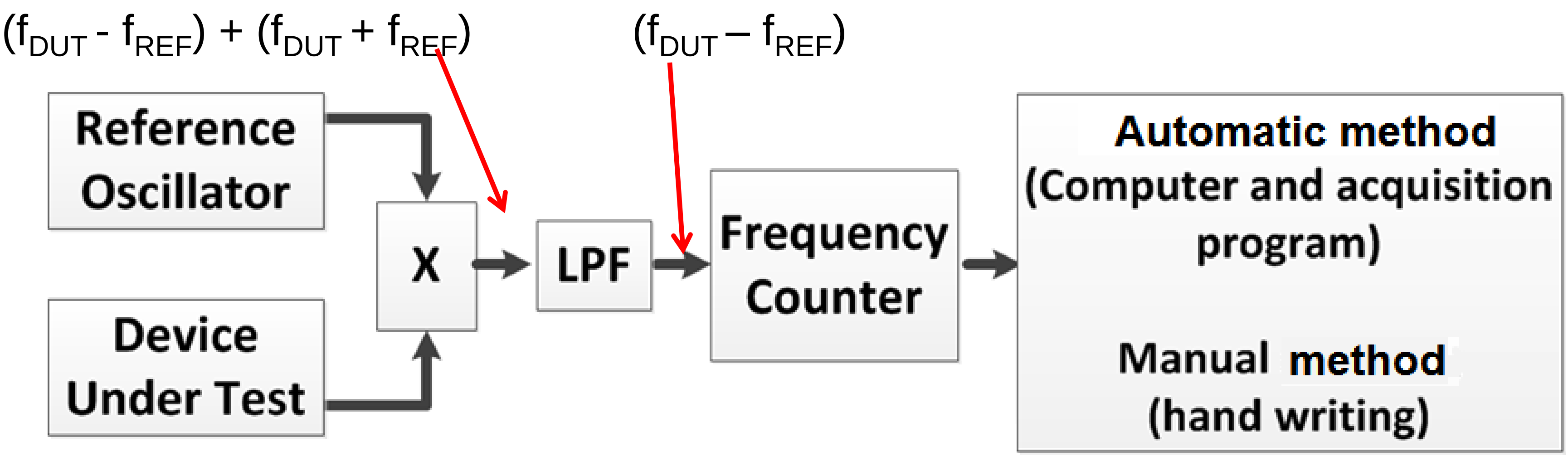
Calibration Method

- Using a time interval counter you can acquire directly the time interval difference
- You need only the reference oscillator, a time interval counter, cables, adapters, frequency divisor (if you need) and a PC (if you can automate the time interval counter)



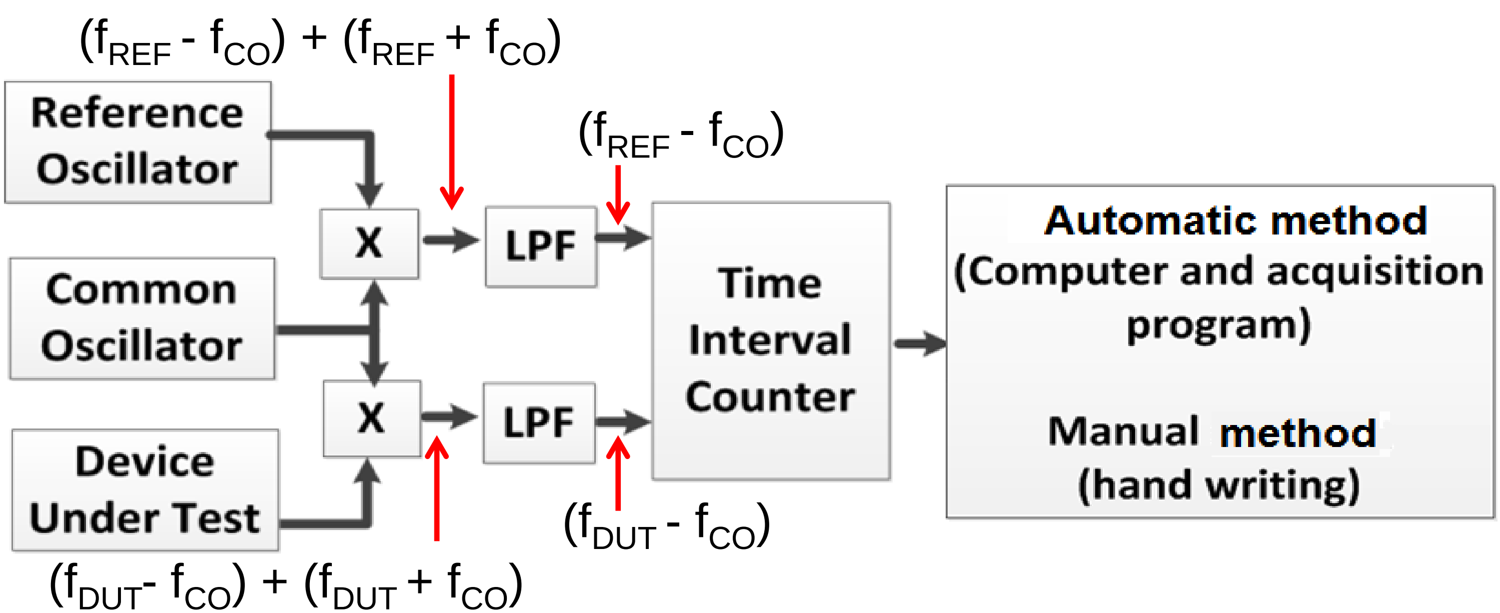
Calibration Method

- If the frequency counter can't register the variations of the DUT you can use a frequency mixer and a Low Pass Filter, improving the frequency counter resolution.
- You need to add the subtracted difference in the post processing results



Calibration Method

- You can improve the single mixer method using a two mixers and two LPF
- Is the most difficulty method to apply. You need another oscillator with a good stability and characterize all the components used



Calibration methods

Devices **(You can used yours own adapted version of the methods for your needs)**	**Recommended method for the DUT**
Frequency meters and counters, waveform and signal generators, devices that display signal visually based in a quartz oscillator	Direct frequency
Quartz oscillators, frequency and counters, waveform and signal generators, devices that display signal visually based in a quartz or rubidium oscillator	Direct phase
Stopwatches, mechanical clocks, quartz oscillators and atomic oscillators	Time interval
Atomic frequency references like rubidium and cesium	Frequency Mixing (single or double)

Calibration results

- The calibration results should be consistent with respect to the use of the device under test
- Try to give the results according to the use, not to the measurement (transformation of units)
- Can you automate the data acquisition? Use the expression of the results in term of the Allan deviation. If not, use the regular statistics that you need to apply the GUM methods

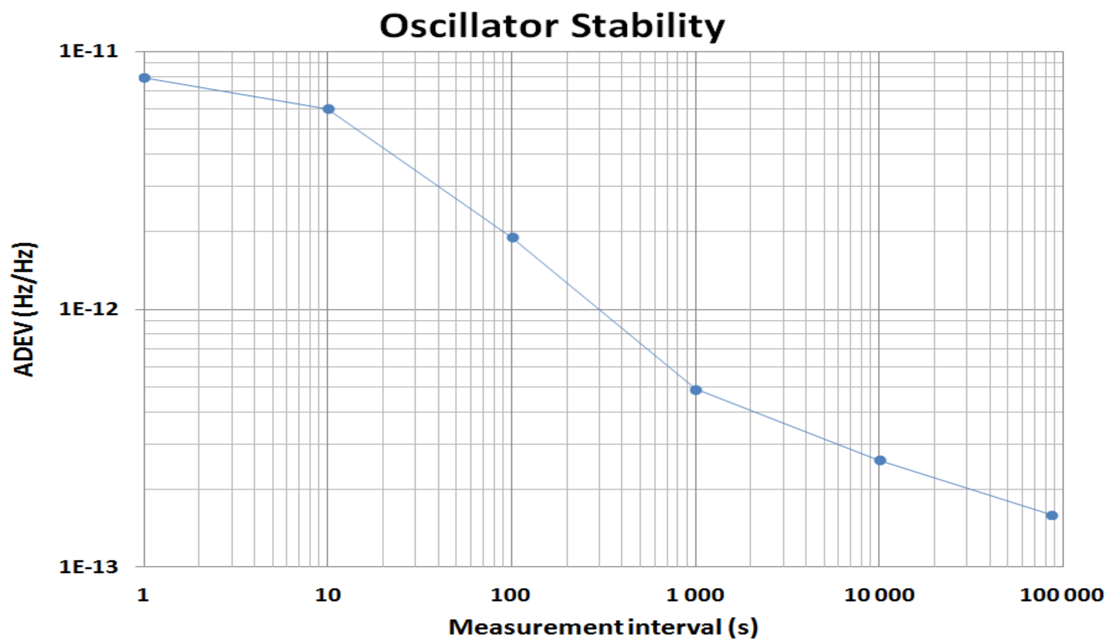
Calibration results

- If you use frequency data to obtain the stability of a clock, you need to transform the results in term of time intervals or phase
- If you use time intervals or phase data to obtain the stability of a oscillator (frequency reference, signals generators etc.) you need to transform the results in term of frequency

$$\Theta_{AB} = \frac{1}{2 \cdot \pi \cdot f} \qquad T = \frac{1}{f} \qquad \frac{\Delta f}{f} = -\frac{\Delta t}{T}$$

Calibration results examples

Results	
Tau (s)	ADEV (Hz/Hz)
1	7,90E-12
10	6,00E-12
100	1,90E-12
1 000	4,90E-13
10 000	2,60E-13
86 400	1,60E-13
$\frac{\Delta f}{f}$ =+6,60E-13 Hz/Hz	



- **The oscillator reference in the calibration period had the following behavior:**

$$\frac{\Delta f}{f} = +1{,}21\text{E-}13 \text{ Hz/Hz}$$

$$\sigma_y(\tau) = \frac{2\times 10^{-12}}{\sqrt{\tau}} \text{ Hz/Hz} \quad 1 \le \tau \le 100000$$

Calibration results examples

Nominal frequency (Hz)	Measurement error (Hz)	Calibration uncertainty (Hz)
1	-0,000 01	0,000 04
10	-0,000 1	0,000 1
100	-0,001	0,002
1 000	-0,01	0,03
10 000	-0,1	0,2
100 000	-1	1
1 000 000	-10	10

The error of the reference was 0 Hz for all of the calibration range

The uncertainty of the reference was 0,000 001 Hz in all of the calibration range

Some useful documents

- ITU-R-TF.686-2 “***Glossary and definitions of time and frequency terms***”.
- NIST Time and Frequency from A to Z: ***An Illustrative glossary***.
- International vocabulary of metrology – Basic and general concepts and associated terms (VIM).
- ITU-R-TF.538 : Measures for random instabilities in frequency and time (phase)

Some useful documents

- ITU-R-REC-TF.1876 “Trusted time source for Time Stamp Authority”
- ITU-T-REC-G.811 “Timing characteristics of primary reference clocks”.
- ITU-T-REC-G.8272-201210 “Timing characteristics of primary reference time clocks”
- http://tf.nist.gov/sim/papers.htm

Thanks

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