Electrical Metrology

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Barry Wood

Piotr Filipski

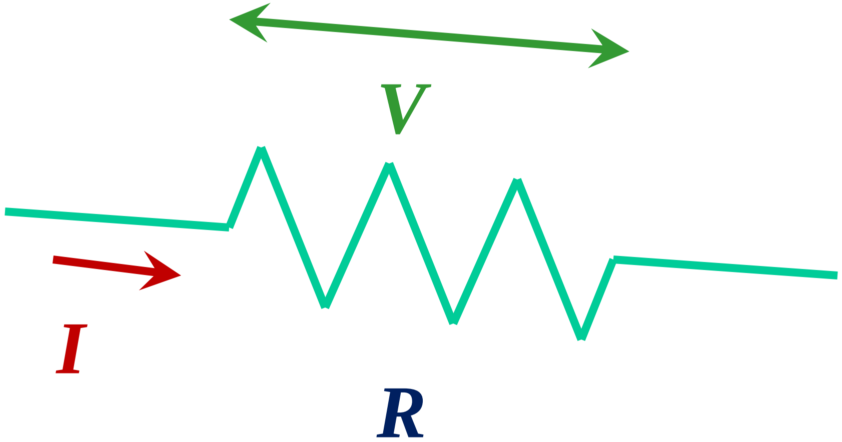
Ken Kochav

for help with some slides and photographs.

Outline

- **Electrical units within the SI**
- **Electrical units in an NMI or DI**
- **Realisation of electrical units**
- **Traceability of electrical units**
- **Measurement methods**
- **Measurement issues.**

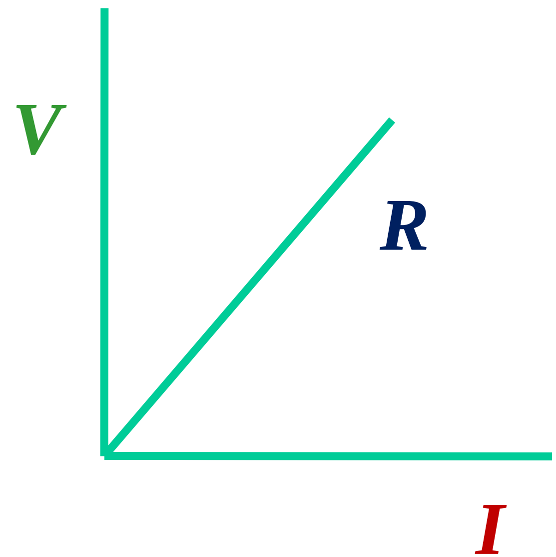
Extra #1



R – resistance

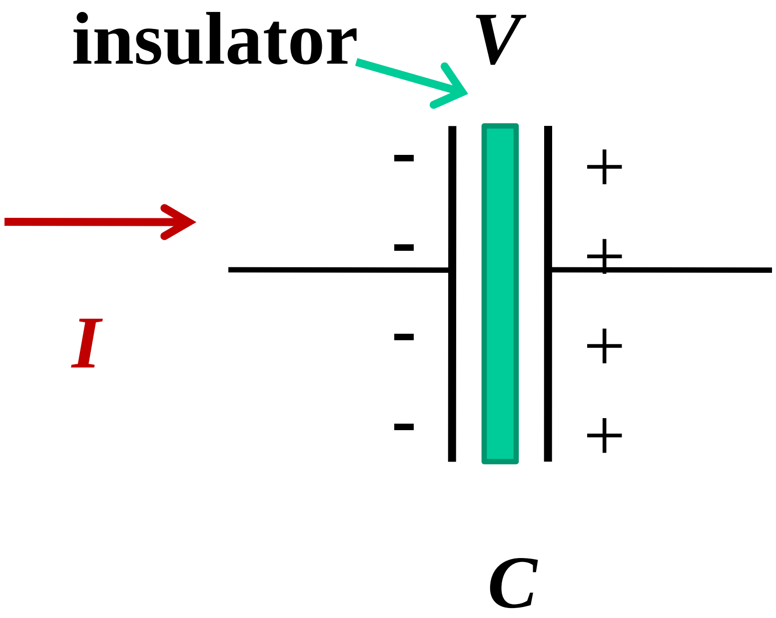
V –potential difference, voltage

I - current



"Ohm's Law": $I = V / R$

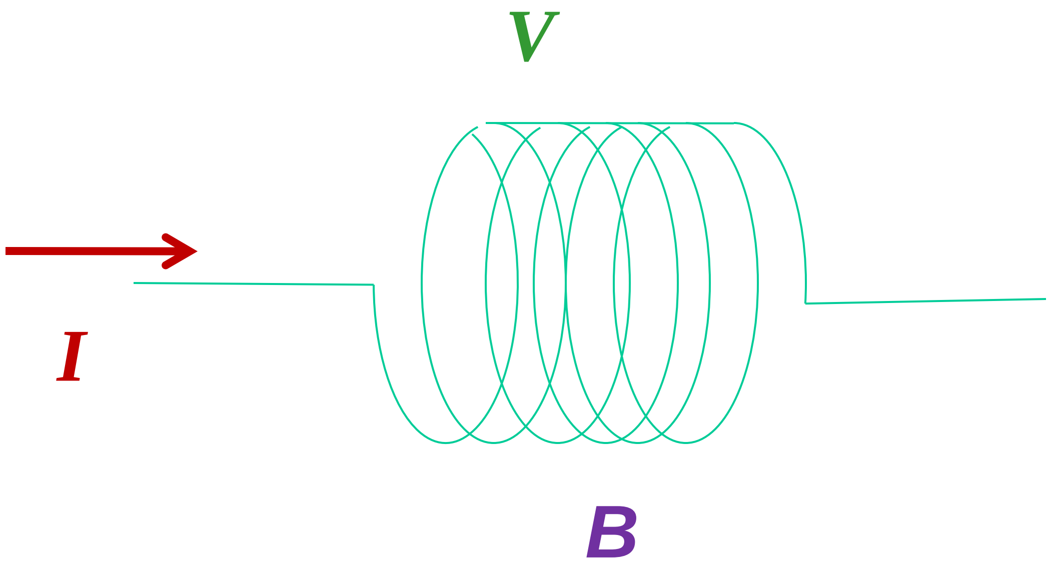
Extra #2



C - capacitance
A – area
D – distance
E - permittivity

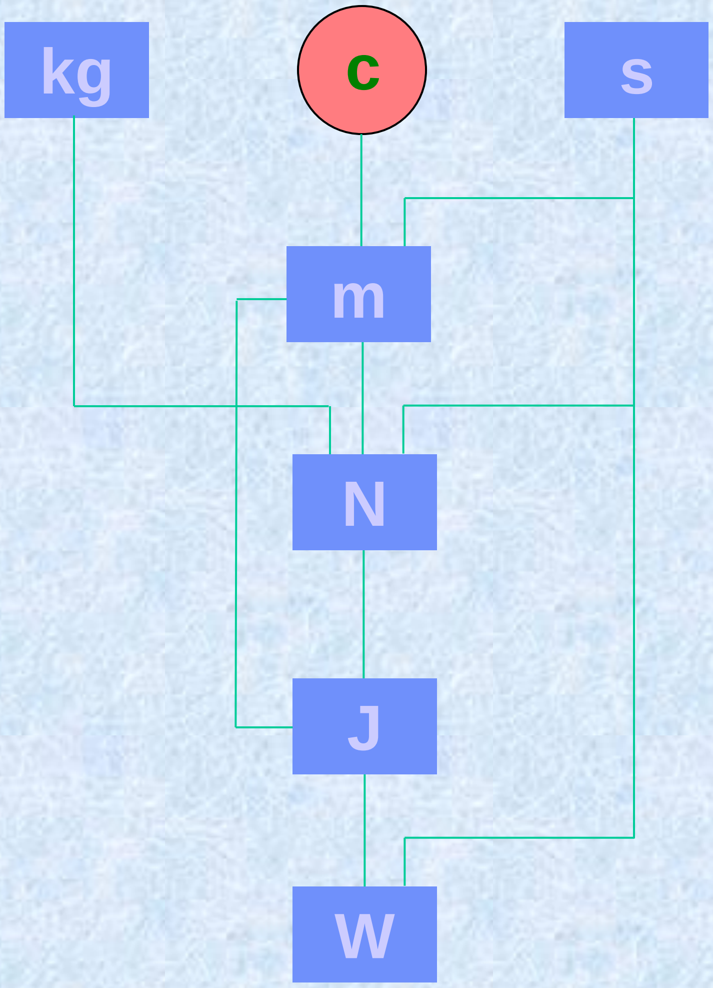
$$C = \varepsilon \frac{A}{d}$$

Extra #3



B – the magnetic field induced in the coil, which will oppose the change of current producing it.

SI Unit Hierarchy - Mechanical Units



Fundamental Constants

c - speed of light

SI Mechanical Units

kg - mass of PtIr at BIPM

s - hyperfine splitting of Cs

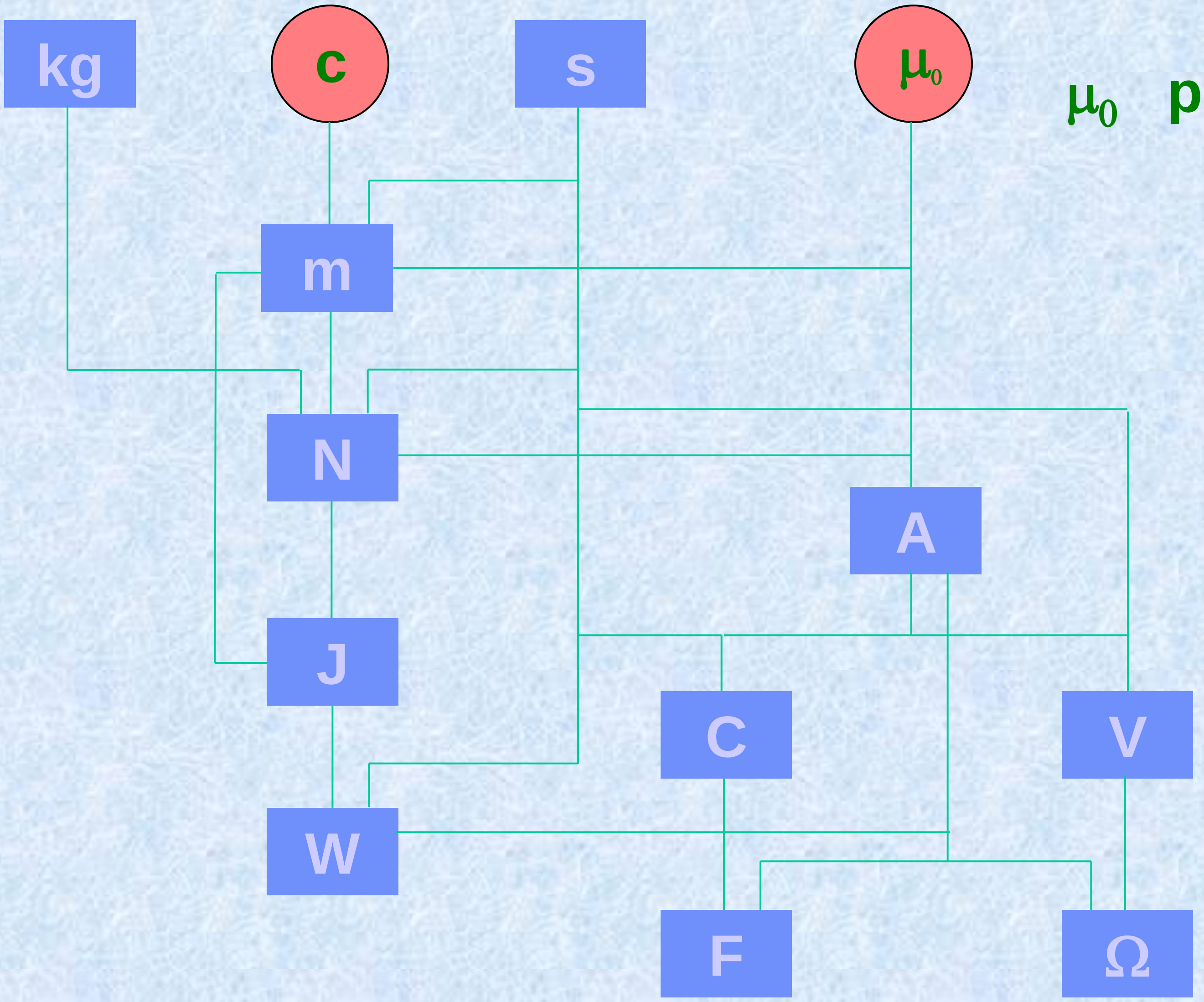
m - meter of length

N - newton of force

J - joule of work

W - watt of energy

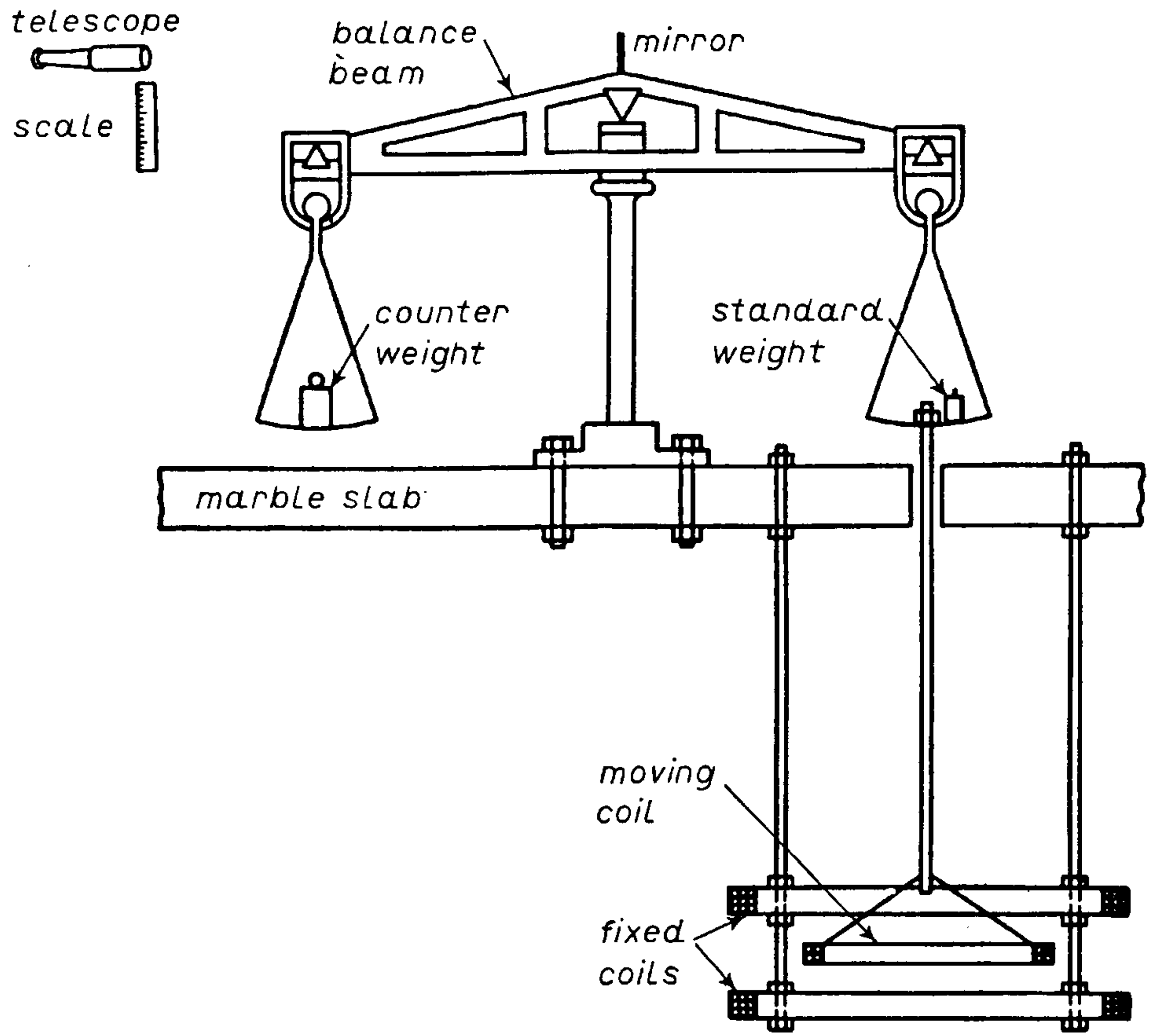
SI Unit Hierarchy & Electrical Units



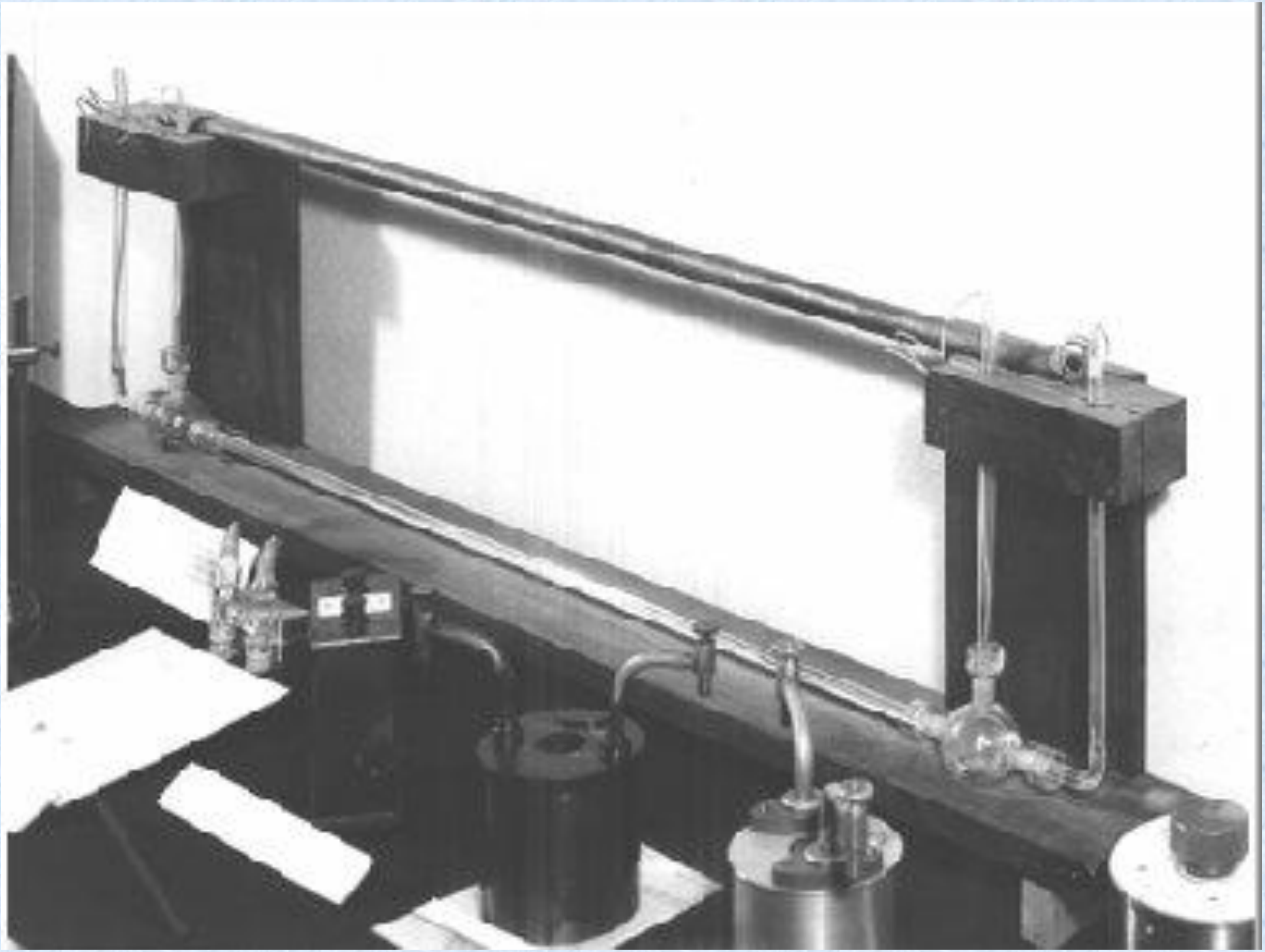
μ_0 **permeability of vacuum**

(1948)

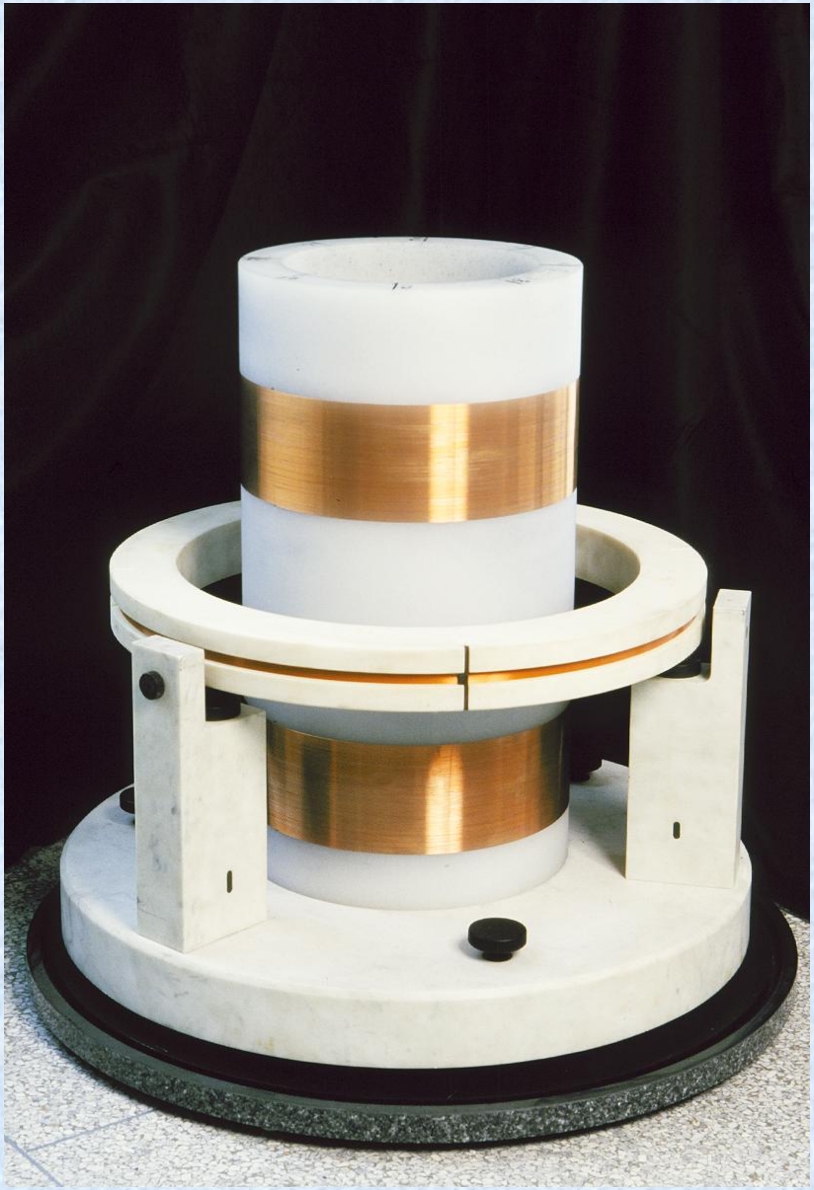
Rayleigh Ampere Balance



Mercury column - 1908 Absolute Ohm



Campbell Mutual Inductor 1950s



1907 a calculation of inductance from dimensional properties

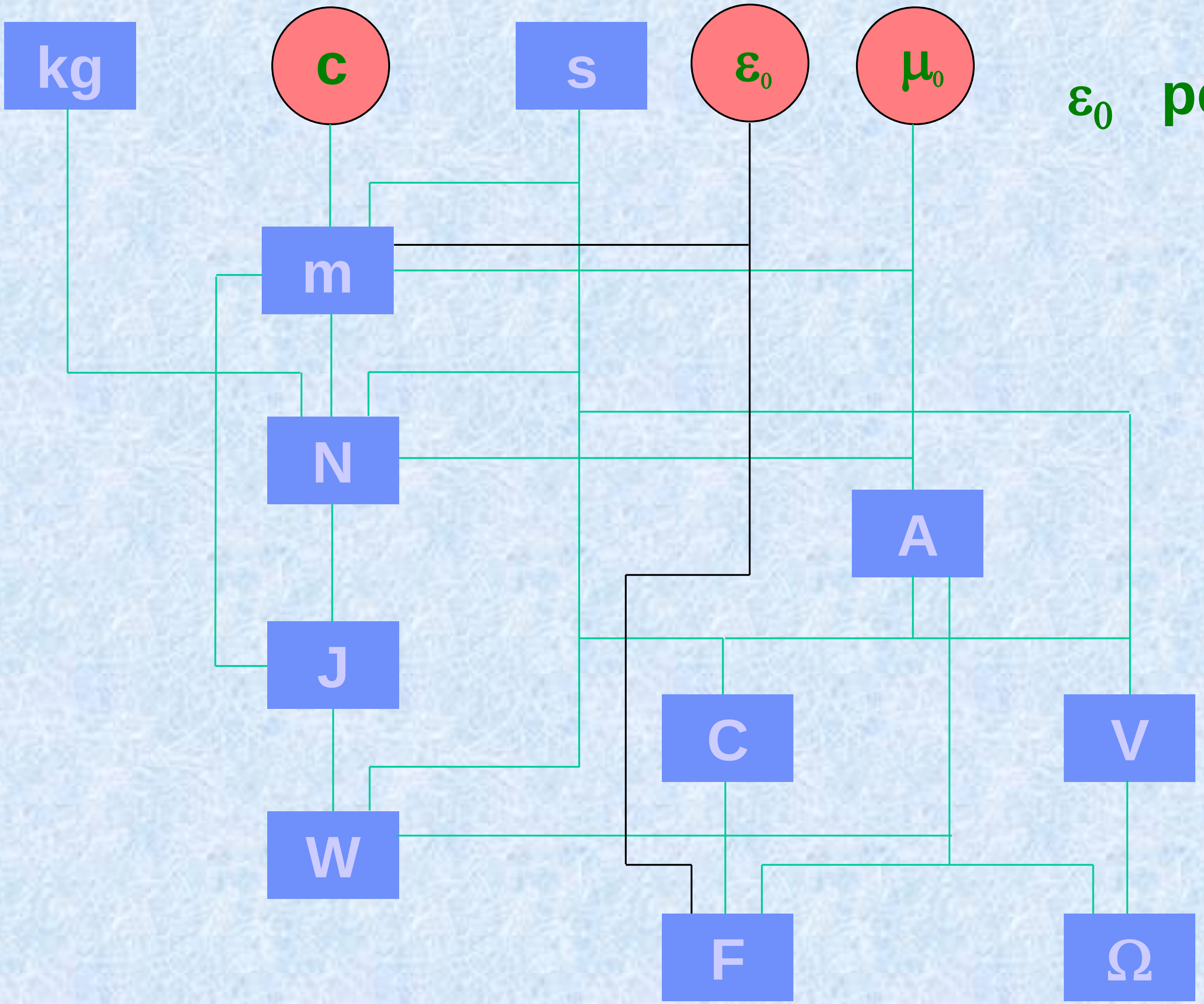
Ratio techniques to relate to the ohm

Popular in the 1950-60s

Problems with current distribution, distortion of wire,...

A few ppm accuracy

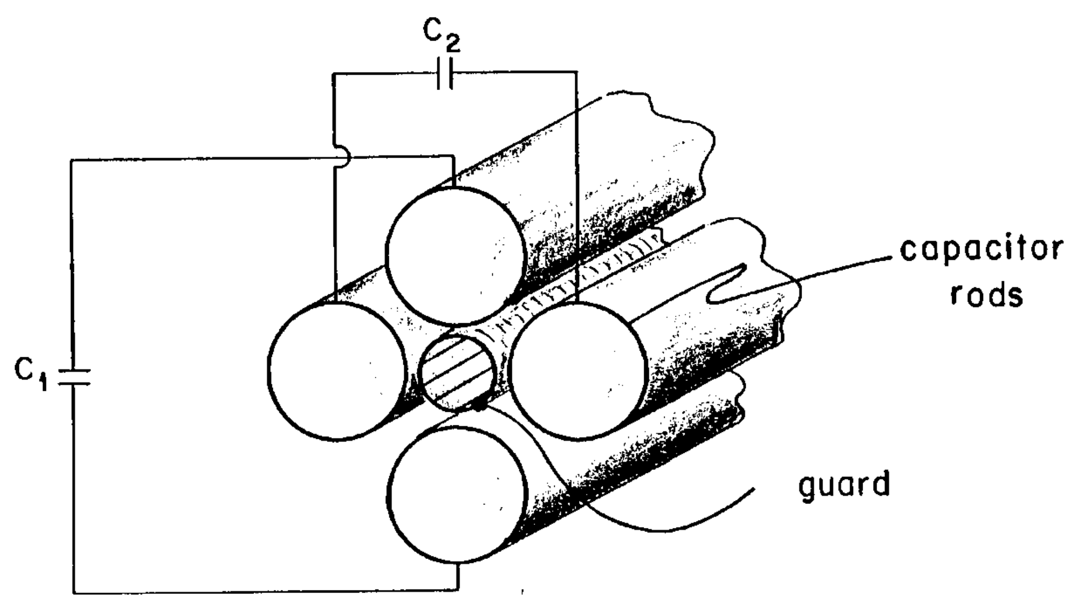
SI Unit Hierarchy & Calculable Capacitor



ε_0 **permittivity of vacuum**

Calculable Capacitor 1970 -

Thompson Lampard 1956



$$C = \frac{\varepsilon_0}{\pi}\ln(2) \quad F/m$$

1 kHz, Δ*l* ≈ 0.2 m, Δ*C* ≈ 0.4 pF (but see *NIST* special)

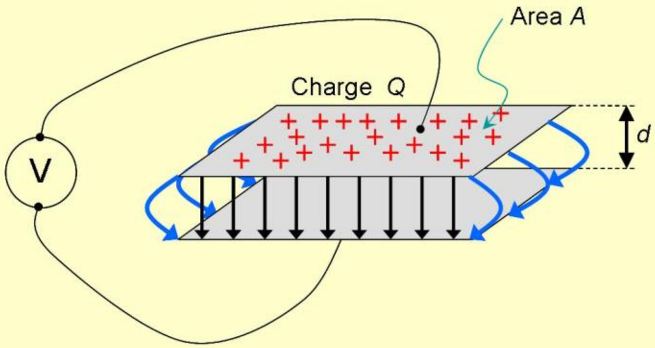
Quadrature bridge - resistance

10pF @ ~ 0.01 ppm

Now used mostly for capacitance

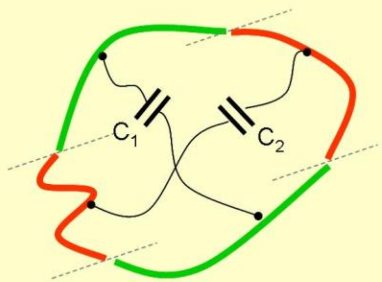
Difficult to build

Extra #4



Simple parallel Plate capacitor

$$C = Q/V \qquad C = \frac{\varepsilon_0 A}{d}$$

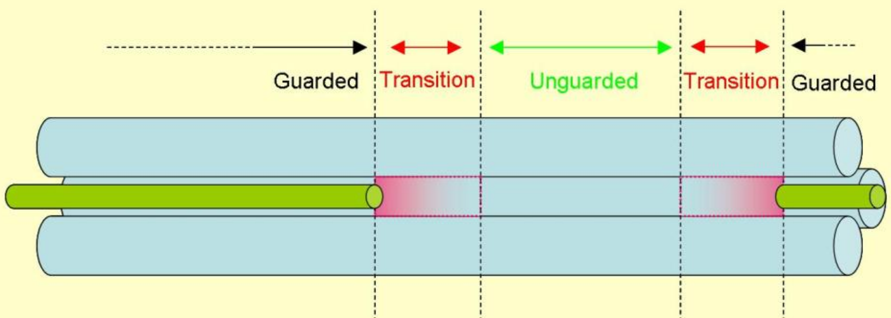


Thompson Lampard Theorem:

System of infinite parallel conductors
C_1 and C_2 are cross capacitances per unit length

$$\exp\left(-\pi C_1/\varepsilon_0\right) + \exp\left(-\pi C_2/\varepsilon_0\right) = 1$$

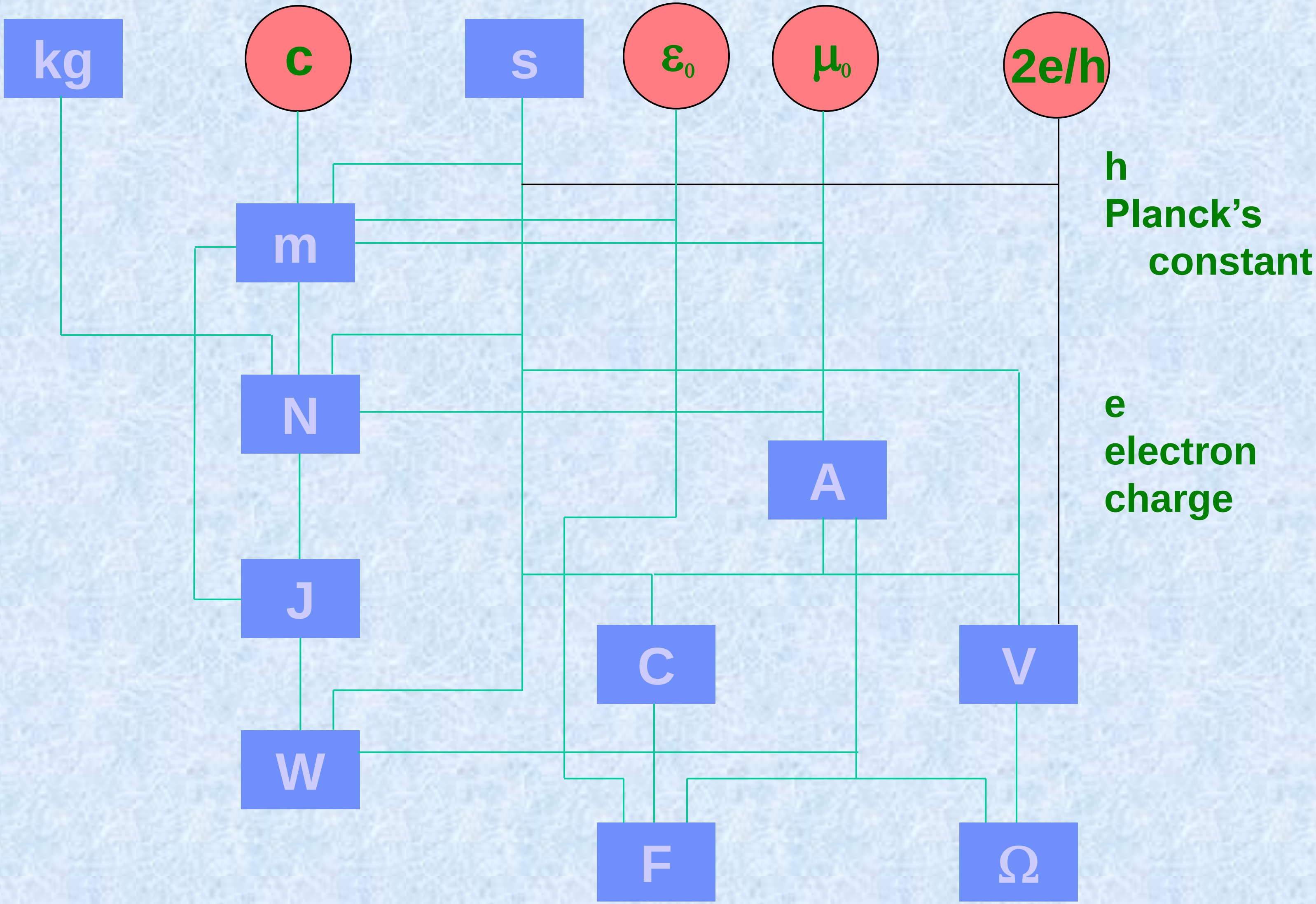
Practical realisation of a calculable cross capacitor



4 cylindrical bars and a central guard bar

BIPM web-site
Oct. 2013

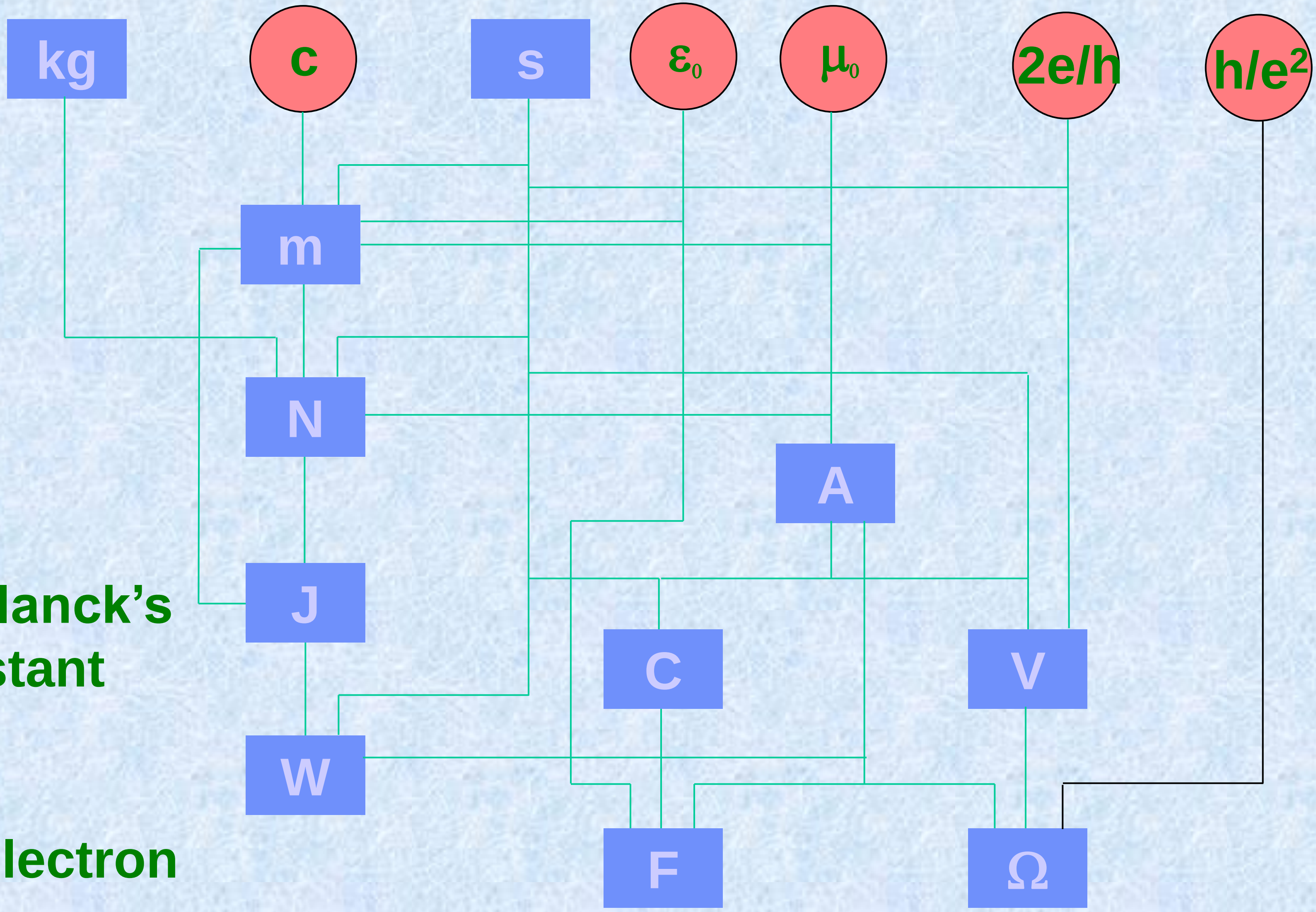
SI Unit Hierarchy & Josephson Effect



The Josephson representation of the SI volt

SI Unit Hierarchy today

ε_0 permittivity of vacuum

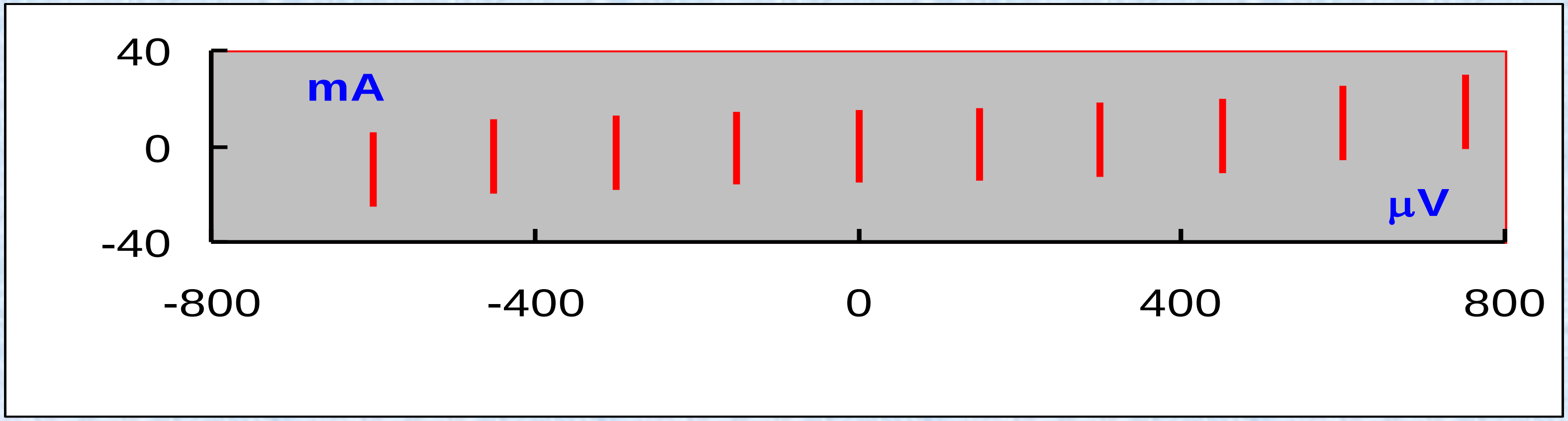


h Planck's constant

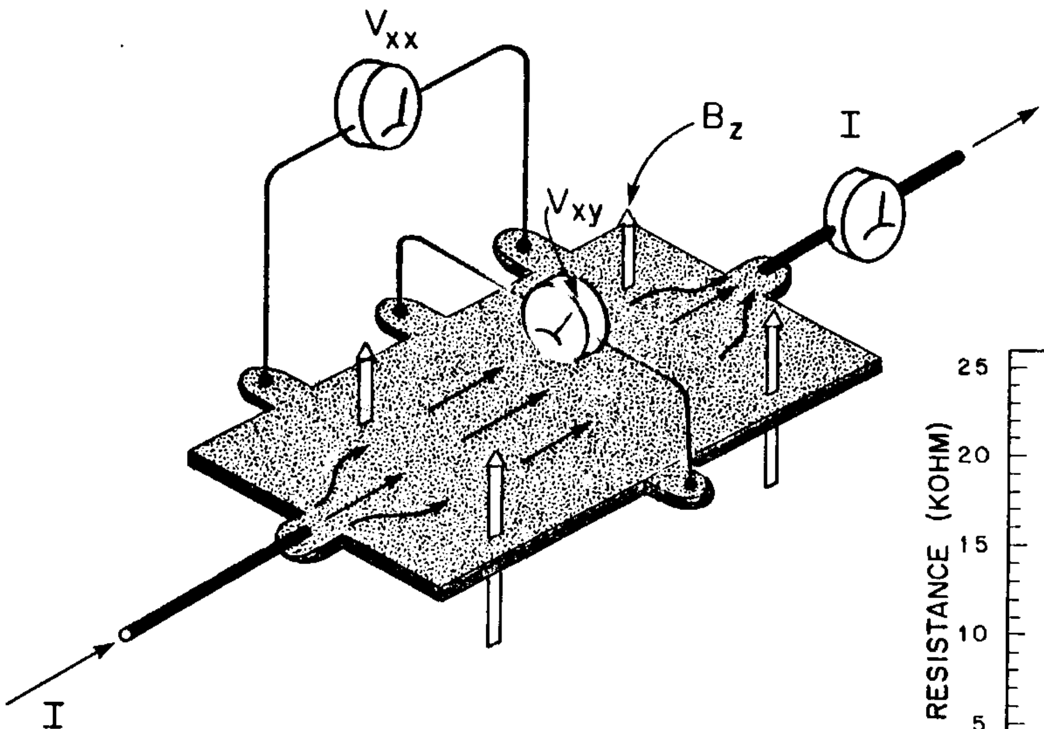
e electron charge

Josephson Effect Voltage Standard

- A Josephson junction is formed when two superconductors are separated by a weak link, which allows tunneling of electron pairs
- A bias current is applied, and the junction is irradiated with microwave radiation
- *IV* curve for the junction shows a series of regularly-spaced steps in the voltage



Quantum Hall Resistance 1990-

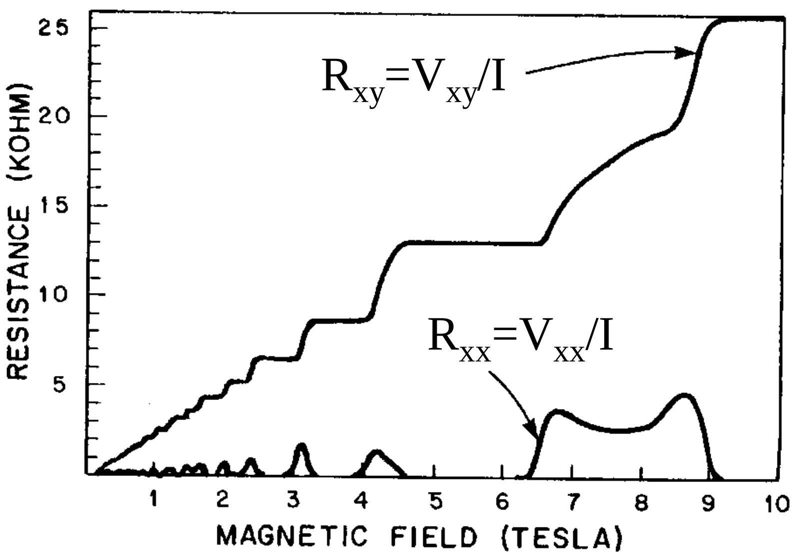


Internationally accepted

Easier to realize

Accurate < 0.003 ppm

***R*$_{K\text{-}90}$ = 25812.807 Ω**



VOLTAGE

DC Voltage: 0 → 1000 V

Josephson Voltage Representation (1990)

Standard cells, zeners

Calibrators, DVMs,

(Hamon divider for scaling)

The Josephson Voltage Standard

$V_n = n.(h/2e) f$ **2e/h = Josephson constant**

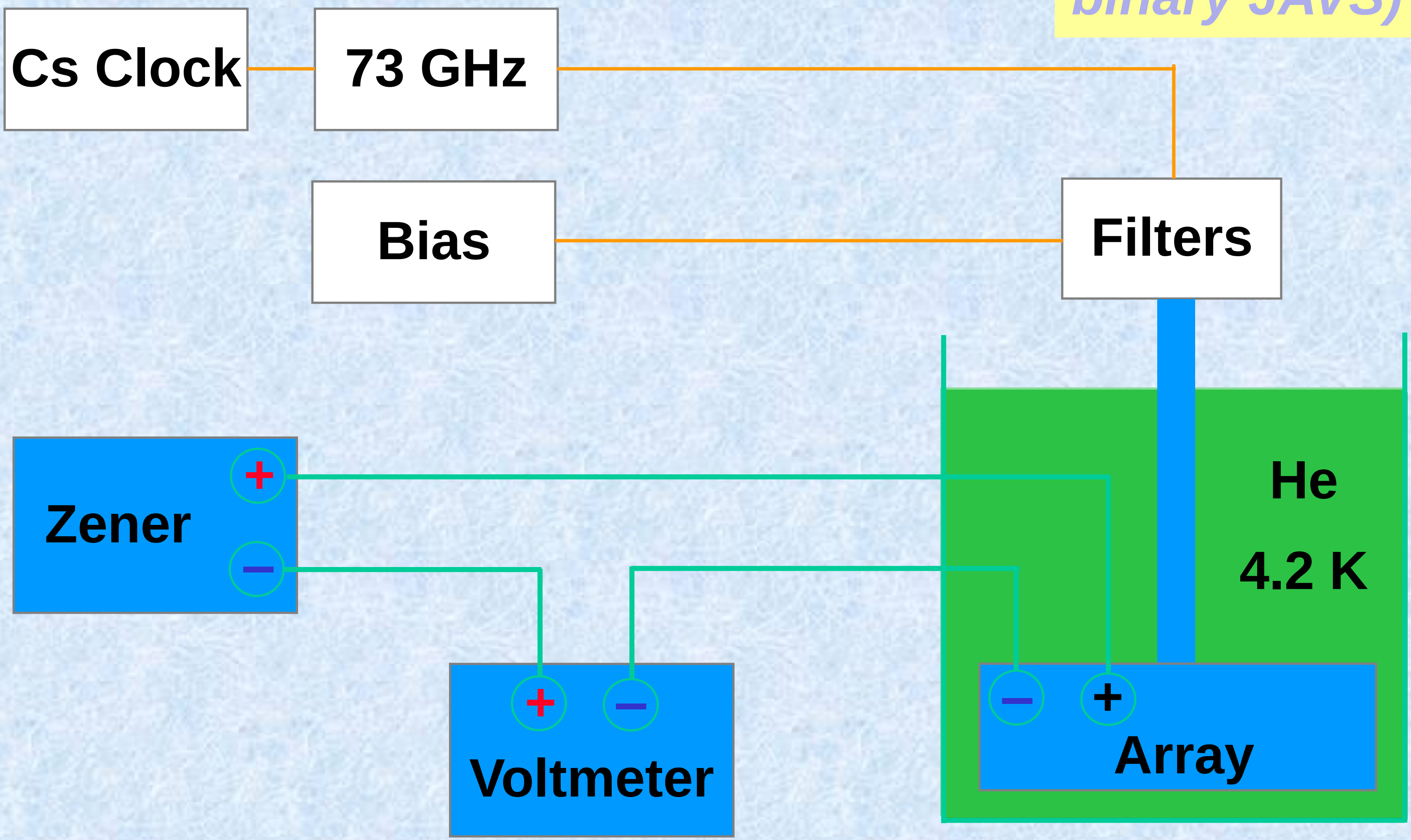
$$K_{J\text{-}90} = 483597.9 \text{ GHz/V}$$

- Steps are vertical, no thermal EMF's in the JJ, no IR drops within the JJ.
- Independent of JJ size, construction, temperature …
- Many steps; V_n=150 μV @ 75 GHz, adjustable to the same resolution as the frequency, f.

The Josephson effect is believed to be independent of correction to about 10^{-17}

The NRC JAVS Schematic

0 – 10 V
Zeners, DMMs
(Also acJAVS, binary JAVS)

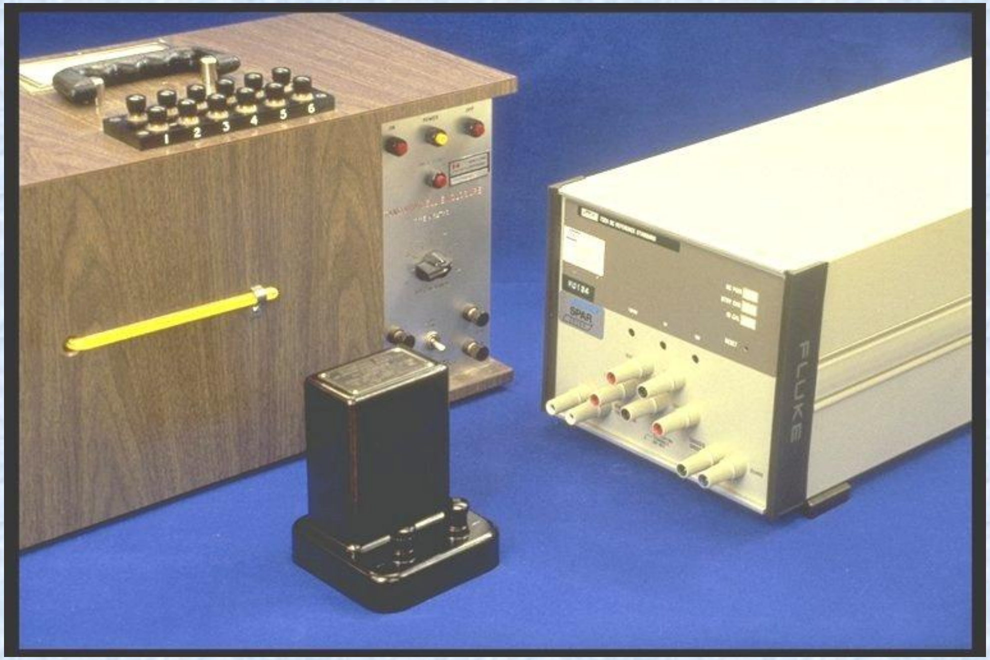


Secondary Voltage Standards

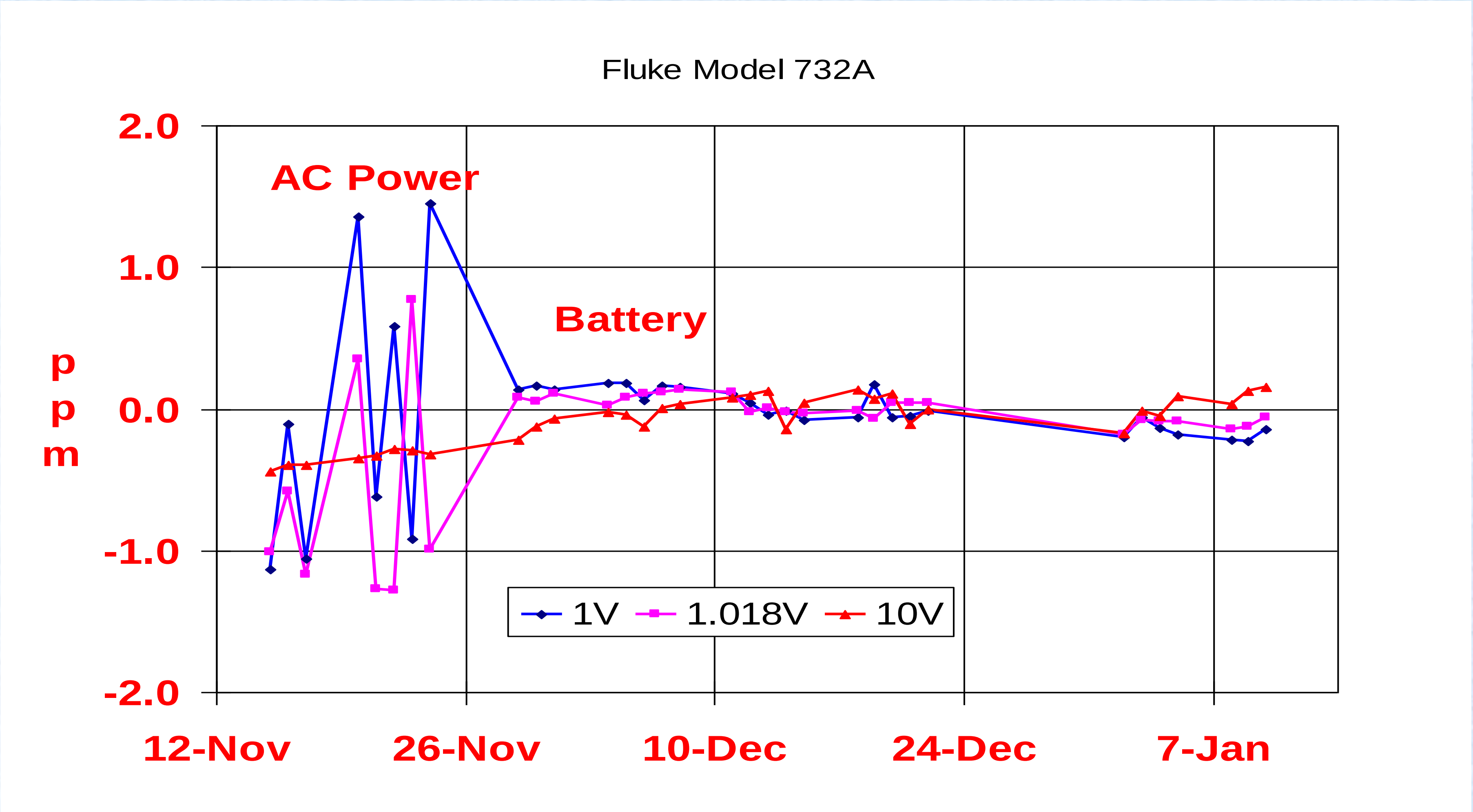
Standard cells used to be primary standards but are gradually falling into dis-use.

Zeners are now the most common voltage standards.

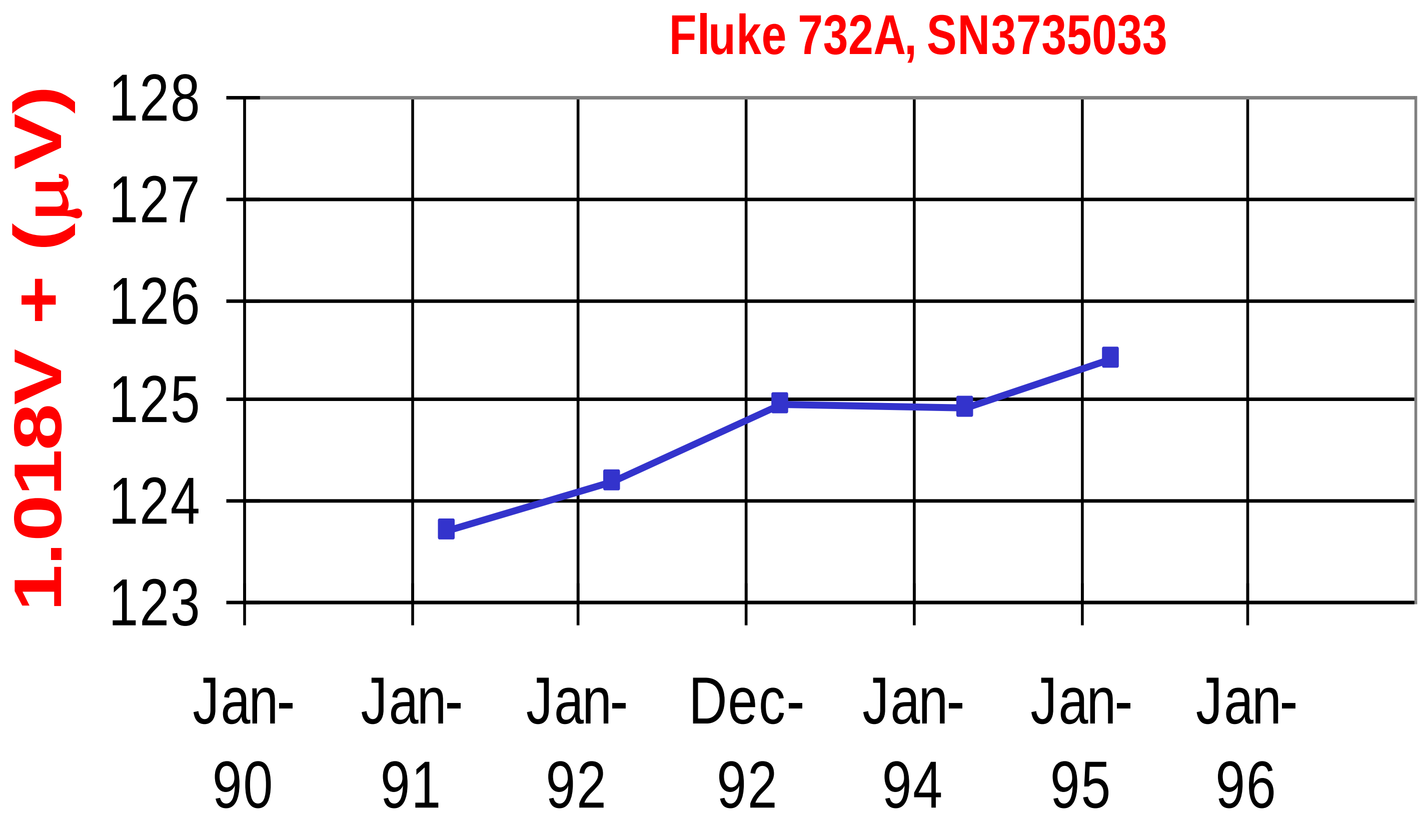
Increasingly industry uses calibrators and DVMs with internal zeners for primary dc voltage reference.



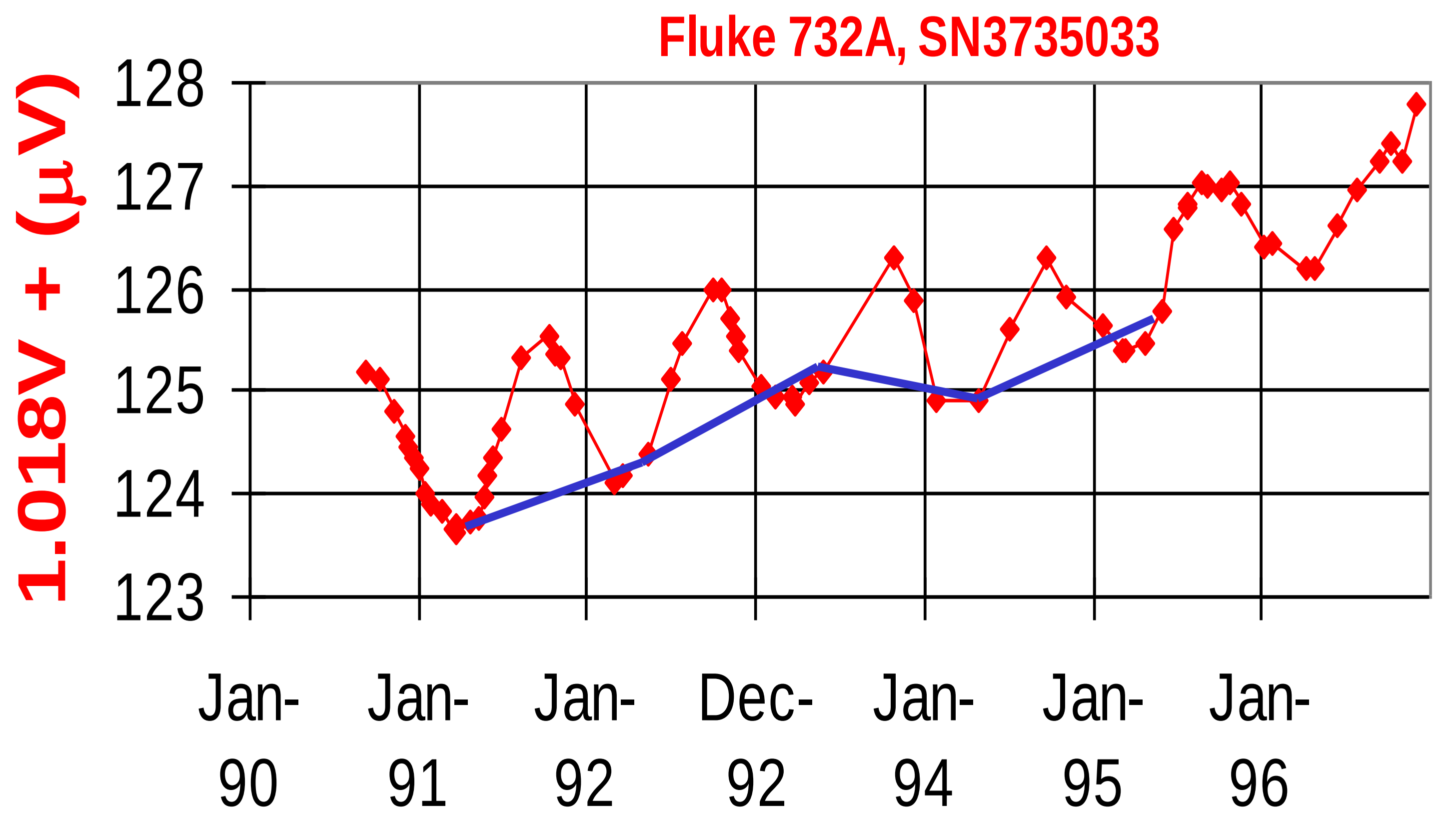
Zener stability - AC Power or Battery



Zener stability - time



Zener stability - time



DC Voltage Summary

Primary standards	Josephson Arrays 10 V (0.001×10^{-6})
Secondary standards	standard cells, zeners(Fluke 732B), DVMs and calibrators 1 mV to 1000 V ($1 \times 10^{-6} – 0.02 \times 10^{-6}$)
Detectors	EM Amplifiers, HP3458A, Keithley NanoVoltmeters
Scaling	JJ Arrays to 10V, Resistive Dividers, Bootstrap techniques for higher voltages Scaling to 1 MV is possible

RESISTANCE

Resistance

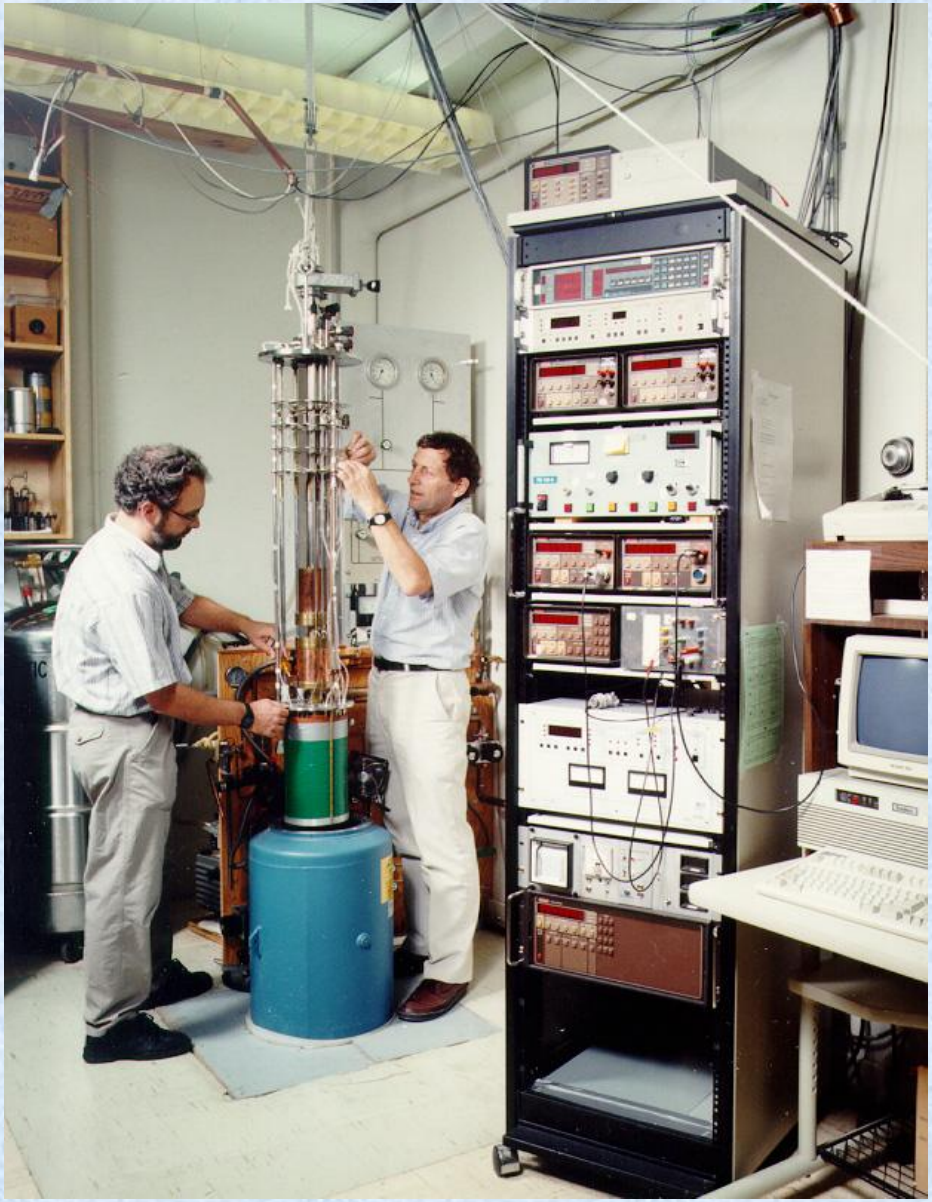
- Easily realizable **10 μΩ to 10 PΩ**

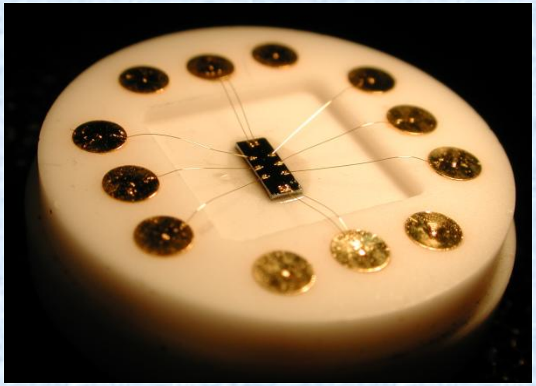
  (10^{22} orders of magnitude)

Teflon	$10^{13} – 10^{18}$ Ωcm	
Sapphire, Quartz	$10^{15} – 10^{18}$ Ωcm	
Cu	1.67 10^{-6} Ω cm	0.0068 /ºC
Pb	20.6 10^{-6} Ω cm	0.0034 /ºC
Evanohm	134 Ωcm	0.00001 /ºC

- Sensors: temperature, pressure, force, optical intensity, strain ……..

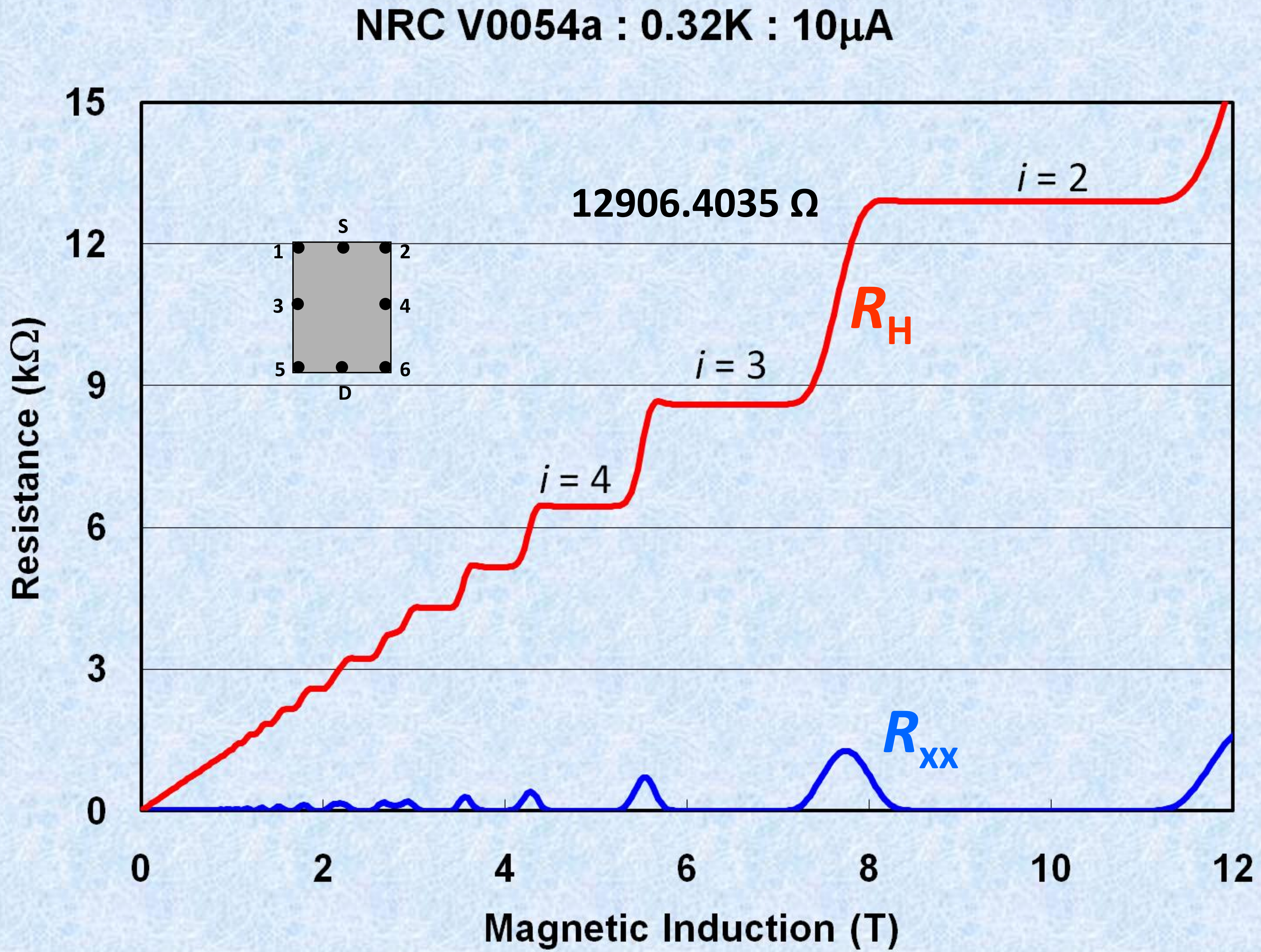
DC Quantum Hall



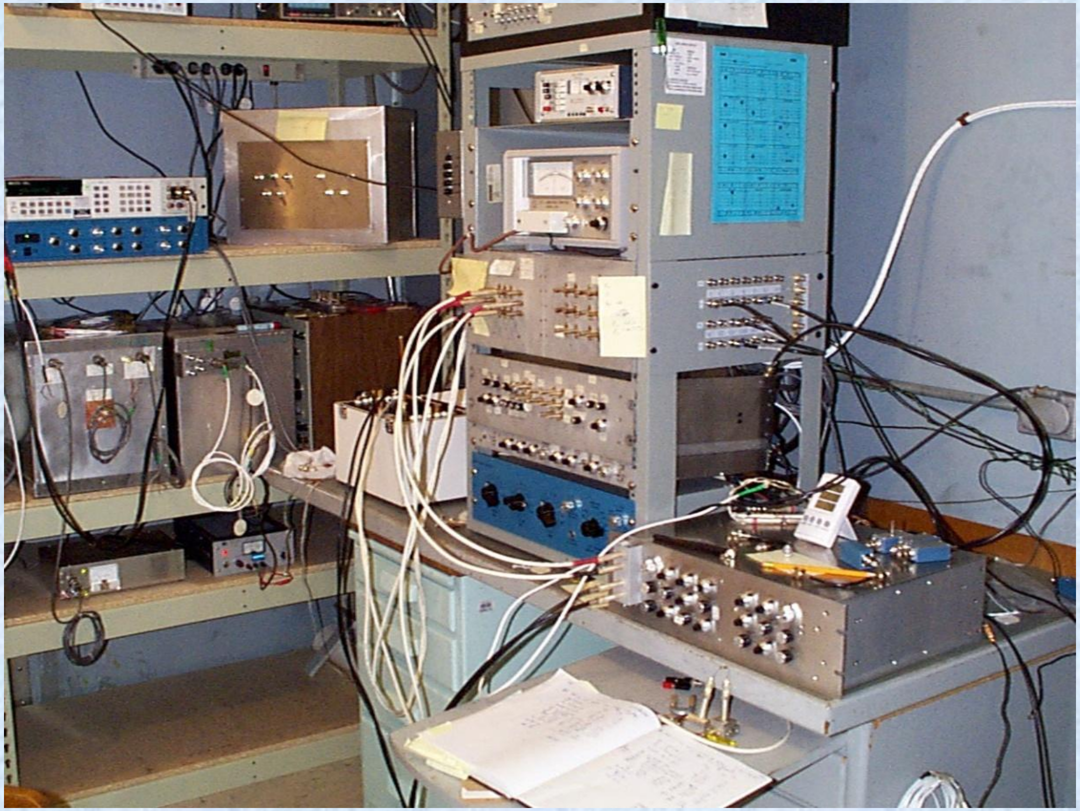


Primary Representation

- **GaAs/AlGaA heterostructure**
- **(also Si Mosfet, II-IV structures, graphene)**
- **14T magnet**
- **Pumped helium cryostat**
- **CCC Bridge, JAVS potentiometer…**


NRC V0054a : 0.32K : 10μA
15
12
9
6
3
0
Resistance (kΩ)
12906.4035 Ω
i = 2
i = 3
i = 4
R_H
R_xx
S
1
2
3
4
5
6
D
0
2
4
6
8
10
12
Magnetic Induction (T)

CCC Resistance Bridge



Primary resistance ratio bridge

measures QHR on steps 2,3,4 & 6..

ratios of 1:1, 10:1, 100:1 and others

0.1Ω - 100kΩ

In many NMIs the CCC resistance bridge is the primary dc ratio bridge

• for the QHR to resistors

• but scaling 0.1Ω - 1MΩ

•Noise optimized for step #2 to 100 Ω.

•Bridge leakage effects are exceptionally small.

•Redundant ways to get the same ratio.

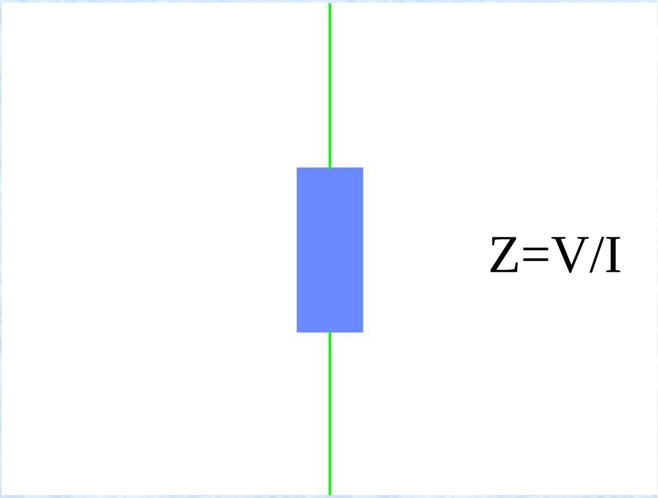
Secondary Standards of Resistance

Coil or thin film
2 or 4 terminal
Oil: 25 C, air 23C, or …….

10 μΩ – 1 Ω
1 Ω – 100 kΩ
100 kΩ – 10 PΩ

Impedances

2 terminal impedance



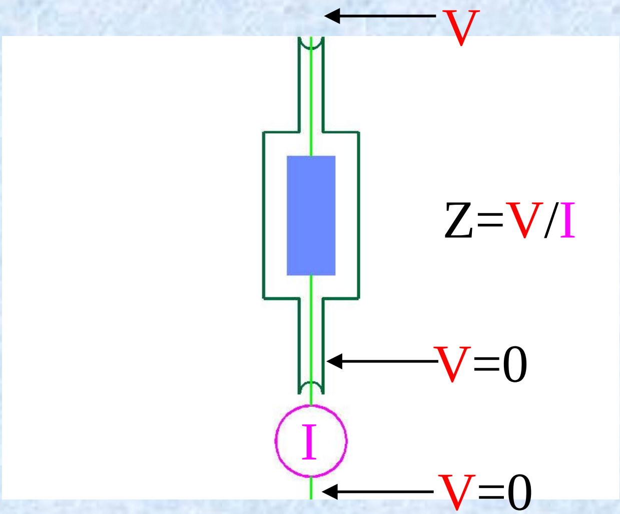
Voltage across and Current through

Internally impedances are 2-terminal devices.

Choice of materials determines external influence on the element: temperature, pressure, humidity.

External parameters, generally electrical, can also have a significant effect in improving the accurate and repeatable measurement of impedance.

3 Terminal Impedances



Voltage across and
Current out

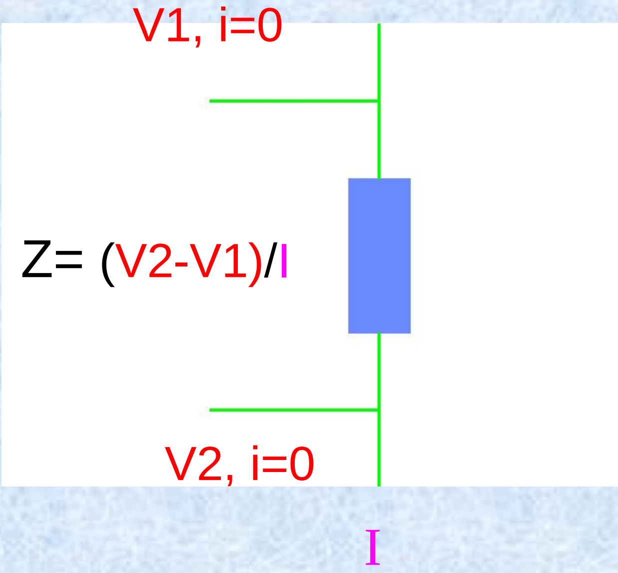
High value impedances are very susceptible to currents from the environment, instruments…

A conductive electrostatic shield surrounding the impedance and connected through a low impedance to a fixed (preferably 0) potential eliminates any external currents and stabilizes internal leakage currents.

This ‘3 terminal’ impedance is defined as the voltage across the impedance divided by the current out of the lower potential lead. It is used for most capacitance and high value resistance measurements.

4 Terminal Impedances

4 terminal impedance



Voltage across and Current through

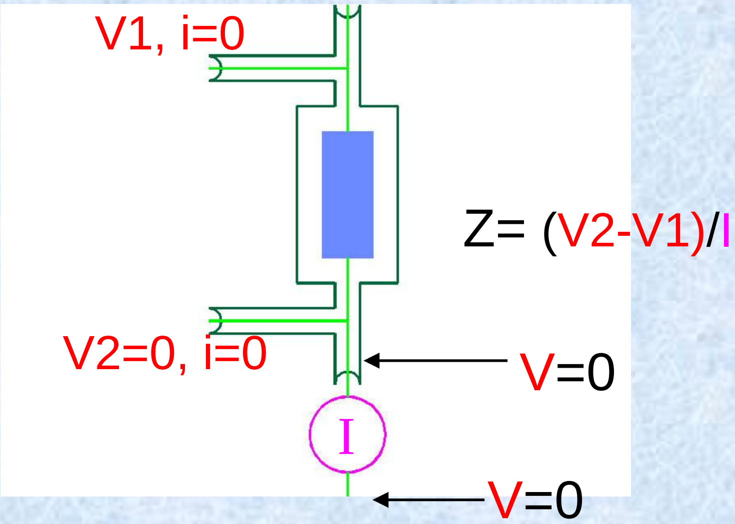
Low value impedances suffer from poorly defined potentials, especially if current is flowing along the leads that are measuring the potential.

Measure the potential with leads that carry no current. Define the potential junction to the impedance with low impedances that are invariant to the current.

This is a '4 terminal' impedance and is commonly used for resistances less than 100 kΩ.

4 Terminal Impedances: Electrostatic Considerations

4 terminal coaxial pair impedance



Voltage across and Current out

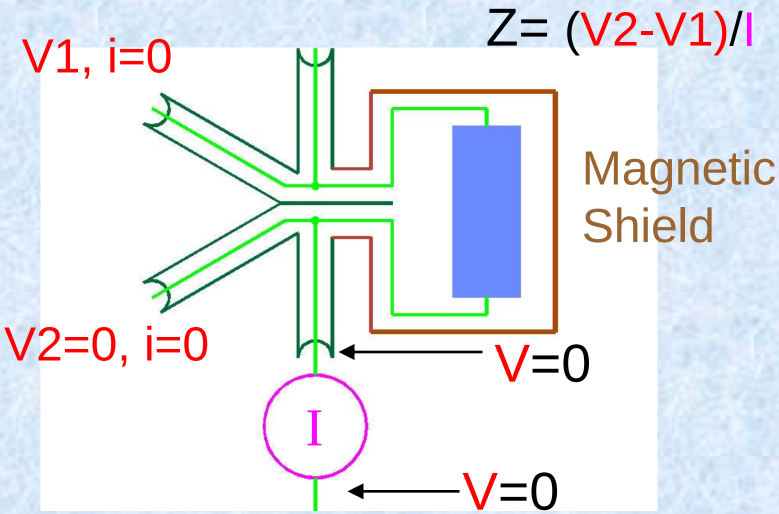
Combines last 2 concepts

- improved potential definitions
- electrostatic shielding
- stable internal leakages to the shield

4 Port impedances are used for resistances < 100 kΩ and capacitances > 100pF where the highest accuracy and frequency dependence are important.

4 Terminal Impedances: Magnetic Considerations

4 Terminal Impedance



Voltage across and Current out

For highest accuracy & frequencies < 100 kHz, control the magnetic leakages as well.

•Coaxial cables with equal and opposite currents in the inner and outer conductors ⇒ magnetically astatic

•Use magnetic shields wherever the coaxial features are lost.

•Orthogonal current and potential lead placement ⇒ magnetically astatic


FLUKE
Ls OVERLOAD
Rs OVERLOAD
OHMF
Power Source
"54"
12 V Power Ports
NOM
Programmer Manual

Secondary Standards of Resistance

2 terminal
air bath
EMI issues
Voltage coefficient
leakage

4 terminal
oil bath
Joule heating
connection sensitive

100 TΩ

100 GΩ

1 GΩ

100 MΩ

1 MΩ

100 kΩ

10 kΩ

QHR ★

1 Ω

0.1mΩ

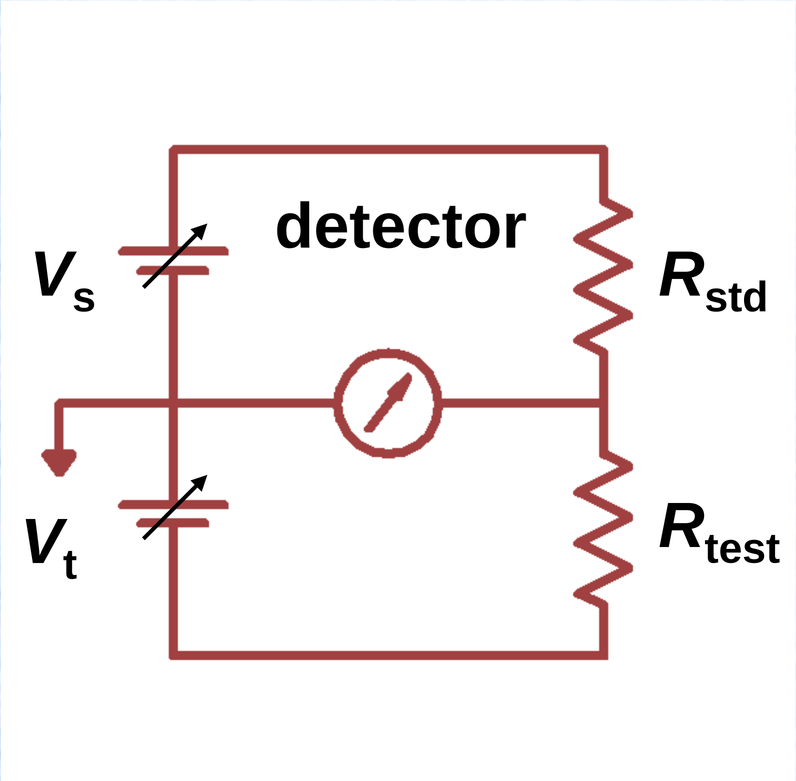
teraohmmeter

dual-source bridge

automated wheatstone bridge

dc current comparator + range extender

Resistance Bridge



For precision values, unknown compared with known resistor

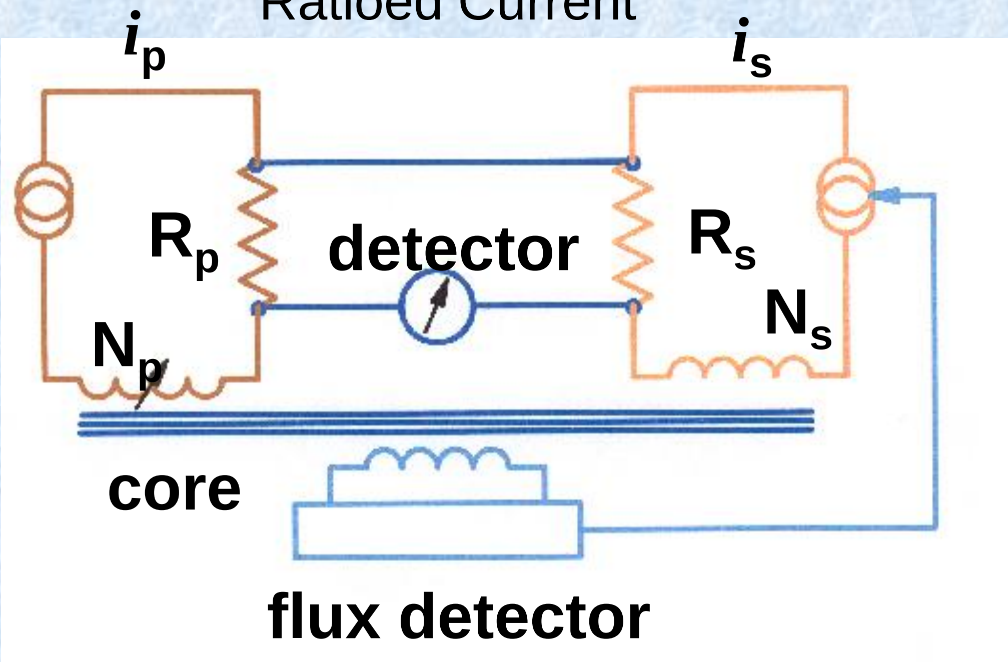
voltages to 1100V

null detector, grounded

reversing sources nulls offsets

DC CURRENT COMPARATOR

Equal Voltage &
Ratioed Current



- N_p varied to zero the difference in voltage drops across resistors
- Then $i_pR_p = i_sR_s$, hence $R_s/R_p = N_s/N_p$
- 4 terminal resistance ratio bridge
- Good linearity - depends on numbers of turns
- 1 Ω →10 kΩ
- Range extender (high current source) down to <10 μΩ
- Commercial bridges available and are extensively used in NMIs and industry.

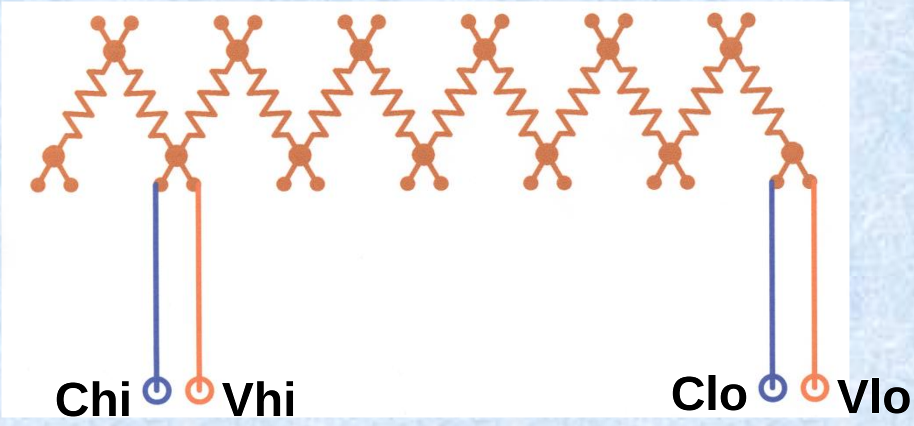
Hamon Resistor – series & parallel configurations

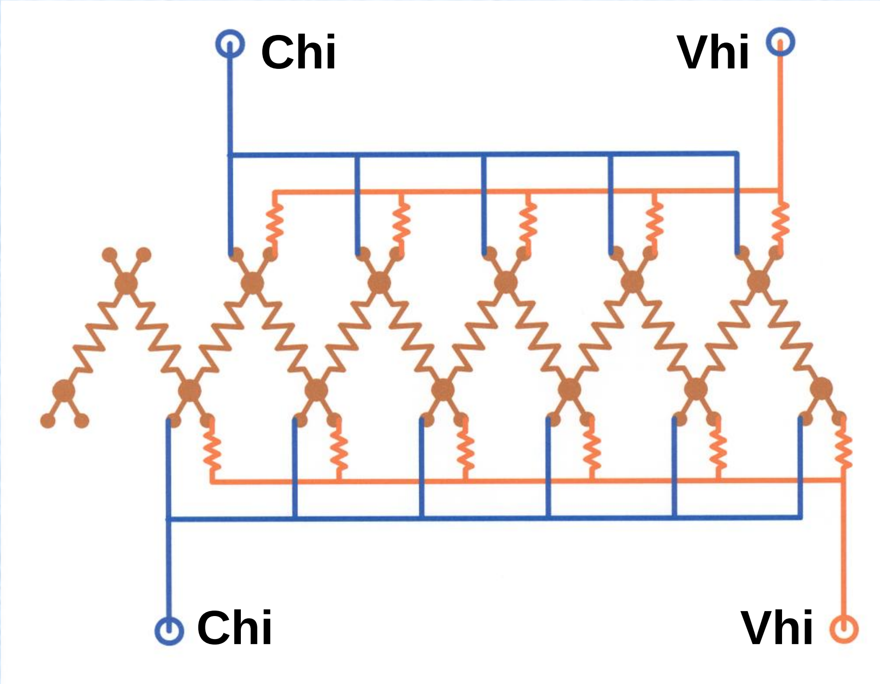
Parallel

Total Resistance = $\frac{R}{n} + \Sigma\frac{\delta R}{n}$

Series

Total Resistance = $nR + \Sigma\delta R$





Ratio of Series/Parallel = $n^2 \quad \delta^2$

Hamon Resistor – Applications

Fluke 752A Reference Divider

Available from 1 Ω → 1GΩ

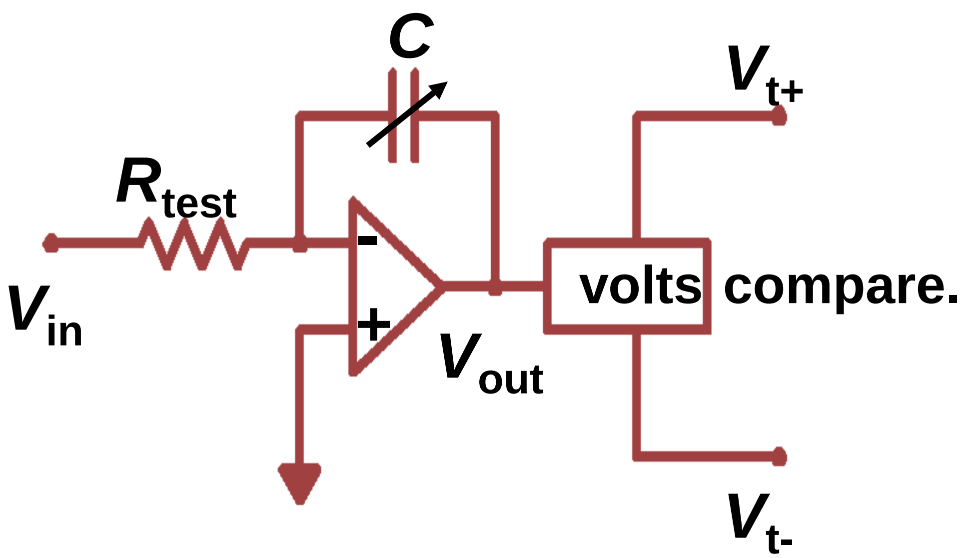
- Voltage Ratios, especially to >1kV
- 10:1 and 100:1 Resistance Ratios
- high resistance as ratio standards

  (in the last decade replaced by CCC)
- Power Coefficient determinations

  the power per Hamon element resistor, *R*, is kept constant and with a current independent 10:1 bridge yields the power coefficient.

Teraohmmeter

A timed capacitive charging technique



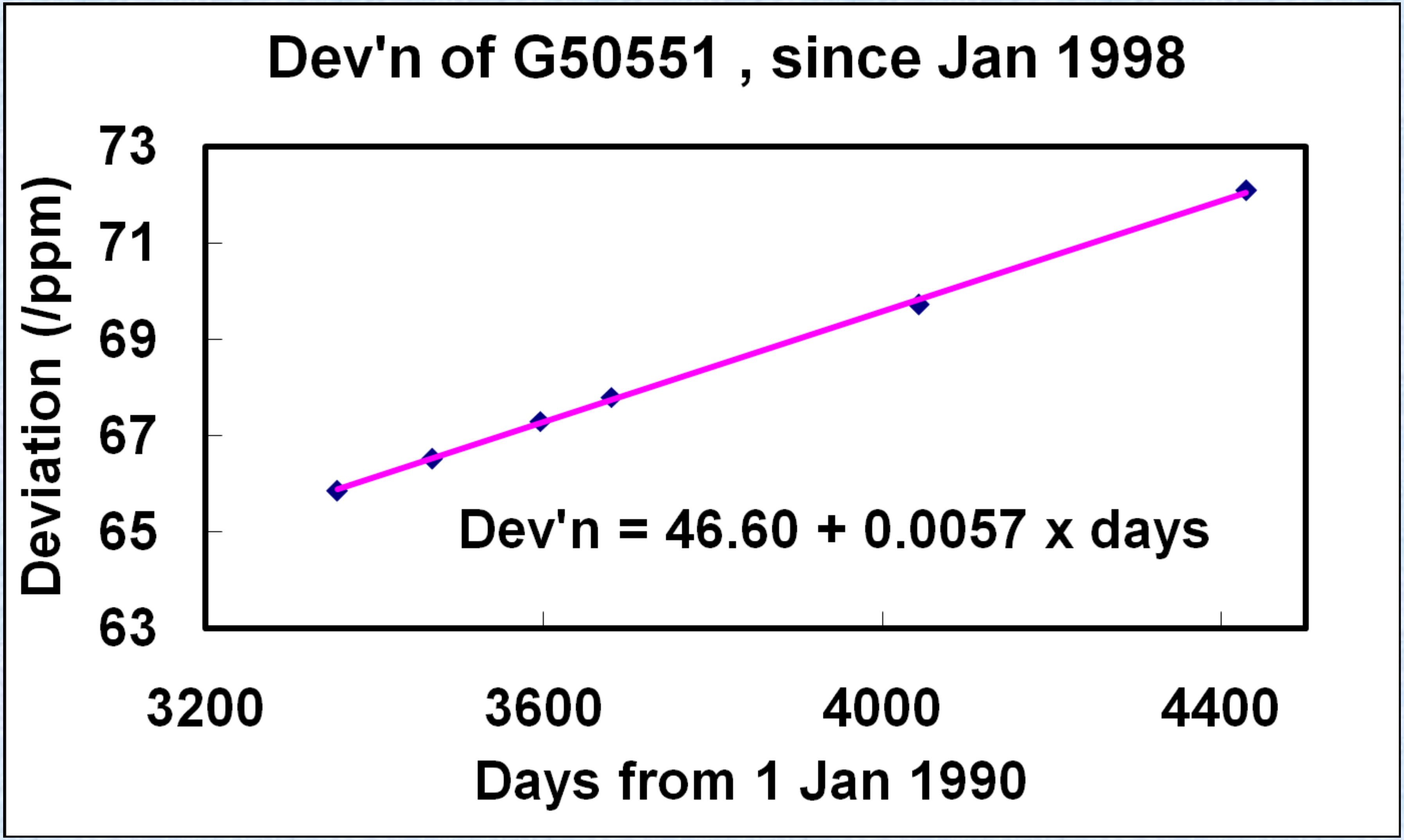
$$100\ \text{G}\Omega \longrightarrow 10\ \text{P}\Omega$$

$$R_{test} = \frac{V_{in}\,\Delta t}{C\,\Delta V_t}$$

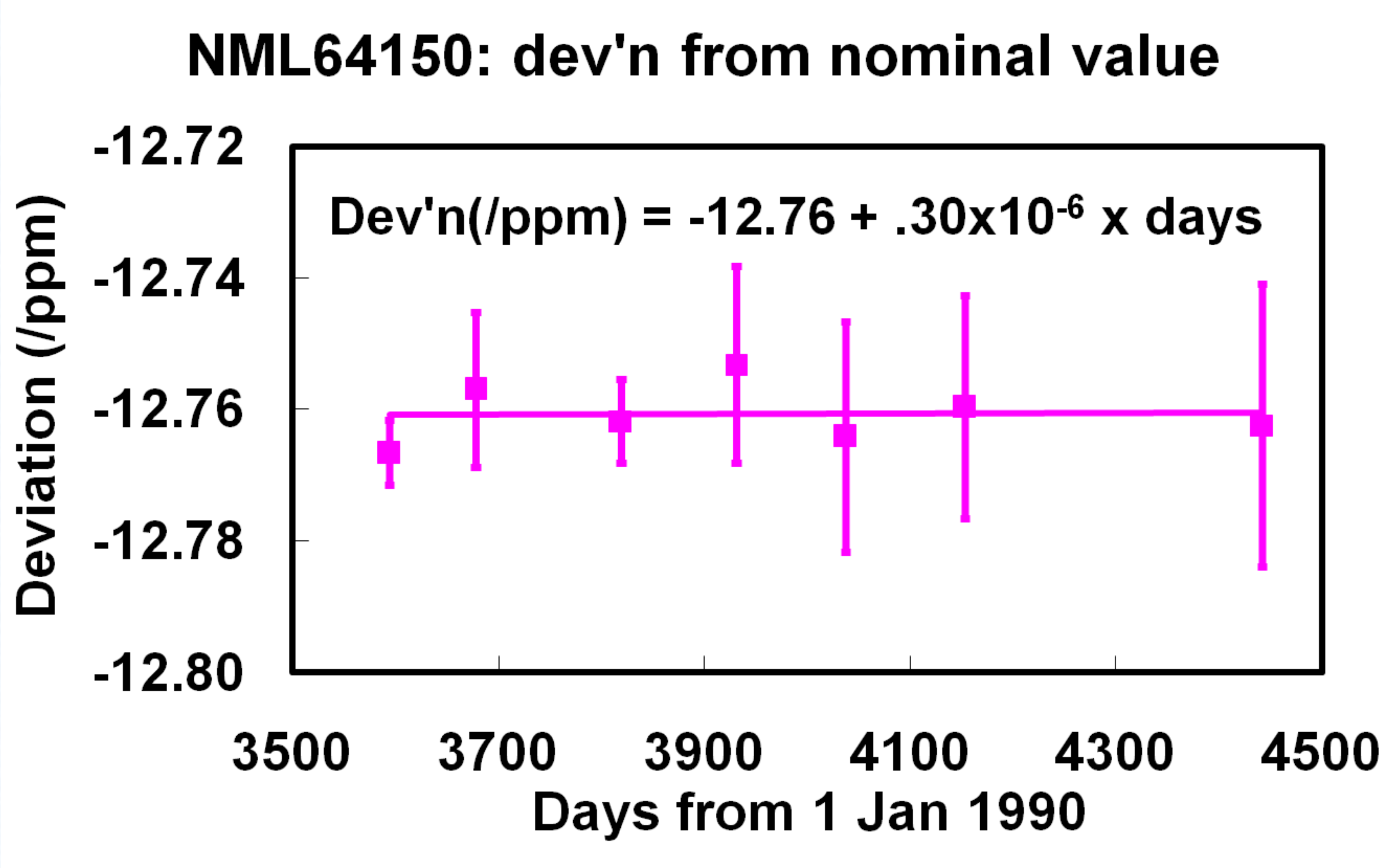
$V_t = 0.1, 1.0$ or 10 V

C = 27, 270, 2700 pF

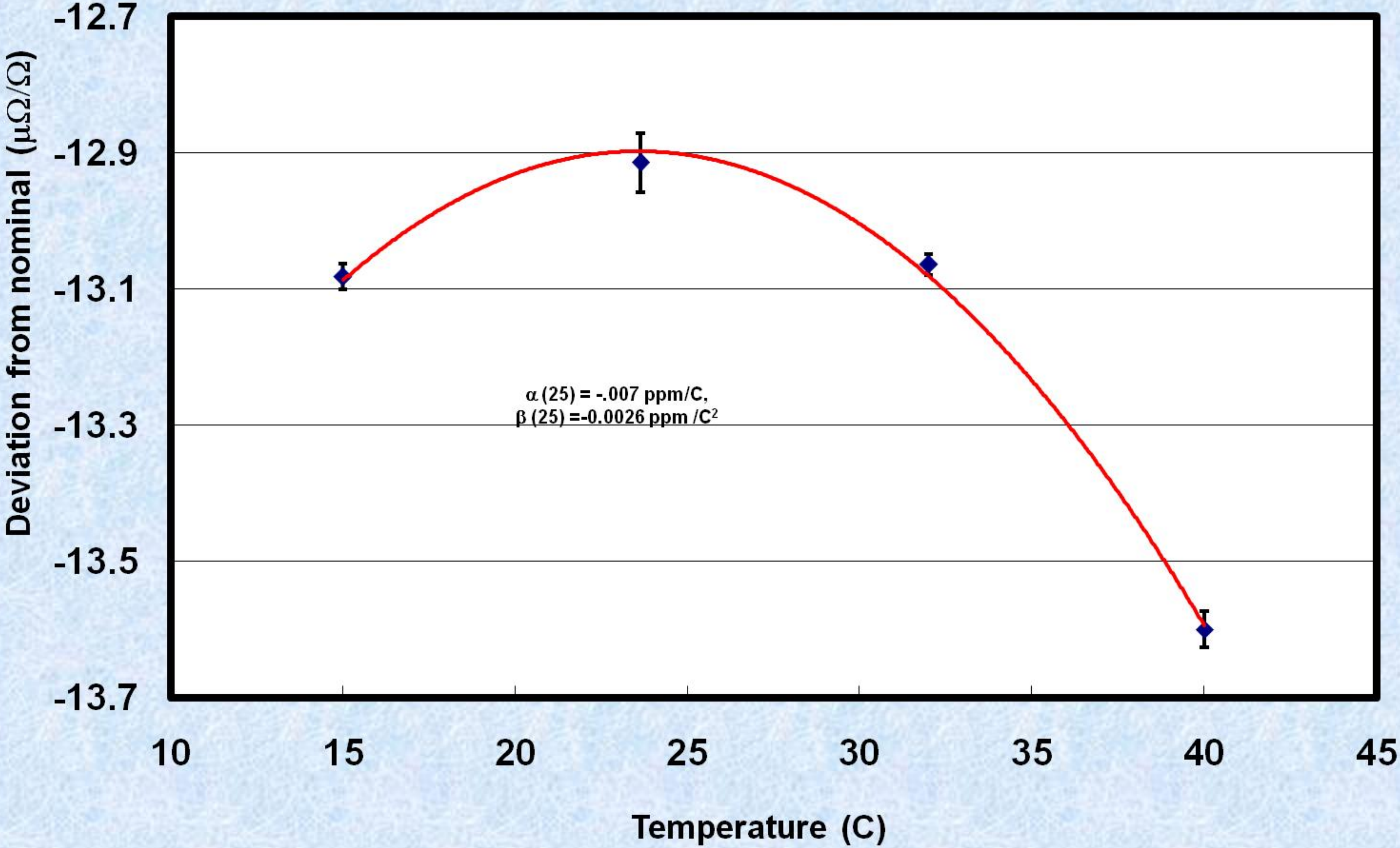
Temporal shift of resistance



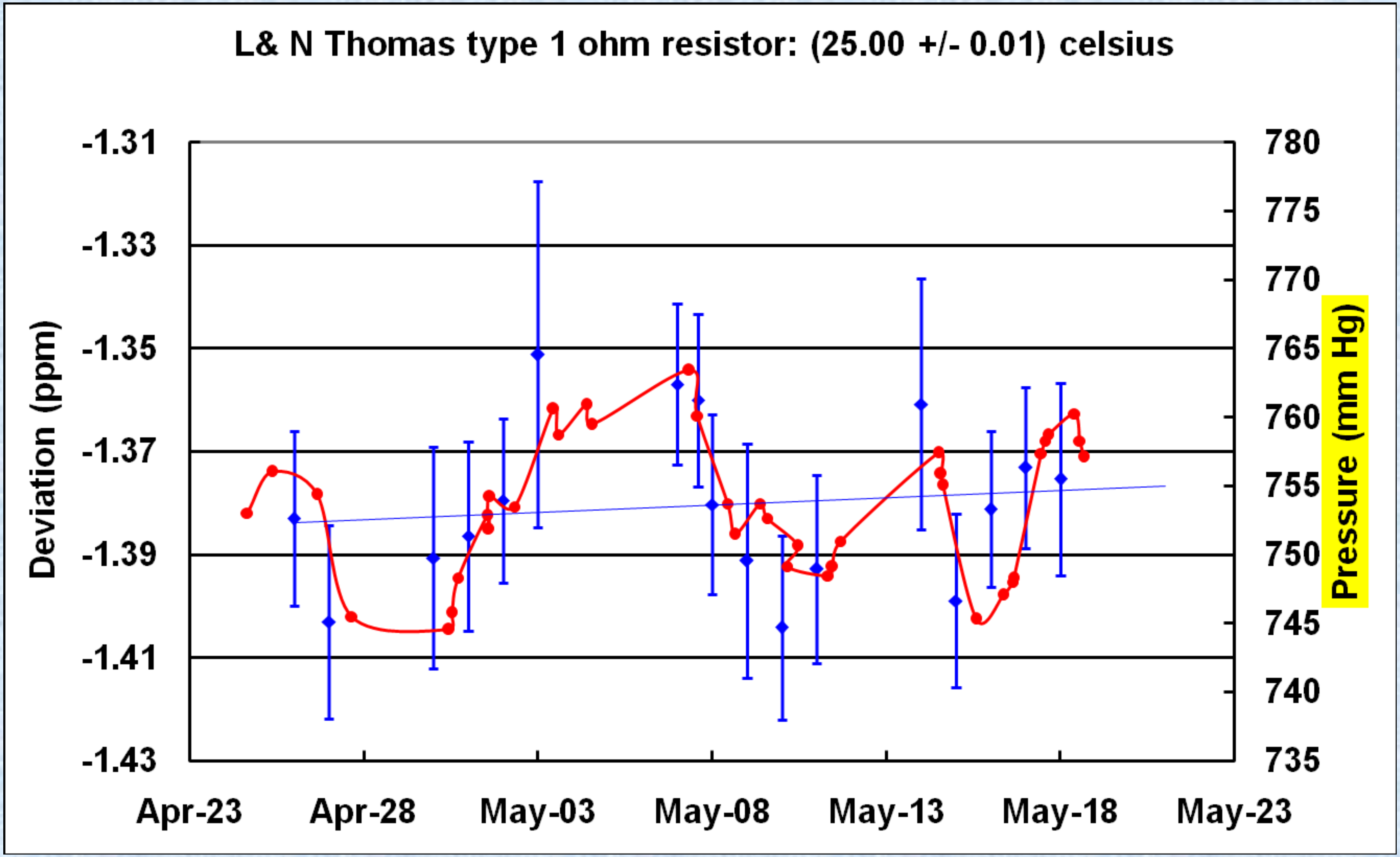
Temporal shift of resistance



Thermal deviation of NML64150 from nominal



Pressure variation of resistance



Capacitance

Capacitance

Primary standard - calculable capacitor

Calculable capacitors to a ~ 0.01 pp
Direct 0.02-1 pF from

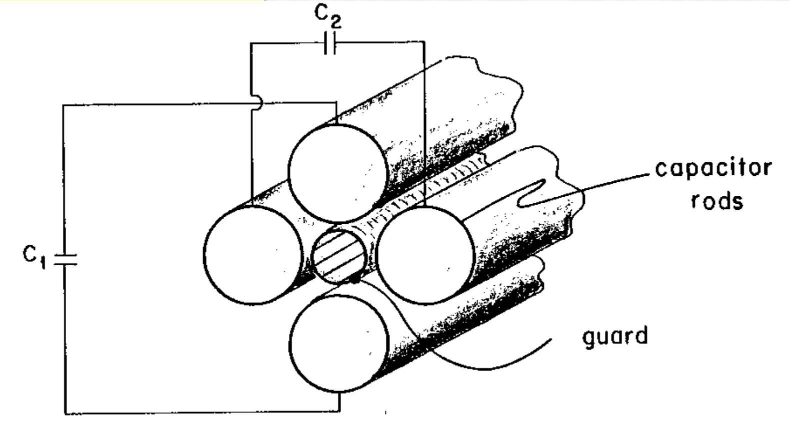
length measurements

NML, NIST, LNE, BIPM

(NIM, NRC)

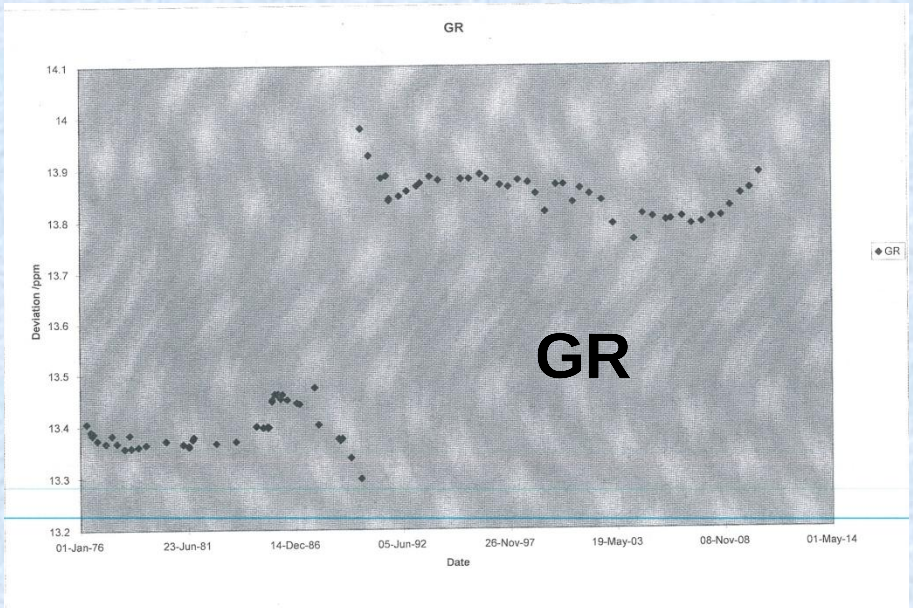
Secondaries at 10, 100 pF

Since 1990 determination via QHR has been an option

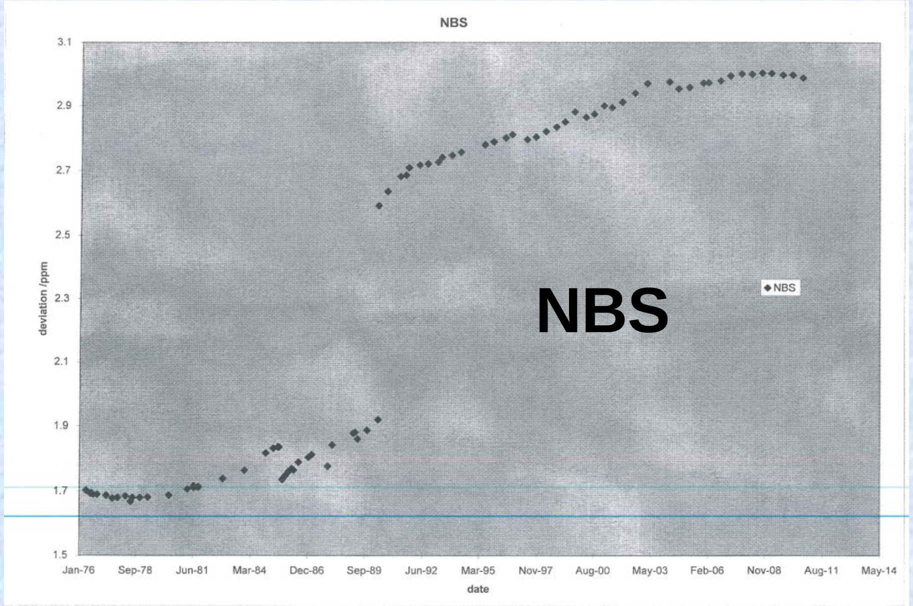


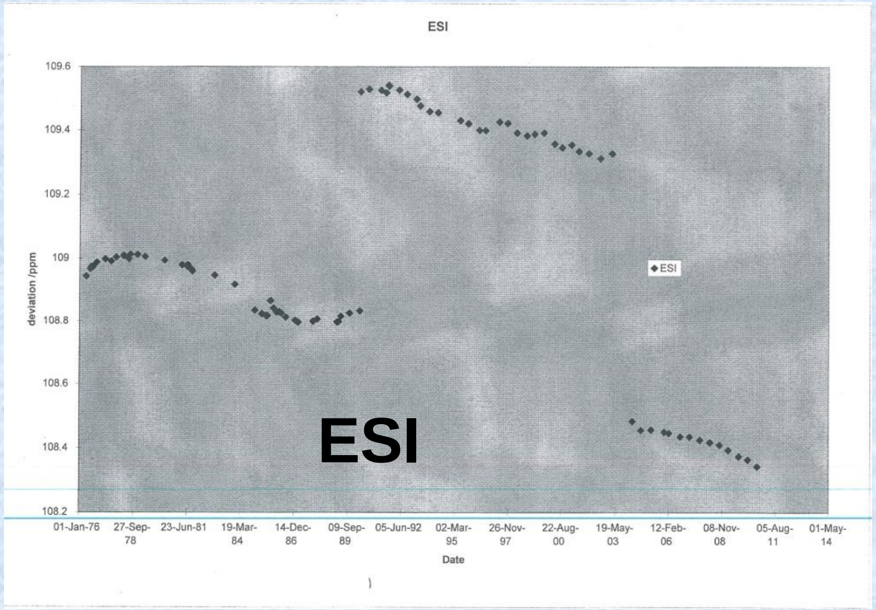
Calculable Capacitor

Primary standards : stability with time, and temperature

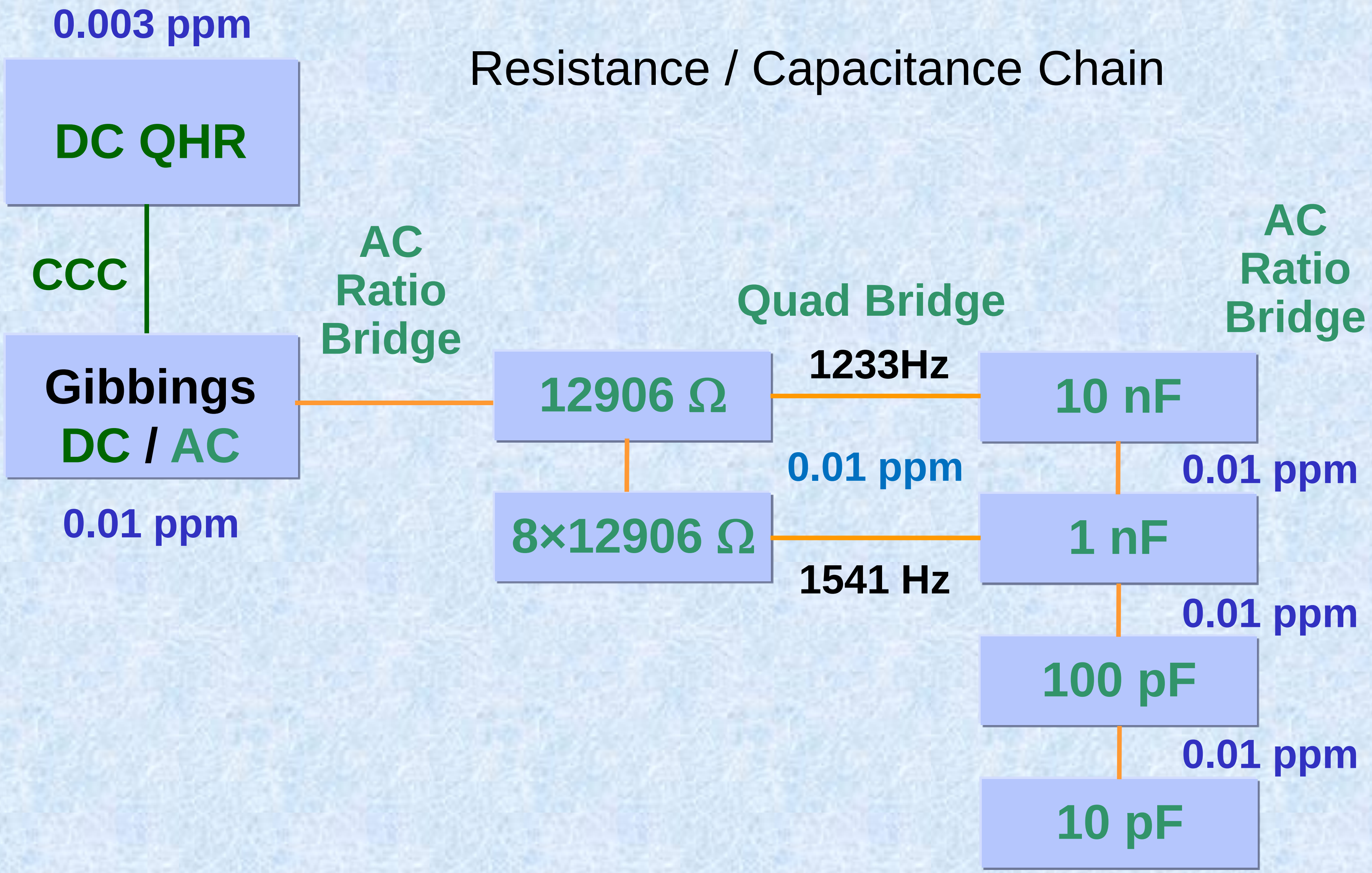


3 x 10 pF
Controlled airbath
35 yrs drift:
GR 0.2 ppm
NBS 0.9 ppm
1990 step is SI change
ESI drop – 2003 power outage





Capacitance traceability via the QHR



Changing from DC Bridges to AC Bridges

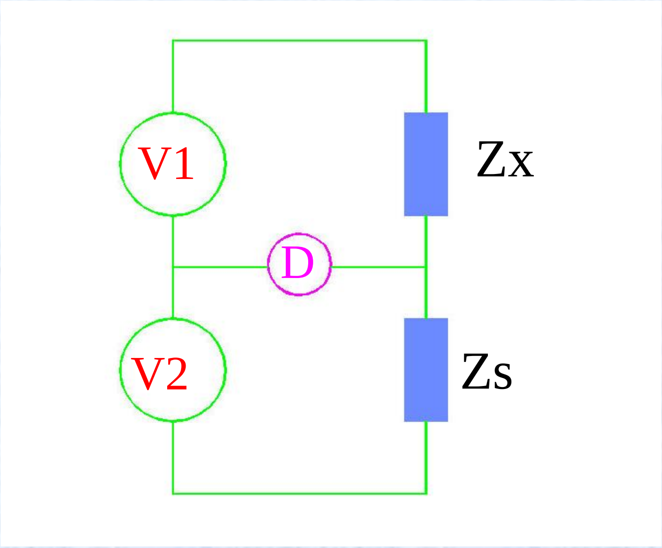
Voltage sources - transformers and IVDs

Null current detectors - injection/detection transformers

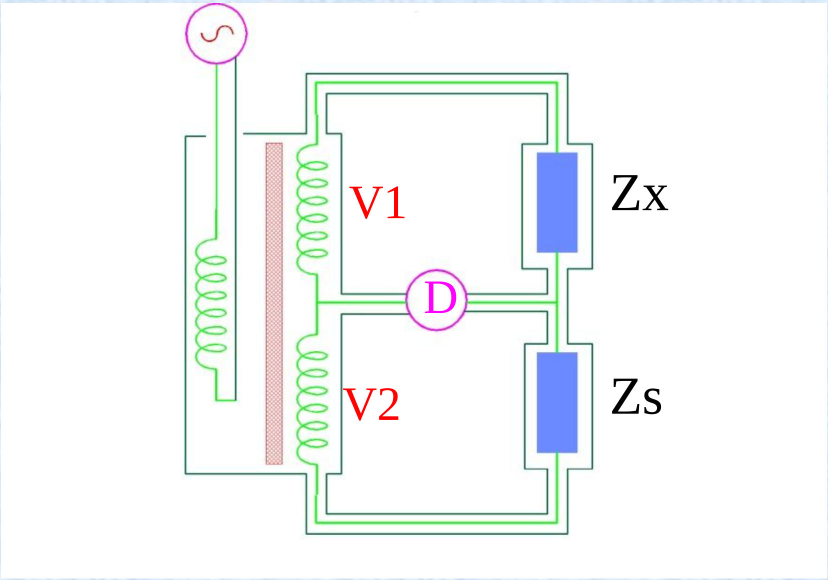
Single wires - coaxial cables perhaps with current equalizers (coaxial chokes)

Voltage null detectors - phase sensitive Lock-in amplifiers

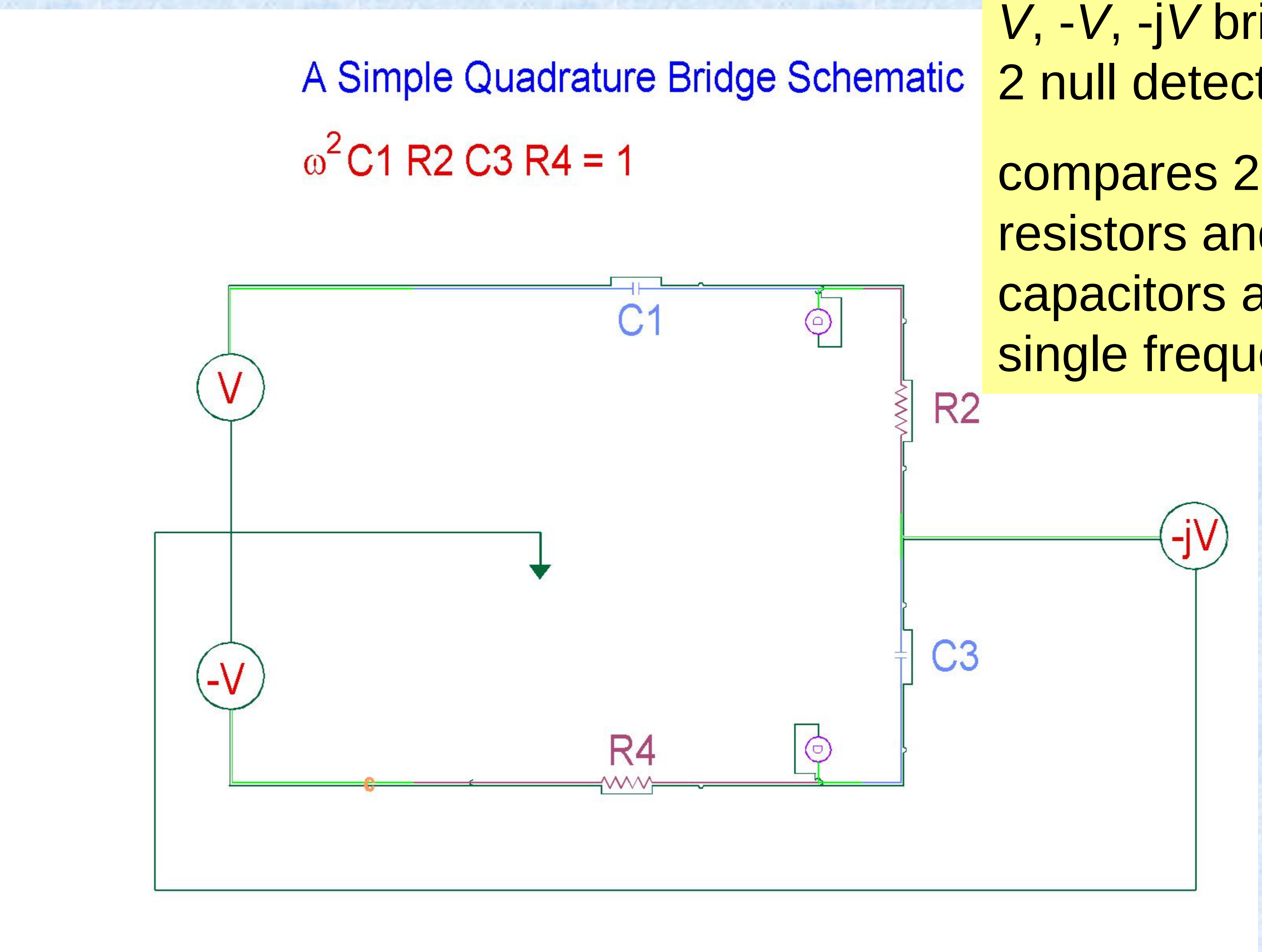
DC 2 Port Ratio Bridge



AC 2 Port Ratio Bridge



AC Quadrature Bridge



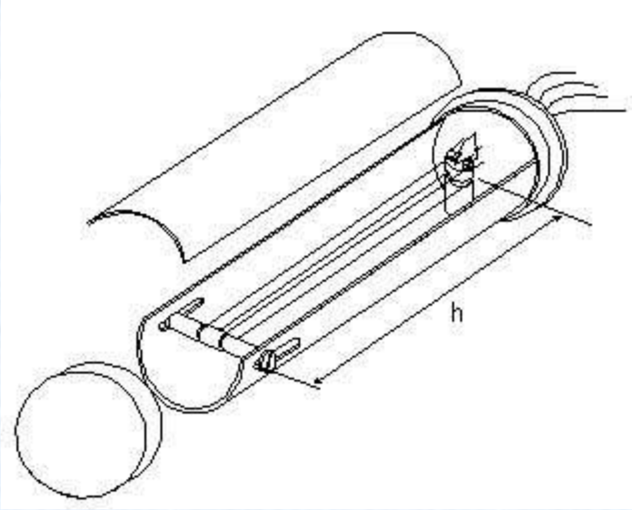
V, -*V*, -j*V* bridge
2 null detectors

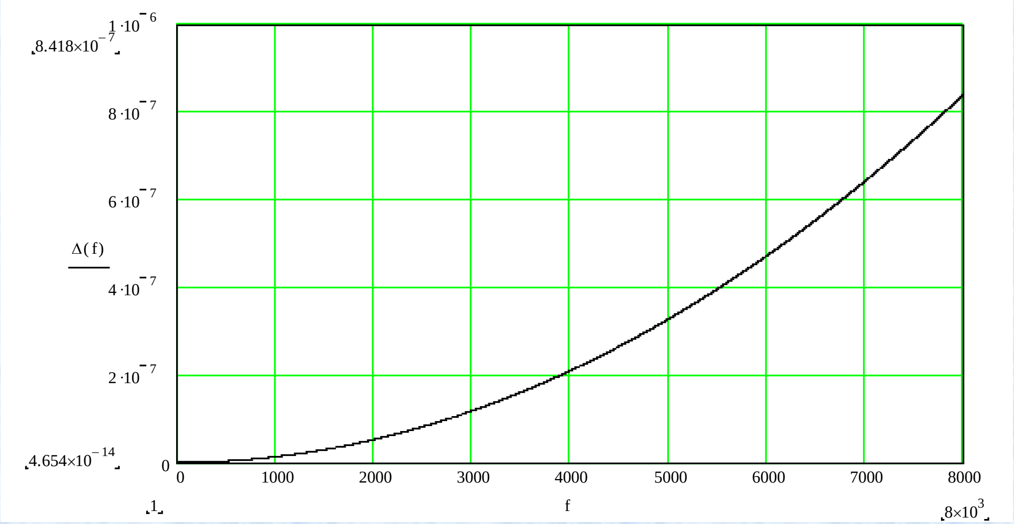
compares 2 resistors and 2 capacitors at a single frequency.

Frequency Calculable Resistors

There are frequency dependence models for two common types of precision resistors; the coaxial or Haddad type and the reversed quadrifilar or Gibbings type.

Comparisons of different designs show that, these models and their resistors can used up to 10 kHz and for resistors < 20 kΩ. At 1592 Hz / 1 kΩ the accuracy can be <0.01 ppm





Commercial Capacitance Bridges

Andeen Hagerling 2700A

50 Hz – 20 kHz, 5 ppm, 8 digits

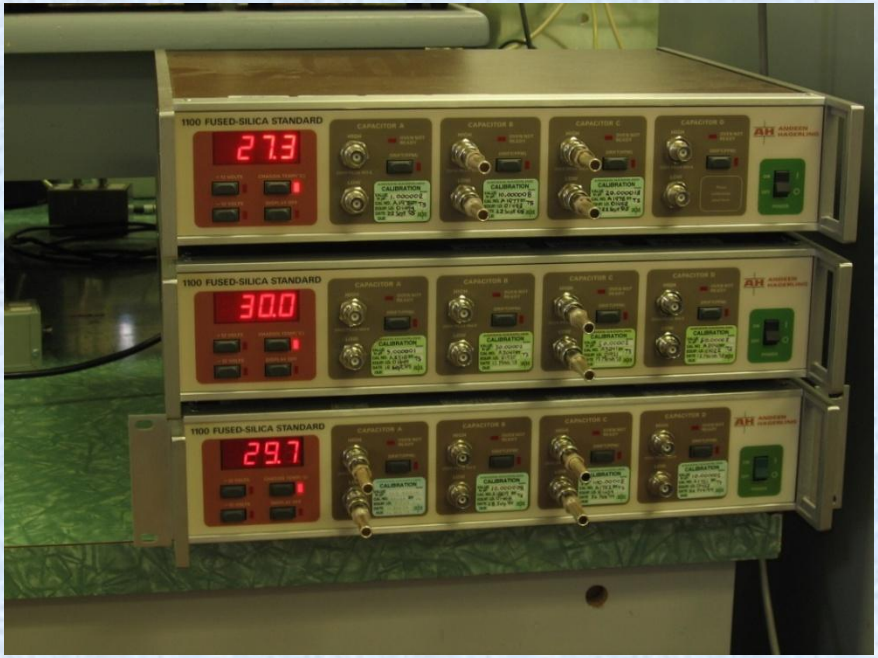
QuadTech 7600 10Hz – 2MHz

0.05% accuracy, 7 digits

Agilent 4284A 20Hz – 1MHz

0.05% accuracy, 6 digits

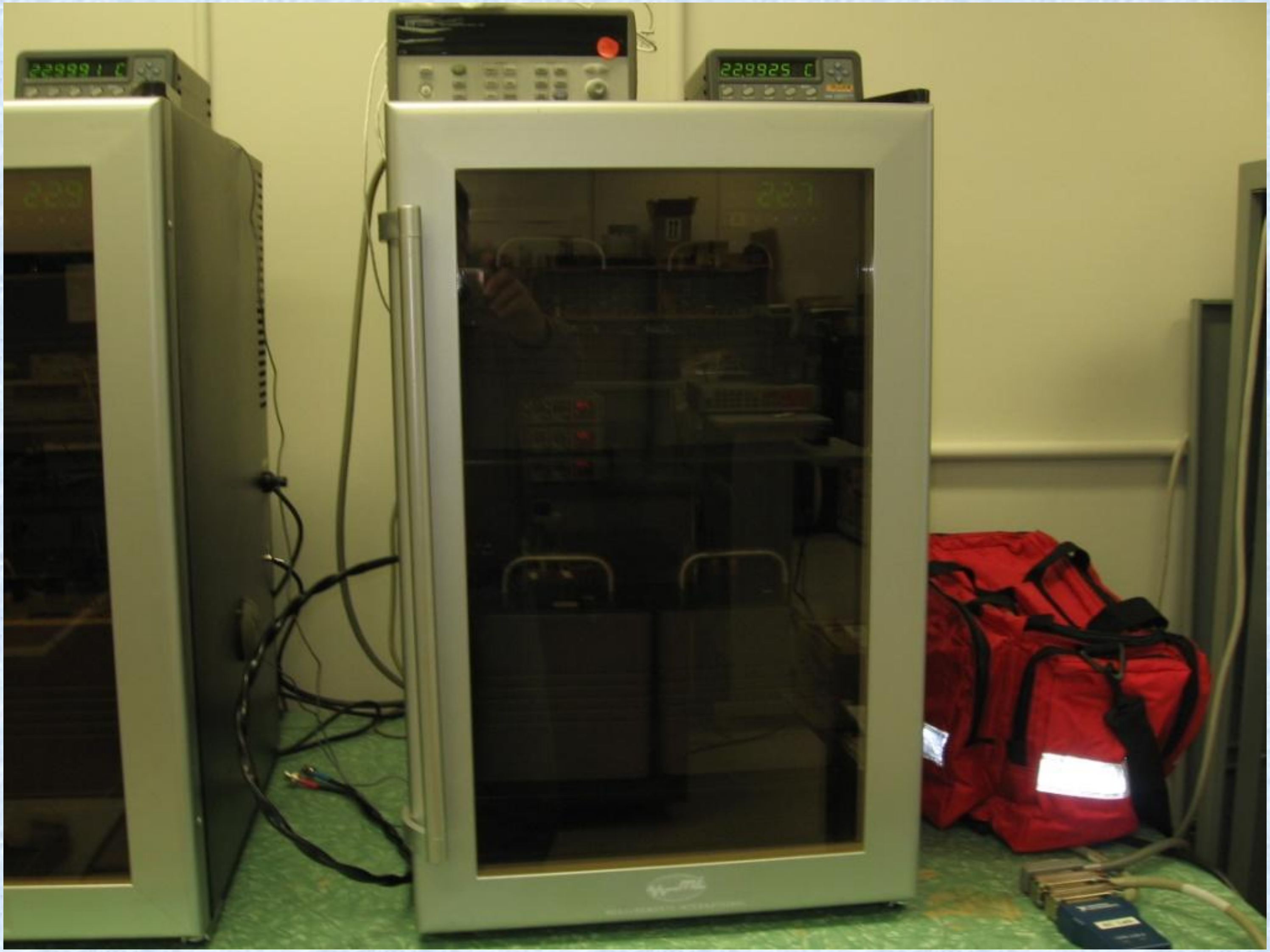
Capacitance – secondary standards



Fused Silica
oven to stabilise
1 kHz, 0.4 ppm
Drift $1x10^{-7}$ over 10 years

Decade values, air or mica (GR1409)
Thermally unstable
Air 1pF (15V) 3.2 ppm
1 μF (0.25 V) 100 ppm

Capacitor stabilisation



Inductance

Calibration of inductors

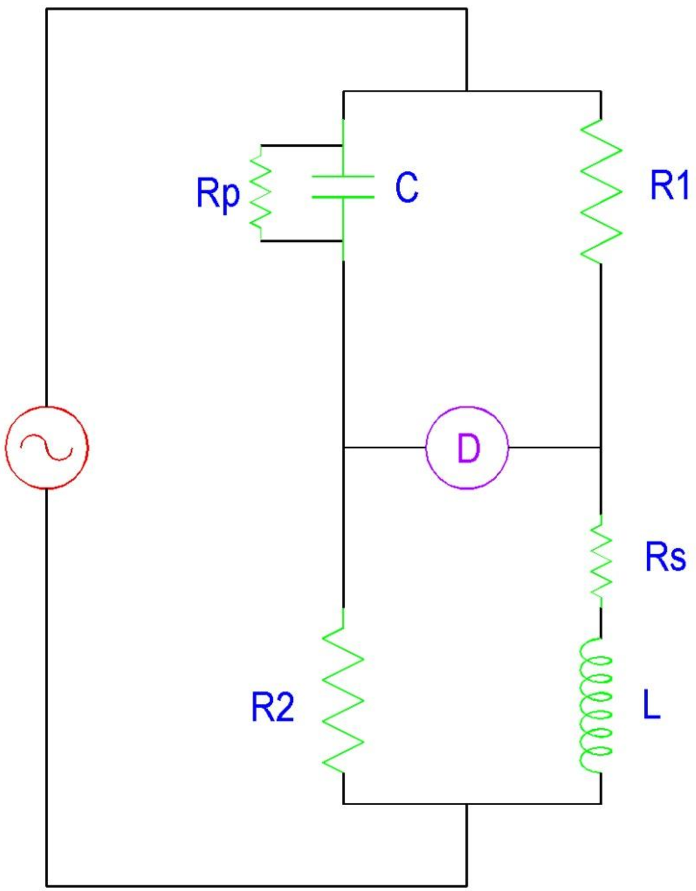
Link to SI using Maxwell-Wien bridge; with traceable capacitors and resistors
Calibrate clients with commercial LCR meter

100 uH (ohms)
→ 10 H (kohms)
Measure resistance (HP3458A) – use with temperature dependance
Use LCR meter as transfer standard
Gives 100 ppm at 1 mHm
500 ppm at 10 H

1 mH 2 mH

50 µH

Maxwell-Wein Bridge - Inductance



$$\frac{Rp\,(Rs + j\omega L)}{1 + j\omega CRp} = R1R2$$

Determines the value and equivalent series resistance (or Q) of an inductor from 2 resistors, a capacitor and the frequency.

Usually operated as a 2 port bridge.

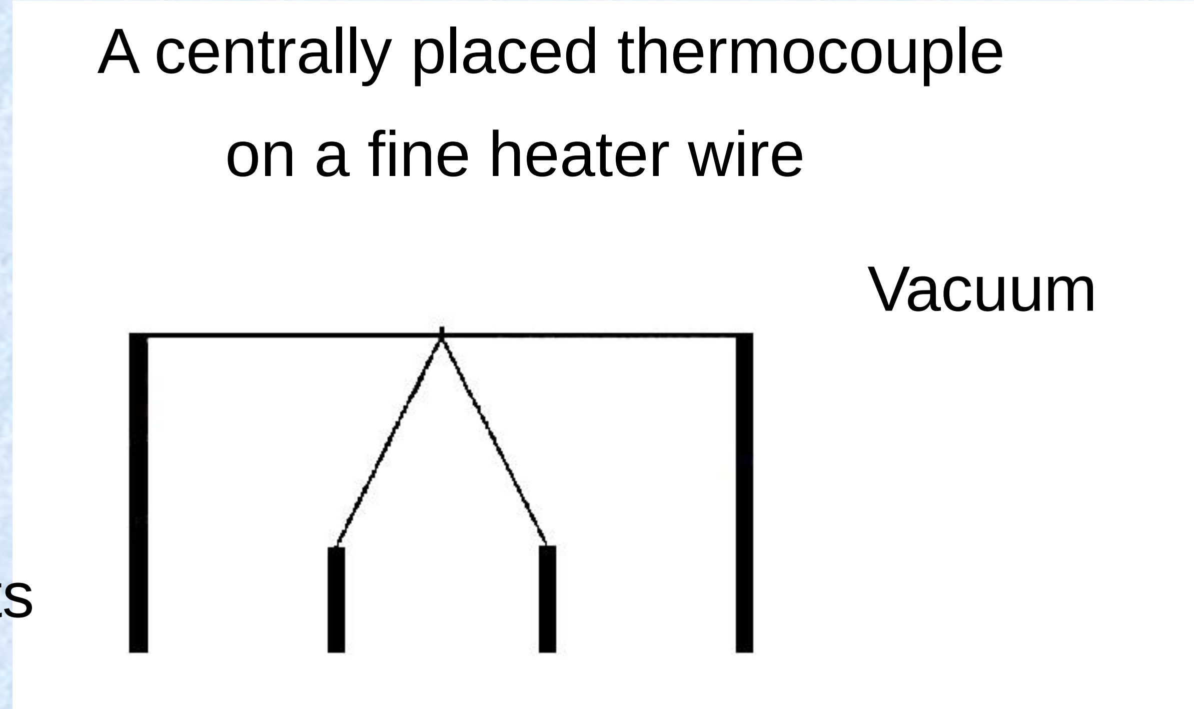
ac/dc transfer

AC/DC Transfer

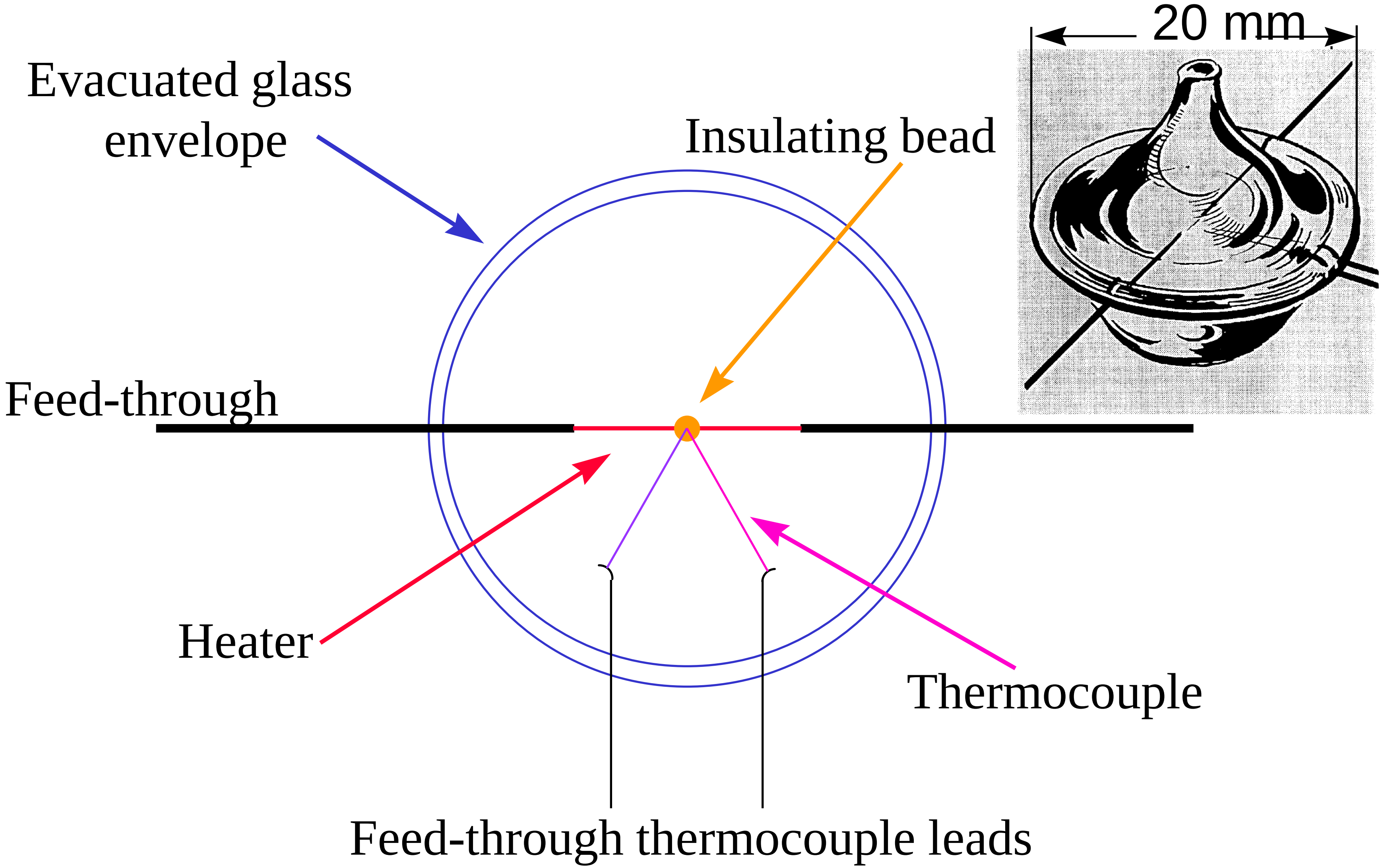
The equivalence of DC and AC quantities is achieved when they produce the same electrical power which in turn must be equivalent to mechanical power. Thus temperature is a good and unambiguous indicator of this equivalence.

ac JAVS are being developed : limited range of voltage and frequency to date. Very few in operation yet.

Primitive Schematic of a Thermal Voltage Converter



UHF-pattern single junction thermal converter (SJTC)



ac-dc voltage transfer difference

The ac-dc voltage transfer difference δ of a transfer standard is defined as:

$$\delta = \frac{V_{ac} - V_{dc}}{V_{dc}} \bigg|_{E_{ac}=E_{dc}}$$

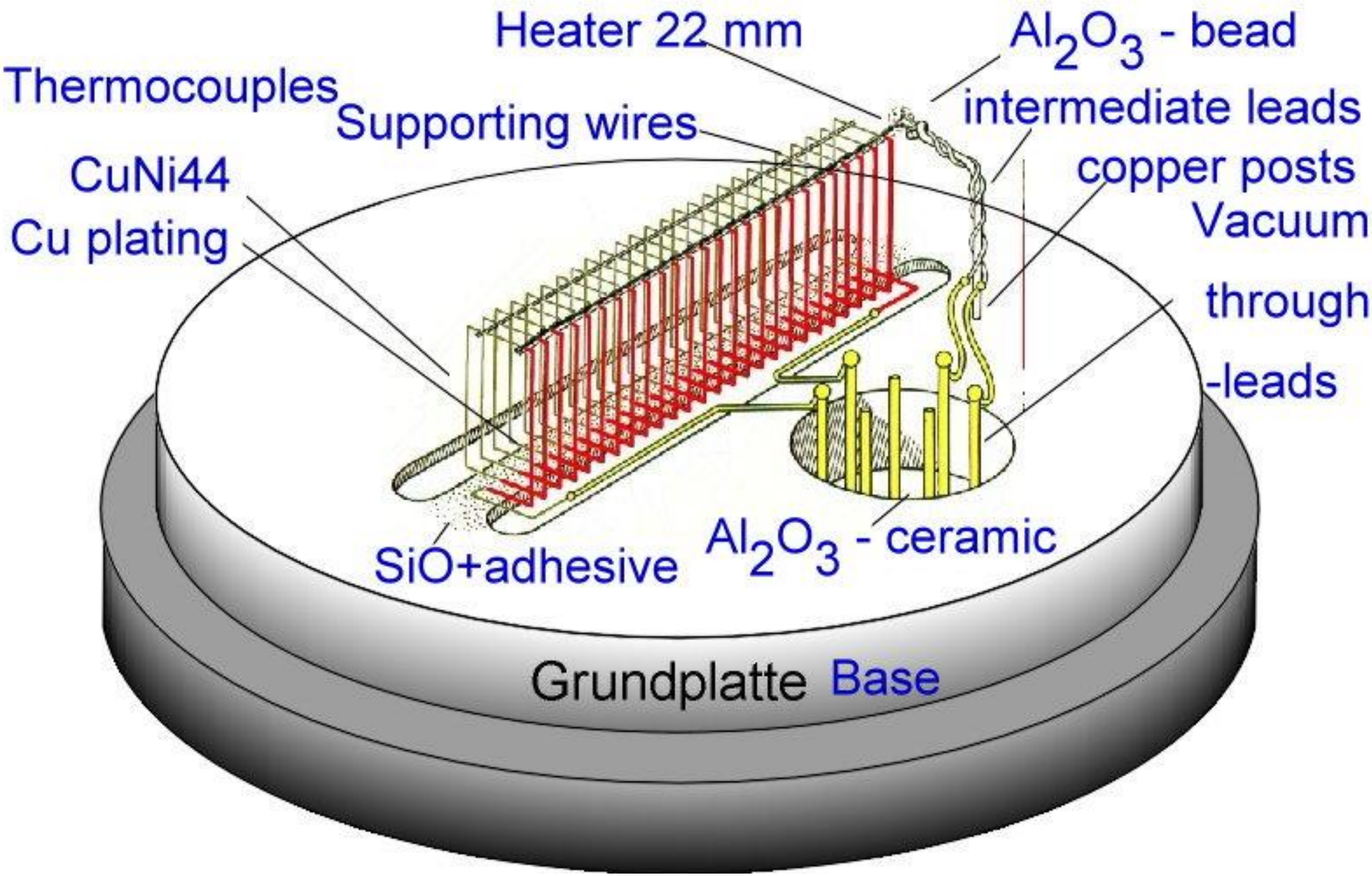
where V_{ac} is the rms value of the ac input voltage, V_{dc} is the dc input voltage, which when reversed produces the same mean output voltage of the transfer standard as V_{ac}. E_{dcN}, E_{dcR} and E_{ac} are the output voltages of the standard when the appropriate voltages have been applied.

$$|V_{dcN}| = |V_{dcR}| = V_{dc}$$

$$E_{dc} = \frac{E_{dcN} + E_{dcR}}{2} = E_{ac}$$

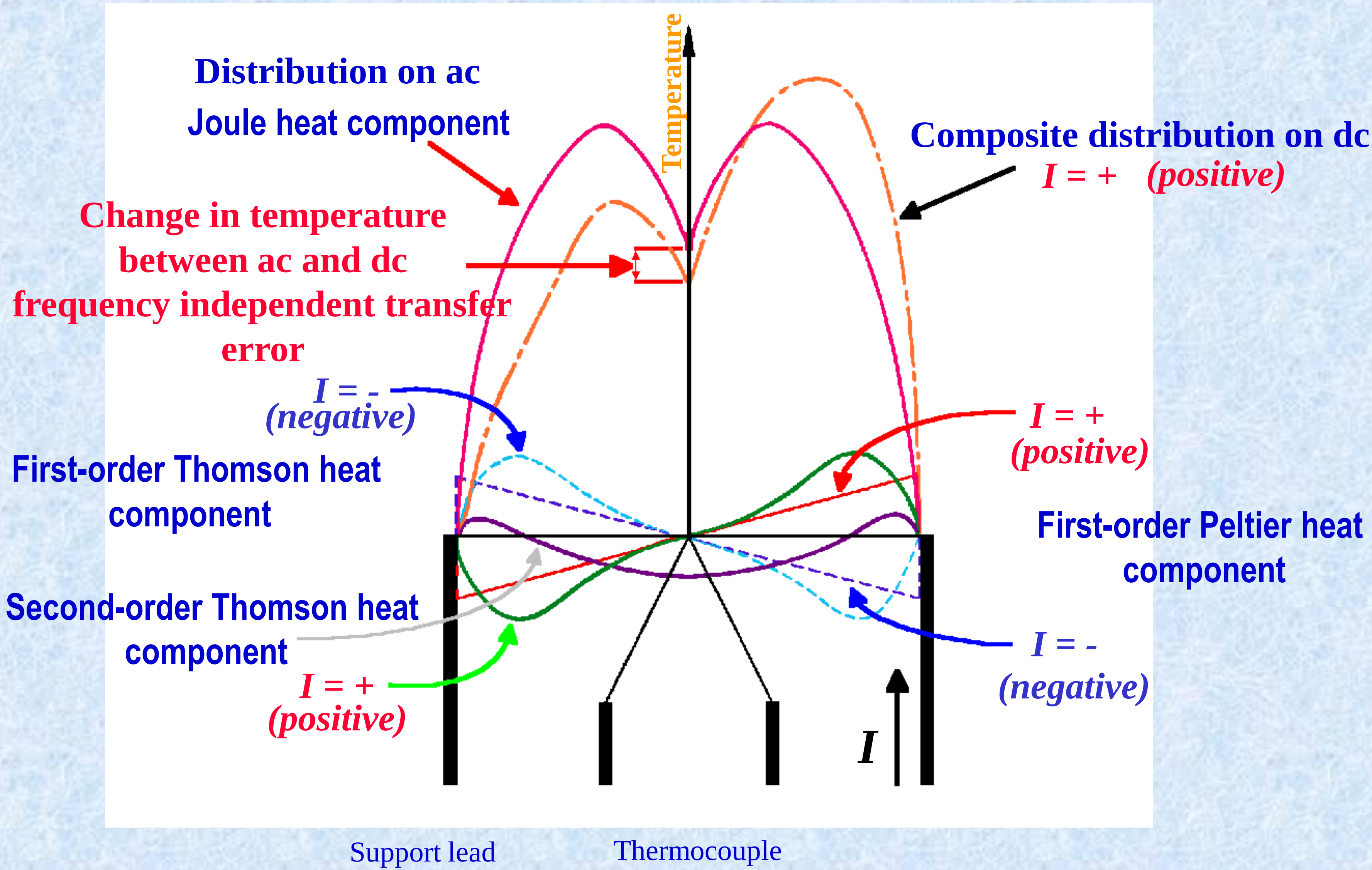
$$V_{ac} \neq V_{dc}$$

PTB 3-dimensional MJTC

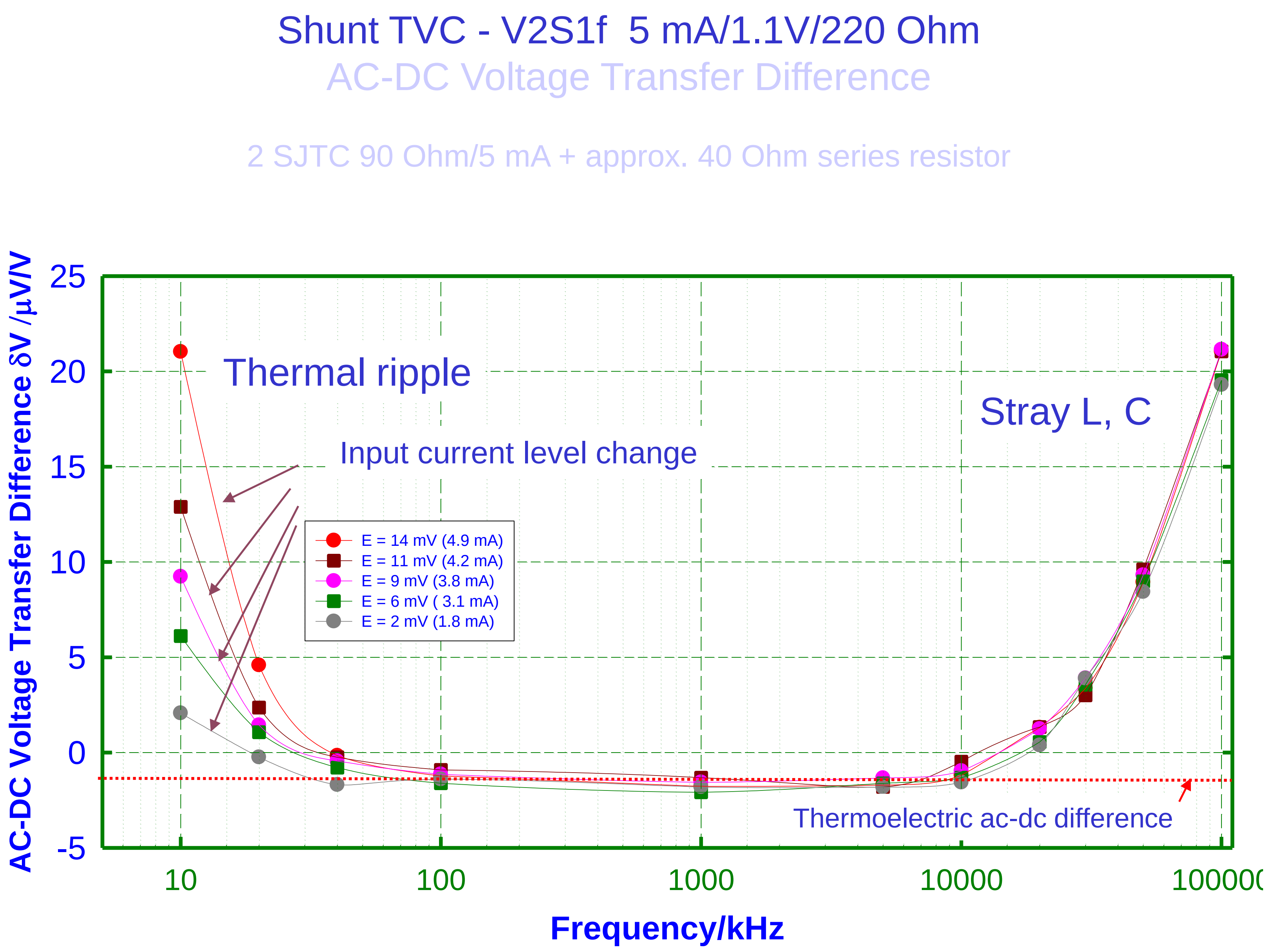


http://www.ptb.de/en/org/2/21/212/mjtc.htm

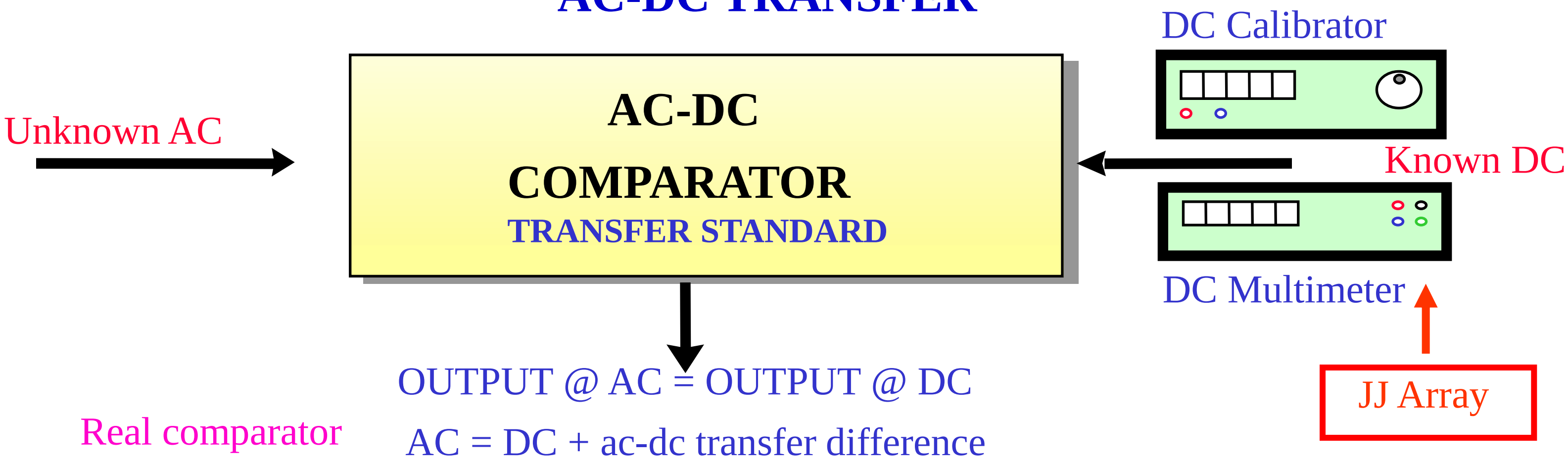
Temperature Distribution of the Heater of a SJTC



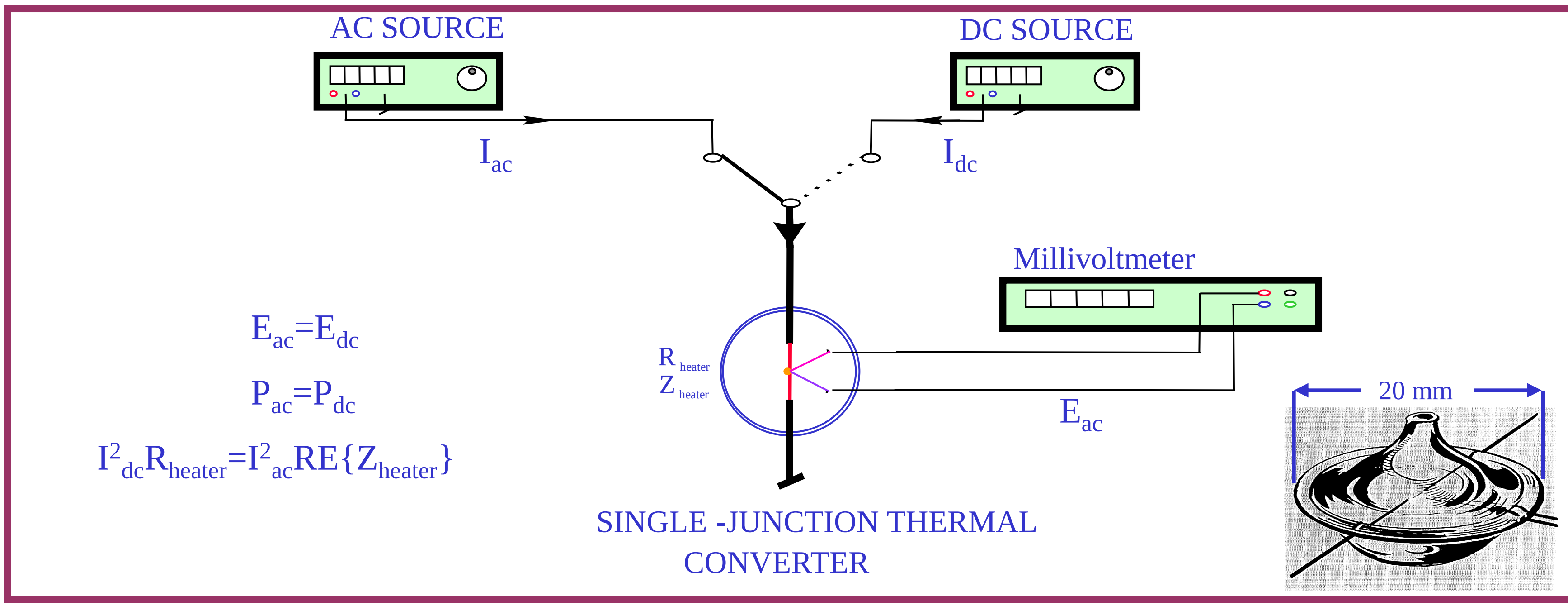
B. D. Inglis, Standards for AC-DC Transfer, *Metrologia*, vol. 29, p. 193, May 1992


Shunt TVC - V2S1f 5 mA/1.1V/220 Ohm
AC-DC Voltage Transfer Difference
2 SJTC 90 Ohm/5 mA + approx. 40 Ohm series resistor
AC-DC Voltage Transfer Difference δV /µV/V
Frequency/kHz
25
20
15
10
5
0
-5
10
100
1000
10000
100000
Thermal ripple
Input current level change
Stray L, C
Thermoelectric ac-dc difference
E = 14 mV (4.9 mA)
E = 11 mV (4.2 mA)
E = 9 mV (3.8 mA)
E = 6 mV (3.1 mA)
E = 2 mV (1.8 mA)

AC-DC TRANSFER



Real comparator



$\delta_{ac\text{-}dc}$ - ac-dc transfer difference

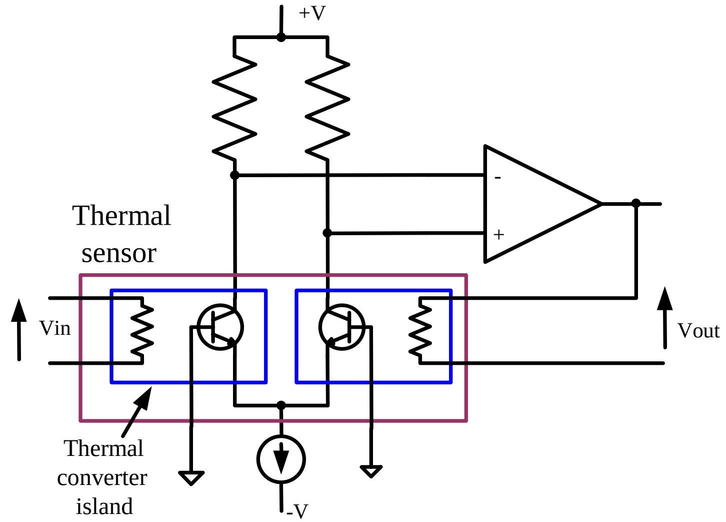
defined at the input

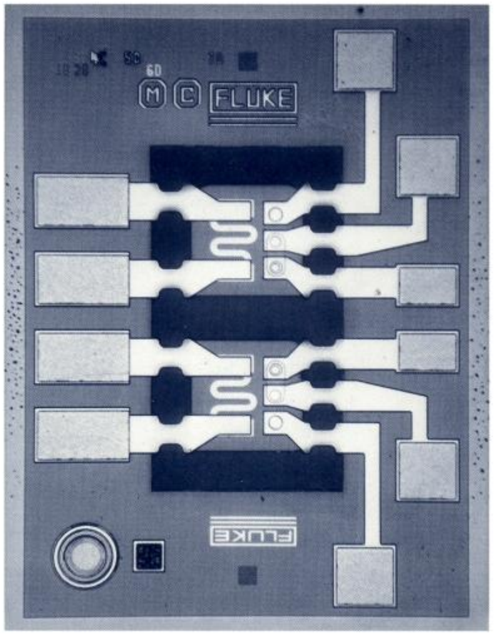
$$\delta_{ac-dc} = \frac{V_{ac} - V_{dc}}{V_{dc}}\Big|_{E_{ac}=E_{dc}}$$

defined at the output

$$\delta_{ac-dc} = -\frac{E_{ac} - E_{dc}}{nE_{dc}}\Big|_{E_{ac}=E_{dc}}$$

Fluke 792A AC-DC transfer standard





The patented Fluke Solid-State RMS Sensor.

Fluke 792A thermal sensor

The 792A consists of four main components: The Transfer Unit, Power Pack, 1000V Range Resistor and Transfer Switch.

Manual switch

Battery pack 792A transfer unit 1000 V range resistor

Comparison of Low Voltage Transfer Standards

Questions??

Feel free to contact me at
dave.inglis@nrc.ca;

Or, since I have left NRC and the above mail won't work forever, at

allan.inglis@sympatico.ca