Applying Orthogonal Array Design Matrices to Experimental Studies for the Halon Replacement Program for Aviation

Gerald Shaughnessy
Department of Mathematics
University of Dayton, Dayton Ohio

(Abstract)

Using Orthogonal Arrays to design an experimental matrix it is possible to vary a large number of parameters simultaneously in a designed experiment with a relatively small number of experimental runs and be able to estimate separately the effects of each of the parameters on a response variable. This paper will detail the use of a thtrty-two run Orthogonal Array to *study* the *effects* of fifteen variables on the pounds of agent required to extinguish a fire in a realistic dry bay test facility. Two agents (Halon 1301 and Perfluorohexane) were used in the experiments. Using the orthogonal property of the test matrix. separate estimates of amount of agent required to extinguish a fire were determined for Halon and Perfluorohesane.

The paper will introduce the general concept of using Orthogonal ***Arrays*** in designing a test matrix. and the application of thts methodology to fire in a realistic *dry* bay facility. Details of the data analysis will be **discussed** including use of the data to check for the reasonableness of the underlying assumptions and how to determine when a transformation of the original response variable is indicated.

KEY WORDS: Orthogonal Arrays; Halon 1301; Data analysis; Designed experiment.

1. What is an Orthogonal array design matrix?

Suppose that it is desired to design an experiment study the effect of three variables: temperature, pressure and catalyst on the yield from a reaction. If each of the variables is to be studied at two levels (settings), there are eight different combinations of variable settings.

Condition	Temperature	Pressure	Catalyst
1	120"	50 psi	A
2	120"	50 psi	B
3	120"	100 psi	A
4	120"	100 psi	B
5	190°	50 psi	A
6	190°	50 psi	B
7	190"	100 psi	A
8	190°	100 psi	B

If the variable settings are coded as follows:

Temperature	Pressure	Catalyst
120" (-1)	50 psi (-1)	A (-1)
190° (1)	100 psi (1)	B (1)

The table of variable conditions can be expressed as:

Condition	Temperature	Pressure	Catalyst
1	-1	-1	-1
2	-1	-1	1
3	-1	1	-1
4	-1	1	1
5	1	-1	-1
6	1	-1	1
7	1	1	-1
8	1	1	1

This is an orthogonal array. It is usually of interest to calculate the "Effect of a factor". The Effect of a factor is defined to be the difference between the mean of all data points obtained when the factor is set at it's high setting and the mean of all data points obtained when the factor is set at it's low setting. The effect of temperature on the response is calculated as follows: the mean of the four conditions run at the low temperature would be subtracted from the mean of the four conditions run at the high temperature. An examination of the experimental conditions in table 2 shows the effect of the orthogonality or "balance". The four conditions run at the high temperature have two at the low pressure and two at the high pressure and have two using catalyst A and two using catalyst B. The four conditions run at the low temperature have two at the low pressure and two at the high pressure and have two using catalyst A and two using catalyst B. This balance means that when the temperature effect is calculated the influences of the other factors are averaged out. A check of the table of conditions shows that this balance is also present for the other two variables: pressure and catalyst.

Table 1. Design Matrix using All Eight Experimental Conditions

Condition	Temperature	Pressure	Catalyst
1	-1	-1	-1
2	-1	-1	1
3	-1	1	-1
4	-1	1	1
5	1	-1	-1
6	1	-1	1
7	1	1	-1
8	1	1	1

If only four experimental conditions were run it is still possible to retain the orthogonal array structure.

Table 2. Design Matrix using Half of the Eight Experimental Conditions

Condition	Temperature	Pressure	Catalyst
1	-1	-1	-1
4	-1	1	1
6	1	-1	1
7	1	1	-1

The effect of temperature can be estimated from these four experimental runs. An examination of the design matrix in table 2 shows that the two experimental runs at the low temperature contain one run at the low pressure and one run at the high pressure, also one run using catalyst A and one run using catalyst B. Therefore, when the mean response for the experimental runs at the low temperature is calculated the potential influence of the other two factors is "averaged out" over both a high and a low setting. When the mean response for the two runs at the lugh temperature is calculated the same "balance" with respect to pressure and catalyst is present.

A further examination of the design matrix shows that this same balance of factors is true when the mean response is calculated for the low and high settings for pressure and for the mean response for catalyst A and catalyst B. Because of tlus structure of the design matrix it is possible to use the same four data points to estimate the effect of all three factors: temperature, pressure and catalyst on the observed response variable. The price that is paid for not running all of the experimental conditions is that interaction effects between variables become confounded (mixed up) with each other or even possibly with the main effect of a variable. The extent to which interaction effects are confounded and the effects with wluch they are confounded are known for each orthogonal design matrix.

What is *an* "interaction effect"

Two factor studied in an experiment are said to "interact" if the effect of one factor on the response variable is affected by the setting of the second factor.

Figure 1.

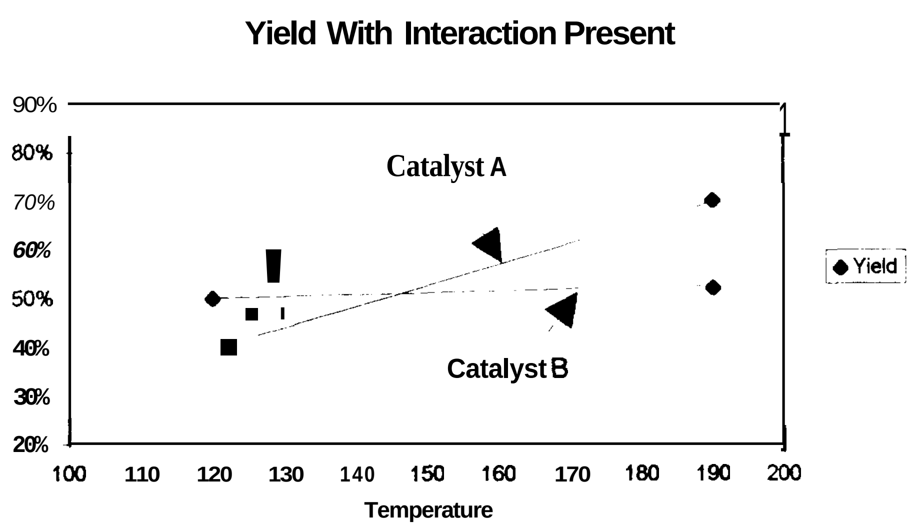
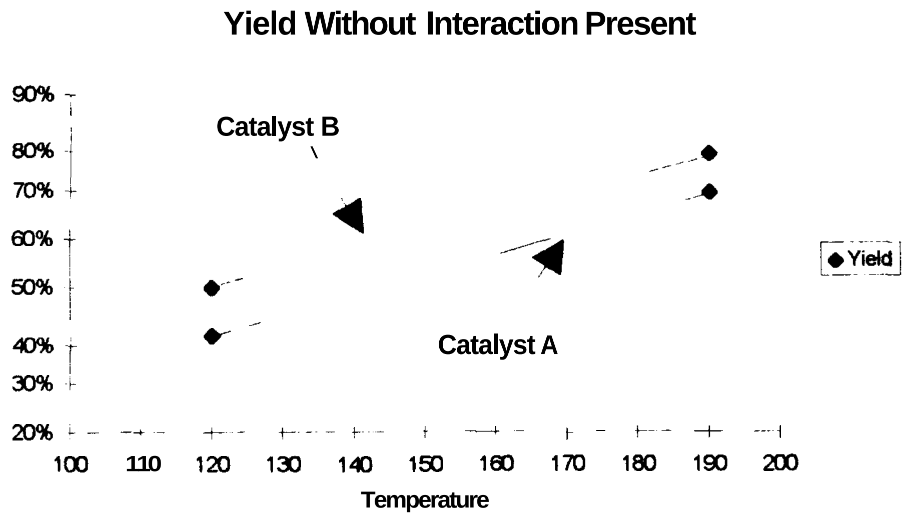


Figure 2.



In both figure 1 and figure 2 the points that are plotted on the charts represent the mean yield at the indicated conditions. Figure one shows an interaction between catalyst and temperature. When the temperature is increased from 120° to 190° the yield increases from 40% to about 70% when catalyst A is used. However when catalyst B is used the increase in temperature from 120° to 190° produces very little change in the yield of the process. If someone were to ask: "What is the effect on yield when the temperature is increased", the answer would have to be that it depends on which catalyst is being used. Figure 2 represents a situation where no interaction is present between catalyst and temperature. When the temperature is increased from 120" to 190° the yield increases at about the same rate for both catalyst A and catalyst B.

2. Application to Halon testing

An experiment was conducted to study fifteen variables in an aircraft dry bay test facility. The tests were performed at the Aircraft Survivability Range Facility (ASRF) at Wright Patterson Air Force Base, Ohio. An aircraft dry bay is defined as a void volume within the mold lines of an airplane, excluding air inlets, engine compartments, and exhaust nozzles. Dry bays may contain fluid lines such as fuel, hydraulic and others. They may contain avionics, flight control actuators and other equipment. Dry bays are normally free of flammable liquids and vapors, but combat damage or equipment failure may release flammable liquids into the dry bay. If an ignition source is present combustion may result.

Experimental dry bay test facilities were constructed and the following fifteen variables were studied in an orthogonal test matrix. The purpose of the experiment was to determine which of the variables in an aircraft dry bay fire have the largest influence on the amount of agent needed to extinguish a fire.

Table 3. The factors used and the settings for each level.

Factor	Abbreviation	Low Setting (-1)	High Setting (+1)
Agent	Agnt	Perflourohexane	Halon 1301
External Airflow Rate	Ext A. F.	0 ktas	400 ktas
Total Zone Volume	Vol	11 ft^3	100 ft'
Pre-Burn Time	Preb	5 msec	20 msec
Fuel Temperature	**Ftmp**	100^0 F	150^0 F
Clutter	**Clut**	33%	66%
Bottle Location	Loc	One at end	Two at 2/3
Bottle Pressure	Bprs	350 psig	*600* psig
Compartment Config	conf	1:1 L/D	4:1 L/D
Compartment Damage	**Dam**	12"x12"	**7"x7"**
Fuel Tank Level	Levl	4" JP-8	7" JP-8
Hydraulic Line Pressure	**Hydr**	**off**	on
Internal Airflow Rate	Inte	500 cfm	1000 cff
Fixture Orientation	Ornt	0^0	90"
Agent Temperature	Atemp	-20^0	150^0

The response variable (called Value) was determined by an iterative bracketing method. For a given **set** of conditions an initial weight of agent would be used in the first test run, if the fire was extinguished the weight of agent would be halved for the next run, however if the fire was not extinguished the weight of agent would be doubled. When an experimental run that extinguished the fire followed a run that did not extinguish the fire the next run would use the average of the two weights. This protocol was continued for at least four additional test runs. The recorded response variable (Value) for a given set of conditions was the average of the least weight of agent that extinguished the **fire** and the maximum weight that failed to extinguish the fire. For example a set of test runs could be **as** follows:

	Test 1	Test 2	Test **3**	Test **4**	Value
Weight	**5** Ibs.	2.5 lbs.	**3.75** lbs.	**4.38** Ibs.	1.69 **lbs**
Test result	Fire out	Fire not out	Fire not out	Fire not out	

The **data** analysis was **also** performed using the least weight of agent **that** extinguished the fire as the response variable and the conclusions were the same.

With fifteen factors each at two settings 2^{15} =**32,768** experimental **runs** would be required to run each possible conibination of variable settings. The orthogonal design **matrix** given in table **4** was employed in the experiment. This design **matrix** requires 32 experimental runs.

Table 4. The thirty two experimental runs

Ext A. F.	Vol	Conf	Loc	Clut	Ornt	Hydr	Dam	Inte	Atmp	Levl	Bprs	Preb	Agnt	Ftmp	Value
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0.47
-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	0.03
-1	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	0.34
-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	0.06
-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	0.09
-1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	0.41
-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	1	1	-1	0.09
-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	0.81
-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	0.61
-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	4.5
-1	1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	1	0.42
-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	5.63
-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	4.06
-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1.87
-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1	1	4.06
-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	0.34
1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1.18
1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	7.04
1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	6.5
1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	22
1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	32.5
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.75
1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	7.5
1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	2.25
1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	3.75
1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	0.23
1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1.63
1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	0.12
1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	0.57
1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	0.71
1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	0.2
1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	0.47

ANALYSIS OF TEE FACTORIAL EXPERIMENT

The data was first analyzed using "Yates' Algorithm" to calculate effect size and sum of squares for each factor and interaction between factors. The sum of squares for each factor is a measure of the variability between the mean response at the low setting of a variable compared to the mean response at the high setting. The sum of squares for each factor was then expressed as a percent of total variability. The larger the percent of total variability for any factor, the stronger the indication from the data that the effect of that factor on the response is of sufficient size to stand out from the experimental error or "noise".

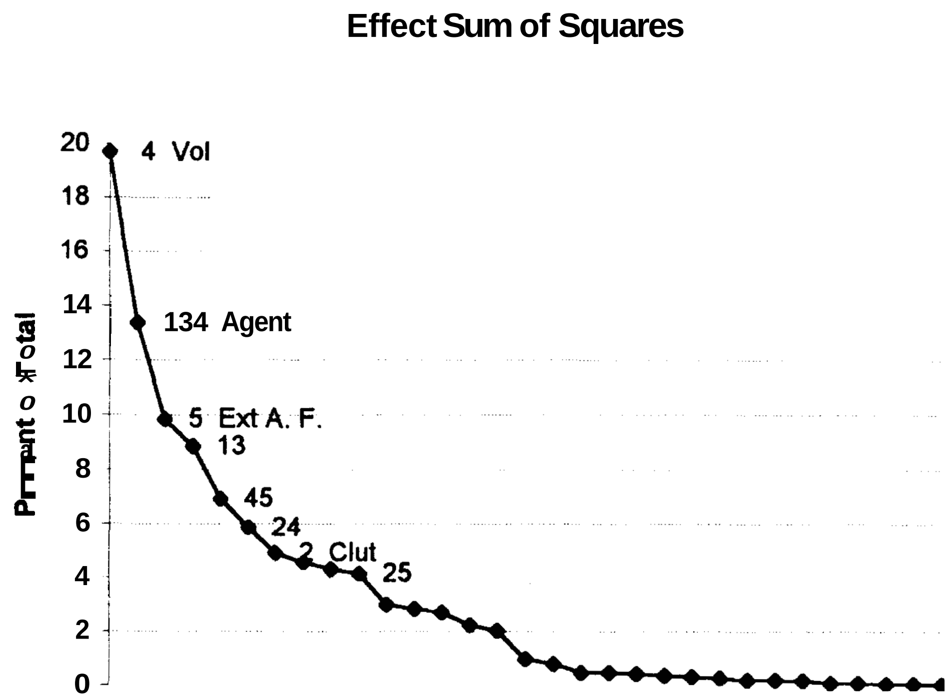
Table 5. Analysis of the dry bay experiment

ID Yates Order	EFFECT	SUM OF SQUARES	PERCENT OF TOTAL
MEAN	3.56844		
1 Ftmp	-0.68063	3.706	0.262934
2 **Clut**	2.94563	69.4137	4.92476
12	-1.17688	11.0803	0.786124
3 Conf	0.899375	6.471	0.459105
13	-3.94813	124.702	8.84733
23	0.155625	0.193753	.0137
123 Inte	-2.22688	39.6718	2.81463
4 Vol	**5.88937**	277.478	**19.6865**
14	-1.29812	13.481	0.956453
24	3.21062	82.4649	5.85072
124 **Bprs**	-0.91188	6.65213	0.471955
34	0.364375	1.06215	.0754
134 **Agent**	**-4.84812**	**188.035**	**13.3407**
234 Damg	0.548125	2.40353	0.170526
1234	-1.88938	28.5579	2.02613
5 Ext A. F.	**4.16313.**	**138.653**	**9**.a3715
15	-0.32188	0.828828	.0588
25	2.69938	58.293	4.13577
125 Levl	-0.79063	5.0007	0.35479
35	0.733125	4.29978	0.305061
135 Preb	-2.29188	42.0215	2.98135
235 Hydr	0.266875	0.569778	.0404
I235	-2.18312	38.1283	2.70513
45	3.49062	97.4757	6.91571
Dummy	-0.85938	5.9082	0.419175
245 Ornt	2.83938	64.4964	4.57589
1245	-0.58563	2.74365	0.194657
345 Loc	0.323125	0.835278	.0593
1345	-2.75188	60.5825	4.29821
2345	0.584375	2.73195	0.193827
12345 Atemp	-1.98563	31.5417	2.23782
TOTAL		1409.48	

The effects with the largest percent of variability are: **Vol (20%),** Agent **(13%), Ext** AF. **(10%).** There are no other effects with more than **10%** percent of total variability. There are, however some two factor interactions **(1&3, 2&4, 4&5)** with percent of total variability between 5% and 10%.

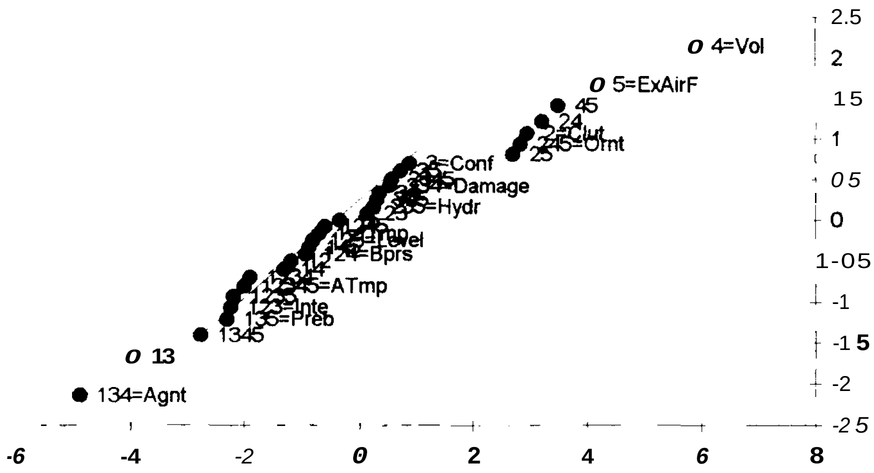
A plot of the effect sum of squares ranked in size order is shown in figure 3.

Figure 3.



To further examine the data a "Normal plot" of the effects (a plot on Normal graph paper) was constructed With this type of design there is no replication of experimental conditions to provide an estimate of experimental error. *An* analysis method that is often used to separate real effects from noise is a Normal plot of the effects. Assuming that the data is approximately Normally distributed, the effects of the factors that have little or no influence on the response variable should plot to be a straight line on the Normal plot. Point that fall considerably off the line formed by the majority of plotted values *suggest* that those effects are having a stronger influence on the response. *An* examination of *the* Normal plot dearly shows factors 4 (Vol), *5* (Ext AF.), and at the low end 134 (Agent) are well of the line formed by the majority of points. The plot also seems to indicate that perhaps factor 2 (Clut) and some two factor interactions are "off the line".

Figure 4. Normal Plot of Effects



The effects that appear **to** lie on a line are "pooled" into a term to estimate experimental error. This pooling produced the following Analysis of Variance **(ANOVA)** table.

Table 6. Analysis of variance Table

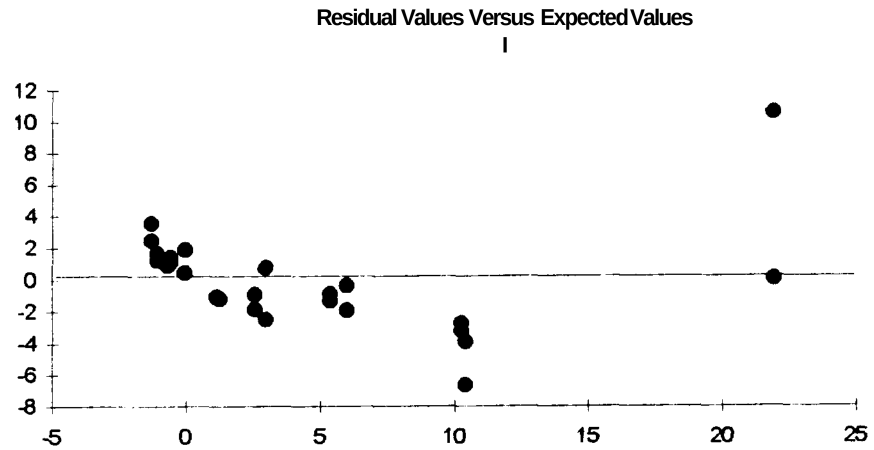
EFFECT	D. F.	S. S.	M. S.	F
2 Clut	1	69.4137	69.4137	4.95054
13	1	124.702	124.702	8.89363
4 Vol	1	277.478	277.478	19.7895
24	1	82.4649	82.4649	5.88134
Agent	1	188.0	188.0	13.4105
5Ext A. F	1	138.653	138.653	9.88863
25	1	58.293	58.293	4.15742
45	1	97.4757	97.4757	6.9519
245 Omt	1	64.4964	64.4964	4.59984
ERROR	22	308.472	14.0214	

All of **these** effects are statistically significant at the .05 level of significance.

Transformation of the Response Variable

When performing an analysis of **data,** it is **often** the case that the underlying assumptions of the **data** analysis are better satisfied **by** using a transformation of the response variable rather than the original metric in which the **data** is reported. The model assumed for this **type** of analysis is of the form: response = $\beta_o + \beta_1 x_1 + \beta_2 x_2 + \ldots\ldots\ldots\ldots\ldots\ldots \beta_k x_k$, where β_o is the mean of the **data** each β_i is one-half the effect for factor i and x_i is **1** if factor i is at the high setting and **-1** if factor i is at the low setting. Common statistical practice would indicate that an analysis of the data using a logarithm of the response should be considered when the range of the **data** is large. That is, if the largest **data** value is more than **10** times the smallest value. To determine if a transformation of the **data** is needed, a plot of the residuals verses the predicted values is constructed. If the plot show a purely random **pattern** about zero a transformation is not indicated. **A** plot of the residuals verses the predicted values **was** constructed. The plot is shown below.

Figure 5



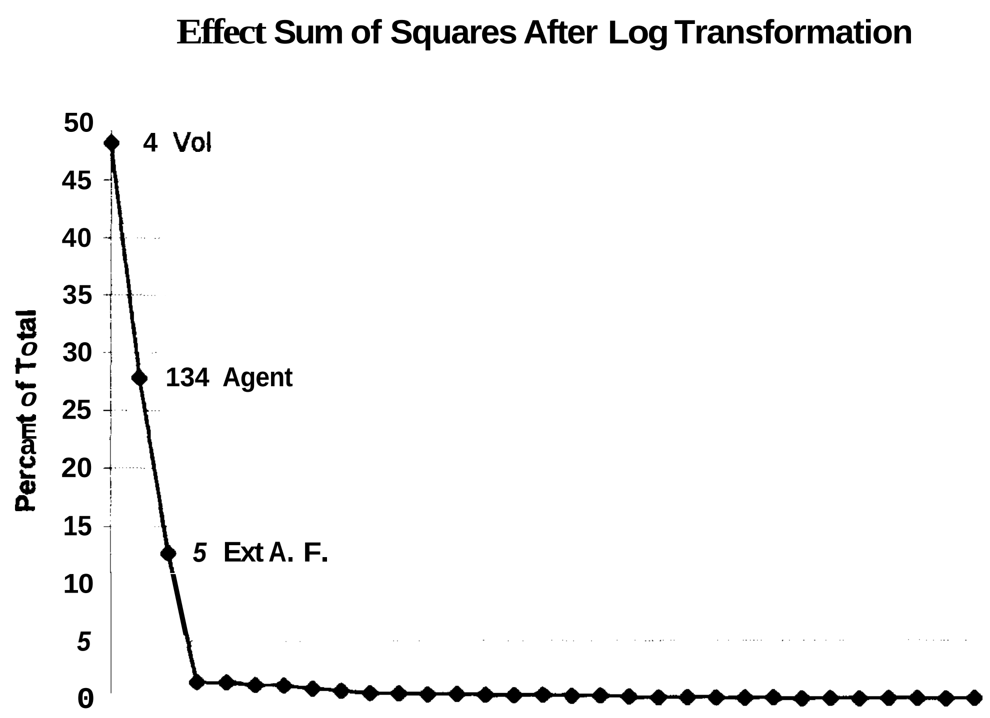
This plot **does** not show the characteristics of a "random" scatter about zero that would be expected if the underlying assumptions of the analysis were being **satisfied. The** plot indicates that an analysis should be considered using some transformation of the **original response. A** log transformation on the response was performed and the **data** reanalyzed.

Table 7. ANALYSIS OF THE FACTORIAL EXPERIMENT AFTER LOG TRANSFORMATION

ID Yates Order	EFFECT	SUM OF SQUARES	PERCENT OF TOTAL
MEAN	-0.00600011		
1 Ftmp	0.11605	0.107741	0.114837
2 Clut	0.377689	1.14119	1.21636
12	-0.18976	0.288058	0.307032
3 Conf	0.237176	0.450019	0.479661
13	0.028501	0.0065	0.00693
23	-0.41977	1.40966	1.5
123 lntc	-0.11227	0.100838	0.10748
4 Vol	**2.37809**	**45.2423**	**48.2224**
14	-0.32816	0.861504	0.91825
24	0.419618	1.40863	1.50142
124 Bprs	-0.28829	0.664879	0.708674
34	-0.22846	0.417538	0.445
134 Agent	**-1.80575**	**26.0859**	**27.8042**
234 Damg	0.0795	0.0505	0.0538
1234	0.202128	0.326844	0.348373
5 Ext A. F.	**1.21952**	**11.8977**	**12.6814**
15	0.379964	1.15498	1.23
25	0.222182	0.394919	0.420932
125 Levl	0.055985	0.0251	0.0267
35	0.0176	0.00248	0.00265
135 Preb	-0.01531	0.00188	0.002
235 Hydr	-0.05675	0.0258	0.0275
1235	-0.0385	0.0118	0.0126
45	0.0916	0.0671	0.0715
Dummy	-0.25739	0.529979	0.564889
215 Ornt	0.186231	0.277457	0.295733
1245	0.0415	0.0138	0.0147
345 Loc	0.110881	0.0984	0.105
1345	0.200364	0.321165	0.34232
2345	0.179921	0.258973	0.276031
12345 Atemp	-0.14854	0.176508	0.188134
TOTAL	93.8202		

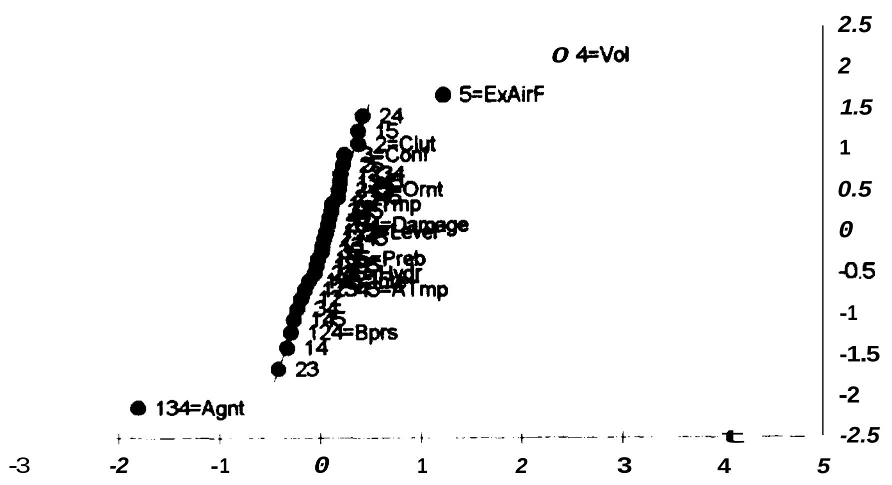
The effects with the largest percent of variability are: **Vol (48%),** Agent **(28%), Ext A.F. (13%).** There arc no other effects with more than 5% percent of total variability.

Figure 6



Next a Normal plot of the effects after the Log transformation was constructed.

Figure 7 Normal Plot of the Effects



Now the Normal plot is much easier to interpret. Only Vol (4), E d A.F. (5), and, at the bottom of the plot. Agent (134) are seen to standout from the line formed by the other effects. The conclusion from this plot is that the data give strong evidence to indicate that only these three effects are having an influence on the response variable large enough to clearly stand out from the experimental error or "noise".

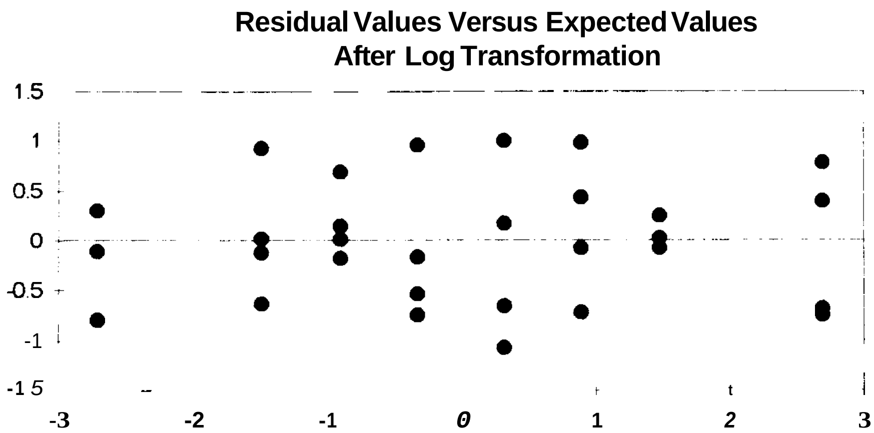
The remaining effects are "pooled into a term to estimate experimental error. This pooling produced the following Analysis of Variance (ANOVA) table.

Table 8. Analysis of variance Table After Log Transformation

EFFECT	D. F.	S. S.	M. S.	F
Vol	1	45.2423	45.2423	119.574
Agent	1	26.0859	26.0859	68.9441
Ext A.F.	1	11.8977	11.8977	31.4453
ERROR	28	10.5942	0.378364	
TOTAL	31	93.8202		

Next a plot of residuals versus predicted values is made to check on the fit of the model.

Figure 8



The residual plot now looks much more like a random scatter plot of points about zero.

3. Conclusion

The data analysis was performed on the original response variable (Weight of agent required to extinguish fire) and on the Logarithm of the response. The conclusions were similar for both analyses. The three most important factors influencing the response were: External Airflow Rate, Total Zone Volume, and Agent. Without the logarithm transformation it appeared that some two factor interactions and possibly clutter may be standing out from the Noise also. However, the residual plot gave a strong indication that a transformation was necessary for the assumptions underlying the analysis to be satisfied The analysis of the data after the log transformation confirmed the value of the transformation. The three effects: External Airflow Rate, Total Zone Volume, and Agent *stood* out more clearly as the only effects having a substantial influence on the response. The residual plot after making the log transformation gave a strong indication that the residual terms were randomly distributed and were independent of the mean response.