



Feedforward and Feedback Methods for Improving Robotic Accuracy

Douglas Bristow

William Walker III Professor of Mechanical Engineering

Director, CAMT

Missouri University of Science and Technology

dbristow@mst.edu

Workshop on Standards for Robot Absolute Accuracy and Performance Assessment

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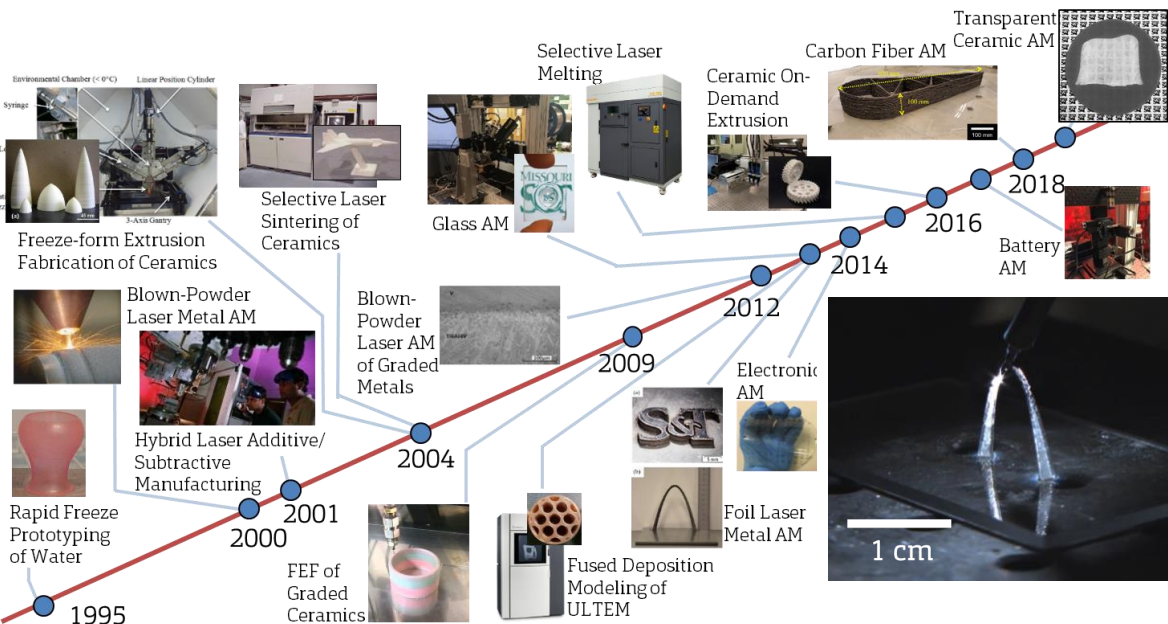
Center for Aerospace Manufacturing Technologies

camt.mst.edu

Who are we?

An academic-industry partnership with a mission to address critical needs of the aerospace manufacturing industry through technology development and workforce training since 2003.

A legacy of Additive Manufacturing Innovation...



Key Technologies...

- **Additive Manufacturing** (metal powder/wire hybrid DED, PBF, foil; ceramic extrusion, DED; large-scale FDM)
- **Robotic Manufacturing** (machining, grinding, incremental sheet forming)
- **Modeling** (multi-scale physics, additive manufacturing, composites)
- **Metrology** (metrology-in-the-loop control, high-order calibration, reverse engineering)
- **Repair** (automated reverse engineering + hybrid DED)

2024 Membership



The Missouri Protoplex

Launch Partners:

Boeing
Lockheed Martin
Caterpillar
Army

Facility:

40,000 sq. ft. High Bay
80,000 sq. ft. Lab and Office

Equipment:

DMG-Mori LASERTEC 65
DMG-Mori LASERTEC 4300
SLM 500 Quad Laser 700W LPBF
SPEE3D Cold-Spray
VRC Gen IV Cold-Spray
Bond GL7 Friction Stir
Wire-Arc AM + Machining Cell
Robotic Machining Cell
Alpha Laser Micro Welding 900W
Impossible Objects Composites AM



Dr. Richard Billo

Distinguished Professor, Mechanical and Aerospace Engineering
Director, Missouri Protoplex

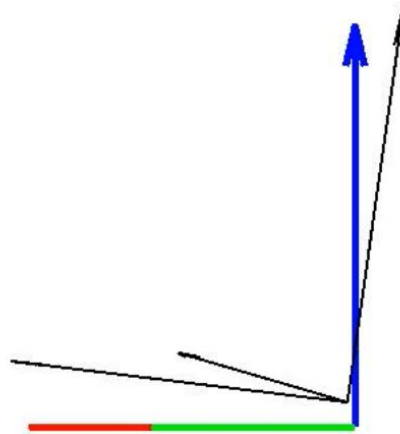
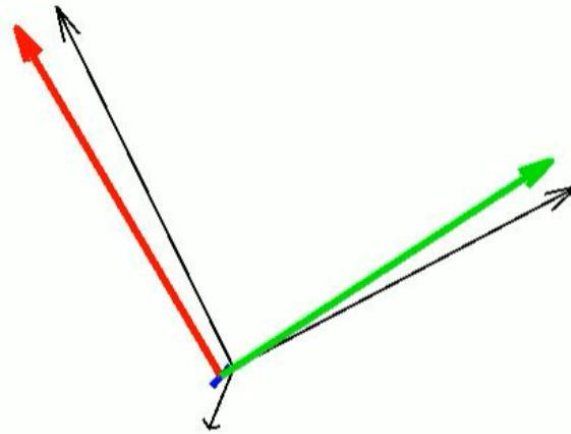
Opens Spring 2026

What Do Kinematic Errors Look Like?

Direct measurement of 6 DoF motion error projected onto Joint 2.

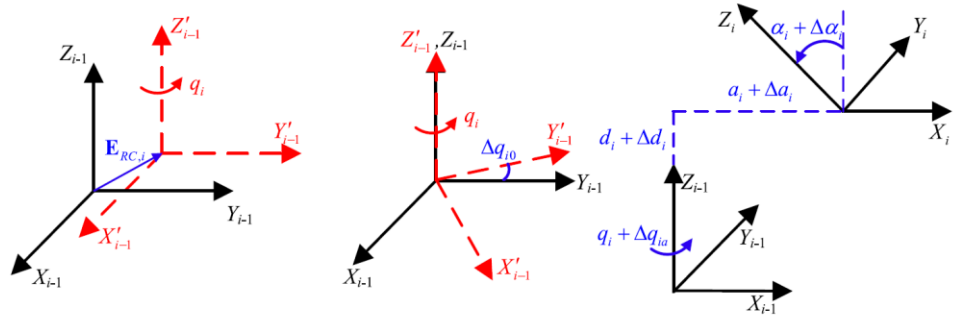
Orientation error amplified for clarity.

Error motion is complicated.



Kinematic Error Modeling

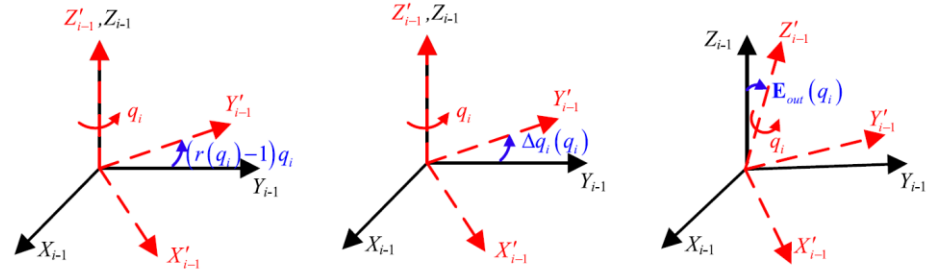
Error Taxonomy



(a) Rotating Center Offset

(b) Mastering Error

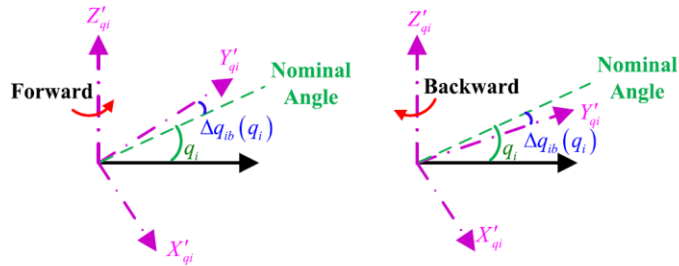
(c) Link Length and Assembly Error



(d) Pitch Error

(e) Strain Wave Gearing Error

(f) Out of Plane Error



(g) Backlash Error

Kinematic Model

Nominal Kinematics

$$F_a(\mathbf{q}) = \mathbf{E}_1^0(q_1, s_1) \mathbf{T}_1^0(q_1) \mathbf{E}_2^1(q_2, s_2) \mathbf{T}_2^1(q_2) \dots \mathbf{E}_n^{n-1}(q_n, s_n) \mathbf{T}_n^{n-1}(q_n)$$

“Lumped” Kinematics Errors

$$\mathbf{E}_i^{i-1}(q_i, s_i) \approx \begin{bmatrix} 1 & -\varepsilon_Z(q_i, s_i) & \varepsilon_Y(q_i) & \delta_X(q_i) \\ \varepsilon_Z(q_i, s_i) & 1 & -\varepsilon_X(q_i) & \delta_Y(q_i) \\ -\varepsilon_Y(q_i) & \varepsilon_X(q_i) & 1 & \delta_Z(q_i) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 6 DoF joint-angle dependent error
- “Cam-like” motion error
- Independent forward and backward motion models for rotation around joint angle; i.e., hysteresis

Model Identification

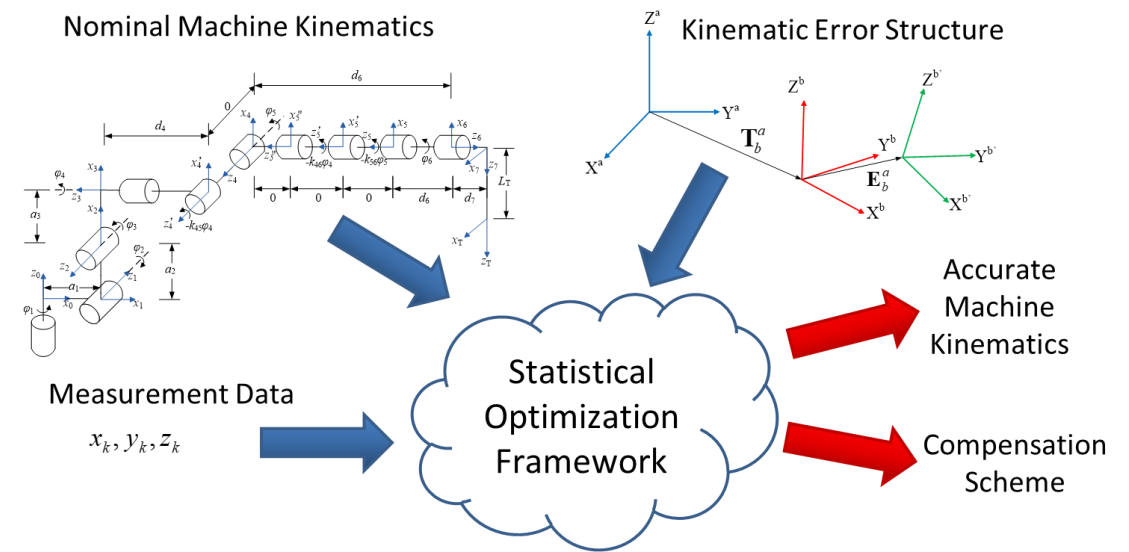
Identification Challenges

- Robot repeatability and measurement noise are:
 - Individually significant; on the order of 10% or more of desired accuracy.
 - Different in different directions and in different poses.
- Throughput is limited to 1 measurement every 10-15 seconds (motion + measure).
- Thermal variations over measurement period can be a factor.



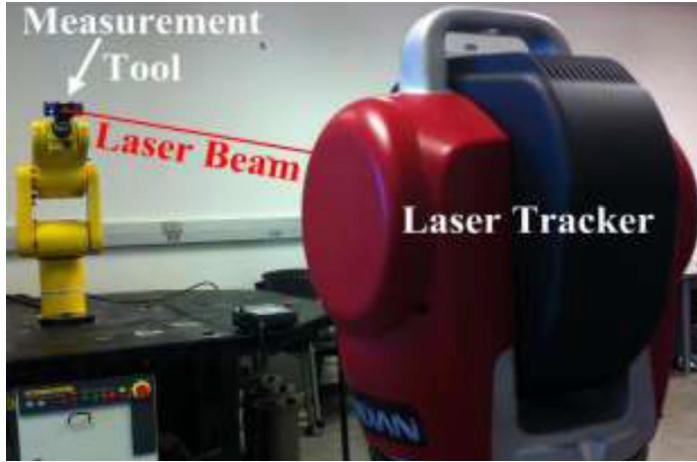
The Right Solution

- Measure/model variances
- Statistical optimization
 - Manages non-homogenous variances
 - **Avoids over-fitting data**

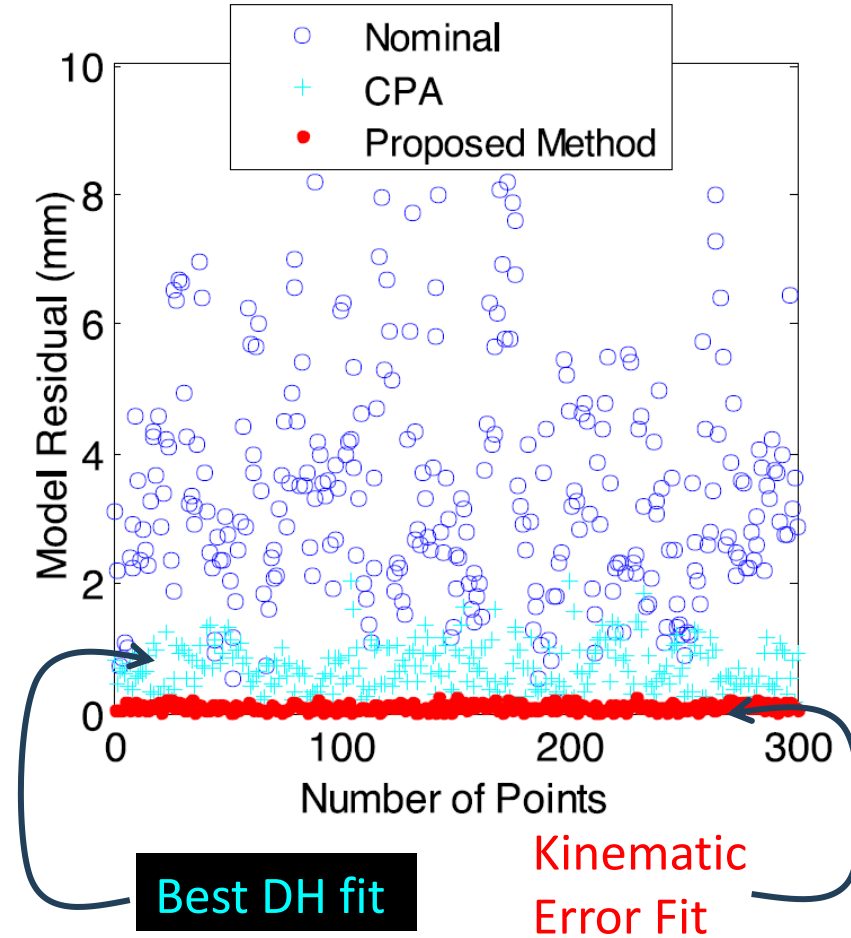
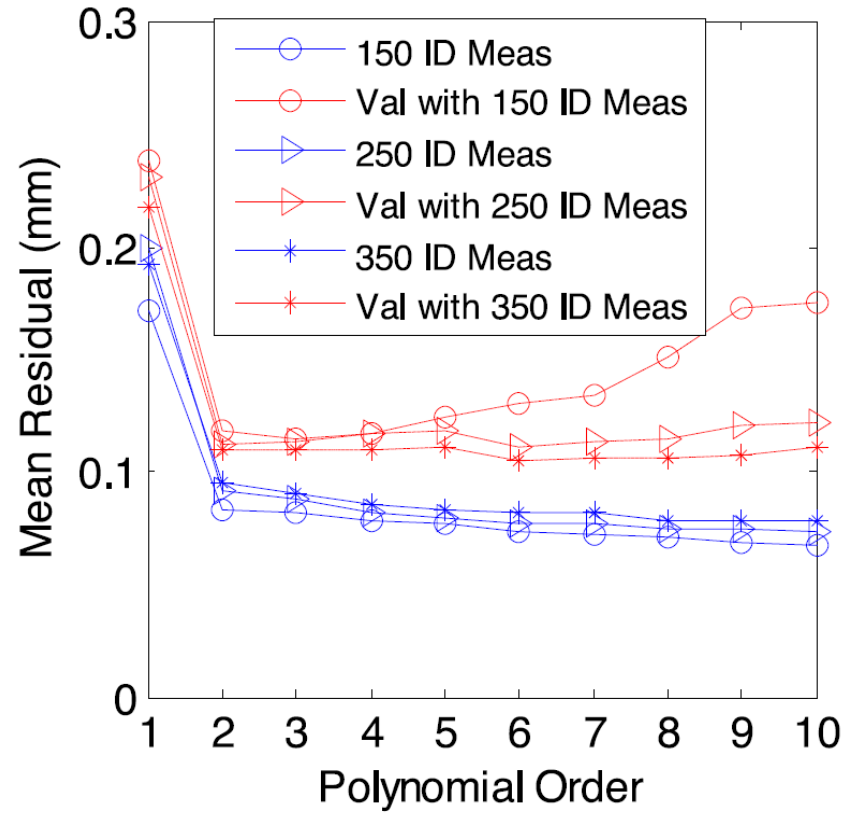


Results

Fanuc LR-Mate 200i



Does not overfit



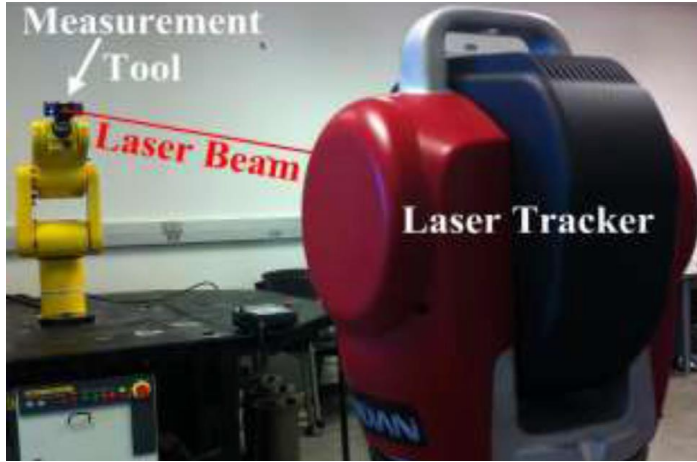
Validation Data

	Mean residual (mm)	Maximum residual (mm)	Gamma 99% (mm)
Nominal	3.418	8.210	8.721
CPA method	0.705	2.061	1.875
Proposed method	0.104	0.261	0.277

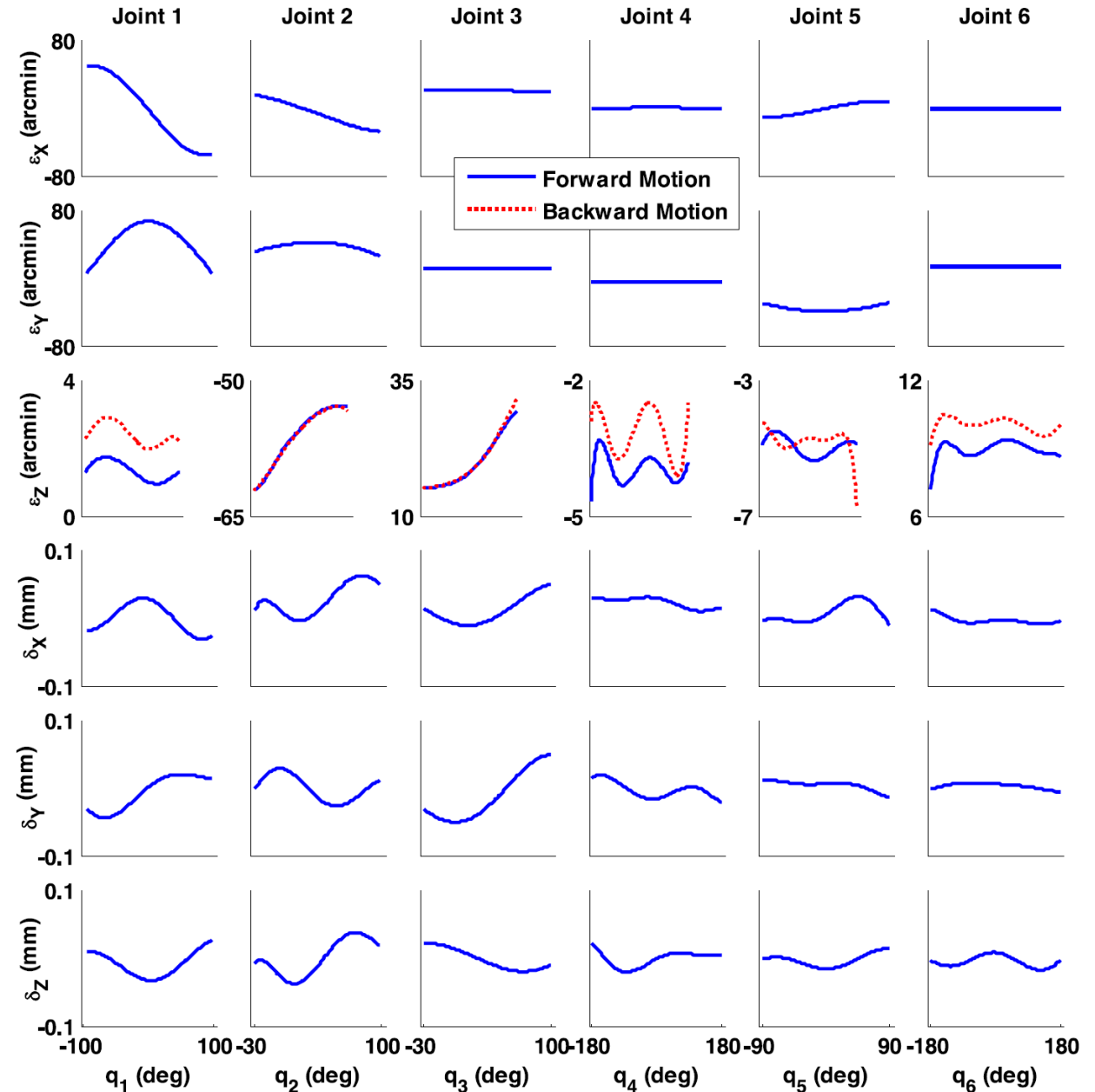
10x Better than DH fit

Results

Fanuc LR-Mate 200i



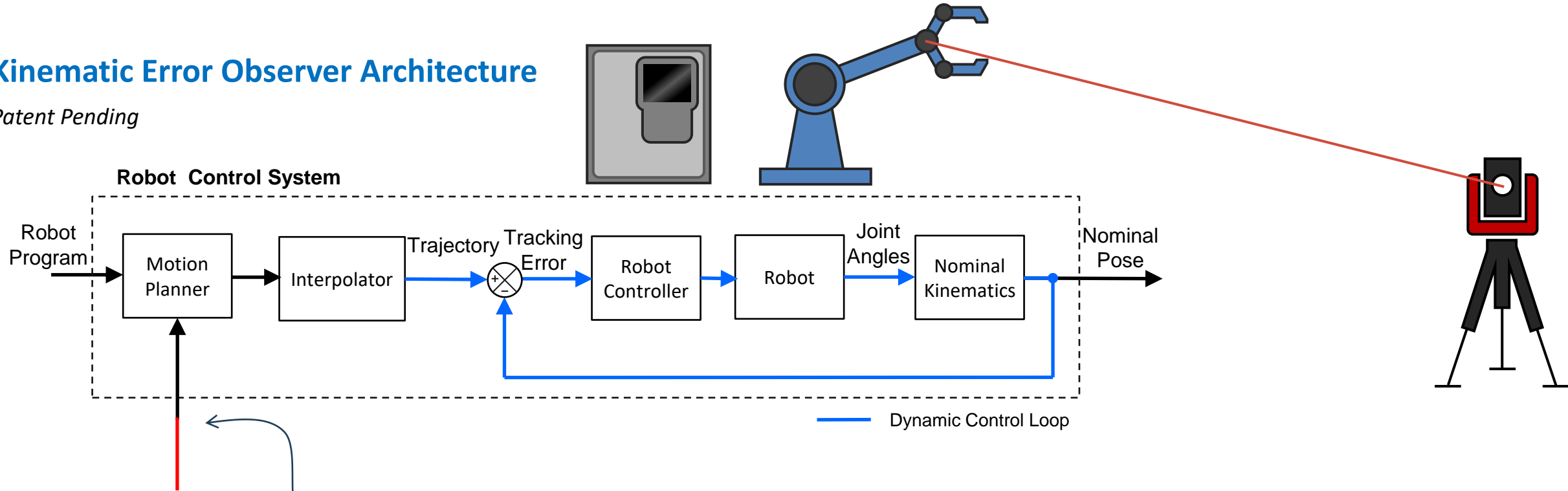
Error Kinematic Model



Metrology in the Loop (MitL)

Kinematic Error Observer Architecture

Patent Pending



Key Challenge:

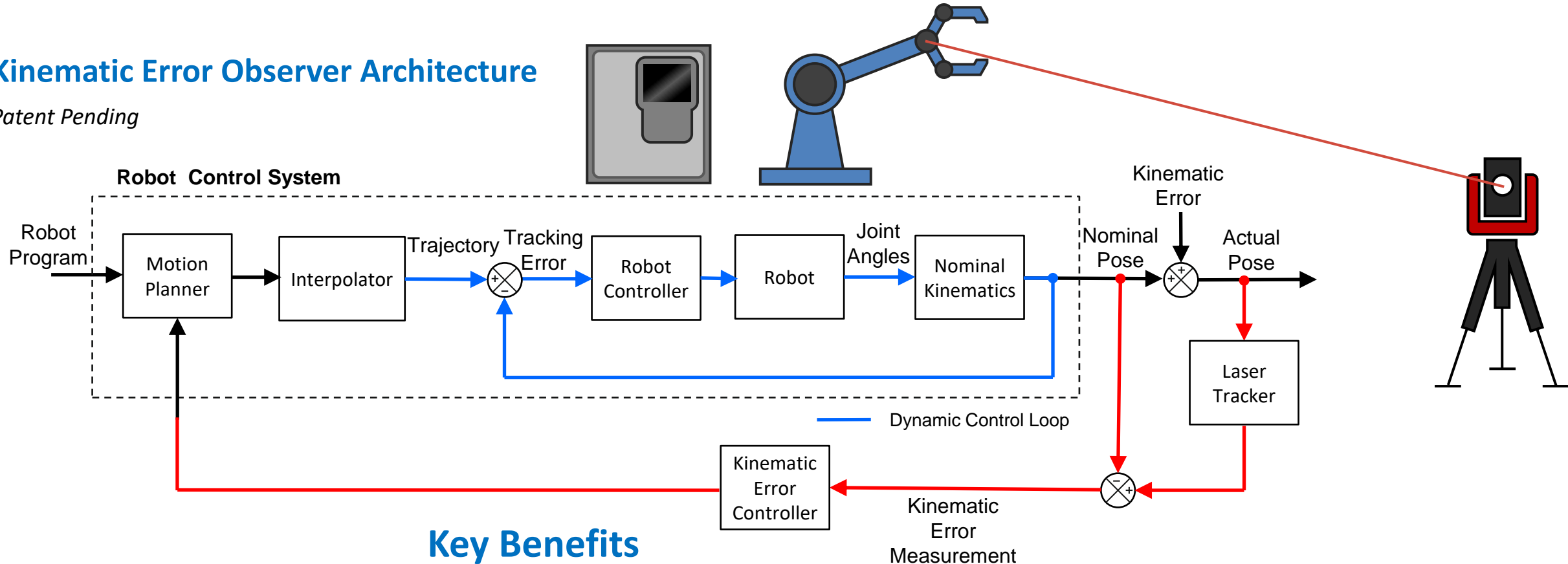
The position-offset channel has up to **250 ms of delay** on robot controllers!

Closed-loop bandwidth through this channel limited to 1-2 Hz. Too low for tracking.

Metrology in the Loop (MitL)

Kinematic Error Observer Architecture

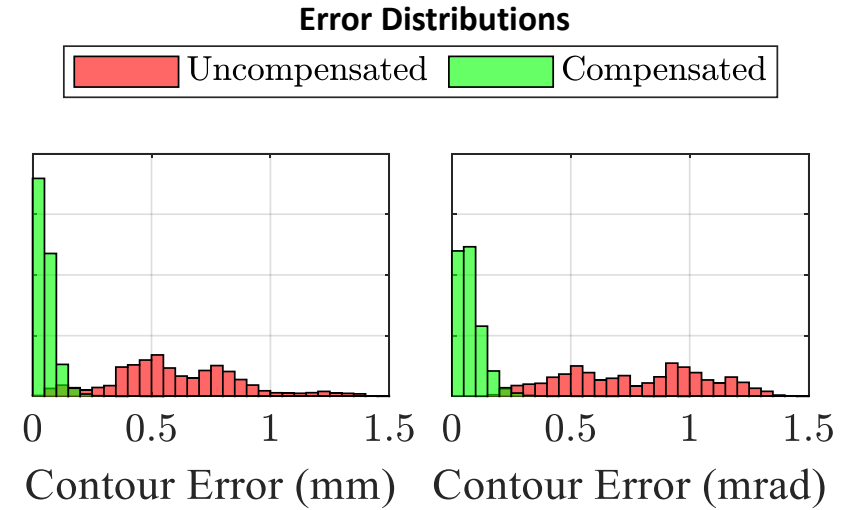
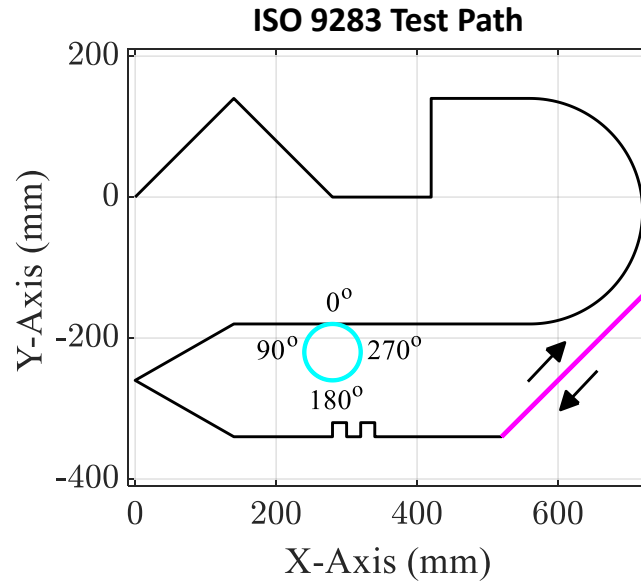
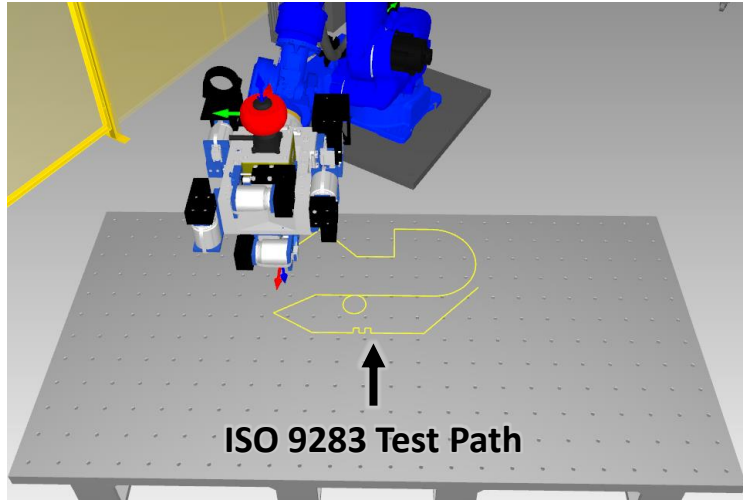
Patent Pending



Key Benefits

- **Non-Invasive** → Works with existing robot control system, not in place of it.
- **Simple Activation** → Enabled with a single command.
- **High Performance** → Submillimeter accuracy
- **Extremely Robust** → Minimal instability risk

Path Accuracy



- Reduced avg. error by **order of magnitude**
- Max errors at direction changes (hysteresis)
 - Can reduce with careful programming
 - Potential to reduce with feedforward design – current investigation underway

Position Error Statistical Characteristics

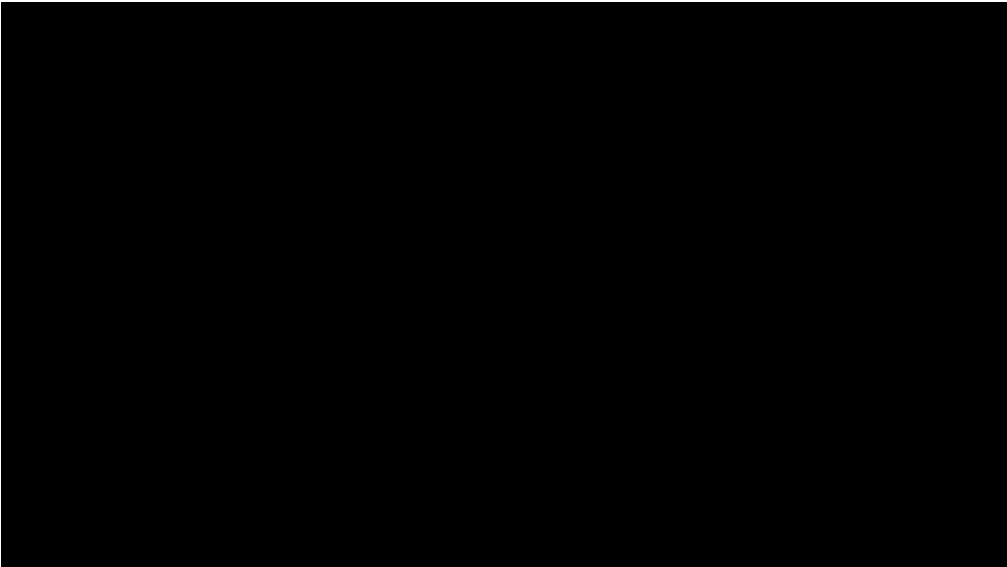
	Mean (mm)	Std. Dev. (mm)	Max (mm)
Out-of-the-Box	0.601	0.266	1.473
MitL	0.055	0.039	0.525

Orientation Error Statistical Characteristics

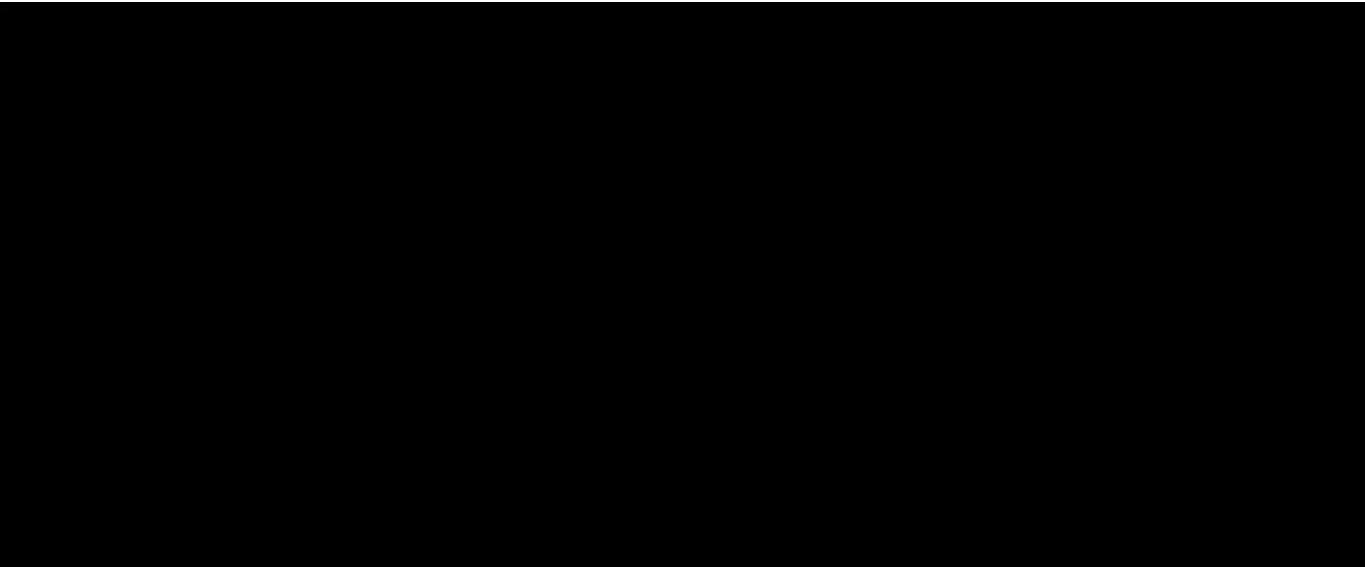
	Mean (mrad)	Std. Dev. (mrad)	Max (mrad)
Out-of-the-Box	0.771	0.293	1.457
MitL	0.078	0.056	0.964

MitL Simple Machining

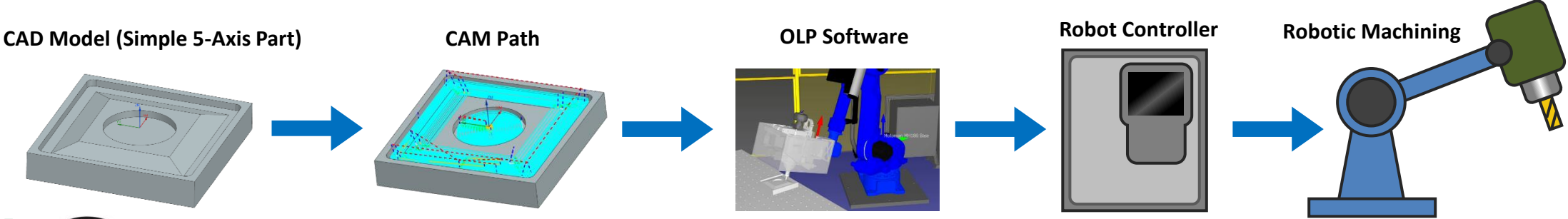
Face Milling



3+2-Axis Machining



Programming Pipeline

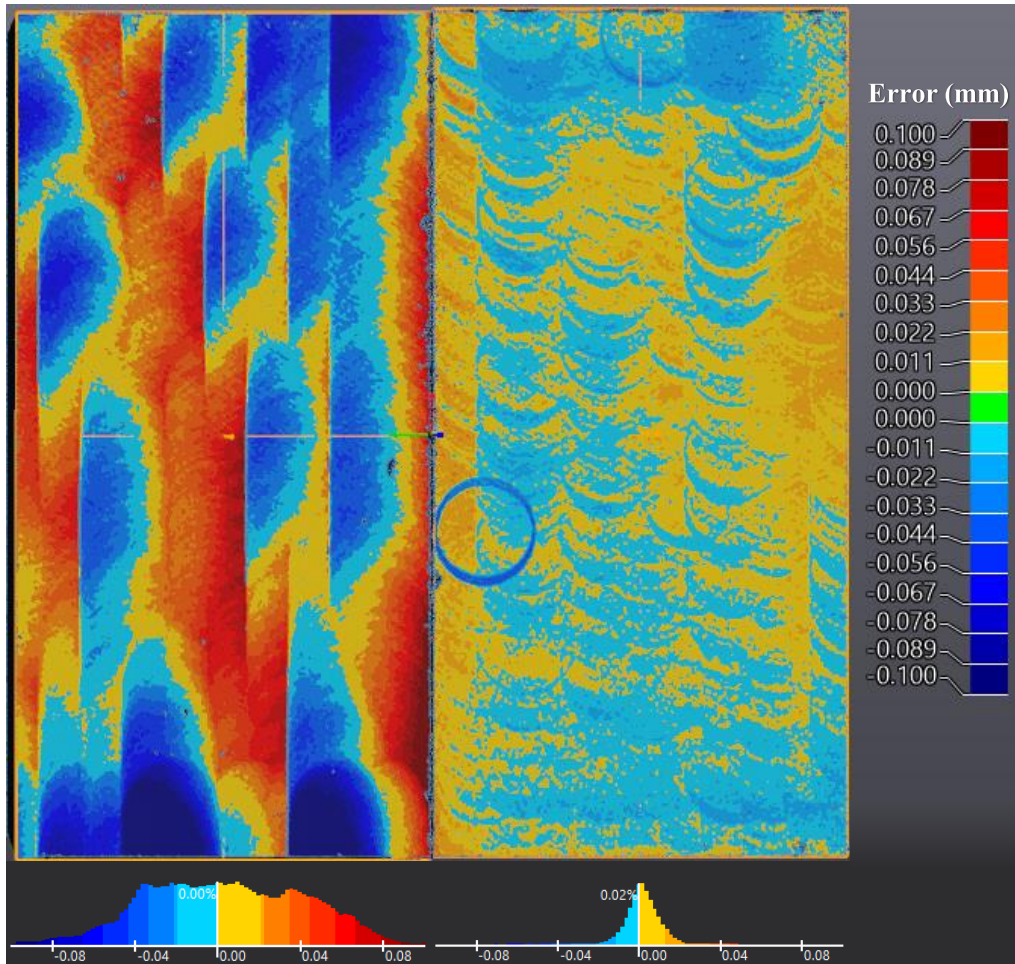


MitL Simple Machining Results

Face Milling

Out-of-the-Box

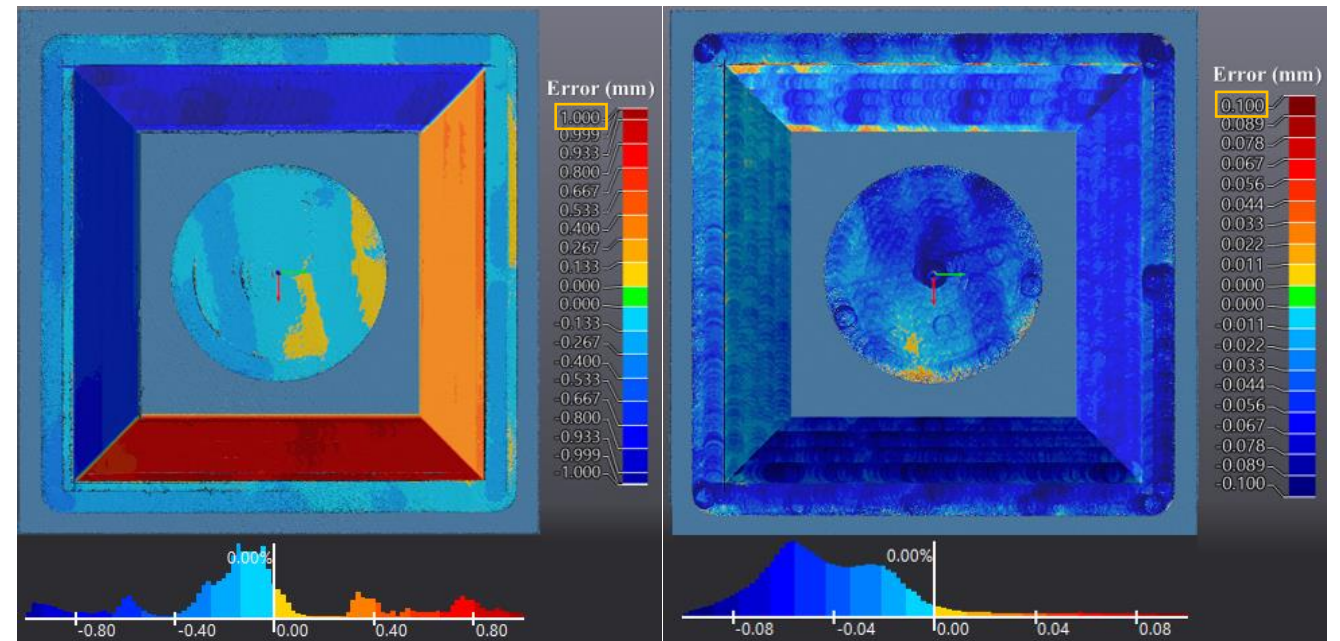
MitL



3+2-Axis Machining

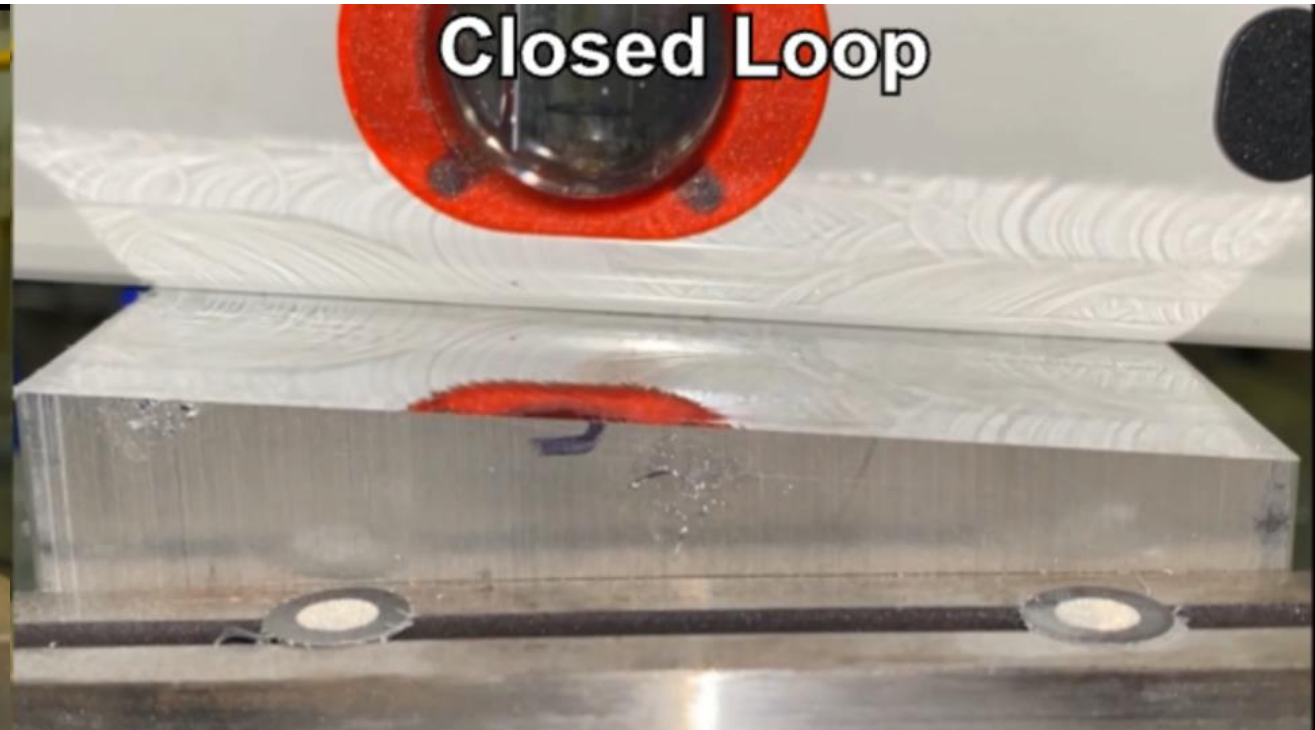
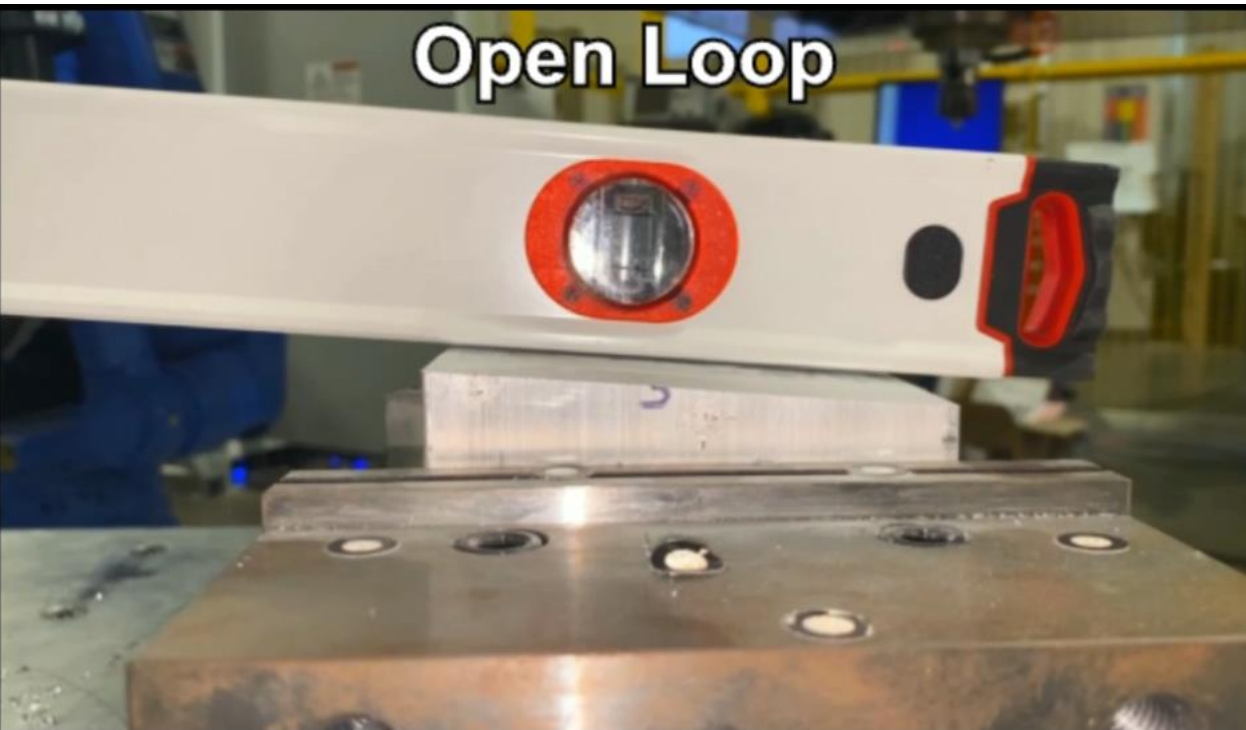
Out-of-the-Box

MitL



MitL improved 5-axis machining accuracy by **order of magnitude** and achieved a flatness of 25 μm !

MitL Large Part Accuracy

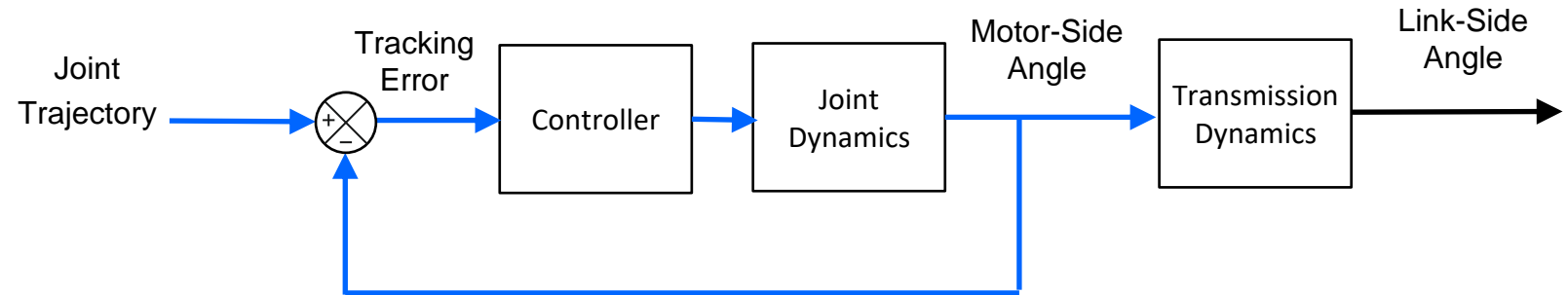


Can I Use MitL Strategy on Dual-Encoders?

What makes dual-encoders challenging?

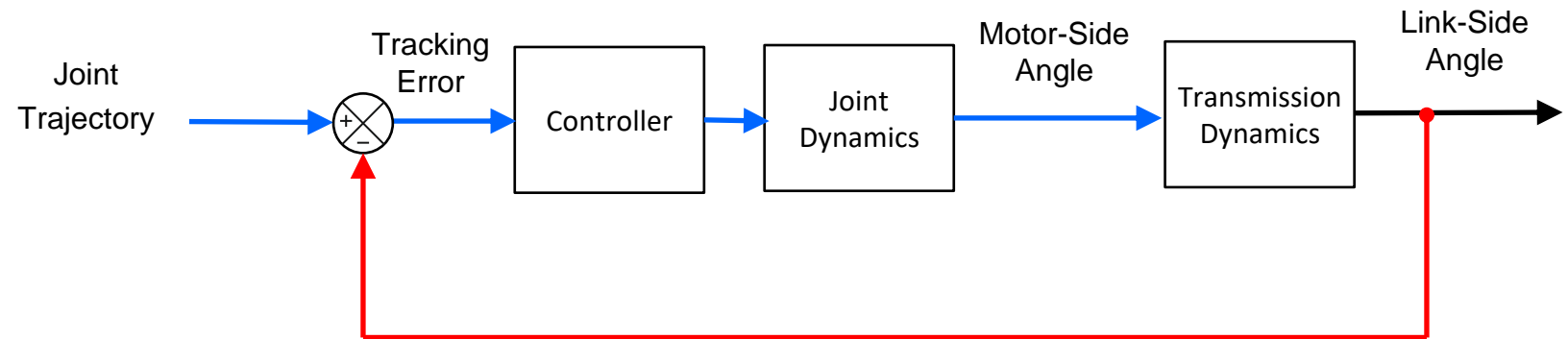
Standard Configuration

- Transmission dynamics actually *add* to loop phase increasing stability.



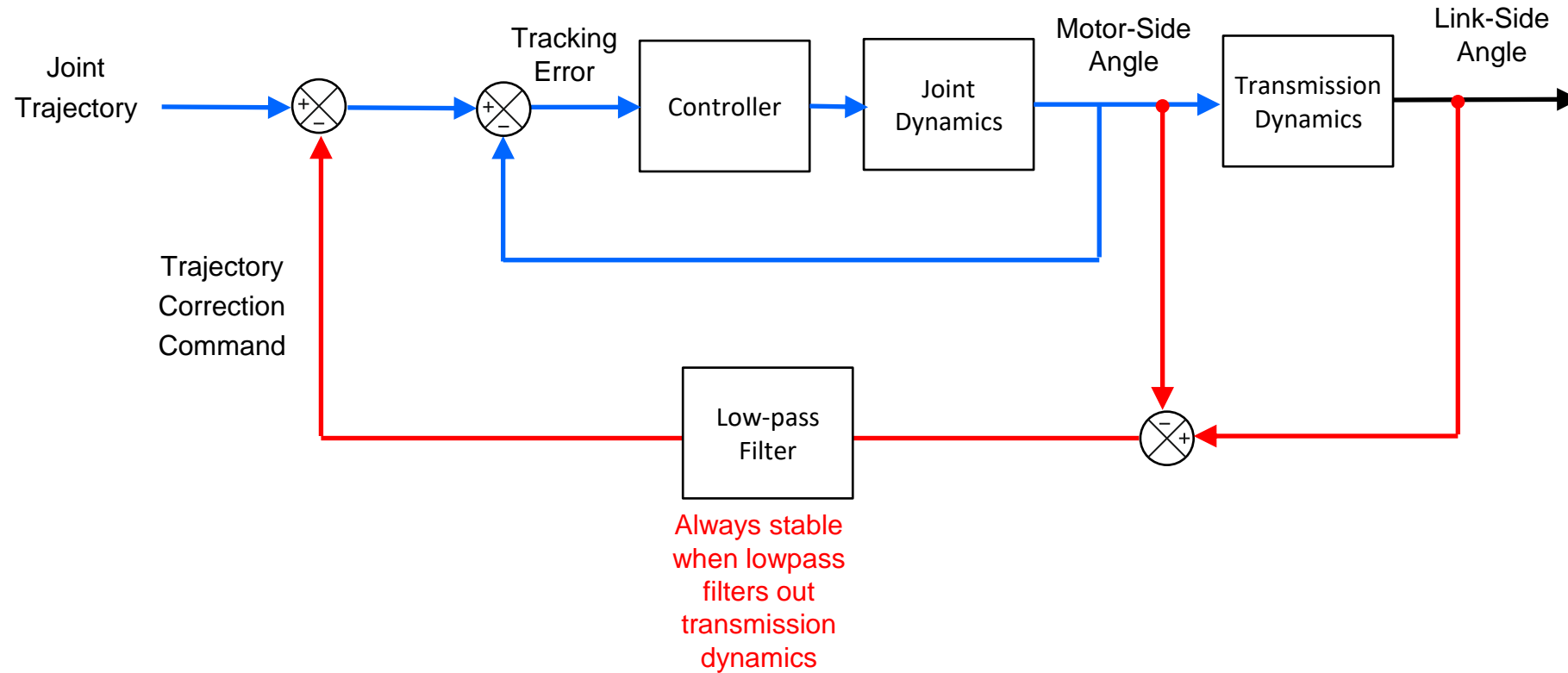
Link-Side Feedback

- Significant phase loss in loop, increases instability.
- Low-resolution on link-side reduces opportunity for high-gain derivative stabilization.

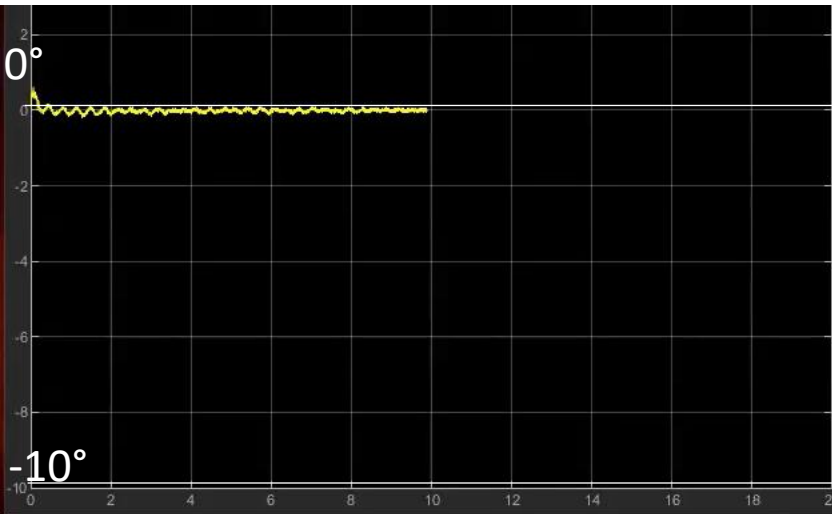


Can I Use MitL Strategy on Dual-Encoders?

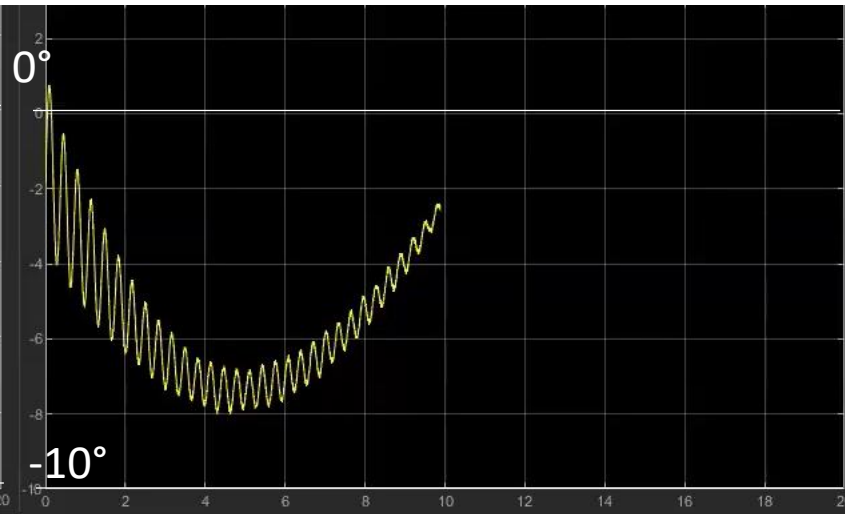
Yes!



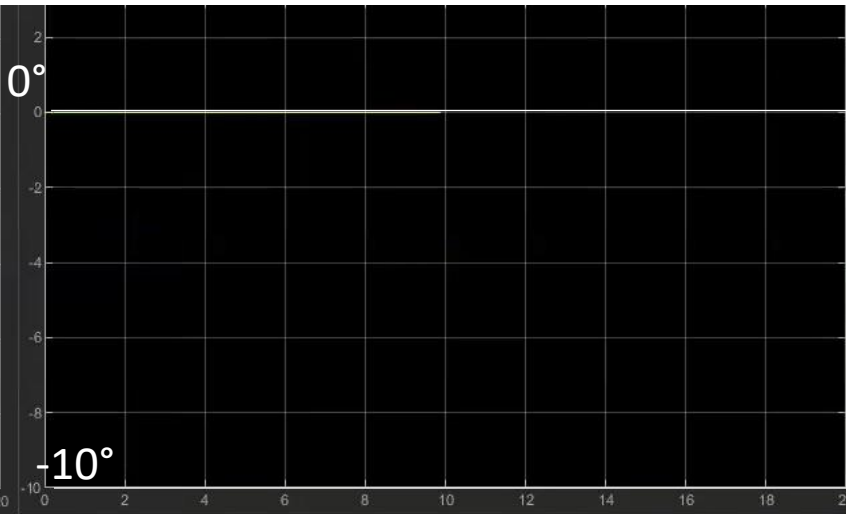
Motor Side Error



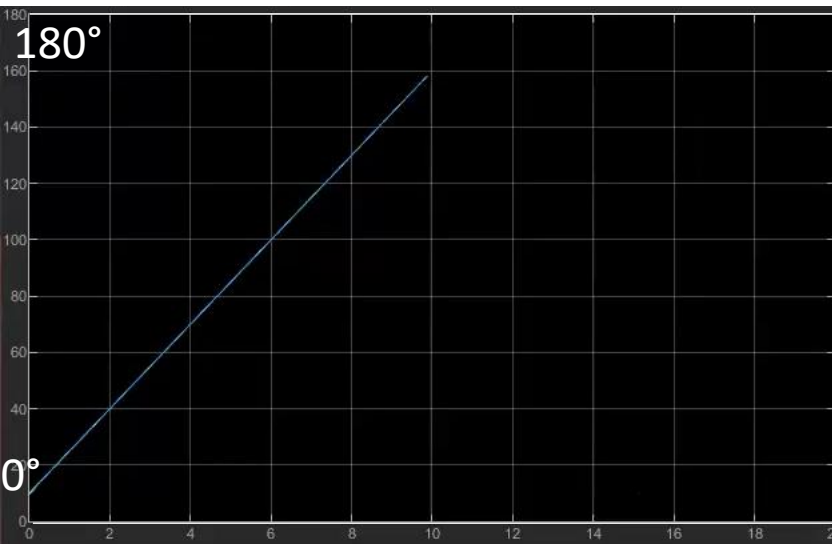
Link Side Error



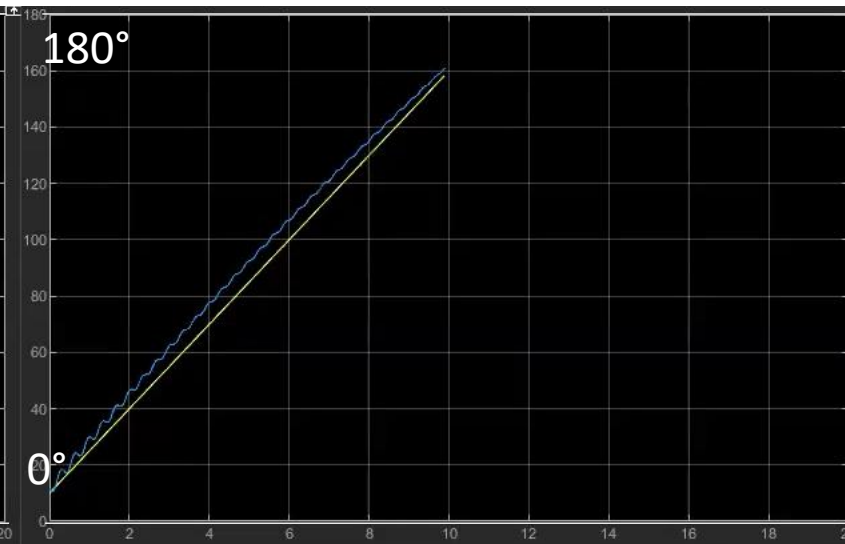
Reference Correction



Motor Side Tracking



Link Side Tracking



Collaborators

Robert Landers, Notre Dame

Philip Freeman, Boeing

Joshua Johnson, Boeing

Kam Lau, Automated Precision

Levi Armstrong, SWRI

Graduate Students

Patrick Sammons

Jennifer Creamer

Li He

Patrick Bazzoli

Mitch Woodside

Grant Bergstrom

Sponsors



Questions?