A system for quantum state transfer and entanglement generation includes a quantum system including a plurality of qubits, a processor, and a memory. The memory includes instructions stored therein, which, when executed by the processor, cause the quantum system to: access a signal of the quantum system; encode unknown coefficients in one qubit of the plurality of qubits; initialize each of the remaining qubits of the plurality of qubits in state 0; group the plurality of qubits into a plurality of subsystems; in each of the plurality of subsystems: encode quantum information into Greenberger-Horne-Zeilinger-like (GHZ-like) states using nearest-neighbor interactions; and apply a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems.
\[ |\psi| = a|0\rangle + b|1\rangle \]
\[ |+\rangle = \sqrt{\frac{1}{2}} (|0\rangle + |1\rangle) \]
300

Start

110

Encode unknown coefficients in one qubit

120

Encode quantum information into GHZ-like states using nearest-neighbor interactions

130

Apply a generalized controlled-phase gate between the plurality of hypercubes to merge the GHZ-like states into an entangled state between of the plurality of subsystems

140

Rotate the entangled state into a desired GHZ-like state by concentrating an entanglement in each hypercube of the plurality of hypercubes onto one qubit $c_j$ of each of the plurality of hypercubes

150

Apply a single-qubit rotation to $c_j$

160

Redistribute the entanglement to the plurality of hypercubes as a resulting GHZ-like state

FIG. 3
Apply $H = \frac{1}{\gamma_{\text{max}}} \sum_{i,j} |1\rangle \langle 1| \otimes |1\rangle \langle 1|$

Total Time $\sim \frac{\gamma_{\text{max}}}{V^2}$

After Time $t = \frac{\pi}{\gamma_{\text{max}}} V^2$

$V = 3 = \text{Volume of Each Small GHZ States}$

$\gamma_{\text{max}}$

Unentangle

Single-Qubit Rotation

Re-entangle

FIG. 4
(a)

\[ N = 3 \]
\[ N = 2 \]
\[ N = 1 \]
\[ N = 0 \]

\[ |m_0\rangle \]

\[ |m_1\rangle \]
\[ |m'_1\rangle \]

\[ \Omega_0 \]
\[ \Omega_+ \]

\[ E = 0 \]

FIG. 5A

(b)

\[ J_z > 0 \]

\[ E \neq 0 \]

\[ N = 2 \]
\[ N = 1 \]
\[ N = 0 \]

\[ |m_1\rangle \]

\[ |m_0\rangle \]

FIG. 5B
\[ J_z < 0 \]

\[ E \neq 0 \]

\[ N = 1 \]

\[ N = 0 \]

\[ |m'_1\rangle \]

\[ |m_0\rangle \]

\[ \Delta \]

\[ \Omega \]

FIG. 5C
SYSTEMS AND METHODS FOR OPTIMAL STATE TRANSFER AND ENTANGLEMENT GENERATION IN POWER-LAW INTERACTING SYSTEMS

CROSS-REFERENCE TO RELATED APPLICATIONS/CLAIM OF PRIORITY

[0001] This application claims the benefit of, and priority to, U.S. Provisional Patent Application No. 63/262,085, filed on Oct. 4, 2021, and U.S. Provisional Patent Application No. 63/378,210, filed on Oct. 3, 2022, the entire contents of which are hereby incorporated herein by reference.

GOVERNMENT SUPPORT

[0002] This invention was made with government support under W911NF1610349 awarded by the Department of the Army; DGE1840340 awarded by the National Science Foundation; and DMR1420541 awarded by the National Science Foundation (NSF). The government has certain rights in the invention.

TECHNICAL FIELD

[0003] The present disclosure relates generally to the field of quantum operations. More specifically, the present disclosure provides at least a system and method for encoding an arbitrary qubit into a multiqubit and subsequently transferring the quantum state.

BACKGROUND

[0004] Harnessing entanglement between many particles is key to a quantum advantage in applications including sensing and timekeeping, secure communication, and quantum computing. Encoding quantum information into a multiqubit Greenberger-Horne-Zeilinger-like state is particularly desirable as a subroutine in many quantum applications including metrology, quantum computing, anonymous quantum communication, and quantum secret sharing.

[0005] Accordingly, there is interest in encoding quantum information into a multiqubit Greenberger-Horne-Zeilinger-like state to improve harnessing entanglement.

SUMMARY

[0006] An aspect of the present disclosure provides a system for quantum state transfer and entanglement generation. The system includes a quantum system including a plurality of qubits, a processor, and a memory. The memory includes instructions stored thereon, which, when executed by the processor, cause the quantum system to: access a signal of the quantum system; encode unknown coefficients in one qubit of the plurality of qubits; initialize each of the remaining qubits of the plurality of qubits in state |0⟩; group the plurality of qubits into a plurality of subsystems; and applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems.

[0007] In accordance with aspects of the disclosure, the instructions, when executed by the processor, may further cause the quantum system to rotate the entangled state into a desired GHZ-like state by concentrating an entanglement in each subsystem of the plurality of subsystems onto one qubit c of each of the plurality of subsystems.

[0008] In an aspect of the present disclosure, the instructions, when executed by the processor, may further cause the quantum system to apply a single-qubit rotation to c.

[0009] In another aspect of the present disclosure, the instructions, when executed by the processor, may further cause the quantum system to redistribute the entanglement to the plurality of subsystems as a resulting GHZ-like state.

[0010] In yet another aspect of the present disclosure, the instructions, when executed by the processor, may further cause the quantum system to repeatedly feed the resulting GHZ-like state back into applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between of the plurality of subsystems to yield larger GHZ-like states.

[0011] In accordance with further aspects of the present disclosure, the system may further include a Hadamard gate. The rotation of c is applied using the Hadamard gate.

[0012] In an aspect of the present disclosure, the subsystems may include hypercubes.

[0013] In another aspect of the present disclosure, the unknown coefficients may include a, b.

[0014] In yet another aspect of the present disclosure, the unknown coefficients of the one qubit of the plurality of qubits may be encoded as a|0⟩ + b|1⟩.

[0015] In yet another aspect of the present disclosure, the quantum system may include a power-law interacting Hamiltonian.

[0016] An aspect of the present disclosure provides a method for quantum state transfer and entanglement generation. The method includes: accessing a signal of a quantum system, the quantum system including a plurality of qubits; encoding unknown coefficients in one qubit of the plurality of qubits; initializing each of the remaining qubits of the plurality of qubits in state |0⟩; grouping the plurality of qubits into a plurality of subsystems; and in each of the plurality of subsystems encoding quantum information into Greenberger-Horne-Zeilinger-like (GHZ-like) states using nearest-neighbor interactions; and applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems.

[0017] In another aspect of the present disclosure, the method may further include rotating the entangled state into a desired GHZ-like state by concentrating an entanglement in each subsystem of the plurality of subsystems onto one qubit c of each of the plurality of subsystems.

[0018] In yet another aspect of the present disclosure, the method may further include applying a single-qubit rotation to c.

[0019] In yet another aspect of the present disclosure, the method may further include redistributing the entanglement to the plurality of subsystems as a resulting GHZ-like state.

[0020] In accordance with further aspects of the present disclosure, the method may further include repeatedly feeding the resulting GHZ-like state back into applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between of the plurality of subsystems to yield larger GHZ-like states.

[0021] In another aspect of the present disclosure, the subsystems may include hypercubes.
[0022] In yet another aspect of the present disclosure, the unknown coefficients may include a, b.
[0023] In a further aspect of the present disclosure, the unknown coefficients of the one qubit of the plurality of qubits may be encoded as a(0) + b(1).
[0024] In a further aspect of the present disclosure, the rotation of ε may be applied using a Hadamard gate.
[0025] An aspect of the present disclosure provides a method for quantum state transfer and entanglement generation. The method includes: accessing a signal of a quantum system, the quantum system including a plurality of qubits; encoding a(0) + b(1) in one qubit of the plurality of qubits; initializing each of the remaining qubits of the plurality of qubits in state 10; grouping the plurality of qubits into a plurality of hypercubes; in each of the hypercubes of the plurality of hypercubes encoding quantum information into Greenberger-Horne-Zeilinger-like (GHZ-like) states; merging the GHZ-like states into an entangled state between the plurality of hypercubes; rotating the entangled state into a desired GHZ-like state; applying a single-qubit rotation to one bit of one of the plurality of hypercubes; and redistributing the entanglement to the plurality of hypercubes.
[0026] Further details and aspects of exemplary embodiments of the present disclosure are described in more detail below with reference to the appended figures.

BRIEF DESCRIPTION OF THE DRAWINGS

[0027] A better understanding of the features and advantages of the present disclosure will be obtained by reference to the following detailed description that sets forth illustrative embodiments, in which the principles of the present disclosure are utilized, and the accompanying drawings of which:

[0028] FIG. 1 is a diagram of an exemplary quantum system diagram, in accordance with examples of the present disclosure;
[0029] FIG. 2 is a diagram of a method for quantum state transfer and entanglement generation for the quantum system of FIG. 1, in accordance with examples of the present disclosure;
[0030] FIG. 3 illustrates a flow diagram of the method for encoding an arbitrary qubit into a multiqubit and subsequently transferring the quantum state for the quantum system of FIG. 1, in accordance with examples of the present disclosure;
[0031] FIG. 4 is a diagram illustrating quantum state transfer of FIG. 1, in accordance with examples of the present disclosure;
[0032] FIGS. 5A-C is a diagram illustrating a scheme for realizing dipole-dipole Ising interactions using polar molecules, in accordance with examples of the present disclosure;
[0033] FIGS. 6A and 6B, illustrate a dressing scheme for realizing H_m and H_m', via microwave dressing, with \( |i\rangle = (|s\rangle \pm |p\rangle, |\overline{s}\rangle \pm |\overline{p}\rangle) \sqrt{2} \) and \( |i\rangle = (|s\rangle \pm |p\rangle, |\overline{s}\rangle \pm |\overline{p}\rangle) \sqrt{2} \), in accordance with examples of the present disclosure;
[0034] FIG. 7 is a diagram illustrating the effective van der Walls (vdW) interactions between controls and targets for negative Ising interactions, in accordance with examples of the present disclosure; and

[0035] FIG. 8 is a schematic diagram of an exemplary processing system diagram for use with the system of FIG. 1, in accordance with examples of the present disclosure.

DETAILED DESCRIPTION

[0036] The present disclosure relates generally to the field of quantum operations. More specifically, the present disclosure provides a method for encoding an arbitrary qubit into a multiqubit and subsequently transferring the quantum state.
[0037] Embodiments of the present disclosure are described in detail with reference to the drawings wherein like reference numerals identify similar or identical elements.
[0038] Although the present disclosure will be described in terms of specific examples, it will be readily apparent to those skilled in this art that various modifications, rearrangements, and substitutions may be made without departing from the spirit of the present disclosure. The scope of the present disclosure is defined by the claims appended hereto.
[0039] For purposes of promoting an understanding of the principles of the present disclosure, reference will now be made to exemplary embodiments illustrated in the drawings, and specific language will be used to describe the same. It will nevertheless be understood that no limitation of the scope of the present disclosure is thereby intended. Any alterations and further modifications of the novel features illustrated herein, and any additional applications of the principles of the present disclosure as illustrated herein, which would occur to one skilled in the relevant art and having possession of this disclosure, are to be considered within the scope of the present disclosure.
[0040] Referring to FIG. 1, a diagram of an example quantum system 100 (e.g., hypercube C), is shown. The quantum system 100 may include two or more qubits 102. Qubits 102 may be arranged in a one-or more dimensional system 100. For example, the qubits 102 may be grouped into hypercubic lattices. Although qubit systems are used as an example, it is contemplated that the disclosed technology may be used in qudit systems (arbitrary finite-level systems) as well.
[0041] Harnessing entanglement between many particles is key to a quantum advantage in applications including sensing and timekeeping, secure communication, and quantum computing. For example, encoding quantum information into a multiqubit Greenberger-Horne-Zeilinger-like (GHZ-like) state is particularly desirable as a subroutine in many quantum applications, including metrology, quantum computing, anonymous quantum communication, and quantum secret sharing.
[0042] The speed at which one can unitarily encode an unknown qubit state a(0) + b(1) into a GHZ-like state a(00 . . . 0) + b(11 . . . 1) of a large system is constrained by Lieb-Robinson bounds and depends on the nature of the interactions in the system. In systems with finite-range interactions and power-law interactions decaying with distance r as r^d for all \( \alpha \leq d < 4 \), where d is the dimension of the system, the Lieb-Robinson bounds imply a linear light cone for the propagation of quantum information. Consequently, in such systems, the linear size of a GHZ-like state that can be prepared from entangled particles cannot grow faster than linearly with time.
The disclosed method enables the encoding of a qubit into a GHZ-like state in a one-dimensional quantum system 100. Although a one-dimensional system is used as an example, systems of larger dimensionality are contemplated.

Referring to FIGS. 1, 2, 3, and 8, a method 300 and illustrative schematic 800 for quantum state transfer and entanglement generation for the quantum system of FIG. 1 is shown. The system 800 for quantum state transfer and entanglement generation may include a processor system 810 (FIG. 8) and a memory 811, including instructions stored thereon, which when executed by the processor 810, cause the quantum system 100 to perform the steps of method 300.

The processor 810 may be connected to a computer-readable storage medium or a memory 811. The computer-readable storage medium or memory 811 may be a volatile type of memory, e.g., RAM, or a non-volatile type of memory, e.g., flash media, disk media, etc. In various aspects of the disclosure, the processor 810 may be any type of processor such as a digital signal processor, a microprocessor, an ASIC, a graphics processing unit (GPU), a field-programmable gate array (FPGA), or a central processing unit (CPU).

In aspects of the disclosure, the memory 811 can be random access memory, read-only memory, magnetic disk memory, solid-state memory, optical disc memory, and/or another type of memory. In some aspects of the disclosure, the memory 811 can be separate from the processor and can communicate with the processor through communication buses of a circuit board and/or through communication cables such as serial ATA cables or other types of cables. The memory 811 includes computer-readable instructions that are executable by the processor 810 to operate the processor. In other aspects of the disclosure, the system 800 may include a network interface to communicate with other computers or to a server. A storage device may be used for storing data.

Initially at step 110, the unknown coefficients a, b are encoded in one qubit 104 while the other qubits 102 are each initialized in state |0⟩. The next step 120 of the method 300 assumes the ability to encode information into GHZ-like states in subsystem 106 using, for example, nearest-neighbor interactions. Next at step 130, a generalized controlled-phase gate (Equation 6) is applied between the subsystems 106 to “merge” the GHZ-like states into an entangled state between all sites (FIGS. 2 and 4).

At step 140, the entangled state is rotated into the desired GHZ-like state by concentrating the entanglement in each subsystem 126 onto one qubit 152 (FIGS. 2 and 4).

At step 150, single-qubit rotations are applied. For example, qubit 152 c of each subsystem 126 may be rotated.

At step 160 the entanglement is redistributed to the rest of the quantum system 100. Repeatedly feeding the resulting GHZ-like state back into step 130 of the method 300 yields larger and larger GHZ-like states.

The Lieb-Robinson bounds become less stringent for longer-range interactions, i.e., those with a<2d+1. The bounds theoretically allow quantum information to travel a distance r in time t that scales sublinearly with r. However, no method in the present literature can saturate these bounds. In particular, existing methods for a∈(d,2d] are exponentially slower than what is allowed by the corresponding bounds. Up until now, the existence of this gap between the Lieb-Robinson bounds and the achievable methods has meant that at least one of the two is not yet optimal, hinting at either a tighter Lieb-Robinson bound or the possibility of speeding up many quantum information processing tasks.

The disclosed technology closes the gap for all a∈(d,2d+1] in d≥1 by designing method 300 for encoding an arbitrary qubit into a multiqubit GHZ-like state and, subsequently, transferring information at the limits imposed by the Lieb-Robinson bounds. There are at least three implications of the method. First, within these regimes of a, the disclosed technology establishes the tightness of the Lieb-Robinson bounds, up to sub-polynomial corrections, and effectively puts an end to the fifteen-year search for a tighter bound. The disclosed technology may provide optimal designs for power-law interacting systems, including trapped ions (a∈[0,3]) in one and two dimensions, ultracold atoms in photonic crystals, van-der-Waals interacting Rydberg atoms (a=6) in three dimensions, as well as the very common case of dipolar interactions in nitrogen-vacancy centers, polar molecules, and dipole-dipole interacting Rydberg atoms (a=3) in two dimensions. Finally, the disclosed technology implies a lower bound on the gate count in simulating power-law interacting systems on a quantum computer, providing a benchmark for the performance of quantum simulation algorithms.

For simplicity, a d-dimensional hypercubic lattice A and a two-level system located at every site of the lattice is considered. It is contemplated that the disclosed technology generalizes straightforwardly to all regular lattices. Without loss of generality, it is assumed that the lattice spacing is one. A power-law interacting Hamiltonian H(t) =∑i,j∈Λ hi,j(t), where hi,j(t) is a Hamiltonian supported on sites i, j such that, at all times t and for all i,j, provides |hi,j(t)|≤dist(i,j)da+b, where dist(i,j) is the distance between i, j, and |⋅| is the operator norm, and a+b is a constant. (GHZ(a,b)0) is used to denote the GHZ-like state over sites in S∈Λ:

\[ |\text{GHZ}(a,b)\rangle = \bigotimes_{i=0}^{d-1} |0\rangle_{i} |1\rangle_{d} \]

where |0⟩, |1⟩ ∈ |x⟩ ∈ |x⟩0 (x=0,1) are product states over all sites in S and a, b are complex numbers such that |a|^2+|b|^2=1. In particular, |GHZ⟩ is used to denote the symmetric state a=b=1/√2.

Given a d-dimensional hypercube CΛ of length rz1, encode a possibly unknown state a|0⟩+b|1⟩ of a site c E C into the GHZ-like state |GHZ(a,b)⟩ over C, assuming that all sites in C, except for c, are initially in the state |0⟩. Specifically, a time-dependent, power-law interacting Hamiltonian H(t) that generates U(t)=U^T exp (−i∫^t_0 dH(s)) is constructed that satisfies:

\[ U(\gamma|0⟩0+b|1⟩0)\bigotimes_{c∈C} |0⟩_{c} = a|0⟩_{c} \bigotimes_{b} |1⟩_{c} \]

at time:

\[ t(\gamma) ≤ K_{\gamma} = \begin{cases} \log_{d+1} r & \text{if } 0 < a ≤ 2d, \\ r^\frac{a}{2(2d)} & \text{if } a = 2d, \text{ and} \\ r^\frac{a}{2d+1} & \text{if } 2d < a ≤ 2d+1 \end{cases} \]

Here, γ=3√d, K_{\gamma}, and K_{\gamma} are constants independent of t and r. Additionally, by reversing the unitary in Eqn. 2 to “concentrate” the information in |GHZ(a,b)⟩ onto a differ-
ent site in C, a quantum state may be transferred from ceC to any other site c′eC in time 2t

[0058] The disclosed method recursively builds the GHZ-like state in a large hypercube from the GHZ-like states of smaller hypercubes. For the base case, hypercubes of finite lengths, i.e., rs×s for some fixed r, can always be generated in times that satisfy Eqn. 3 for some suitably large (but constant) prefactor Kp. Assuming that information can be encoded into a GHZ-like state in hypercubes of length r, in time t, satisfying Eqn. 3, the following subroutine encodes information into a GHZ-like state in an arbitrary hypercube C of length r=mr, containing c, the site initially holding the phase information a, b. Here m is an a-dependent number to be chosen later.

[0059] At step 120, the hypercube C is divided into ms smaller hypercubes C1, . . . , Cms, each of length r. Without loss of generality, C1 is the hypercube that contains c. Let V=r be the number of sites in each Cj, a, b are simultaneously encoded into (GHZ(a,b))Cj and prepare (GHZ)Cj for all j=2, . . . , ms, which, takes time

\[ t = Kp \times \begin{cases} \log_2 r & \text{if } d < a < 2d, \\ \sqrt{r} & \text{if } a = 2d, \text{and} \\ r^{-2d} & \text{if } 2d < a < 2d + 1 \end{cases} \]  

(Eqn. 4)

[0060] By the end of this step, the hypercube C is in the state

\[ |a⟩|b⟩|Δ⟩_{j=1}^s \otimes |1⟩_{j=1}^s \]  

(Eqn. 5)

[0061] Next, at step 130 the following Hamiltonian is applied to the hypercube C:

\[ H_2 = \frac{1}{(m)^2} \sum_{j=2}^{ms} \sum_{c_1} \sum_{c_2} \sum_{c_1} |1⟩_c \otimes |L⟩_{c_1} \otimes |1⟩_{c_2} \tag{Eqn. 6} \]

This Hamiltonian effectively generates the so-called controlled-PHASE gate between the hypercubes, with C1 being the control hypercube and C2, . . . , Cms being the target hypercubes. The interactions between qubits in Eqn. 6 are selected to be identical for simplicity. If the interactions were to vary between qubits, the interaction between C1 and Cj may be turned off once the total phase accumulated by C1 reaches π. The prefactor 1/(m)r ensures that this Hamiltonian satisfies the condition of a power-law interacting Hamiltonian. It is straightforward to verify that, under this evolution, the state of the hypercube C evolves to:

\[ |a⟩|b⟩|Δ⟩_{j=1}^s \otimes |1⟩_{j=1}^s \]  

(Eqn. 7)

[0062] after time t=pi/2r(m)r/N2.

[0063] To obtain the desired state (GHZ(a,b))C, it remains to apply a Hadamard gate on the effective qubit \(|0⟩_j \) for j=2, . . . , ms. The Hadamard gate is applied in the following three steps by first concentrating the information stored in hypercube Cj onto a single site cj\in Cj (Step 3), then applying a Hadamard gate on Cj (Step 4), and then unfolding the information back onto the full hypercube Cj (Step 5).

[0065] Next, at step 140, for each hypercube Cj (j=2, . . . , ms) and given a designated site, a \in Cj, there exists a (time-dependent) Hamiltonian Hj that generates a unitary Uj such that

\[ \phi_0|0⟩ + \phi_1|1⟩ \rightarrow \phi_j|0⟩ + \phi_j|1⟩ \]  

(Eqn. 8)

[0066] for all complex coefficients \( \phi_0, \phi_1 \). In the state t=1 satisfying Eqn. 4. By linearity, this property applies even if Cj is entangled with other hypercubes. Consequently, backward time evolution under Hj generates Uj, which "undoes" the GHZ-like state of the jth hypercube:

\[ \phi_0|0⟩ + \phi_1|1⟩ \rightarrow \phi_j|0⟩ + \phi_j|1⟩ \]  

(Eqn. 9)

[0067] for any \( \phi_0, \phi_1 \). In this step, Uj is simultaneously applied to Cj for all j=2, . . . , ms. These unitaries rotate the state of C to:

\[ α|0⟩_j |1⟩_j + β|1⟩_j |0⟩_j \]  

(Eqn. 10)

where \( |1⟩ = |0⟩ \pm |1⟩ \)/√2.

[0068] Next, at step 150 a Hadamard gate is applied,

\[ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \]  

(Eqn. 11) to the site c, of each hypercube Cj; j=2, . . . , ms. These Hadamard gates can be implemented arbitrarily fast since any constraints are not assumed on the single-site terms of the Hamiltonian. The state of C by the end of this step is:

\[ α|0⟩_j |0⟩_j + β|1⟩_j |1⟩_j \]  

+Eqs. 12

[0069] Next at step 160, Uj is applied again to each hypercube Cj (j=2, . . . , ms) to obtain the desired GHZ-like state:

\[ α|0⟩_j |0⟩_j + β|1⟩_j |1⟩_j \]  

(Eqn. 13)

\[ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \]  

(Eqn. 14)

At the end of this routine, the unitary satisfying Eqn. 2 in time t=3t1+r=3t1+pi/2r(m)r/N2 (Eqn. 14) has been implemented.
Three cases corresponding to different ranges of $m$ are considered and it is shown that if $t_i(r_i)$ satisfies Eqn. 3, then $t(r)$ also satisfies Eqn. 3.

For $ae(2d, 2d+1)$, $t_i < k r_i^{-2d}$. Selecting $m = 1$ to be a constant integer provides:

$$t \leq \left( \frac{k_a}{m^{3/2d}} + n r_i^{-2d} \right) m r_i^{-2d} \leq k_a r_i^{-2d}, \quad (\text{Eqn. 15})$$

where $m > 3^{(2^{r_i} - 2d)}$ and $k_a \equiv \frac{n r_i^{-2d}}{1 - m^{-3/2d}}$. \(\text{Eqn. 16}\)

For $ae(d, 2d)$, $m$ is chosen to scale with $r_i$ such that $r_i^{-(d+1)} < m < r_i^{-(2d+1)}$ where $a = 2d/a$. The length of the larger cube $C$ is then $r_i^{-d} r_i^{-(d+1)}$, and, therefore, the total time is:

$$t \leq 3 k_a \log m \sum_i r_i + \Delta \left( 2 \sqrt{\log m} \right)^{d+1} \quad (\text{Eqn. 17})$$

$$+ \frac{4 k_a}{m^{3/2d}} \log r_i = k_a \log m r_i, \quad (\text{Eqn. 18})$$

where $K_a = \log 4 / \log(2d/a)$ is selected and $K_a \log r_i = 2 \sqrt{\log m}$ is assumed to simplify the expression. The factor $\log 4$ in the definition of $K_a$ may be made arbitrarily close to $\log 3$ by increasing $K_a$.

Finally, for $ae = 2d$, $m$ is chosen such that

$$\exp\left( \frac{\gamma}{2d} \sqrt{\log r_i} \right) \leq m \leq 2 \exp\left( \frac{\gamma}{2d} \sqrt{\log m} \right),$$

where $\gamma = 3d$. Substituting $t_i < k_a \exp(\sqrt{\log r_i})$ into Eqn. 14:

$$t_i \leq 3 k_a t_i \sum_i \exp(\sqrt{\log r_i}), \quad (\text{Eqn. 19})$$

Assuming $r_i \leq \exp(8/d)$, it is straightforward to prove that $\sqrt{\sum_i \log(\log m)} \leq 2$. Applying this condition on the above inequality:

$$t \leq \frac{1}{e} \left( \frac{3 k_a}{n^{2d/2}} \right)^{2d/2} \leq k_a \log m r_i, \quad (\text{Eqn. 20})$$

where $r_i = m r_i$ is the length of the resulting GHZ-like state and $K_a \geq 2^{\pi d/2} \sqrt{n^2 - 3}$. \(\text{Eqn. 20}\)

In the disclosed method, quantum information is simultaneously encoded into the GHZ-like state over $C_1$ and the symmetric GHZ states are created over other multiqubit subsystems $C_2, \ldots, C_m$. As a result, the implementation of the controlled operations in step 130 (FIG. 1) is enhanced quadratically by the volume of each subsystem.

In aspects, the size of the resulting GHZ-like state may be maximized at the end of each iteration by allowing $m$ to depend on $a$ and on the size of the existing GHZ-like states. When the disclosed method is used for state transfer, this strategy results in most of the qubits between the source and the target sites participating in the transfer, significantly speeding up the disclosed method.

First, the disclosed method enables encoding an unknown qubit into a multiqubit GHZ-like state and, subsequently, performing state transfer at unprecedented speeds. For $d < 2d$, which applies, for example, to dipole-dipole interactions ($a = 3$) in two dimensions and to the effective interactions between trapped ions ($a = 0, 3$) in one and two dimensions, the disclosed method encodes information into GHZ-like states and transfers information in polylogarithmic time, exponentially faster than methods available in the literature. Even for the seemingly weak long-range interactions with $a = 2d$, such as van der Waals interactions between Rydberg atoms ($a = 6$) in three dimensions, the disclosed method still takes only sub-polynomial time to entangle an entire system and to transfer a quantum state. When applied to the preparation of GHZ states, these speedups enable potential improvements to quantum sensors built from nitrogen-vacancy centers, Rydberg atoms, and polar molecules, as well as to atomic clocks based on trapped ions.

[0081] The optimality of the disclosed systems and methods for $ae(1,3)$ in one dimension and $ae(d, 2d)$ in $d > 1$ dimensions also lays the foundation for optimal quantum information processing in power-law interacting systems. Using quantum state transfer between auxiliary qubits and encoding qubits into large GHZ-like states as subroutines, the disclosed systems and methods leads to optimal implementations of quantum gates between distant qubits in large quantum computers. In particular, the faster encoding of information into a GHZ-like state of ancillary qubits speeds up the implementation of the quantum fanout—a powerful multiquubit quantum gate. At the same time, the faster state transfer speeds up the constructions of multiscale entanglement renormalization ansatz (MERA) states, commonly used to represent highly entangled states, such as topologically ordered states. Specifically, a fanout gate may be implemented on qubits in a hypercube of volume $n$ and prepare a MERA state on these qubits in time $t = \text{polylog}(n)$ for $ae(2d, 2d+1)$.

In practice, using single-site Hamiltonians to implement the echocoupling technique the controlled-PHASE gate in step 130 of the disclosed systems and methods can be realized starting from time-independent power-law interactions between all sites of the system. The method therefore does not require explicit time-dependent control of individual two-qubit Hamiltonians, making it appealing for implementation on available experimental platforms. However, because the diameter of the GHZ-like state increases by more than twofold in every iteration of the method, the scaling in Eq. 3 may only be observed in large systems.

Information-propagation speed limits... Conceptually, since the disclosed systems and methods saturate (up to sub-polynomial corrections) the Lieb-Robinson bounds for...
of smaller a. The Frobenius bound, which considers information propagation from the operator-spreading perspective, constrains information-propagation tasks that are more demanding than the tasks that saturate the Lieb-Robinson bound, and therefore has a more stringent light cone. For example, quantum state transfer given intermediate qubits in arbitrary initial states (i.e., universal state transfer) is constrained by the Frobenius light cone, whereas state transfer assuming initialized intermediate qubits is constrained by the Lieb-Robinson bound and can actually violate the Frobenius light cone. The presently disclosed methods prove for the first time that the task of encoding information into GHZ-like state, which is at least as hard as state transfer with initialization—is not constrained by the Frobenius light cone, but is instead tightly constrained (up to sub-polynomial corrections) by the Lieb-Robinson bound. In particular, when $d < 2d$, the disclosed systems and methods prove that state transfer with initialization can be implemented exponentially faster than state transfer without initialization, which is constrained by polynomial light cones in this regime. Furthermore, since the disclosed systems and methods for encoding into a GHZ-like state can also be used to prepare a known GHZ-like state, the disclosed systems and methods also prove for the first time that preparing a known GHZ-like state is not constrained by the Frobenius light cone.

In aspects, the disclosed method can be generalized to the regime $0 < a < 2d$, where there are still substantial gaps between the Lieb-Robinson bounds and achievable methods. The bounds suggest that, in addition to the distance, the information-propagation time also depends on the total number of sites on the lattice.

The disclosed systems and methods also give the first known example of a lower bound on the gate count in simulating power-law systems on a quantum computer: it takes $\Omega(n)$ elementary quantum gates to simulate an n-qubit power-law system evolving for time $2t_n$, where:

\[
L_n = \begin{cases} 
\Theta(d^{\log^2 d}) & \text{if } d < a < 2d, \\
\Theta(d^{\frac{\log n}{\log \log n}}) & \text{if } a = 2d, \text{ and} \\
\Theta(a^{\frac{d-1}{d-2}}) & \text{if } 2d < a \leq 2d + 1 
\end{cases}
\]  

(23)

(24)

(25)

(26)

(27)

(28)

(29)

(30)

(31)

(32)

(33)

(34)

(35)

(36)
\[ E_0 = \begin{cases} \sqrt{d^{2+2\alpha}} & \text{if } d < a \leq 2d, \\ \sqrt{d^{2-\alpha}} & \text{if } 2d < a \leq 2d + 1 \end{cases} \] (Eqn. 25)

The gap between this state-of-the-art upper bound and the lower bound \( \Omega(n) \) of the disclosed systems and methods hints at the possibility of a more efficient algorithm for simulating power-law systems.

Regarding FIGS. 5A-C, a diagram illustrating a scheme for realizing dipole-dipole Ising interactions using polar molecules is shown. In the state transfer method, there are three types of qubits: control qubits, target qubits, and uninvolved qubits. The uninvolved qubits, which are confined to a single state, can generally be prepared easily by storing them in a state which does not interact with the other two types of qubit. For the control and target qubits, a Hamiltonian with long-range Ising interactions between control and target qubits is used. However, the control qubits must not interact with themselves, and similarly the target qubits must not interact with themselves. However, this behavior does not generally occur and must be engineered.

The spin-echo-like approach is used. This approach relies on the ability to realize a Hamiltonian of the form

\[ H_{\text{Is}} = \frac{1}{4} \sum_{ij} J_{ij} \sigma_i \sigma_j. \] (Eqn. 26)

as well as its negative \(-H_{\text{Is}}\), where \( Z_i \) denotes the Pauli-Z matrix for the i-th qubit. By evolving under \( H_{\text{Is}} \) for time \( t \), applying a Pauli-X gate to either the control or the target qubits, and evolving under \(-H_{\text{Is}}\) for a time \( t/\omega \), the evolution from the control-target interactions remains, as desired. In light of this, most of the following discussion will be centered around how to realize both signs of the Ising interactions. Local control may be achieved in tweezer arrays of Rydberg atoms and polar molecules or by using quantum gas microscopes for polar molecules and dysprosium.

Referring to FIG. 5A, by either dressing rotational states with \( \Omega_{\alpha}(\imath 1, 2, 3) \) or \( \Omega_{\alpha}(\imath 1, 2, 3) \), both positive and negative Ising interactions may be realized. With reference to FIG. 5B, by turning on an electric field, the molecular states gain permanent dipole moments, resulting in positive interactions. Referring to FIG. 5C, by combining the application of an electric field and dressing, negative Ising interactions can be realized. In FIGS. 5B and 5C, N denotes which rotational states the molecular states are adiabatically connected to in E.

The state-transfer procedures may be implemented using dipole-dipole interactions in a system of polar alkali-metal dimers. A negligible tunneling of molecules and a single molecule per lattice site may be assumed. Three different example schemes for realizing the desired interactions are presented. In all three, the electronic and vibrational ground states, can be described in terms of the angular momentum operator N. In the first, both positive and negative Ising interactions can be realized at the cost of involving large rotational states (N=5). In the second, only positive Ising interactions can be realized, but only N=2 rotational states are needed. In the third, only negative Ising interactions can be realized, but only N=1 rotational states are needed. These are shown in FIGS. 5A, 5B, and 5C, respectively.

In the first scheme, the \( |N=2, \alpha\rangle \) and \( |N=3, \alpha\rangle \) states are dressed with no electric field. In order to realize positive interactions, only \( \Omega_{\alpha} \) is turned on, which resonantly drives the \( \alpha \) transitions. Although any pair of states dressed by \( \Omega_{\alpha} \), \( |N=2, M=\pm 2\rangle \) and \( |N=3, M=\pm 2\rangle \) is used as an example. The corresponding dipole-dipole interaction is:

\[ H_{\text{dd}} = \sum_{\langle ij \rangle} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} \langle \mu_i \mu_j \rangle_{\alpha}, \] (Eqn. 27)

where \( r_{ij} \) is the distance between molecules i and j, \( \theta_{ij} \) is the angle between the displacement vector of molecules i and j, and \( \langle \mu_i \mu_j \rangle_{\alpha} \) is the corresponding transition dipole moment, where \( \mu_{ij} = \Sigma_{\alpha} \epsilon_{ij} \alpha \) is a component of the dipole operator \( \hat{\alpha} \) and \( \Sigma_{\alpha} \epsilon_{ij} \alpha = (\Sigma_{\alpha} \Sigma_{\alpha} \epsilon_{ij} \alpha) \sqrt{2} \). If \( \mathbf{m}_0 \parallel \mathbf{0} \) and \( \mathbf{m}_1 \parallel \mathbf{1} \), then the dipole-dipole interactions in the \( |\mathbf{m}_0 \rangle \parallel \mathbf{m}_1 \rangle \) basis becomes:

\[ H_{\text{dd}} = \sum_{\langle ij \rangle} \frac{\mu_i^2 + \mu_j^2}{4} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} \langle 1 - Z_i 1 - Z_j \rangle, \] (Eqn. 28)

thus realizing the desired positive Ising interactions. Note that in addition to the Ising interactions, the above Hamiltonian also has terms proportional to Z. However, this is not an issue, as upon applying the spin-echo procedure using \(-H_{\text{Is}}\), these terms will disappear. Additionally, because \( N=2,3 \) is used rather than \( N=1,2 \), there are no interactions associated with \( \mathbf{m}_0 \parallel \mathbf{0} \) due to selection rules. Finally, note that for the above interactions to hold, a strong drive \( \Omega_{\alpha} > \hbar \Delta \) must be used.

To realize the negative interactions, only \( \Omega_{\alpha} \) is turned on, driving the \( \sigma_- \) transitions. Considering only the states \( |1, 2, 3 \rangle \parallel 1, 3, 2 \rangle \), the corresponding dipole-dipole interactions take the form:

\[ H_{\text{dd}} = \sum_{\langle ij \rangle} \frac{\mu_i^2 + \mu_j^2}{2} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} \langle 1 - Z_i 1 - Z_j \rangle, \] (Eqn. 29)

where \( \mu_{ij} = (1, 2, 3) \parallel (1, 3, 2) \) is the corresponding transition dipole moment. In this case, \( \mathbf{m}_0 \parallel (1, 2, 3) \parallel 1, 3, 2 \rangle \), and \( \mathbf{m}_1 \parallel 0 \parallel 0 \) are encoded, leading to the resulting interaction Hamiltonian in the \( |\mathbf{m}_0 \rangle \parallel |\mathbf{m}_1 \rangle \) basis:

\[ H_{\text{dd}} = \sum_{\langle ij \rangle} \frac{\mu_i^2 + \mu_j^2}{8} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} (1 - Z_i)(1 - Z_j), \] (Eqn. 30)

thus realizing the desired negative Ising interactions.

In the second scheme, an electric field may be applied. Defining \( \psi_{\alpha} \parallel 0 \parallel \mathbf{E} \) as the states which are adiabatically connected (as E is turned on) to \( |N, M \parallel |\mathbf{m}_0 \rangle = |\psi_{\alpha} \parallel \mathbf{E} \rangle \) and \( |\mathbf{m}_1 \rangle \parallel 0 \parallel \mathbf{E} \) are encoded. Due to the electric field E, these two states gain permanent dipole moments \( \mu_{\alpha} \) and \( \mu_{\alpha} \). Additionally, because the electric field does not couple 1,2
[0108] providing positive Ising interactions. Note that negative interactions cannot be achieved in this way, so another scheme must be used in combination with this approach.

[0109] In addition to an electric field \( E \), the states \( |0\rangle, |p\rangle \) and \( |0\rangle, |\bar{p}\rangle \) are dressed, encoding \( |m\rangle = |\bar{p}\rangle |0\rangle + \sqrt{1 - \lambda} |\bar{p}\rangle |\bar{p}\rangle \) and \( |m\rangle = |0\rangle |0\rangle + \sqrt{1 - \lambda} |0\rangle |\bar{p}\rangle \), where \( \lambda \) is defined by \( \Omega, \Delta \). For any electric field strength \( E \), the value of a can always be tuned so that there are only negative Ising interactions. Unlike the second scheme, positive Ising interactions can be achieved, so by combining the two schemes, one may avoid the complications associated with using high N in the first scheme.

[0110] Thus, by transferring the \( |m\rangle, |\bar{p}\rangle \) population to the \( |m\rangle, |\bar{p}\rangle \) state and changing which of the above schemes is used, it is possible to implement the state transfer method via polar molecules.

[0111] In aspects, the state transfer method may be implemented using Rydberg atoms, which will share several similarities with the disclosed implementation in polar molecules. In order to do so, \( |m\rangle, |\bar{p}\rangle \) is encoded in linear combinations of four different states \( |s\rangle, |\bar{s}\rangle, |l\rangle, |\bar{l}\rangle \) corresponding to a ground-state, a Rydberg state, and two Rydberg p states. In the case of the latter two states, \( \theta \) is the angle between the polarization of the drive needed to couple the s state to the corresponding p state. Additionally, the p states may have similar values of the principal quantum number \( n \) compared to the s state, leading to strong dipole-dipole interactions between s and p states. These states are illustrated in FIGS. 6A and 6B. The disclosed systems and methods enable much stronger interactions, providing faster state transfer with fewer errors due to dissipation.

[0112] In order to realize \( H_{dd} \), the \( |s\rangle, |\bar{s}\rangle \) state may be strongly dressed with the \( |l\rangle, |\bar{l}\rangle \) state with a strong resonant microwave field. In the absence of any external fields, these two states interact according to the dipole-dipole interaction

\[
H_{dd} = \sum_{i,j} \mu_{ij} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} \langle |l\rangle, |\bar{l}\rangle |s_i, l_j\rangle \langle s_i, l_j | |l\rangle, |\bar{l}\rangle \rangle.
\]

[0113] where \( r_{ij} \) is the distance between atoms i and j, \( \theta_{ij} \) is the angle the displacement vector makes with the quantization axis, and \( \mu_{ij} = e \cdot d_{ij} \) is the corresponding transition dipole moment, where \( d_{ij} = \bar{d} \) is a component of the dipole operator \( \bar{d} \). Using the strong resonant microwave field, \( |m\rangle, |\bar{m}\rangle \) are encoded in \( |s\rangle, |\bar{s}\rangle \) \( |l\rangle, |\bar{l}\rangle \) \( |l\rangle, |\bar{l}\rangle \). In the limit of a strong microwave Rabi frequency \( \Omega_{m} \), the dipole-dipole interactions in the \( |m\rangle, |\bar{m}\rangle \) basis may be expressed as

\[
H_{dd} = \sum_{i,j} \mu_{ij} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} \langle |l\rangle, |\bar{l}\rangle |s_i, l_j\rangle \langle s_i, l_j | |l\rangle, |\bar{l}\rangle \rangle.
\]

[0114] In order to realize \( -H_{dd} \), the same approach as for \( H_{dd} \) may be used. However, rather than dressing the \( |p\rangle \) state with \( |\bar{p}\rangle \), the \( |s\rangle \) state is resonantly dressed with \( |p\rangle \). In this case, the corresponding dipole-dipole interaction is

\[
H_{dd} = \sum_{i,j} \frac{1 - 3 \cos^2 \theta}{2r_{ij}^3} |l, p_i\rangle \langle p_j, l_j | |l, p_i\rangle \langle p_j, l_j |.
\]

[0115] where \( \mu_{ij} = |l, p_i\rangle \langle p_j, l_j | \). Note that, aside from a constant factor, the dipole-dipole interaction here only differs by a sign from \( H_{dd} \). As such, if \( |m\rangle, |\bar{p}\rangle = |s\rangle + |p\rangle \) \( \sqrt{2} \) is encoded and again the limit of a strong microwave Rabi frequency \( \Omega_{m} \), \( H_{dd} \) is taken, the resulting interactions are

\[
H_{dd} = -\sum_{i,j} \frac{\mu_{ij}^2}{8} \frac{1 - 3 \cos^2 \theta}{r_{ij}^3} (1 - Z_i)(1 - Z_j).
\]

[0116] which differs from \( H_{dd} \) by a negative constant. Thus, by transferring the \( |m\rangle, |\bar{p}\rangle \) population to the \( |m\rangle, |\bar{p}\rangle \) state and changing the polarization of the microwave field, it is possible to implement the state transfer method via Rydberg atoms. Note that when the polarization of the drive is changed, one must ensure that the qubit state is temporarily not encoded in \( |s\rangle, |p\rangle \) \( |\bar{s}\rangle, |\bar{p}\rangle \).

[0117] Using similar principles to the above, it is also possible to realize the desired control-target interactions without the spin-echo approach. This is achieved by turning on both drives \( \Omega_{dd} \), with detunings \( \Delta_{dd} \), and using different principal quantum numbers \( n \) for \( |l\rangle, |\bar{l}\rangle \). With a proper choice of parameters, two of the resulting dressed states \( |s\rangle, |\bar{s}\rangle \) \( |l\rangle, |\bar{l}\rangle \) interact with each other but not themselves. Thus, by forming the control qubits with \( |l\rangle \) \( |\bar{l}\rangle \) and the target qubits with \( |s\rangle \) \( |\bar{s}\rangle \), the desired pure control-target interactions can be realized, although a spin-echo sequence must be applied to all qubits to eliminate an effective longitudinal field. The details of engineering this form of interactions can be generalized to polar molecules and magnetic atoms as well.

[0118] FIGS. 6A and 6B, illustrate a dressing scheme for realizing (FIG. 6A) \( H_{dd} \) and (FIG. 6B) \( H_{dd} \) via microwave dressing, with \( |m\rangle, |\bar{m}\rangle = |s\rangle + |p\rangle \) \( \sqrt{2} \) and \( |l\rangle, |\bar{l}\rangle = |s\rangle + |p\rangle \) \( \sqrt{2} \). In each case, \( \Omega_{dd} \) may be used so that the microwave-dressed basis may be used.

[0119] In aspects, \( 1/e^6 \) vdw interactions may be used for state transfer in 3D. While this will not be as fast as using \( 1/r^6 \) dipole-dipole interactions, it provides the benefit of being less complicated since the vdw interactions will have no significant angular dependence and there will be no microwave drive necessary. Moreover, it provides a natural means of experimentally demonstrating that vdw interactions are long-range in three dimensions.

[0120] One option to realize the desired sign change in the vdw interactions is by inducing a Förster resonance for a s state via an external magnetic field. Provided one keeps the atoms sufficiently far apart outside of the dipole-dipole interaction regime, where the interaction causes pairs of s states to hybridize with pairs of p states, the sign of \( C_G \) will be opposite for either state of the Förster resonance.

[0121] Alternatively, there is another approach which does not require the use of external fields. Here, \( |m\rangle, |\bar{m}\rangle = |s\rangle, |\bar{s}\rangle \).
where $C_{60}$, $C_{6}$, denote the vdw interaction coefficients for the interaction between the $s$ states, the $s'$ interactions, and the cross interaction between $s$ and $s'$. The resulting interactions for $|m_{0}\{s\}| |m_{1}\{s'\}$ takes the form:

$$H_{\text{int}} = -\frac{1}{4} \sum_{s,s'} \frac{C_{60}}{\rho_{6}} (1 - Z_{s})(1 - Z_{s'})$$  \hspace{1cm} (Eqn. 37)

naturally realizing the desired Ising interactions. In contrast, the resulting interactions for $|m_{0}\{s\}| |m_{1}\{s'\}$ are:

$$H'_{\text{int}} = -\frac{1}{4} \sum_{s,s'} \frac{C_{60}}{\rho_{6}} (1 - Z_{s})(1 - Z_{s'}) +$$

$$\frac{C_{6}}{\rho_{6}} (1 - Z_{s})(1 + Z_{s'}) + \frac{C_{6}}{\rho_{6}} (1 - Z_{s'})(1 + Z_{s}) + \frac{C_{6}}{\rho_{6}} (1 + Z_{s})(1 + Z_{s'})$$  \hspace{1cm} (Eqn. 38)

In this case, the resulting Ising interactions have strength $-(C_{60}/4 + C_{6}/4 - C_{6}/2)\rho_{6}$. Thus, in order to ensure that $H'_{\text{int}}$ differs by a negative sign from $H_{\text{int}}$, then $C_{60} + C_{6} - 2C_{6} < 0$, which is achievable for reasonable choices of Rydberg states. One minor complication arises via the fact that the coefficients of the $Z$ terms relative to the Ising interactions are not the same for each Rydberg. However, this is easily remedied by applying an X gate to all atoms at once as in the usual spin-echo protocol. Since it is applied to all atoms, it will not affect the Ising interactions themselves, but it will reverse the evolution due to the $Z$ terms, allowing one to ensure that their contribution is 0 at the end of the evolution.

FIG. 7 illustrates the effective van der Walls (vdW) interactions between controls and targets for negative Ising interactions, normalized to the case of positive interactions. The Rydberg states used are $n$ and $n+1$.

In aspects, the state transfer method may be implemented in a two-dimensional lattice of ultracold dysprosium atoms, in particular, bosonic isotopes of dysprosium (such as $^{162}$Dy, $^{164}$Dy). The nuclear spin may be ignored. The computation states $m_{0}$, $m_{1}$ in the Zeeman manifold of the electronic ground state of the dysprosium may be encoded. The interactions between a pair of dysprosium atoms $i$, $j$ are given by magnetic dipole-dipole interactions (in units of $\mu_{0}\langle j_{x}j_{y}\rangle/4\pi$):

$$H_{ij} = -\frac{1}{\rho_{6}} (1 - 3\cos^{2}(\theta)) \left[J_{ij} - \frac{1}{4}(J_{i}^{x}J_{j}^{x} + J_{i}^{y}J_{j}^{y})\right]$$  \hspace{1cm} (Eqn. 38)

[0128] Here, $J = 8$ is the electronic angular momentum of the ground state, and $\theta, \phi$ are the polar and azimuthal angles with respect to the quantization axis (which is set by a magnetic field which also breaks the degeneracy of the Zeeman levels $M_{s}$, $-8 \leq M_{s} \leq 8$).

[0129] The simplest way of realizing the diagonal interactions that are used is to make use of the $J_{x}J_{y}$ term in Eqn. 38. For instance, the choice $m_{0} = -8$, $m_{1} = 8$ yields the interaction

$$H_{\text{int}} = \sum_{s,s'} \frac{1}{\rho_{6}} (1 - 3\cos^{2}(\theta)) Z_{s}Z_{s'}$$

up to potential longitudinal terms ($Z_{s}$, $Z_{s'}$) depending on the energies of the two states $m_{0}$, $m_{1}$.

[0130] In order to realize the reverse Hamiltonian, i.e., the same Hamiltonian but with opposite sign, the flip-flop interactions in Eqn. 38 may be used. Using microwave and optical dressing, for example, the following dressed states may be selected: $m_{0} = (4 - 2i) \sqrt{2}$, and $m_{1} = 8 + 7c + 8$, where $a$, $b$, $c$ amplitudes are tunable via the dressing parameters. This choice yields $m_{0}m_{1} = 0$ and by properly tuning $a$, $b$, $c$ that $m_{i}J_{m_{i}} = 0$. In this case, only the flip-flop interactions from Eqn. 38 contribute and, in fact, the only nonzero matrix element is $m_{0}m_{1}J_{m_{0}m_{1}}$, thus realizing the same $Z_{s}Z_{s'}$ interactions, as before, but with the opposite sign. In general, this would also give rise to longitudinal terms that depend on both the interactions strength and the energy splitting of the states. These can be dealt with the spin echo procedure as explained previously.

[0131] Certain embeddings of the present disclosure may include some, all, or none of the above advantages and/or one or more other advantages readily apparent to those skilled in the art from the drawings, descriptions, and claims included herein. Moreover, while specific advantages have been enumerated above, the various embeddings of the present disclosure may include all, some, or none of the enumerated advantages and/or other advantages not specifically enumerated above.

[0132] The embodiments disclosed herein are examples of the disclosure and may be embodied in various forms. For instance, although certain embodiments herein are described as separate embodiments, each of the embodiments herein may be combined with one or more of the other embodiments herein. Specific structural and functional details disclosed herein are not to be interpreted as limiting, but as a basis for the claims and as a representative basis for teaching one skilled in the art to variously employ the present disclosure in virtually any appropriately detailed structure. Like reference numerals may refer to similar or identical elements throughout the description of the figures.

[0133] The phrases “in an embodiment,” “in embodiments,” “in various embodiments,” “in some embodiments,” or “in other embodiments” may each refer to one or more of the same or different example embodiments provided in the present disclosure. A phrase in the form “A or B” means “A, (B), or (A and B).” A phrase in the form “at least one
of A, B, or C” means “(A); (B); (C); (A and B); (A and C); (B and C); or (A, B, and C).”

[0134] It should be understood that the foregoing description is only illustrative of the present disclosure. Various alternatives and modifications can be devised by those skilled in the art without departing from the disclosure. Accordingly, the present disclosure is intended to embrace all such alternatives, modifications, and variances. The embodiments described with reference to the attached drawing figures are presented only to demonstrate certain examples of the disclosure. Other elements, steps, methods, and techniques that are insubstantially different from those described above and/or in the appended claims are also intended to be within the scope of the disclosure.

What is claimed is:

1. A system for quantum state transfer and entanglement generation, the system comprising:
a quantum system including a plurality of qubits;
a processor; and
a memory, including instructions stored thereon, which, when executed by the processor, cause the quantum system to:
access a signal of the quantum system;
encode unknown coefficients in one qubit of the plurality of qubits;
initialize each of the remaining qubits of the plurality of qubits in state $|0\rangle$;
group the plurality of qubits into a plurality of subsystems;
in each of the plurality of subsystems: encode quantum information into Greenberger-Horne-Zeilinger-like (GHZ-like) states using nearest-neighbor interactions; and
apply a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems.

2. The system of claim 1, wherein the instructions, when executed by the processor, cause the quantum system to rotate the entangled state into a desired GHZ-like state by concentrating an entanglement in each subsystem of the plurality of subsystems onto one qubit $c_i$ of each of the plurality of subsystems.

3. The system of claim 2, wherein the instructions, when executed by the processor, cause the quantum system to apply a single-qubit rotation to $c_i$.

4. The system of claim 3, wherein the instructions, when executed by the processor, cause the quantum system to redistribute the entanglement to the plurality of subsystems as a resulting GHZ-like state.

5. The system of claim 4, wherein the instructions, when executed by the processor, cause the quantum system to repeatedly feed the resulting GHZ-like state back into applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems to yield larger GHZ-like states.

6. The system of claim 3, wherein the system further includes a Hadamard gate, and wherein the rotation of $c_i$ is applied using the Hadamard gate.

7. The system of claim 1, wherein the subsystems include hypercubes.

8. The system of claim 1, wherein the unknown coefficients include $a$, $b$.

9. The system of claim 8, wherein the unknown coefficients of the one qubit of the plurality of qubits is encoded as $a|0\rangle + b|1\rangle$.

10. The system of claim 1, wherein the quantum system includes a power-law interacting Hamiltonian.

11. A method for quantum state transfer and entanglement generation, the method comprising:
accessing a signal of a quantum system, the quantum system includes a plurality of qubits;
encoding unknown coefficients in one qubit of the plurality of qubits;
initializing each of the remaining qubits of the plurality of qubits in state $|0\rangle$;
grouping the plurality of qubits into a plurality of subsystems;
in each of the plurality of subsystems: encoding quantum information into Greenberger-Horne-Zeilinger-like (GHZ-like) states using nearest-neighbor interactions; and
applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems.

12. The method of claim 11, further comprising rotating the entangled state into a desired GHZ-like state by concentrating an entanglement in each subsystem of the plurality of subsystems onto one qubit $c_i$ of each of the plurality of subsystems.

13. The method of claim 12, further comprising applying a single-qubit rotation to $c_i$.

14. The method of claim 13, further comprising redistributing the entanglement to the plurality of subsystems as a resulting GHZ-like state.

15. The method of claim 14, further comprising repeatedly feeding the resulting GHZ-like state back into applying a generalized controlled-phase gate between the plurality of subsystems to merge the GHZ-like states into an entangled state between the plurality of subsystems to yield larger GHZ-like states.

16. The method of claim 11, wherein the subsystems include hypercubes.

17. The method of claim 11, wherein the unknown coefficients include $a$, $b$.

18. The method of claim 17, wherein the unknown coefficients of the one qubit of the plurality of qubits is encoded as $a|0\rangle + b|1\rangle$.

19. The method of claim 11, wherein the rotation of $c_i$ is applied using a Hadamard gate.

20. A method for quantum state transfer and entanglement generation, the method comprising:
accessing a signal of a quantum system, the quantum system includes a plurality of qubits;
encoding $a|0\rangle + b|1\rangle$ in one qubit of the plurality of qubits;
initializing each of the remaining qubits of the plurality of qubits in state $|0\rangle$;
grouping the plurality of qubits into a plurality of hypercubes;
in each of the hypercubes of the plurality of hypercubes encoding quantum information into Greenberger-Horne-Zeilinger-like (GHZ-like) states;
merging the GHZ-like states into an entangled state between the plurality of hypercubes;
rotating the entangled state into a desired GHZ-like state;
applying a single-qubit rotation to one bit of one of the plurality of hypercubes; and redistributing the entanglement to the plurality of hypercubes.