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Table 9.1 Use of Range to Estimate Standard Deviation

Number of Sets of Replicates k	Factor Degrees of Freedom	Number of Replicates in a Set				
		2	3	4	5	6
1	d_2^* v	1.41	1.91	2.24	2.48	2.67
		1.00	1.98	2.93	3.83	4.68
3	d_2^* v	1.23	1.77	2.12	2.38	2.58
		2.83	5.86	8.44	11.1	13.6
5	d_2^* v	1.19	1.74	2.10	2.36	2.56
		4.59	9.31	13.9	18.4	22.6
10	d_2^* v	1.16	1.72	2.08	2.34	2.55
		8.99	18.4	27.6	36.5	44.9
15	d_2^* v	1.15	1.71	2.07	2.34	2.54
		13.4	27.5	41.3	54.6	67.2
20	d_2^* v	1.14	1.70	2.07	2.33	2.54
		17.8	36.5	55.0	72.7	89.6
∞	d_2^*	1.13	1.69	2.06	2.33	2.53
$s_p = \frac{\bar{R}}{d_2^*}$						

Intermediate values for d_2^* and v may be obtained by interpolation, or from the reference from which this table was adapted. Example: When using a Range chart for volume calibrations, based on two runs for each unknown test item, the d_2^* value is 1.41. If testing 10 points on a tape measure, using 3 runs, the average range value is calculated for all points and d_2^* is 1.72.

Adapted from Lloyd S. Nelson, J. Qual. Tech. 7 No. 1, January 1975. ©American Society for Quality Control, Reprinted by permission.

**Table 9.2 Z-Factors for Two-sided Confidence Intervals
for the Normal Distribution**

Confidence Level (%)	Z Factor
50	0.68
67	1.00
75	1.15
90	1.645
95	1.960
95.28	2.000
99.00	2.575
99.74	3
99.993 4	4
99.999 95	5
100, 10^{-9}	6
100, 10^{-12}	7
100, 10^{-15}	8
100, $10^{-18.9}$	9
100, 10^{-23}	10

Table 9.3 Student t-Variate

*	80 %	90 %	95 %	98 %	99 %	99.73 %
<i>d.f.</i>	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$	$t_{0.9985}$
1	3.078	6.314	12.706	31.821	63.657	235.80
2	1.886	2.920	4.303	6.965	9.925	19.207
3	1.638	2.353	3.182	4.541	5.841	9.219
4	1.533	2.132	2.776	3.747	4.604	6.620
5	1.476	2.015	2.571	3.365	4.032	5.507
6	1.440	1.943	2.447	3.143	3.707	4.904
7	1.415	1.895	2.365	2.998	3.499	4.530
8	1.397	1.860	2.306	2.896	3.355	4.277
9	1.383	1.833	2.262	2.821	3.250	4.094
10	1.372	1.812	2.228	2.764	3.169	3.957
11	1.363	1.796	2.201	2.718	3.106	3.850
12	1.356	1.782	2.179	2.681	3.055	3.764
13	1.350	1.771	2.160	2.650	3.012	3.694
14	1.345	1.761	2.145	2.624	2.977	3.636
15	1.341	1.753	2.131	2.602	2.947	3.586
16	1.337	1.746	2.120	2.583	2.921	3.544
17	1.333	1.740	2.110	2.567	2.898	3.507
18	1.330	1.734	2.101	2.552	2.878	3.475
19	1.328	1.729	2.093	2.539	2.861	3.447
20	1.325	1.725	2.086	2.528	2.845	3.422
25	1.316	1.708	2.060	2.485	2.787	3.330
30	1.310	1.697	2.042	2.457	2.750	3.270
40	1.303	1.684	2.021	2.423	2.704	3.199
60	1.296	1.671	2.000	2.390	2.660	3.130
∞	1.282	1.645	1.960	2.326	2.576	3.00

* Columns to be used in calculating corresponding two-sided confidence interval. From: NIST Handbook 91 p. T-5; Last column from B.J. Joiner, J. Research NIST. (for Two-Sample t-Test for Equal Means)

Table 9.4 Critical values for the F-Test, $F_{0.975}$

n_1 = degrees of freedom for numerator n_2 = degrees of freedom for denominator

$n_1 \backslash n_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18
6	8.81	7.26	6.60	6.23	5.99	5.83	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48

For use for a one-tailed test of equality of standard deviation estimate at 2.5 % level of confidence, or for a two-tailed test at 5 % level of confidence.

Table 9.5 Critical values for the F-Test, $F_{0.99}$

v_1 = degrees of freedom for numerator v_2 = degrees of freedom for denominator

$v_1 \backslash v_2$	1	2	3	4	5	6	7	8	9	10
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
65	7.04	4.95	4.10	3.62	3.31	3.09	2.93	2.80	2.69	2.61
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59
75	6.99	4.90	4.05	3.58	3.27	3.05	2.89	2.76	2.65	2.57
80	6.96	4.88	4.04	3.56	3.25	3.04	2.87	2.74	2.64	2.55
85	6.94	4.86	4.02	3.55	3.24	3.02	2.86	2.73	2.62	2.54
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52
95	6.91	4.84	3.99	3.52	3.22	3.00	2.83	2.70	2.60	2.51
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50
105	6.88	4.81	3.97	3.50	3.20	2.98	2.81	2.69	2.58	2.49
110	6.87	4.80	3.96	3.49	3.19	2.97	2.81	2.68	2.57	2.49
115	6.86	4.79	3.96	3.49	3.18	2.96	2.80	2.67	2.57	2.48
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

For use for a one-tailed test of equality of standard deviation estimate at 1 % level of confidence, or for a two-tailed test at 2 % level of confidence.

**Table 9.6 Factors for Two-sided Tolerance Intervals
for the Normal Distribution**

n \ P	= 0.95					= 0.99				
	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
2	22.858	32.019	37.674	48.430	60.573	114.363	160.193	188.491	242.300	303.054
3	5.922	8.380	9.916	12.861	16.208	13.378	18.930	22.401	29.055	36.616
4	3.779	5.369	6.370	8.299	10.502	6.614	9.398	11.150	14.527	18.383
5	3.002	4.275	5.079	6.634	8.415	4.643	6.612	7.855	10.260	13.015
6	2.604	3.712	4.414	5.775	7.337	3.743	5.337	6.345	8.301	10.548
7	2.361	3.369	4.007	5.248	6.676	3.233	4.613	5.488	7.187	9.142
8	2.197	3.136	3.732	4.891	6.226	2.905	4.147	4.936	6.468	8.234
9	2.078	2.967	3.532	4.631	5.899	2.677	3.822	4.550	5.966	7.600
10	1.987	2.839	3.379	4.433	5.649	2.508	3.582	4.265	5.594	7.129
11	1.916	2.737	3.259	4.277	5.452	2.378	3.397	4.045	5.308	6.766
12	1.858	2.655	3.162	4.150	5.291	2.274	3.250	3.870	5.079	6.477
13	1.810	2.587	3.081	4.044	5.158	2.190	3.130	3.727	4.893	6.240
14	1.770	2.529	3.012	3.955	5.045	2.120	3.029	3.608	4.737	6.043
15	1.735	2.480	2.954	3.878	4.949	2.060	2.945	3.507	4.605	5.876
16	1.705	2.437	2.903	3.812	4.865	2.009	2.872	3.421	4.492	5.732
17	1.679	2.400	2.858	3.754	4.791	1.965	2.808	3.345	4.393	5.607
18	1.655	2.366	2.819	3.702	4.725	1.926	2.753	3.279	4.307	5.497
19	1.635	2.337	2.784	3.656	4.667	1.891	2.703	3.221	4.230	5.399
20	1.616	2.310	2.752	3.612	4.614	1.860	2.659	3.168	4.161	5.312
21	1.599	2.286	2.723	3.577	4.567	1.833	2.620	3.121	4.100	5.234
22	1.584	2.264	2.697	3.543	4.523	1.808	2.584	3.078	4.044	5.163
23	1.570	2.244	2.673	3.512	4.484	1.785	2.551	3.040	3.993	5.098
24	1.557	2.225	2.651	3.483	4.447	1.764	2.522	3.004	3.947	5.039
25	1.545	2.208	2.631	3.457	4.413	1.745	2.494	2.972	3.904	4.985
26	1.534	2.193	2.612	3.432	4.382	1.727	2.469	2.941	3.865	4.935
27	1.523	2.178	2.595	3.409	4.353	1.711	2.446	2.914	3.828	4.888

From: NBS Handbook 91, p T-11

Table 9.7 Factors for Computing Two-sided Confidence Limits for α (Part 1)

Degrees of Freedom	$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.001$	
	S_U	S_L	S_U	S_L	S_U	S_L
1	17.79	0.3576	86.31	0.2969	844.4	0.2480
2	4.859	0.4581	10.70	0.3879	33.29	0.3291
3	3.183	0.5178	5.449	0.4453	11.65	0.3824
4	2.567	0.5590	3.892	0.4865	6.938	0.4218
5	2.248	0.5899	3.175	0.5182	5.085	0.4529
6	2.052	0.6143	2.764	0.5437	4.128	0.4784
7	1.918	0.6344	2.498	0.5650	3.551	0.5000
8	1.820	0.6513	2.311	0.5830	3.167	0.5186
9	1.746	0.6657	2.173	0.5987	2.894	0.5348
10	1.686	0.6784	2.065	0.6125	2.689	0.5492
11	1.638	0.6896	1.980	0.6248	2.530	0.5621
12	1.598	0.6995	1.909	0.6358	2.402	0.5738
13	1.564	0.7084	1.851	0.6458	2.298	0.5845
14	1.534	0.7166	1.801	0.6549	2.210	0.5942
15	1.509	0.7240	1.758	0.6632	2.136	0.6032
16	1.486	0.7808	1.721	0.6710	2.073	0.6116
17	1.466	0.7372	1.688	0.6781	2.017	0.6193
18	1.448	0.7430	1.658	0.6848	1.968	0.6266
19	1.432	0.7484	1.632	0.6909	1.925	0.6333
20	1.417	0.7535	1.609	0.6968	1.886	0.6397
21	1.404	0.7582	1.587	0.7022	1.851	0.6457
22	1.391	0.7627	1.568	0.7074	1.820	0.6514
23	1.380	0.7669	1.550	0.7122	1.791	0.6568
24	1.370	0.7709	1.533	0.7169	1.765	0.6619
25	1.360	0.7747	1.518	0.7212	1.741	0.6668

From: NBS Handbook 91, p T-34

Table 9.7 Factors for Computing Two-sided Confidence Limits for α (Part 2)

Degrees of Freedom	$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.001$	
	S _U	S _L	S _U	S _L	S _U	S _L
26	1.351	0.7783	1.504	0.7253	1.719	0.7130
27	1.343	0.7817	1.491	0.7293	1.698	0.6758
28	1.335	0.7849	1.479	0.7331	1.679	0.6800
29	1.327	0.7880	1.467	0.7367	1.661	0.6841
30	1.321	0.7909	1.457	0.7401	1.645	0.6880
31	1.314	0.7937	1.447	0.7434	1.629	0.6917
32	1.308	0.7964	1.437	0.7467	1.615	0.6953
33	1.302	0.7990	1.428	0.7497	1.601	0.6987
34	1.296	0.8015	1.420	0.7526	1.588	0.7020
35	1.291	0.8089	1.412	0.7554	1.576	0.7052
36	1.286	0.8062	1.404	0.7582	1.564	0.7083
37	1.281	0.8085	1.397	0.7608	1.553	0.7113
38	1.277	0.8106	1.390	0.7633	1.543	0.7141
39	1.272	0.8126	1.383	0.7658	1.533	0.7169
40	1.268	0.8146	1.377	0.7681	1.523	0.7197
41	1.264	0.8166	1.371	0.7705	1.515	0.7223
42	1.260	0.8184	1.365	0.7727	1.506	0.7248
43	1.257	0.8202	1.360	0.7748	1.498	0.7273
44	1.253	0.8220	1.355	0.7769	1.490	0.7297
45	1.249	0.8237	1.349	0.7789	1.482	0.7320
46	1.246	0.8253	1.345	0.7809	1.475	0.7342
47	1.243	0.8269	1.340	0.7828	1.468	0.7364
48	1.240	0.8285	1.335	0.7847	1.462	0.7386
49	1.237	0.8300	1.331	0.7864	1.455	0.7407
50	1.234	0.8314	1.327	0.7882	1.449	0.7427

From: NBS Handbook 91, p. T-34

Table 9.8 Density of Air-free Water in g/cm³ as a Function of Celsius Temperature

T	0.0 °C	0.1 °C	0.2 °C	0.3 °C	0.4 °C	0.5 °C	0.6 °C	0.7 °C	0.8 °C	0.9 °C
0.0	0.999 840	0.999 846	0.999 853	0.999 859	0.999 865	0.999 871	0.999 877	0.999 883	0.999 888	0.999 893
1.0	0.999 899	0.999 903	0.999 908	0.999 913	0.999 917	0.999 921	0.999 925	0.999 929	0.999 933	0.999 937
2.0	0.999 940	0.999 943	0.999 946	0.999 949	0.999 952	0.999 954	0.999 956	0.999 959	0.999 961	0.999 963
3.0	0.999 964	0.999 966	0.999 967	0.999 968	0.999 969	0.999 970	0.999 971	0.999 971	0.999 972	0.999 972
4.0	0.999 972	0.999 972	0.999 972	0.999 971	0.999 971	0.999 970	0.999 969	0.999 968	0.999 967	0.999 965
5.0	0.999 964	0.999 962	0.999 960	0.999 958	0.999 956	0.999 954	0.999 951	0.999 949	0.999 946	0.999 943
6.0	0.999 940	0.999 937	0.999 933	0.999 930	0.999 926	0.999 922	0.999 918	0.999 914	0.999 910	0.999 906
7.0	0.999 901	0.999 896	0.999 892	0.999 887	0.999 881	0.999 876	0.999 871	0.999 865	0.999 860	0.999 854
8.0	0.999 848	0.999 842	0.999 835	0.999 829	0.999 822	0.999 816	0.999 809	0.999 802	0.999 795	0.999 787
9.0	0.999 780	0.999 773	0.999 765	0.999 757	0.999 749	0.999 741	0.999 733	0.999 725	0.999 716	0.999 707
10.0	0.999 699	0.999 690	0.999 681	0.999 672	0.999 662	0.999 653	0.999 643	0.999 634	0.999 624	0.999 614
11.0	0.999 604	0.999 594	0.999 583	0.999 573	0.999 562	0.999 552	0.999 541	0.999 530	0.999 519	0.999 507
12.0	0.999 496	0.999 485	0.999 473	0.999 461	0.999 449	0.999 437	0.999 425	0.999 413	0.999 401	0.999 388
13.0	0.999 376	0.999 363	0.999 350	0.999 337	0.999 324	0.999 311	0.999 297	0.999 284	0.999 270	0.999 256
14.0	0.999 243	0.999 229	0.999 215	0.999 200	0.999 186	0.999 172	0.999 157	0.999 142	0.999 128	0.999 113
15.0	0.999 098	0.999 083	0.999 067	0.999 052	0.999 036	0.999 021	0.999 005	0.998 989	0.998 973	0.998 957
16.0	0.998 941	0.998 925	0.998 908	0.998 892	0.998 875	0.998 858	0.998 841	0.998 824	0.998 807	0.997 890
17.0	0.998 773	0.998 755	0.998 738	0.998 720	0.998 702	0.998 684	0.998 666	0.998 648	0.998 630	0.998 612
18.0	0.998 593	0.998 575	0.998 556	0.998 537	0.998 519	0.998 500	0.998 480	0.998 461	0.998 442	0.998 422
19.0	0.998 403	0.998 383	0.998 364	0.998 344	0.998 324	0.998 304	0.998 284	0.998 263	0.998 243	0.998 222
20.0	0.998 202	0.998 181	0.998 160	0.998 139	0.998 118	0.998 097	0.998 076	0.998 055	0.998 033	0.998 012
21.0	0.997 990	0.997 968	0.997 947	0.997 925	0.997 903	0.997 881	0.997 858	0.997 836	0.997 814	0.997 791
22.0	0.997 768	0.997 746	0.997 723	0.997 700	0.997 677	0.997 654	0.997 630	0.997 607	0.997 584	0.997 560
23.0	0.997 536	0.997 513	0.997 489	0.997 465	0.997 441	0.997 417	0.997 392	0.997 368	0.997 344	0.997 319
24.0	0.997 294	0.997 270	0.997 245	0.997 220	0.997 195	0.997 170	0.997 145	0.997 119	0.997 094	0.997 068
25.0	0.997 043	0.997 017	0.996 991	0.996 966	0.996 940	0.996 913	0.996 887	0.996 861	0.996 835	0.996 808
26.0	0.996 782	0.996 755	0.996 728	0.996 702	0.996 675	0.996 648	0.996 621	0.996 593	0.996 566	0.996 539
27.0	0.996 511	0.996 484	0.996 456	0.996 428	0.996 401	0.996 373	0.996 345	0.996 316	0.996 288	0.996 260
28.0	0.996 232	0.996 203	0.996 175	0.996 146	0.996 117	0.996 088	0.996 060	0.996 031	0.996 001	0.995 972
29.0	0.995 943	0.995 914	0.995 884	0.995 855	0.995 825	0.995 795	0.995 765	0.995 736	0.995 706	0.995 676
30.0	0.995 645	0.995 615	0.995 585	0.995 554	0.995 524	0.995 493	0.995 463	0.995 432	0.995 401	0.995 370
31.0	0.995 339	0.995 308	0.995 277	0.995 246	0.995 214	0.995 183	0.995 151	0.995 120	0.995 088	0.995 056
32.0	0.995 024	0.994 992	0.994 960	0.994 928	0.994 896	0.994 864	0.994 831	0.994 799	0.994 766	0.994 734
33.0	0.994 701	0.994 668	0.994 635	0.994 602	0.994 569	0.994 536	0.994 503	0.994 470	0.994 436	0.994 403
34.0	0.994 369	0.994 336	0.994 302	0.994 268	0.994 234	0.994 201	0.994 167	0.994 132	0.994 098	0.994 064
35.0	0.994 030	0.993 995	0.993 961	0.993 926	0.993 891	0.993 857	0.993 822	0.993 787	0.993 752	0.993 717
36.0	0.993 682	0.993 647	0.993 611	0.993 576	0.993 541	0.993 505	0.993 469	0.993 434	0.993 398	0.993 362
37.0	0.993 326	0.993 290	0.993 254	0.993 218	0.993 182	0.993 146	0.993 109	0.993 073	0.993 036	0.993 000
38.0	0.992 963	0.992 926	0.992 889	0.992 852	0.992 815	0.992 778	0.992 741	0.992 704	0.992 667	0.992 629
39.0	0.992 592	0.992 554	0.992 517	0.992 479	0.992 442	0.992 404	0.992 366	0.992 328	0.992 290	0.992 252

Based on the work of H. Wagenbreth and W. Blanke, PTB - Mitteilungen 6-71. Reference equations for CIPM adopted, air-saturated distilled water should be used and are provided in GLP 10 as published in NISTIR 7383, see latest edition.

Table 9.9 Density of Air, g/cm³, as Function of Temperature and Air Pressure*

Barometric Pressure			Temperature (°C)					
kPa	mbar	mmHg	18	20	22	24	26	28
77.33	773	580	0.000 92	0.000 92	0.000 91	0.000 91	0.000 90	0.000 90
79.99	800	600	0.000 95	0.000 95	0.000 94	0.000 93	0.000 93	0.000 92
82.66	827	620	0.000 99	0.000 98	0.000 97	0.000 96	0.000 96	0.000 95
85.33	853	640	0.001 02	0.001 01	0.001 00	0.001 00	0.000 99	0.000 98
87.99	880	660	0.001 05	0.001 04	0.001 03	0.001 03	0.001 02	0.001 01
90.66	907	680	0.001 08	0.001 07	0.001 07	0.001 06	0.001 05	0.001 04
93.33	933	700	0.001 11	0.001 10	0.001 10	0.001 09	0.001 08	0.001 07
95.99	960	720	0.001 14	0.001 14	0.001 13	0.001 12	0.001 12	0.001 10
98.66	987	740	0.001 18	0.001 17	0.001 16	0.001 15	0.001 14	0.001 13
101.32	1013	760	0.001 21	0.001 20	0.001 19	0.001 18	0.001 17	0.001 17
103.99	1040	780	0.001 24	0.001 23	0.001 22	0.001 21	0.001 21	0.001 20
106.66	1067	800	0.001 27	0.001 26	0.001 25	0.001 24	0.001 24	0.001 23

Computed for air at 50 % relative humidity using Option A in SOP 2. Conversions of barometric pressure are not exact. See NIST Special Publication 811 for exact conversions.

*Equations to compute the density of air at any pressure, temperature, and relative humidity are given in Section 8 of the Appendix to SOP No. 2 and Option B is preferred for all precision mass and volume calibrations.

Table 9.10 Density and Coefficient of Expansion

Material	Density (g/cm ³)	Coefficient of Expansion		
		Linear (/ °C)	Cubical (/ °C)	Cubical (/ °F)
Aluminum (<i>not suitable for provers</i>)	2.7	0.000 024	0.000 069	0.000 038
Brass	8.4 at 0 °C 8.3909 at 20 °C	0.000 019	0.000 054	
Cast iron	7.0 7.2	0.000 010	0.000 030	
Copper		0.000 017	0.000 050	
Diamond		0.000 001 18	0.000 003 5	
Fused silica (quartz)		0.000 000 5	0.000 001 6	
Glass, borosilicate (T1CA)			0.000 010	
Glass, borosilicate (T1CB)			0.000 015	
Glass, soda-lime			0.000 025	
Gold	18.0		0.000 043	
Invar		0.000 000 4	0.000 001 2	
Lead	11.34	0.000 029	0.000 087	
Length bench		0.000 010 6		
Mercury			0.000 018	
Nichrome	8.39		0.000 039	
Nichrome V	8.5		0.000 039	
Platinum		0.000 011	0.000 027	
Polycarbonate plastic			0.000 45	
Polypropylene plastic			0.000 24	
Polystyrene plastic			0.000 21	
Steel, stainless (mass)	7.84, 7.95, 8.0, 8.03		0.000 045	
Steel, stainless (provers)			0.000 047 7	0.000 026 5
Steel, 304 stainless (provers)			0.000 051 8	0.000 028 8
Steel, tape, mild		0.000 0116	0.000 033 5	
Steel, pressure vessel, low carbon			0.000 028 8	0.000 016
Steel, prover, low carbon		0.000 012	0.000 033 5	0.000 018 6
Steel, terne plate		0.000 012	0.000 035	0.000 019 5
Tantalum	16.6		0.000 020	
Water (20 °C)			0.000 21	

Reference values for cubical coefficient of expansion given in per °F are provided as reference for prover calibrations used for petroleum products where the reference temperature is 60 °F.

Table 9.11 A Short Table of Random Numbers

46	96	85	77	27	92	86	26	45	21	89	91	71	42	64	64	58	22	75	81	74	91	48	46	18
44	19	15	32	63	55	87	77	33	29	45	00	31	34	84	05	72	90	44	27	78	22	07	62	17
34	39	80	62	24	33	81	67	28	11	34	79	26	35	34	23	09	94	00	80	55	31	63	27	91
74	97	80	30	65	07	71	30	01	84	47	45	89	70	74	13	04	90	51	27	61	34	63	87	44
22	14	61	60	86	38	33	71	13	33	72	08	16	13	50	56	48	51	29	48	30	93	45	66	29
40	03	96	40	03	47	24	60	09	21	21	18	00	05	86	52	85	40	73	73	57	68	36	33	91
52	33	76	44	56	15	47	75	78	73	78	19	87	06	98	47	48	02	62	03	42	05	32	55	02
37	59	20	40	93	17	82	24	19	90	80	87	32	74	59	84	24	49	79	17	23	75	83	42	00
11	02	55	47	48	84	74	36	22	67	19	20	15	92	53	37	13	75	54	89	56	73	23	39	07
10	33	79	26	34	54	71	33	89	74	68	48	23	17	49	18	81	05	52	85	70	05	73	11	17
67	59	28	25	47	89	11	65	65	20	42	23	96	41	64	20	30	89	87	64	37	93	36	96	35
93	50	75	20	09	18	54	34	68	02	54	87	23	05	43	36	98	29	97	93	87	08	30	92	98
24	43	23	72	80	64	34	27	23	46	15	36	10	63	21	59	69	76	02	62	31	62	47	60	34
39	91	63	18	38	27	10	78	88	84	42	32	00	97	92	00	04	94	50	05	75	82	70	80	35
74	62	19	67	54	18	28	92	33	69	98	96	74	35	72	11	68	25	08	95	31	79	11	79	54
91	03	35	60	81	16	61	97	25	14	78	21	22	05	25	47	26	37	80	39	19	06	41	02	00
42	57	66	76	72	91	03	63	48	46	44	01	33	53	62	28	80	59	55	05	02	16	13	17	54
06	36	63	06	15	03	72	38	01	58	25	37	66	48	56	19	56	41	29	28	76	49	74	39	50
92	70	96	70	89	80	87	14	25	49	25	94	62	78	26	15	41	39	48	75	64	69	61	06	38
91	08	88	53	52	13	04	82	23	00	26	36	47	44	04	08	84	80	07	44	76	51	52	41	59
68	85	97	74	47	53	90	05	90	84	87	48	25	01	11	05	45	11	43	15	60	40	31	84	59
59	54	13	09	13	80	42	29	63	03	24	64	12	43	28	10	01	65	62	07	79	83	05	59	61
39	18	32	69	33	46	58	19	34	03	59	28	97	31	02	65	47	47	70	39	74	17	30	22	65
67	43	31	09	12	60	19	57	63	78	11	80	10	97	15	70	04	89	81	78	54	84	87	83	42
61	75	37	19	56	90	75	39	03	56	49	92	72	95	27	52	87	47	12	52	54	62	43	23	13
78	10	91	11	00	63	19	63	74	58	69	03	51	38	60	36	53	56	77	06	69	03	89	91	24
93	23	71	58	09	78	08	03	07	71	79	32	25	19	61	04	40	33	12	06	78	91	97	88	95
37	55	48	82	63	89	92	59	14	72	19	17	22	51	90	20	03	64	96	60	48	01	95	44	84
62	13	11	71	17	23	29	25	13	85	33	35	07	69	25	68	57	92	57	11	84	44	01	33	66
29	89	97	47	03	13	20	86	22	45	59	98	64	53	89	64	94	81	55	87	73	81	58	46	42
16	94	85	82	89	07	17	30	29	89	89	80	98	36	25	36	53	02	49	14	34	03	52	09	20
04	93	10	59	75	12	98	84	60	93	68	16	87	60	11	50	46	56	58	45	88	72	50	46	11
95	71	43	68	97	18	85	17	13	08	00	50	77	50	46	92	45	26	97	21	48	22	23	08	32
86	05	39	14	35	48	68	18	36	57	09	62	40	28	87	08	74	79	91	08	27	12	43	32	03
59	30	60	10	41	31	00	69	63	77	01	89	94	60	19	02	70	88	72	33	38	88	20	60	86
05	45	35	40	54	03	98	96	76	27	77	84	80	08	64	60	44	34	54	24	85	20	85	77	32
71	85	17	74	66	27	85	19	55	56	51	36	48	92	32	44	40	47	10	38	22	52	42	29	96
80	20	32	80	98	00	40	92	57	51	52	83	14	55	31	99	73	23	40	07	64	54	44	99	21
13	50	78	02	73	39	66	82	01	28	67	51	75	66	33	97	47	58	42	44	88	09	28	58	06
67	92	65	41	45	36	77	96	46	21	14	39	56	36	70	15	74	43	62	69	82	30	77	28	77
72	56	73	44	26	04	62	81	15	35	79	26	99	57	28	22	25	94	80	62	95	48	98	23	86
28	86	85	64	94	11	58	78	45	36	34	45	91	38	51	10	68	36	87	81	16	77	30	19	36
69	57	40	80	44	94	60	82	94	93	98	01	48	50	57	69	60	77	69	60	74	22	05	77	17
71	20	03	30	79	25	74	17	78	34	54	45	04	77	42	59	75	78	64	99	37	03	18	03	36
89	98	55	98	22	45	12	49	82	71	57	33	28	69	50	59	15	09	25	79	39	42	84	18	70
58	74	82	81	14	02	01	05	77	94	65	57	70	39	42	48	56	84	31	59	18	70	41	74	60
50	54	73	81	91	07	81	26	25	45	49	61	22	88	41	20	00	15	59	93	51	60	65	65	63
49	33	72	90	10	20	65	28	44	63	95	86	75	78	69	24	41	65	86	10	34	10	32	00	93
11	85	01	43	65	02	85	69	56	88	34	29	64	35	48	15	70	11	77	83	01	34	82	91	04
34	22	46	41	84	74	27	02	57	77	47	93	72	02	95	63	75	74	69	69	61	34	31	92	13

From: NBS Handbook 91, p T-82

Table 9.12 Percentiles of the F Distribution $F_{0.95}(n_1, n_2)$

n_1 = degrees of freedom for numerator n_2 = degrees of freedom for denominator

n_2	n_1														
	1	2	3	4	5	6	7	8	9	10	20	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	248.0	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.45	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.66	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.80	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.56	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.87	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.44	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.15	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	2.94	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.77	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.65	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.54	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.46	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.39	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.33	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.28	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.23	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.19	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.16	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.12	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.10	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.07	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.05	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.03	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.01	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	1.99	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	1.97	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	1.96	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	1.94	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	1.93	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.84	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.75	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.66	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.57	1.39	1.32	1.22	1.00

Created with Excel function: =ROUND(F.INV(0.95, Numerator,Denominator),2); validated with NBS Handbook 91.

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Table 9.13 Critical Values for the F-test, $F_{\max 0.05}$

	Number of treatments, levels, groups										
<i>n-1</i>	2	3	4	5	6	7	8	9	10	11	12
2	39.00	87.50	142.00	202.00	266.00	333.00	403.00	475.00	550.00	626.00	704.00
3	15.40	27.80	39.20	50.70	62.00	72.90	83.50	93.90	104.00	114.00	124.00
4	9.60	15.50	20.60	25.20	29.50	33.60	37.50	41.10	44.60	48.00	51.40
5	7.20	10.80	13.70	16.30	18.70	20.80	22.90	24.70	26.50	28.20	29.90
6	5.82	8.38	10.40	12.10	13.70	15.00	16.30	17.50	18.60	19.70	20.70
7	4.99	6.94	8.44	9.70	10.80	11.80	12.70	13.50	14.30	15.10	15.80
8	4.43	6.00	7.18	8.12	9.03	9.78	10.50	11.10	11.70	12.20	12.70
9	4.03	5.34	6.31	7.11	7.80	8.41	8.95	9.45	9.91	10.30	10.70
10	3.72	4.85	5.67	6.34	6.92	7.42	7.87	8.28	8.66	9.01	9.34
12	3.28	4.16	4.75	5.30	5.72	6.09	6.42	6.72	7.00	7.25	7.43
15	2.86	3.54	4.01	4.37	4.68	4.95	5.19	5.40	5.59	5.77	5.95
20	2.46	2.95	3.29	3.54	3.76	3.94	4.10	4.24	4.37	4.49	4.59
30	2.07	2.40	2.61	2.78	2.91	3.02	3.12	3.21	3.29	3.36	3.39
60	1.67	1.85	1.96	2.04	2.11	2.17	2.22	2.26	2.30	2.33	2.36
∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

If the F-ratio is close to 1 in the calculations, the variation among the samples/replicates is not statistically different. Look up the number of levels and the degrees of freedom (number of items in each level minus 1) in the F_{\max} table. For example, if there were 11 replicate observations ($n = 11$), then the degrees of freedom is $11 - 1$ or 10 and with two treatments (replicates) for a k of 2 (k is the number of levels/groups and not the coverage factor used for uncertainties), the table shows an F_{\max} value of 3.72.