Polarization efficiency correction

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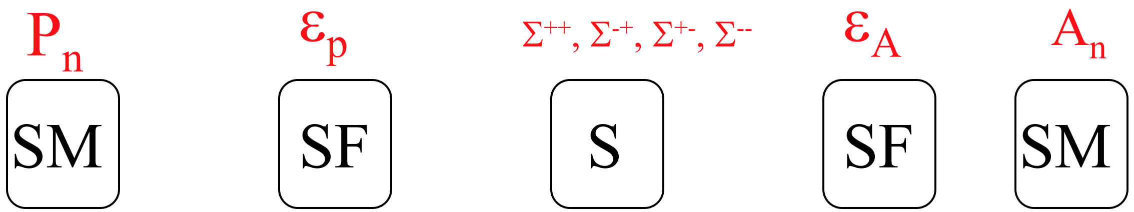
Polarization efficiency correction (in general)



In general

Nomenclature: in general

SM: Supermirror;
SF: neutron spin flipper;
S: sample;
P_n: neutron polarization provided by the polarizer;
P: percentage of neutrons in the "+" state from the polarizer and $P_n=2P-1$
A_n: neutron polarization provided by the analyzer;
A: percentage of neutrons in the "+" state from the analyzer and $A_n=2A-1$
ε_P: the percentage of neutrons flipped by the flipper before the sample;
ε_A: the percentage of neutrons flipped by the flipper after the sample;
f_P: measured flipping ratio with the flipper before the sample;
f_A: measured flipping ratio with the flipper after the sample;

Here I ignored the spin transport loss and can be added into the matrix operation as an additional polarizing element.

Polarization efficiency correction (^{3}He analyzer)

P_n	ε_p	$\Sigma^{++}, \Sigma^{-+}, \Sigma^{+-}, \Sigma^{--}$	T^+, T^-
SM	SF	S	^{3}He

^{3}He analyzer & flipper

Nomenclature: ^{3}He polarizer related

T_n: transmission of unpolarized neutrons through a polarized ^{3}He cell;

T_0: transmission of unpolarized neutrons through a unpolarized ^{3}He cell;

T_E: transmission of unpolarized neutrons through ^{3}He cell windows;

$T^{\pm}$: transmission if neutron spin is parallel (antiparallel) to ^{3}He nuclear spin;

F: flipping ratio produced by a polarized ^{3}He cell;

$P_{he}(t)$: ^{3}He polarization at elapse time t in hours;

P_{He}^0: ^{3}He polarization at time t=0;

T_1: ^{3}He polarization relaxation time on the neutron beam line in hours;

l: ^{3}He cell path length;

λ: neutron wavelength;

n: number density of ^{3}He gas;

σ: the 1/v neutron capture cross section given by $\sigma(\lambda) \approx \sigma_0\lambda$;

Polarized ^{3}He related equations

$$P_n = \frac{T^+ - T^-}{T^+ + T^-} = \tanh\left[\sigma(\lambda) n_{He} L P_{He}\right] = \sqrt{1 - \left(\frac{T_0}{T_n}\right)^2}$$

$$T_n = T_E \exp(-\sigma n_{He} L) \cosh(\sigma n_{He} L P_{He})$$

$$T_n = T_0 \cosh(\sigma n l P_{He})$$

$$T_0 = T_E e^{-\sigma n l}$$

$$T^{\pm} = T_E e^{-\sigma n l (1 \mp P_{He})}$$

$$F = T^+ / T^- = e^{2\sigma n l P_{He}}$$

$$P_{He}(t) = P_{He}^0 \exp\left(-t / T_1\right)$$

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Spin transport loss & sample depolarization issue

- Assume spin transport loss and/or sample depolarization affects the spin-up and spin-down neutrons in the same way; and assume the sample depolarization is equivalent to the transport loss;
- It can be shown that the overall transport loss is a product of each individual transport loss;
- It can be shown that all spin transport loss and/or sample depolarization could be lumped to either the polarizer;

SM & flipper efficiency (^{3}He analyzer & flipper)

Four cross section method to determine SM and flipper efficiency

With a ^{3}He analyzer and a NMR based flipper

$$P_n = \frac{I^{uu}\left(T_{ud}^{+} + T_{ud}^{-}\right) - I^{ud}\left(T_{uu}^{+} + T_{uu}^{-}\right)}{I^{uu}\left(T_{ud}^{+} - T_{ud}^{-}\right) + I^{ud}\left(T_{uu}^{+} - T_{uu}^{-}\right)}$$

Where $P_n = 2P - 1$

$$1 - 2\varepsilon_p = \frac{I^{du}\left(T_{dd}^{+} + T_{dd}^{-}\right) - I^{dd}\left(T_{du}^{+} + T_{du}^{-}\right)}{P_n\left(I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right)}$$

where

T: transmission of the neutrons passing through the polarized 3He.

+(-): neutron spin is parallel (antiparallel) to ^{3}He nuclear spin;

U and D: neutron spin states, UP(U) or DOWN (D); uu, ud, du, and dd correspond four different cross sections.

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Error propagation to SM & flipper efficiency

Four cross section method

$$(\delta P_n)^2 = \left(\frac{\delta P_n}{\delta I^{uu}}\right)^2 \left(\delta I^{uu}\right)^2 + \left(\frac{\delta P_n}{\delta I^{ud}}\right)^2 \left(\delta I^{ud}\right)^2$$

$$+ \left(\frac{\delta P_n}{\delta T^{+}_{uu}}\right)^2 \left(\delta T^{+}_{uu}\right)^2 + \left(\frac{\delta P_n}{\delta T^{-}_{uu}}\right)^2 \left(\delta T^{-}_{uu}\right)^2 + \left(\frac{\delta P_n}{\delta T^{+}_{ud}}\right)^2 \left(\delta T^{+}_{ud}\right)^2 + \left(\frac{\delta P_n}{\delta T^{-}_{ud}}\right)^2 \left(\delta T^{-}_{ud}\right)^2$$

$$\frac{\delta P_n}{\delta I^{uu}} = \frac{2I^{ud}\left(T^{+}_{ud}T^{+}_{uu} - T^{-}_{ud}T^{-}_{uu}\right)}{\left[I^{uu}\left(T^{+}_{ud} - T^{-}_{ud}\right) + I^{ud}\left(T^{+}_{uu} - T^{-}_{uu}\right)\right]^2} \qquad \frac{\delta P_n}{\delta I^{ud}} = \frac{-2I^{uu}\left(T^{+}_{ud}T^{+}_{uu} - T^{-}_{ud}T^{-}_{uu}\right)}{\left[I^{uu}\left(T^{+}_{ud} - T^{-}_{ud}\right) + I^{ud}\left(T^{+}_{uu} - T^{-}_{uu}\right)\right]^2}$$

For $I^{uu}=100$ and $I^{ud}=10$, $T_{uu}^{+} = T_{ud}^{+} = 0.2$ and $T_{uu}^{-} = T_{ud}^{-} = 0.01$

$$\frac{\delta P_n}{\delta I^{uu}} = 1.6\%$$

We can do the same thing for other terms.

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Error propagation to SM & flipper efficiency

$$\frac{\delta P_n}{\delta T_{ud}^{+}} = \frac{-2I^{uu}\left(I^{uu}T_{ud}^{-} - I^{ud}T_{uu}^{+}\right)}{\left[I^{uu}\left(T_{ud}^{+} - T_{ud}^{-}\right) + I^{ud}\left(T_{uu}^{+} - T_{uu}^{-}\right)\right]^2}$$

$$\frac{\delta P_n}{\delta T_{ud}^{-}} = \frac{2I^{uu}\left(I^{uu}T_{ud}^{+} - I^{ud}T_{uu}^{-}\right)}{\left[I^{uu}\left(T_{ud}^{+} - T_{ud}^{-}\right) + I^{ud}\left(T_{uu}^{+} - T_{uu}^{-}\right)\right]^2}$$

$$\frac{\delta P_n}{\delta T_{uu}^{+}} = \frac{2I^{ud}\left(I^{ud}T_{uu}^{-} - I^{uu}T_{ud}^{+}\right)}{\left[I^{uu}\left(T_{ud}^{+} - T_{ud}^{-}\right) + I^{ud}\left(T_{uu}^{+} - T_{uu}^{-}\right)\right]^2}$$

$$\frac{\delta P_n}{\delta T_{uu}^{-}} = \frac{-2I^{ud}\left(I^{uu}T_{uu}^{+} - I^{ud}T_{ud}^{-}\right)}{\left[I^{uu}\left(T_{ud}^{+} - T_{ud}^{-}\right) + I^{ud}\left(T_{uu}^{+} - T_{uu}^{-}\right)\right]^2}$$

Error propagation to SM & flipper efficiency

Four cross section method

$$(\delta\varepsilon_p)^2 = \left(\frac{\delta\varepsilon_p}{\delta P_n}\right)^2 (\delta P_n)^2 + \left(\frac{\delta\varepsilon_p}{\delta I^{du}}\right)^2 \left(\delta I^{du}\right)^2 + \left(\frac{\delta\varepsilon_p}{\delta I^{dd}}\right)^2 \left(\delta I^{dd}\right)^2$$

$$+\left(\frac{\delta\varepsilon_p}{\delta T_{dd}^{+}}\right)^2 \left(\delta T_{dd}^{+}\right)^2 + \left(\frac{\delta\varepsilon_p}{\delta T_{dd}^{-}}\right)^2 \left(\delta T_{dd}^{-}\right)^2 + \left(\frac{\delta\varepsilon_p}{\delta T_{du}^{+}}\right)^2 \left(\delta T_{du}^{+}\right)^2 + \left(\frac{\delta\varepsilon_p}{\delta T_{du}^{-}}\right)^2 \left(\delta T_{du}^{-}\right)^2$$

$$\frac{\delta\varepsilon_p}{\delta P_n} = \frac{-(1-2\varepsilon_p)}{P_n}$$

$$\frac{\delta\varepsilon_p}{\delta I^{dd}} = \frac{-2I^{du}\left(T_{du}^{+}T_{dd}^{+} - T_{du}^{-}T_{dd}^{-}\right)}{P_n\left[I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right]^2}$$

$$\frac{\delta\varepsilon_p}{\delta I^{du}} = \frac{2I^{dd}\left(T_{du}^{+}T_{dd}^{+} - T_{du}^{-}T_{dd}^{-}\right)}{P_n\left[I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right]^2}$$

Error propagation to SM & flipper efficiency

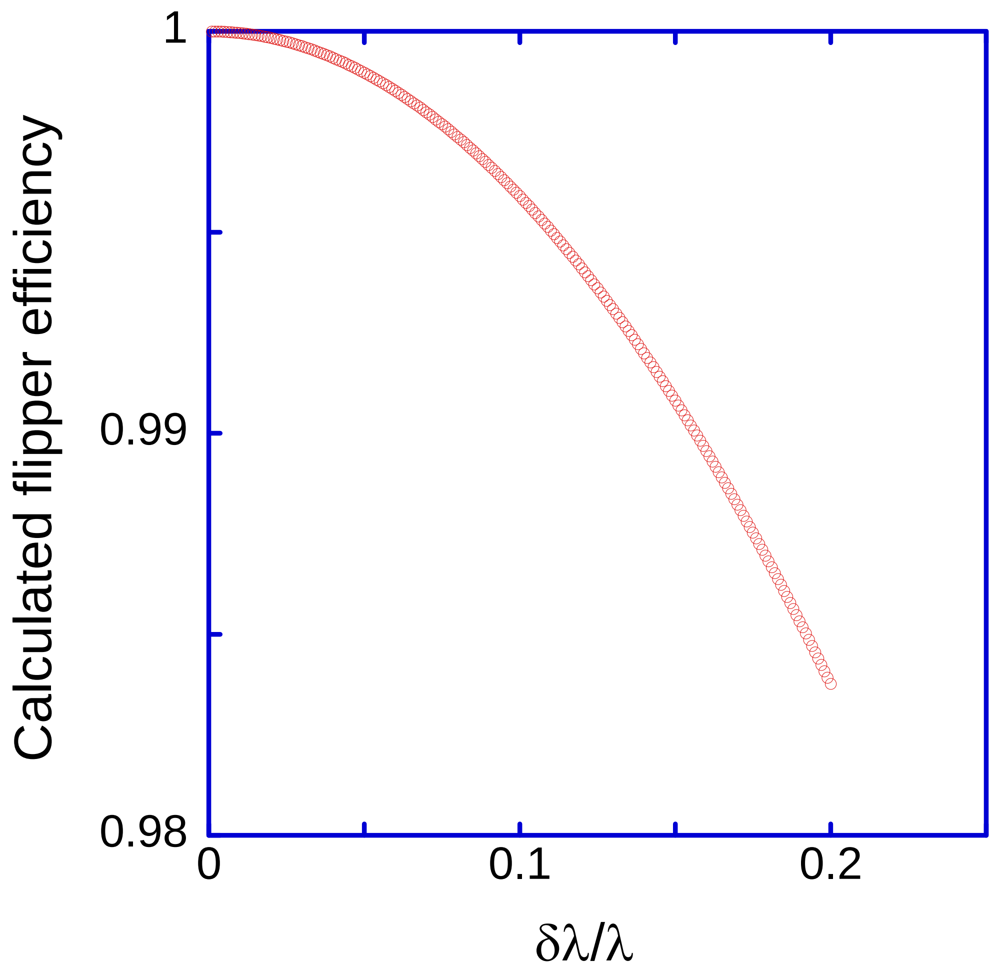
$$\frac{\delta\varepsilon_p}{\delta T_{du}^{+}} = \frac{2I^{dd}\left(I^{dd}T_{du}^{-} - I^{du}T_{dd}^{+}\right)}{P_n\left[I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right]^2}$$

$$\frac{\delta\varepsilon_p}{\delta T_{du}^{-}} = \frac{-2I^{dd}\left(I^{dd}T_{du}^{+} - I^{du}T_{dd}^{-}\right)}{P_n\left[I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right]^2}$$

$$\frac{\delta\varepsilon_p}{\delta T_{dd}^{+}} = \frac{-2I^{du}\left(I^{du}T_{dd}^{-} - I^{dd}T_{du}^{+}\right)}{P_n\left[I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right]^2}$$

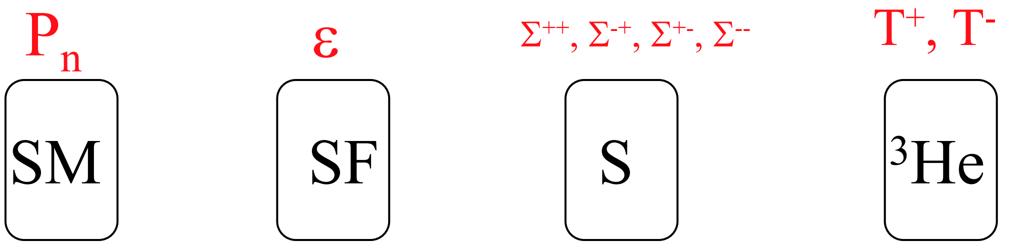
$$\frac{\delta\varepsilon_p}{\delta T_{dd}^{-}} = \frac{2I^{du}\left(I^{dd}T_{dd}^{+} - I^{du}T_{du}^{-}\right)}{P_n\left[I^{dd}\left(T_{du}^{+} - T_{du}^{-}\right) + I^{du}\left(T_{dd}^{+} - T_{dd}^{-}\right)\right]^2}$$

Flipper efficiency vs wavelength spread



- Mezei precession spin flipper.
- Large wavelength spread for SANS, 10%-20%.
- Measured flipping probability of higher than 0.985.
- Calculated flipping probability slightly higher (0.991 @ 15% wavelength spread).

Data reduction--polarization efficiency correction



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$$I^{++} = (1+P_n)\Sigma^{++}T^{+} + (1+P_n)\Sigma^{+-}T^{-} + (1-P_n)\Sigma^{-+}T^{+} + (1-P_n)\Sigma^{--}T^{-}$$

$$I^{-+} = [1+(1-2\varepsilon)P_n]\Sigma^{++}T^{+} + [1+(1-2\varepsilon)P_n]\Sigma^{+-}T^{-} + [1-(1-2\varepsilon)P_n]\Sigma^{-+}T^{+} + [1-(1-2\varepsilon)P_n]\Sigma^{--}T^{-}$$

flipped

$$I^{+-} = (1+P_n)\Sigma^{++}T^{-} + (1+P_n)\Sigma^{+-}T^{+} + (1-P_n)\Sigma^{-+}T^{-} + (1-P_n)\Sigma^{--}T^{+}$$

$$I^{--} = [1+(1-2\varepsilon)P_n]\Sigma^{++}T^{-} + [1+(1-2\varepsilon)P_n]\Sigma^{+-}T^{+} + [1-(1-2\varepsilon)P_n]\Sigma^{-+}T^{-} + [1-(1-2\varepsilon)P_n]\Sigma^{--}T^{+}$$

$$\begin{pmatrix} I^{++} \\ I^{-+} \\ I^{+-} \\ I^{--} \end{pmatrix} = T \begin{pmatrix} \Sigma^{++} \\ \Sigma^{-+} \\ \Sigma^{+-} \\ \Sigma^{--} \end{pmatrix}$$

Note: $T = P_a F P_n$, P_a, F and P_n are 4x4 matrix for the ^{3}He analyzer (T^+ and T^- defined before), the spin flipper (efficiency of ε) and the SM polarizer (efficiency of P_n). Σ^{++}, Σ^{-+}, Σ^{+-}, and Σ^{--} are the four scattering cross sections for the sample, including the sample holder and the cell background.

Data reduction--polarization efficiency correction

Polarization efficiency corrected background

P_n	ε	$\Sigma^{++}, \Sigma^{-+}, \Sigma^{+-}, \Sigma^{--}$	T^+, T^-
SM	SF	S	^{3}He

$$B^{++} = B^{--}$$

$$B^{+-} = B^{-+} = 0$$

unflipped

$$I^{++} = (1+P_n)B^{++}T^{+} + (1-P_n)B^{--}T^{-} = B^{++}\left[(1+P_n)T^{+} + (1-P_n)T^{-}\right]$$

$$I^{-+} = \left[1+(1-2\varepsilon)P_n\right]B^{++}T^{+} + \left[1-(1-2\varepsilon)P_n\right]B^{--}T^{-} = B^{++}\left\{\left[1+(1-2\varepsilon)P_n\right]T^{+} + \left[1-(1-2\varepsilon)P_n\right]T^{-}\right\}$$

flipped

$$I^{+-} = (1+P_n)B^{++}T^{-} + (1-P_n)B^{--}T^{+} = B^{++}\left[(1+P_n)T^{-} + (1-P_n)T^{+}\right]$$

$$I^{--} = \left[1+(1-2\varepsilon)P_n\right]B^{++}T^{-} + \left[1-(1-2\varepsilon)P_n\right]B^{--}T^{+} = B^{++}\left\{\left[1+(1-2\varepsilon)P_n\right]T^{-} + \left[1-(1-2\varepsilon)P_n\right]T^{+}\right\}$$

$$B^{++} = \frac{I^{++}}{(1+P_n)T^{+} + (1-P_n)T^{-}}$$

or

$$B^{++} = \frac{I^{-+}}{[1+(1-2\varepsilon)P_n]T^{+} + [1-(1-2\varepsilon)P_n]T^{-}}$$

or

$$B^{++} = \frac{I^{+-}}{(1+P_n)T^{-} + (1-P_n)T^{+}}$$

or

$$B^{++} = \frac{I^{--}}{[1+(1-2\varepsilon)P_n]T^{-} + [1-(1-2\varepsilon)P_n]T^{+}}$$

Note: B=the sample holder and the cell background.

Flipping ratio monitoring

No sample

3He unflipped (original state)

$$FR = \frac{(1+P_n)T^+ + (1-P_n)T^-}{[1+(1-2\varepsilon)P_n]T^+ + [1-(1-2\varepsilon)P_n]T^-}$$

3He flipped

$$FR = \frac{[1+(1-2\varepsilon)P_n]T^- + [1-(1-2\varepsilon)P_n]T^+}{(1+P_n)T^- + (1-P_n)T^+}$$

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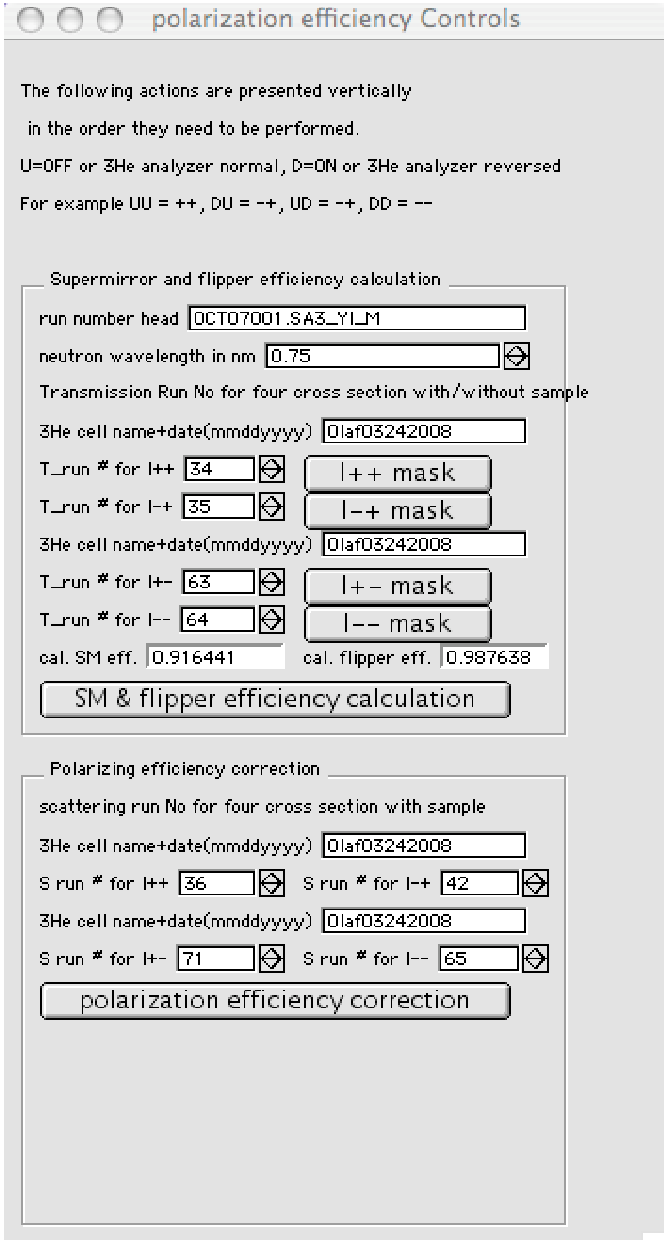
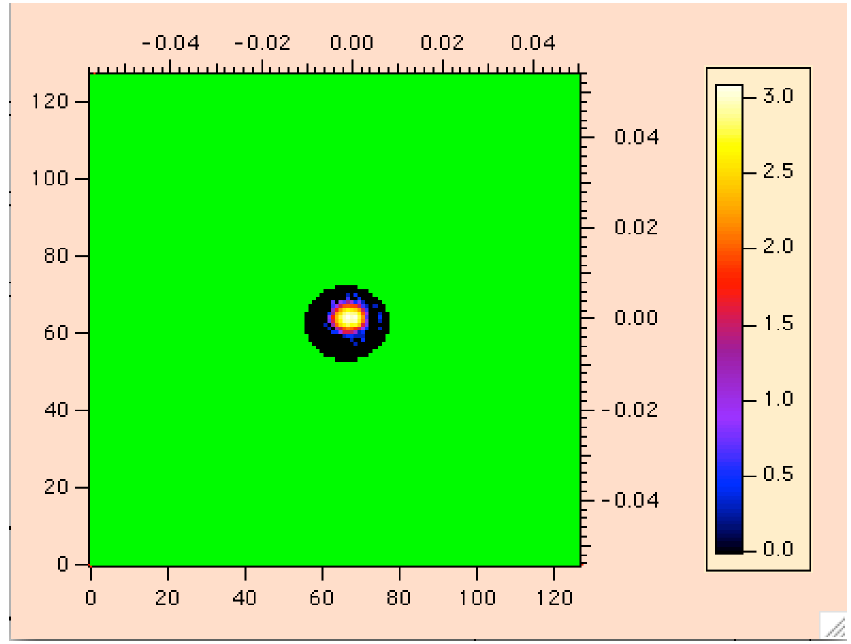
An interface for polarization efficiency correction





- IGOR based compatible with SANS software;
- Implemented SM and flipper efficiency calculation;
- Polarization efficiency correction to sample not implemented yet.

^{3}He polarization decay online monitoring tools

- online monitoring of ^{3}He polarization decay by NMR.
- Periodically monitoring of flipping ratio.
- Periodically monitoring of ^{3}He polarization by neutron measurements.

How T1 uncertainties affect polarization efficiency correction?

$$\frac{\partial T_1}{T_1} = 10\% \Rightarrow$$ 2.5% in SF and ~2.5% in NSF

$$\frac{\partial T_1}{T_1} = 20\% \Rightarrow$$ 5.6% in SF and ~5.6% in NSF

$$\frac{\partial Transmission}{Transmission} = \sigma n l P_{He}(t) \frac{\partial T_1}{T_1} \frac{t}{T_1}$$ Transmission change ~ a few %

Online monitoring tools -- FID NMR

Reflectometry

How T1 uncertainties affect polarization efficiency correction?

$$\frac{\partial T_1}{T_1} = 10\% \Rightarrow$$ 2.5% in SF and ~2.5% in NSF

$$\frac{\partial T_1}{T_1} = 20\% \Rightarrow$$ 5.6% in SF and ~5.6% in NSF

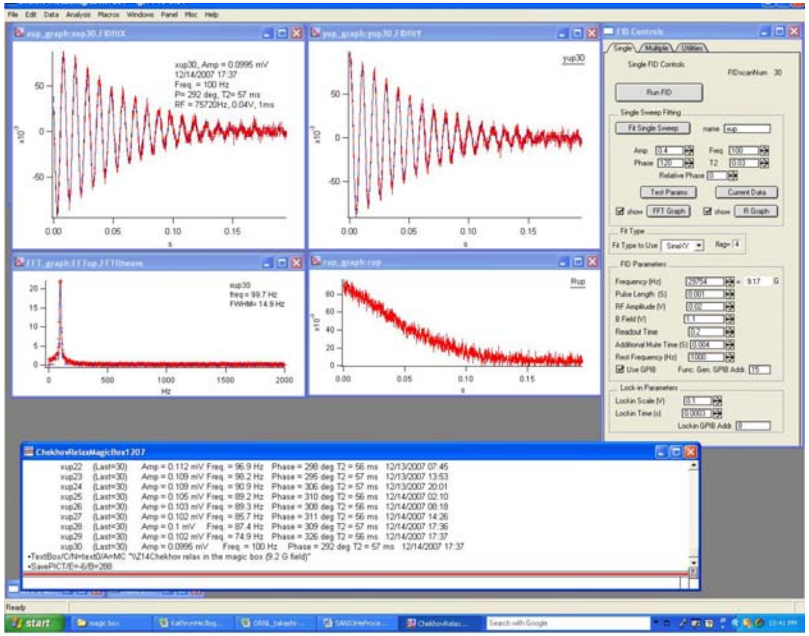
$$\frac{\partial Transmission}{Transmission} = \sigma n l P_{He}(t) \frac{\partial T_1}{T_1} \frac{t}{T_1}$$ Transmission change ~ a few %

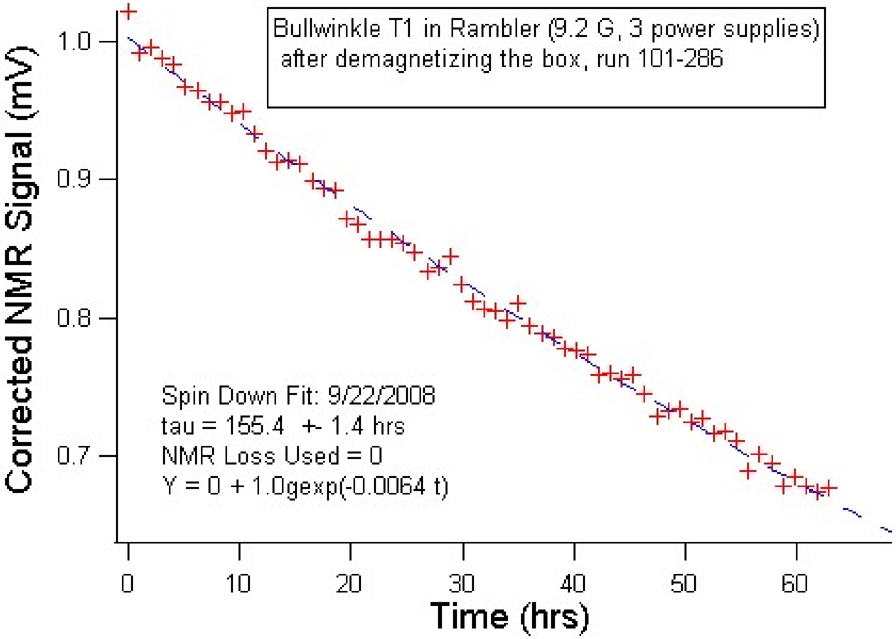
Online monitoring tools -- FID NMR

almost ideal case

$$\frac{\partial T_1}{T_1} < 2\%$$

FID NMR is a direct way to monitor the 3He polarization decay.

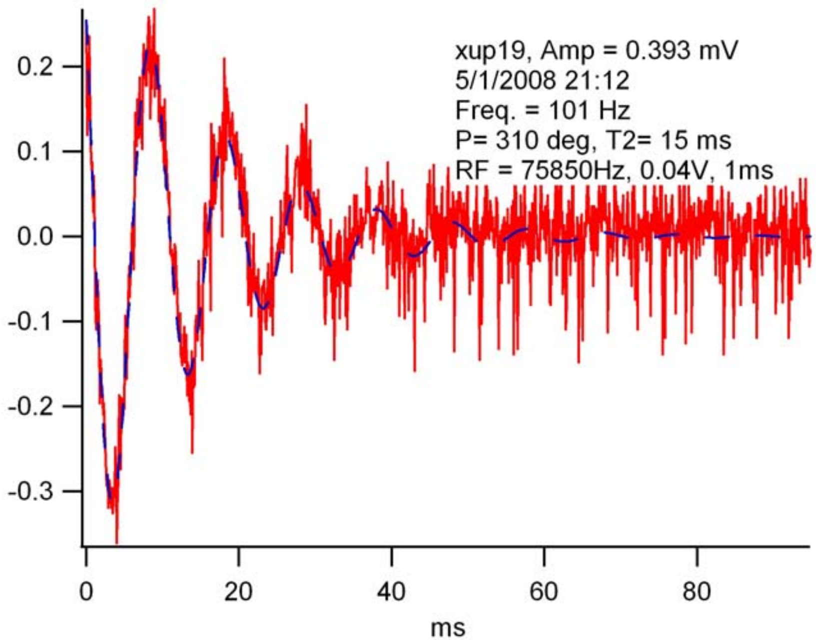


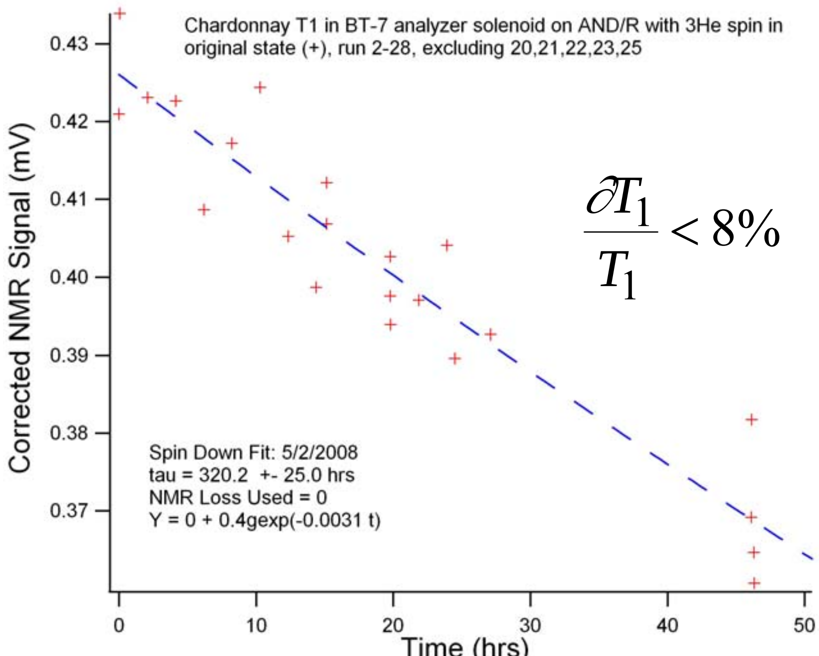


Online monitoring tools -- FID NMR

What's limiting our capability for reflectometry?

Some limitation from FID NMR measurements because of not good field homogeneity around the NMR probe due to restricted space on the instrument for our device. NMR probe is typically forced to the location where the field homogeneity is worst for the magnetically shielded solenoid.

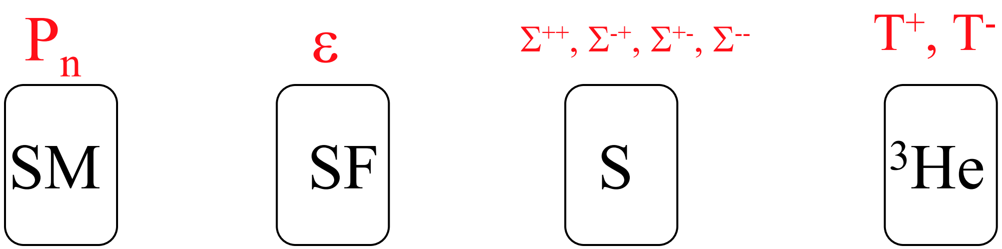




$$\frac{\partial T_1}{T_1} < 8\%$$

Online monitoring tools -- flipping ratio

Flipping ratio can also be used to monitor the 3He polarization although it is not straight forward and we do not prefer because it involves in polarization efficiency correction. However, unpolarized neutron transmission measurements are much better.



unflipped

$$\begin{cases} I^{++} = (1+P_n)\Sigma^{++}T^{+} + (1+P_n)\Sigma^{+-}T^{-} + (1-P_n)\Sigma^{-+}T^{+} + (1-P_n)\Sigma^{--}T^{-} \\ I^{-+} = [1+(1-2\varepsilon)P_n]\Sigma^{++}T^{+} + [1+(1-2\varepsilon)P_n]\Sigma^{+-}T^{-} + [1-(1-2\varepsilon)P_n]\Sigma^{-+}T^{+} + [1-(1-2\varepsilon)P_n]\Sigma^{--}T^{-} \end{cases}$$

flipped

$$\begin{cases} I^{+-} = (1+P_n)\Sigma^{++}T^{-} + (1+P_n)\Sigma^{+-}T^{+} + (1-P_n)\Sigma^{-+}T^{-} + (1-P_n)\Sigma^{--}T^{+} \\ I^{--} = [1+(1-2\varepsilon)P_n]\Sigma^{++}T^{-} + [1+(1-2\varepsilon)P_n]\Sigma^{+-}T^{+} + [1-(1-2\varepsilon)P_n]\Sigma^{-+}T^{-} + [1-(1-2\varepsilon)P_n]\Sigma^{--}T^{+} \end{cases}$$

Online monitoring tools -- flipping ratio

3He unflipped (original state)

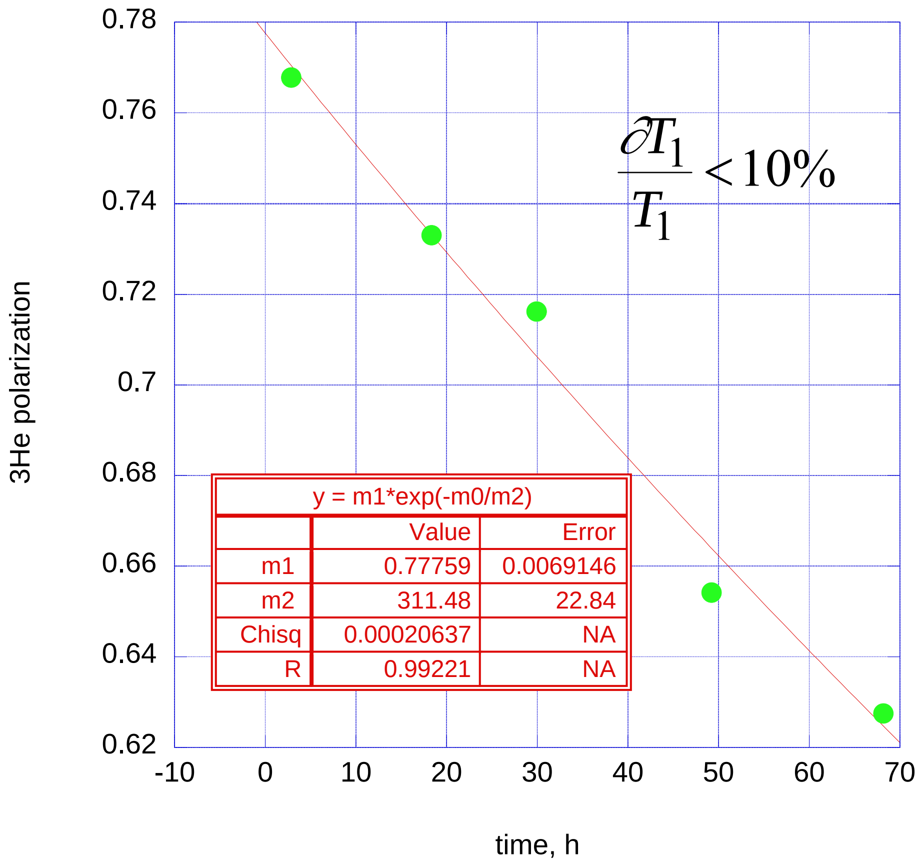
$$\begin{cases} FR = \dfrac{(1+P_n)T^+ + (1-P_n)T^-}{[1+(1-2\varepsilon)P_n]T^+ + [1-(1-2\varepsilon)P_n]T^-} \\ P_{He}(t) = \dfrac{1}{2\sigma nl}\ln\left(\dfrac{FR[1+(2\varepsilon-1)P_n]-(1-P_n)}{(1+P_n)-FR[1-(2\varepsilon-1)P_n]}\right) \end{cases}$$

3He flipped

$$\begin{cases} FR = \dfrac{[1+(1-2\varepsilon)P_n]T^- + [1-(1-2\varepsilon)P_n]T^+}{(1+P_n)T^- + (1-P_n)T^+} \\ P_{He}(t) = \dfrac{1}{2\sigma nl}\ln\left(\dfrac{FR(1+P_n)-[1-(2\varepsilon-1)P_n]}{[1+(2\varepsilon-1)P_n]-FR(1-P_n)}\right) \end{cases}$$

Online monitoring tools -- flipping ratio

Reflectometry



- Not ideal.
- Use spread sheet to match FRs and estimate T1.
- Can calculate 3He polarization from FR, but need good FR measurements and good knowledge of SM polarization efficiency and flipper efficiency.
- Better to have a slit scan at very beginning.

Online monitoring tools -- FID NMR

SANS

How T1 uncertainties affect polarization efficiency correction?

$$\frac{\partial T_1}{T_1} = 10\% \Rightarrow$$ 2.5% in SF and ~2.5% in NSF

$$\frac{\partial T_1}{T_1} = 20\% \Rightarrow$$ 5.6% in SF and ~5.6% in NSF

$$\frac{\partial Transmission}{Transmission} = \sigma n l P_{He}(t) \frac{\partial T_1}{T_1} \frac{t}{T_1}$$

Transmission change ~ a few %

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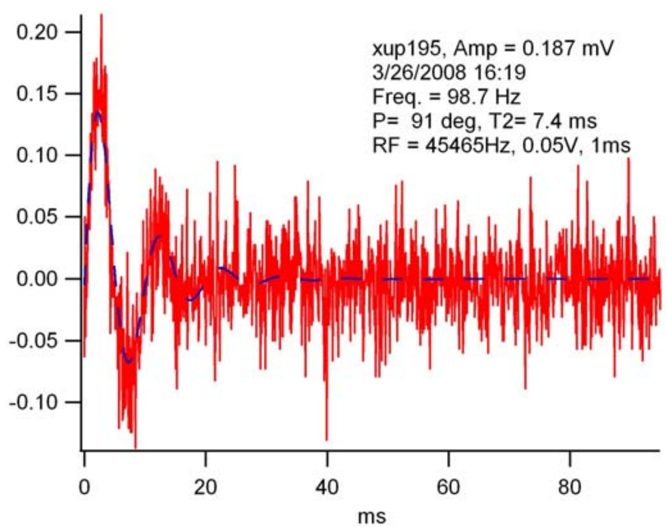
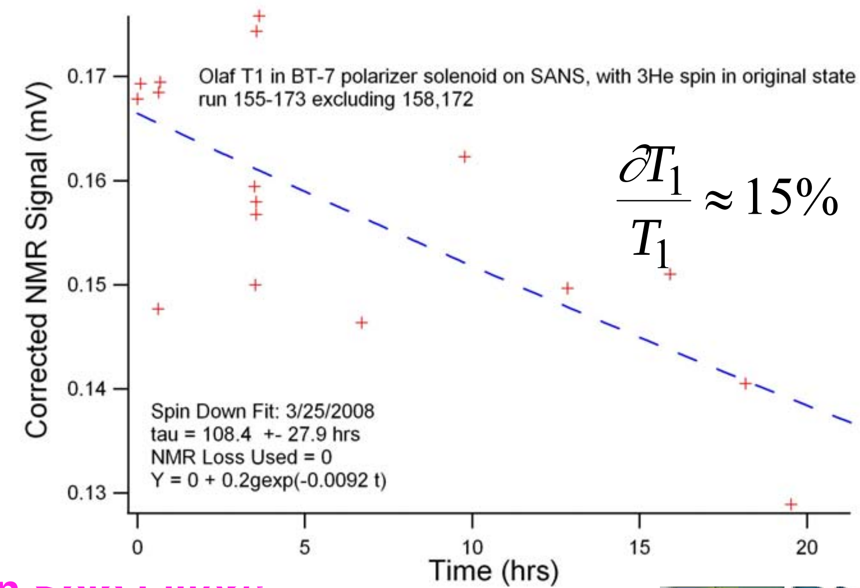
Online monitoring tools -- FID NMR

What's limiting our capability for SANS?

Severe limitation from FID NMR measurements because of really bad field homogeneity around the NMR probe due to very restricted space on the instrument. NMR probe is typically forced to the location where the field homogeneity is worst for the magnetically shielded solenoid. Changing the sample field also changes the NMR signal. The measurement time is typically much shorter than T, making it difficult to monitor T1 from NMR.





Online monitoring tools -- flipping ratio

Limitation and requirement for SANS

- Not ideal.
- Use spread sheet to match FRs and estimate T1.
- Can calculate 3He polarization from FR, but need good FR measurements and good knowledge of SM polarization efficiency and flipper efficiency.
- Need SM and flipper efficiencies to determine 3He polarization, hence polarization decay.
- Need a program to communicate the SANS data.
- Different sample depolarization at different sample.

^{3}He polarization decay-- neutron measurements

P_n ε_p $\Sigma^{++}, \Sigma^{-+}, \Sigma^{+-}, \Sigma^{--}$ T^+, T^-

SM SF S ^{3}He

^{3}He analyzer & flipper

Nomenclature: ^{3}He polarizer related

T_n: transmission of unpolarized neutrons through a polarized ^{3}He cell;

T_0: transmission of unpolarized neutrons through a unpolarized ^{3}He cell;

T_E: transmission of unpolarized neutrons through ^{3}He cell windows;

$T^{\pm}$: transmission if neutron spin is parallel (antiparallel) to ^{3}He nuclear spin;

F: flipping ratio produced by a polarized ^{3}He cell;

$P_{he}(t)$: ^{3}He polarization at elapse time t in hours;

P_{He}^0: ^{3}He polarization at time t=0;

T_1: ^{3}He polarization relaxation time on the neutron beam line in hours;

l: ^{3}He cell path length;

λ: neutron wavelength;

n: number density of ^{3}He gas;

σ: the 1/v neutron capture cross section given by $\sigma(\lambda) \approx \sigma_0\lambda$;

^{3}He polarization decay

T1 determined from unpolarized neutron transmission

$$P_n = \frac{T^+ - T^-}{T^+ + T^-} = \tanh\left[\sigma(\lambda) n_{He} L P_{He}\right] = \sqrt{1 - \left(\frac{T_0}{T_n}\right)^2}$$

$$T_n = T_E \exp(-\sigma n_{He} L)\cosh(\sigma n_{He} L P_{He})$$

$$T_n = T_0 \cosh(\sigma n l P_{He})$$

$$T_0 = T_E e^{-\sigma n l}$$

$$\Rightarrow P_{He}(t) = \frac{1}{\sigma n l} a\cosh\left(\frac{T_n}{T_0}\right)$$

Only at the end of the experiment,
T_0 is measurable.

$$P_{He}(t) = P_{He}^0 \exp\left(-\, t / T_1\right)$$

We need at least three points for this, but they are not convenient from the user experiment. We are typically able to have two points, one at the beginning and the other at the end.

^{3}He polarization and uncertainty

T1 determined from unpolarized neutron transmission (two point)

Method 1

$$T_1 = \frac{\Delta t}{\ln\left(\frac{P_{He}^i}{P_{He}^f}\right)}$$

$$(\partial T_1)^2 = \left(\frac{\partial T_1}{\partial P_{He}^i}\right)^2 \left(\partial P_{He}^i\right)^2 + \left(\frac{\partial T_1}{\partial P_{He}^f}\right)^2 \left(\partial P_{He}^f\right)^2$$

$$\frac{\partial T_1}{\partial P_{He}^i} = \frac{-T_1}{P_{He}^i \ln\left(\frac{P_{He}^i}{P_{He}^f}\right)}$$

$$\frac{\partial T_1}{\partial P_{He}^f} = \frac{T_1}{P_{He}^f \ln\left(\frac{P_{He}^i}{P_{He}^f}\right)}$$

$$\left(\frac{\partial T_1}{T_1}\right)^2 = \left(\ln\left(\frac{P_{He}^i}{P_{He}^f}\right)\right)^{-2} \left(\frac{\partial P_{He}^i}{P_{He}^i}\right)^2 + \left(\ln\left(\frac{P_{He}^i}{P_{He}^f}\right)\right)^{-2} \left(\frac{\partial P_{He}^f}{P_{He}^f}\right)^2$$

$$\frac{\partial P_{He}^i}{P_{He}^i} = \frac{\partial P_{He}^f}{P_{He}^f} = 2\%$$

$P_{He}^{i} = 76.0\%$

& $P_{He}^{i} = 62.7\%$

Δt= 2 days

T1=250 h

$$\partial T_1 = 36.8h$$

$$\frac{\partial T_1}{T_1} \approx 15\%$$

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^{3}He polarization and uncertainty

T1 determined from unpolarized neutron transmission (two point)

^{3}He uncertainty propagation

$$P_{He}(t) = P_{He}^{i} \exp\left[-t / T_1\left(P_{He}^{i}, P_{He}^{f}\right)\right]$$

$$dP_{He}(t) = \frac{\partial P_{He}}{\partial P_{He}^{i}} dP_{He}^{i} + \frac{\partial P_{He}}{\partial T_1} dT_1 = \frac{\partial P_{He}}{\partial P_{He}^{i}} dP_{He}^{i} + \frac{\partial P_{He}}{\partial T_1}\left[\frac{\partial T_1}{\partial P_{He}^{i}} dP_{He}^{i} + \frac{\partial T_1}{\partial P_{He}^{f}} dP_{He}^{f}\right]$$

$$= \left\{\frac{\partial P_{He}}{\partial P_{He}^{i}} + \frac{\partial P_{He}}{\partial T_1}\frac{\partial T_1}{\partial P_{He}^{i}}\right\} dP_{He}^{i} + \frac{\partial P_{He}}{\partial T_1}\frac{\partial T_1}{\partial P_{He}^{f}} dP_{He}^{f}$$

$$(\partial P_{He})^2 = \left\{\frac{\partial P_{He}}{\partial P_{He}^{i}} + \frac{\partial P_{He}}{\partial T_1}\frac{\partial T_1}{\partial P_{He}^{i}}\right\}^2 \left(\partial P_{He}^{i}\right)^2 + \left(\frac{\partial P_{He}}{\partial T_1}\frac{\partial T_1}{\partial P_{He}^{f}}\right)^2 \left(\partial P_{He}^{f}\right)^2$$

$$\Rightarrow \left(\frac{\partial P_{He}}{P_{He}}\right)^2 = \left(1 - \frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^{i}}{P_{He}^{i}}\right)^2 + \left(\frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^{f}}{P_{He}^{f}}\right)^2$$

^3He polarization decay

T1 determined from unpolarized neutron transmission (two point)

Method 2

$$T_1 = \frac{\Delta t}{\ln\left(\frac{P_{He}^i}{P_{He}^f}\right)}$$

$$P_{He}(t) = P_{He}^i \exp(-t / \left(\frac{\Delta t}{\ln\left(\frac{P_{He}^i}{P_{He}^f}\right)}\right) = P_{He}^i \exp\left(-\ln\left(\frac{P_{He}^i}{P_{He}^f}\right)\frac{t}{\Delta t}\right)$$

$$\left(\frac{\partial P_{He}}{P_{He}}\right)^2 = \left(1 - \frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^i}{P_{He}^i}\right)^2 + \left(\frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^f}{P_{He}^f}\right)^2$$

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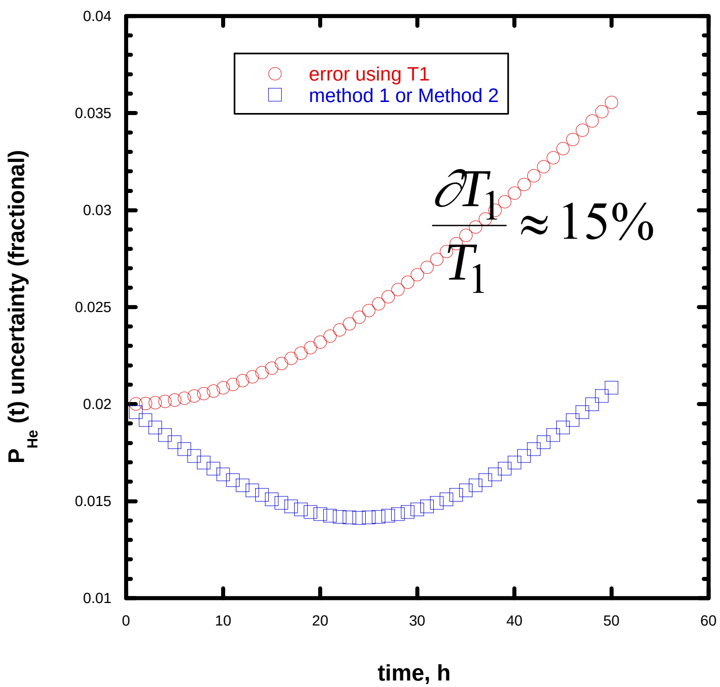
^{3}He polarization uncertainty comparison

Method 1

$$\left(\frac{\partial P_{He}}{P_{He}}\right)^2 = \left(1-\frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^i}{P_{He}^i}\right)^2 + \left(\frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^f}{P_{He}^f}\right)^2$$

Method 2

$$\left(\frac{\partial P_{He}}{P_{He}}\right)^2 = \left(1-\frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^i}{P_{He}^i}\right)^2 + \left(\frac{t}{\Delta t}\right)^2 \left(\frac{\partial P_{He}^f}{P_{He}^f}\right)^2$$



$$\frac{\partial P_{He}^i}{P_{He}^i} = \frac{\partial P_{He}^f}{P_{He}^f} = 2\%$$

$P_{He}^i = 76.0\%$

& $P_{He}^i = 62.7\%$

$\Delta t = 2$ days

T1=250 h

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