SOP No. 6

Recommended Standard Operations Procedure

for

Weighing by Transposition

1. Introduction

1.1. Purpose

This method eliminates possible error in comparison of weights when using an equal-arm balance, due to a small inequality in the length of the arms. While this procedure is written to describe the measurement of masses of weights for calibration purposes, it may be used for general mass determinations, whenever accurate weighing is required. (Note: This is a single - transposition procedure, hence does not eliminate the effects of linear drift).

1.2. Prerequisites

1.2.1. Verify that standard weights are available with valid certificates of calibration.

1.2.2. Verify that the balance to be used is in good working order. This may be verified by maintenance of a valid control chart. Otherwise, a series of trial measurements should be made to ascertain that the balance is functional and that its sensitivity is adequate for the requirements of the measurement.

1.2.3. Verify the ability of the technician to make precision weighings and that he/she has had training in this specific procedure.

2. Methodology

2.1. Scope, Precision, Accuracy

The measurement principle is applicable to weighing with any equal-arm balance. The range of weights that can be calibrated will depend upon the capacity of the balance used. The precision of intercomparison will depend upon the sensitivity of the balance, and the care exercised in making the required weighings. The accuracy will depend on the accuracy of calibration of the standard weights and the precision of intercomparison.

2.2. Summary

The masses of a standard and test weight are intercompared. The loads are then interchanged and a second intercomparison is made. A small calibrated weight is then added to one of the masses and a third weighing is made to determine the sensitivity of the balance under the load conditions.

2.3. Apparatus/Equipment

2.3.1. Equal arm balance of capacity and sensitivity sufficient for the weights tested. The index scale of the balance is conveniently numbered 0 to 20 with 10 as the center division, although other numbering systems such as 0 to 200 are possible. The graduations should be numbered so that addition of a weight to the left arm will increase the balance readings. A system in which the center division is 0 is not recommended since the negative readings that result can cause observational and computational problems.

2.3.2. Standard weights with valid calibrations, traceable to NBS.

2.3.3. Small calibrated weights (usually decimal fractions) to be used as tare weights.

2.3.4. Equipment capable of loading and unloading weights on balance without damage to either (especially important in the case of large weights).

2.4. Symbols

The following symbols are used in this procedure.

S - standard weight
X - weight calibrated
T - counterweight
t - small calibrated. A subscript s or x is used to indicate the larger weight with which it is associated.
sw - small calibrated weight used to evaluate the sensitivity of the balance.
M - the mass of a specific weight. Subscripts s, x, t, sw, are used to identify the weight.
AM - the apparent mass of a specific weight. Subscripts s, x, t, sw together with numerical sub-subscripts as necessary are used to identify the weight.

2.5. Procedure

2.5.1. The following weighing sequences may be used:

Option A

Measurement No.	Left Arm	Right Arm	Observation
1	$X + t_{x_1}$	$S + t_{s_1}$	O_1
2	$S + t_{s_2}$	$X + t_{x_2}$	O_2
3	$S + t_{s_2} + sw$	$X + t_{x_2}$	O_3

Option B

Measurement No.	Left Arm	Right Arm	Observation
1	$S + t_{s_1}$	$X + t_{x_1}$	O_1
2	$X + t_{x_2}$	$S + t_{s_2}$	O_2
3	$X + t_{x_2} + sw$	$S + t_{s_2}$	O_3

2.5.2. Preliminary Procedures

Conduct preliminary measurements (without recording data) to determine the values for t_s and t_x as described in 2.5.3. This will serve to warm up the balance and to facilitate the measurements. In a series of calibrations, this step may be minimized or eliminated after the first series of measurements. Depending on the corrections for S and X, weights t_s and t_x may be unnecessary in some cases.

2.5.3. Measurement Procedure (Option A)

All observations should be recorded on suitable data sheets such as those in the Appendix. Record the laboratory ambient temperature, barometric pressure, and relative humidity.

2.5.3.1. Observation 1. Place the test weight(s) on the left arm and the standard(s) on the right arm. Add t_x, and/or t_s (small calibrated weights) as necessary to obtain an approximate balance. Read and record all data and the sum of the turning points O_1 (see GMP No. 1).

2.5.3.2. Observation 2. Place the standard weight(s) on the left arm and the test weight(s) on the right arm with appropriate small weights, as was done in Observation 1. The tare weights carried with S and X may have to be changed so that O_2 is within one division of O_1 on a 0 to 20 division graduated scale. Read and record all data and the sum of the turning points, O_2.

2.5.3.3. Observation 3. Without disturbing weights already in place from Observation 2, add a small calibrated sensitivity weight (sw) to the left arm, sufficient to displace the turning points 3 to 5 divisions on a 20 division graduated scale. Record the sum of the turning points, O_3.

2.5.4. Measurement Procedure (Option B)

Conduct 3 measurements as in 2.5.3.1, 2.5.3.2, and 2.5.3.3 using the weighing schedule Option B.

3. Calculations

3.1. No Air Buoyancy Correction

The following equations may be used to calculate the corrections required for the test weight(s). The calibrated apparent mass values for t_s, t_x, and sw must be used. C_s is the apparent mass correction for the standard weight(s), S. The symbols N_s and N_x refer to the nominal values of S and X, respectively.

Option A

$$C_x = C_s + \frac{AM_{t_{s_1}} + AM_{t_{s_2}} - AM_{t_{x_1}} - AM_{t_{x_2}}}{2} + \frac{O_1 - O_2}{2} \cdot \frac{AM_{sw}}{O_3 - O_2} + N_s - N_x$$

Option B

$$C_x = C_s + \frac{AM_{t_{s_1}} + AM_{t_{s_2}} - AM_{t_{x_1}} - AM_{t_{x_2}}}{2} + \frac{O_2 - O_1}{2} \cdot \frac{AM_{sw}}{O_3 - O_2} + N_s - N_x$$

3.2. Air Buoyancy Correction

3.2.1. Calculate the air density, ρ_A, as described in Section 8 of the Appendix to SOP No. 2 or obtain from Table 9.9.

3.2.2. Calculate the mass, M_x, of the test weight X, using the mass of the standard weight(s), the tare weights, and the sensitivity weights according to the optional sequence used. It is assumed that the densities of the combination of tare weights used do not differ significantly in the weighing design.

Symbols used: ρ_A = air density
M_i = mass for weight i
ρ_i = density for weight i

3.2.2.1. Optional Sequence A

$$M_x = \frac{M_s\left(1 - \frac{\rho_A}{\rho_s}\right) + \frac{O_1 - O_2}{2} \bullet \frac{M_{sw}\left(1 - \frac{\rho_A}{\rho_{sw}}\right)}{O_3 - O_2} + \frac{M_{t_{s_1}} + M_{t_{s_2}}}{2}\left(1 - \frac{\rho_A}{\rho_{t_s}}\right) - \frac{M_{t_{x_1}} + M_{t_{x_2}}}{2}\left(1 - \frac{\rho_A}{\rho_{t_x}}\right)}{1 - \frac{\rho_A}{\rho_x}}$$

3.2.2.2. Optional Sequence B

$$M_x = \frac{M_s\left(1 - \frac{\rho_A}{\rho_s}\right) + \frac{O_2 - O_1}{2} \bullet \frac{M_{sw}\left(1 - \frac{\rho_A}{\rho_{sw}}\right)}{O_3 - O_2} + \frac{M_{t_{s_1}} + M_{t_{s_2}}}{2}\left(1 - \frac{\rho_A}{\rho_{t_s}}\right) - \frac{M_{t_{x_1}} + M_{t_{x_2}}}{2}\left(1 - \frac{\rho_A}{\rho_{t_x}}\right)}{1 - \frac{\rho_A}{\rho_x}}$$

3.2.2.3. Calculate the mass correction, C_x', as follows:

$$C_x' = M_x - N_x$$

when N_x is the nominal value for X

3.2.2.4. Calculate the apparent mass of X versus the desired reference density of 8.0 g/cm^3 or brass. It is recommended that the apparent mass versus 8.0 g/cm^3 be reported unless otherwise requested. The density of X, ρ_x, must be in g/cm^3.

3.2.2.4.1. Apparent mass versus 8.0 g/cm3

$$AM_x \text{ vs. } 8.0 = M_x \frac{\left(1 - \frac{.0012}{\rho_x}\right)}{0.999850}$$

3.2.2.4.2. Apparent mass versus brass

$$AM_x \text{ vs. brass} = M_x \frac{\left(1 - \frac{.0012}{\rho_x}\right)}{0.999857}$$

4. Assignment of Uncertainty

The limits of uncertainty, U, include estimates of the uncertainty of the mass standards, U_s, plus the uncertainty of measurement, U_m, at the 99.73% level of confidence. The latter is estimated by

$$ts$$

where s is the standard deviation of measurement and t is obtained from Table 9.3.

Then $$U = \pm [U_s + ts]$$

4.1. Precision of Measurement Known from Control Chart Performance (SOP No. 9)

The value for s is obtained from the control chart data for transposition measurements. Statistical control will need to be verified by measurement of at least one check standard while the above measurements are in progress.

Use the value of t (corresponding to a probability level of 99.73%) from Table 9.3 appropriate for the number of degrees of freedom, ν, on which the control limits of the control chart are based.

4.2. Precision Estimated from a Series of Measurements

Measure a stable test object at least 7 times, no two measurements of which may be made on a single day. Calculate the mean and a standard deviation in the conventional manner. The latter is the value of s that is to be substituted in the equation given in Section 4. In this case select the value for t from Table 9.3 based on the number of degrees of freedom involved in computing s.

Note: Repetitive measurements made on the same day evaluate only the short-term standard deviation.

5. Report

5.1. Report results as described in SOP No. 1, Preparation of Calibration/Test Reports

Appendix

Transposition Data Sheet for Equal-Arm Balance (Option A)

Test No.: ______________ Sheet No.: ____________ Date: __________

Item Identification: ____________________________ Balance: _______

Standard Identification: _________________________ Observer: ______

Temperature: ______ Pressure: ___________________ Rel. Hum.: _____

$C_s(C'_s)$= _____________ ± ________________ $AM_{sw}(M_{sw})$= ____________

ρ_s= _______________ ρ_x= ________________ ρ_{sw} = _________________

Time ______________ Balance standard deviation = _________________

Measurement No.	Weights		Turning Points		Sum
	Left Arm	Right Arm	Low	High	
1	$X + t_{x_1}$ t_{x_1}=	$S + t_{s_1}$ t_{s_1}=			O_1=
2	$S + t_{s_2}$ t_{s_2}=	$X + t_{x_2}$ t_{x_2}=			O_2=
3	$S + t_{s_2} + sw$	$X + t_{x_2}$			O_3=

Time _____________

$AM_{t_{s_1}}(M_{t_{s_1}})$= _______ ± _________ $AM_{t_{x_1}}(M_{t_{x_1}})$= ________ ± _________

$AM_{t_{s_2}}(M_{t_{s_2}})$= _______ ± _________ $AM_{t_{x_2}}(M_{t_{x_2}})$= ________ ± _________

ρ_{t_s}= ____________ ρ_{t_x}= ____________

Appendix

Transposition Data Sheet for Equal-Arm Balance (Option B)

Test No.: ______________ Sheet No.: ____________ Date: __________

Item Identification: ____________________________ Balance: _______

Standard Identification: _________________________ Observer: ______

Temperature: _____ Pressure: ________________ Rel. Hum.: _____

$C_s(C_s')=$ _____________ ± ________________ $AM_{sw}(M_{sw})=$ ____________

$\rho_s=$ _______________ $\rho_x=$ ________________ $\rho_{sw}=$ _________________

Time ______________ Balance standard deviation = _________________

Measurement No.	Weights		Turning Points		Sum
	Left Arm	Right Arm	Low	High	
1	$S + t_{s_1}$ $t_{s_1}=$	$X + t_{x_1}$ $t_{x_1}=$			$O_1=$
2	$X + t_{x_2}$ $t_{x_2}=$	$S + t_{s_2}$ $t_{s_2}=$			$O_2=$
3	$X + t_{x_2} + sw$	$S + t_{s_2}$			$O_3=$

Time ______________

$AM_{t_{s_1}}(M_{t_{s_1}})=$ _______ ± _________ $AM_{t_{x_1}}(M_{t_{x_1}})=$ _______ ± ________

$AM_{t_{s_2}}(M_{t_{s_2}})=$ _______ ± _________ $AM_{t_{x_2}}(M_{t_{x_2}})=$ _______ ± ________

$\rho_{t_s}=$ ____________ $\rho_{t_x}=$ ____________

Appendix

Transposition Data Sheet for Equal-Arm Balance (Option A)

Test No.: 135 Sheet No.: 1 Date: 8/26/86

Item Identification: 1000 lb No. 4211 Balance: Russell

Standard Identification: #11 and #22 Observer: HO

Temperature: 23.5°C Pressure: 745.8 mm Hg Rel. Hum.: 50%

$C_s(C'_s)$= 0.0047 lb ± 0.0030 lb $AM_{sw}(M_{sw})$= 0.01 lb

ρ_s= 8.0 g/cm^3 ρ_x= 7.0 g/cm^3 ρ_{sw} = 7.8 g/cm^3

Time 1:15 p Balance standard deviation = 0.0018 lb

Measurement No.	Weights		Turning Points		Sum
	Left Arm	Right Arm	Low	High	
1	$X + t_{x_1}$ t_{x_1}= 3.617lb	$S + t_{s_1}$ t_{s_1} = 3.625lb	4.0 4.1	~~16.3~~ 16.3	O_1= 20.35
2	$S + t_{s_2}$ t_{s_2}=	$X + t_{x_2}$ t_{x_2}= 3.612lb	~~5.6~~ 5.7	~~14.1~~ 14.1 14.0	O_2= 19.75
3	$S + t_{s_2} + sw$	$X + t_{x_2}$	~~8.4~~ 8.4	~~16.0~~ 15.9 15.9	O_3= 24.3

Time 1:40 p

$AM_{t_{s_1}}$ (~~$M_{t_{s_1}}$~~)= 3.625 ± .000075lb $AM_{t_{x_1}}$ (~~$M_{t_{x_1}}$~~)= 3.617 ± negligible

$AM_{t_{s_2}}$ (~~$M_{t_{s_2}}$~~)= 3.625 ± .000075lb $AM_{t_{x_2}}$ (~~$M_{t_{x_2}}$~~)= 3.612 ± negligible

ρ_{t_s}= 8.0 g/cm^3 ρ_{t_x}= 7.8 g/cm^3

$$C_x = C_s + \frac{AM_{t_{s_1}} + AM_{t_{s_2}} - AM_{t_{x_1}} - AM_{t_{x_2}}}{2} + \frac{O_1 - O_2}{2} \bullet \left(\frac{AM_{sw}}{O_3 - O_2} \right)$$

$$C_x = .0047 + \frac{3.625 + 3.625 - 3.617 - 3.612}{2} + \frac{20.35 - 19.75}{2} \bullet \frac{0.01}{24.3 - 19.75}$$

$C_x = .0047 + .0105 + .000659$

$C_x = 0.015859$ lb

Uncertainty = ts + U_s Degrees of freedom in s = 20

Uncertainty = 3.422 (0.0018) + (.0030 + .000075)

Uncertainty = 0.0092346 lb

$C_x = 0.0159 \pm 0.0092$ lb