# **NISTIR XXXX**

# Selected Laboratory and Measurement Practices and Procedures for Length Calibrations (2014 Ed)

Jose A. Torres Ferrer

Office of Weights and Measures (contractor)
Physical Measurement Laboratory

Georgia L. Harris, editor Office of Weights and Measures Physical Measurement Laboratory

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U.S. Department of Commerce *Penny Pritzker, Secretary* 

National Institute of Standards and Technology Willie May, Acting Under Secretary of Commerce for Standards and Technology and Acting Director This page is CODEN page on back of title page or Blank.

#### **Foreword**

This NIST IR of Selected Publications was been compiled as an update for a number of Good Laboratory Practices (GLP), Good Measurement Practices (GMP), Standard Operating Procedures (SOP), Statistical Techniques and Reference Tables. This is the fourth in a series of updates that also include: NISTIR 5672 (2014), NISTIR 6969 (2014), NISTIR 7383 (2013).

This publication covers those GMPs and SOPs required for length calibrations as used in weights and measures applications.

Most of these procedures are updates to procedures that were originally published in NBS Handbook 145, Handbook for the Quality Assurance of Metrological Measurements, in 1986, by Henry V. Oppermann and John K. Taylor. The major changes incorporated 1) uncertainty analyses that comply with current international methods, 2) measurement assurance techniques using check standards or replicate measurements, and 3). No substantive changes were made to core measurement processes or equations.

The following Procedure is new in the 2014 publication, but was prepared by Ken Fraley (retired, OK) and first made available as a draft for laboratories to follow in 2001:

Standard Operating Procedures for: Calibration of Pi Tapes (23)

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# **Good Measurement Practices**

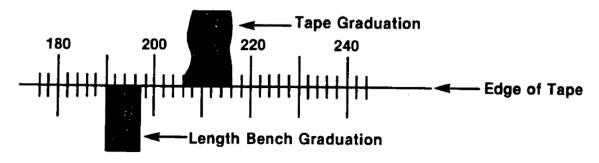


#### GMP 2

#### Good Measurement Practice for Reading the Center of Graduations when using a Microscope

1. The microscope furnished to State laboratories with the length bench has a reticle on which each graduation represents 0.002 inch which must have a suitable calibration from an accredited supplier to ensure metrological traceability to the International System of Units (SI). The difference in length between two tapes is measured from the center of the graduations of interest. The tape should be placed on the length bench such that the reading edge of the tape partially covers the graduations marked on the length bench. Place the microscope so that both the edges of the tape and the edge of the bench graduations are in the field-of-vision of the microscope, preferably near the center, and within reading range of the graduated reticle. Position the microscope so that part of the graduations on the reticle overlaps the bottom portion of the graduation on the tape and part overlaps the graduation on the length bench. See Figure 1 for illustration. Do not move or refocus the microscope once the comparison between the unknown, *X*, and standard, *S*, has begun.

Figure 1. Graduated reticle overlapping tape and bench graduations.



2. Determine the center of a graduation by first determining the width of the graduation, dividing the width by two and adding this value to the reticle reading for the left edge of the graduation. In the illustration above, let the length bench be the standard, S, and the tape be the unknown, X. The left edge of the tape graduation is at 206 (0.206 inch); the right edge is at 216 (0.216 inch). The width of the graduation is 0.010 inch. Adding half this value to 206 (0.206 inch), gives the center of the graduation at 211 (0.211 inch). Following the same procedure for the length bench graduation shows the left edge of the graduation at 190 (0.190 inch) and the right edge at 197 (0.197 inch). The center of the graduation is 193.5 (0.193 5 inch). The difference between the two tapes is 211 - 193.5, which is 17.5 (0.211 inch - 0.193 5 inch = 0.017 5 inch). Since X is longer than S, d is positive. Using the equation X - S = d,

$$X - S = 0.017$$
 5 inch or  $X = S + 0.017$  5 inch

3. If it is necessary to calibrate a rigid rule to the end of the rule, the edge of ends of the rule are reference points for the measurement. In these cases, there will be only a single value read from the microscope reticle for each end of the rule since the center of graduation is assumed to be the end of the rules.

#### **GMP 8**

#### Good Measurement Practice for Reporting Tape Calibrations

Tape calibration results should be reported as described in SOP 1, Preparation of Calibration Certificates. The use of the Calibration Certificate and Test Report Review Checklist, (SOP 1, Appendix B), is highly recommended.

Information pertinent to this type of calibration should include:

- 1. A note that reported lengths may be converted to catenary suspension support lengths by use of the "Equations for Metallic Tapes" (GMP 9);
- 2. Description of the procedure, and the position of the test artifact relative to the observer;
- 3. Tension applied to the tape during the test;
- 4. Type of support, e.g., continuous, catenary (include information regarding spacing);
- 5. The assumed linear coefficient of expansion for the tape material;
- 6. The relationship between the International System of Units (SI) and US customary units;
- 7. The degrees of freedom (effective) associated with the coverage factor *k*;
- 8. AE value, if requested by customer; and
- 9. Weight per Unit Length, if requested by customer.

An example of a Calibration Certificate follows in the Appendix.

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#### **Appendix** – **Example**

#### COMPLIANT CALIBRATION LABORATORY 123 Some Ave. City, State 12312-1231

# CALIBRATION CERTIFICATE FOR 25 foot Iron-Nickel Alloy Tape

Maker: LUFSTARTOYO
Serial No.: C917R
Lab Test No.: TI-14-056
NMI Test No.: 822/1234

#### SUBMITTED BY

#### YOUR CUSTOMER, INC. Customer's Address City, State

This tape has been compared to the standards of Compliant Calibration Laboratory. The horizontal distances between the centers of graduations of the indicated intervals have the following lengths at 20 °C (68 °F) when the tape is subjected to 10 pound horizontally applied tension and supported on a horizontal flat surface. The terminal points of the indicated intervals are the centers of the graduations at the edge of the tape ribbon nearest the observer when the zero graduation is to the left.

Interval	Length	Uncertainty	Interval	Length	Uncertainty
(feet)	(inches)	(inches)	(feet)	(inches)	(inches)
0 to 1	12.000 3	0.001 0	0 to 14	167.999 4	0.001 0
0 to 2	24.000 5	0.001 0	0 to 15	180.000 1	0.001 0
0 to 3	36.000 3	0.001 0	0 to 16	192.000 4	0.001 0
0 to 4	48.000 2	0.001 0	0 to 17	204.000 3	0.001 0
0 to 5	59.999 8	0.001 0	0 to 18	216.000 2	0.001 0
0 to 6	71.999 6	0.001 0	0 to 19	220.000 4	0.001 0
0 to 7	84.000 2	0.001 0	0 to 20	239.999 8	0.001 0
0 to 8	96.000 1	0.001 0	0 to 21	252.000 4	0.001 0
0 to 9	108.000 2	0.001 0	0 to 22	264.000 1	0.001 0
0 to 10	120.000 4	0.001 0	0 to 23	275.999 7	0.001 0
0 to 11	132.000 4	0.001 0	0 to 24	287.999 6	0.001 0
0 to 12	144.000 2	0.001 0	0 to 25	300.000 7	0.001 0
0 to 13	155.999 6	0.001 0			

The data in the above table only applies to those items specifically listed on this report.

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Reported lengths may be converted to catenary suspension support lengths by use of the "Equations for Metallic Tapes" (GMP 9).

The exact relationship between the International System of Units (SI) and the US customary units of length is one foot equals 0.304 8 meter.

#### **Uncertainty statement:**

The combined standard uncertainty includes the standard uncertainty reported for the standard, the standard uncertainty for the measurement process, the standard uncertainty of the coefficient of expansion, the standard uncertainty of the tension weights, the standard uncertainty of the graduated reticle, and a component of uncertainty to account for any observed deviations from NIST values that are less than surveillance limits. The combined standard uncertainty is multiplied by a coverage factor of 2.07, for 35 effective degrees of freedom, to provide an expanded uncertainty, which defines an interval having a level of confidence of approximately 95 percent. The expanded uncertainty presented in this report is consistent with the ISO/IEC Guide to the Expression of Uncertainty in Measurement (2008). The expanded uncertainty is not to be confused with a tolerance limit for the user during application.

#### **Traceability statement:**

The Standards of the Compliant Calibration Laboratory are traceable to the International System of Units (SI) through the National Institute of Standards and Technology, and are part of a comprehensive measurement assurance program for ensuring continued accuracy and measurement traceability within the level of uncertainty reported by this laboratory. The laboratory test number identified above is the unique report number to be used in referencing measurement traceability for artifacts identified in this report only.

#### **Supplemental Information**

#### **Description of artifacts submitted for testing:**

25 foot, iron-nickel alloy (invar), tape. A linear coefficient of thermal expansion of 0.000~000~4~ (°C (0.000 000 22 /°F) was assumed.

#### **Conditions of artifacts submitted for testing:**

Artifact showed evidence of proper handling. No bents or kinks were observed. A light film of oil covered the tape ribbon.

#### **Treatment of artifacts prior to testing:**

Artifact was cleaned with cheesecloth and alcohol to remove the protective oil film. The artifact was kept in the length laboratory, next to the standard, for 36 h to allow for thermal equilibrium.

#### **Equipment & Standards**

**Standard:** Length bench Calibrated: August 2014

**Tension weights:** 10 pound, Class F

**Microscope:** B&L, 0.002 inch graduated reticle

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Procedure used: NISTIRXXXX, SOP 11, Calibration of Metal Tapes, Bench Method

#### **Environmental conditions at time of test:**

Temperature: 20.1 °C to 20.2 °C Barometric Pressure: 752.7 mmHg

Relative Humidity: 43.35 % to 43.40 %

Date artifact was received: September 15, 2014

Date of test: September 17, 2014

Due date per customer's request: September 17, 2016 Date of report preparation: September 18, 2014

Josh Balani, Jr.

Test performed by: Josh Balani, Jr.

Metrology Expert

Member:

NCSLI NCWM ASQ ASTM

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#### GMP 9

#### Good Measurement Practice for Equations for Metallic Tapes

Table 1. Symbols used in this practice.

Symbol	Description					
L	Horizontal straight-line distance					
$L_S$	Calibrated length of the tape interval on a flat surface at $T_S$ and $P_S$					
$L_n$	Designated nominal length of the tape interval					
P	Applied tension					
$P_S$	Standard tension applied to the tape interval for $L_S$					
AE	Average cross-sectional area times Young's Modulus of Elasticity					
T	Temperature					
$T_S$	Standard temperature of the tape interval for $L_S$ , 20 °C (68 °F)					
α	Coefficient of thermal expansion of the tape ribbon					
W	Average weight per unit length of the tape ribbon					
N	Number of equidistant catenary suspensions					
$P_O$	Tension of accuracy (Tension of accuracy is defined as that tension which must be applied to the tape interval to produce its designated nominal length at the observed temperature of the tape.)					
$P_C$	Tension of accuracy while the tape is supported in catenary suspensions					

The horizontal straight-line distance, L, of a tape interval can be computed by the following equation for an applied tension, P, and temperature, T, when the tape is supported for N, number of equidistant catenary suspensions. When both a standard tape and a tape to be calibrated are placed in a catenary suspension condition, the horizontal straight line distance for the compared intervals for each tape must be calculated. Intervals can be compared only at the supported points along the tape length.

$$L = L_S + \frac{L_n (P - P_S)}{AE} + L_n (T - T_S) \alpha - \frac{L_n (W * L_n / N * P)^2}{24}$$
(1)

For simplicity, it is recommended to entirely support the tape on a horizontal flat surface for calibration. When the tape is supported entirely on a horizontal flat surface,  $N = \infty$ , and the general equation is reduced to:

$$L = L_S + \frac{L_n(P - P_S)}{AE} + L_n(T - T_S)\alpha$$
 (2)

The distance, L, of the tape interval can be set to the designated nominal length,  $L_n$ , for determining the tension of accuracy,  $P_O$ , while the tape is supported on a flat surface, by writing equation (2) as follows:

$$L_{n} = L_{S} + \frac{L_{n}(P_{O} - P_{S})}{AE} + L_{n}(T - T_{S})\alpha$$
 (3)

from which the following equations are developed:

$$P_O = P_S + \frac{AE(L_n - L_S)}{L_n} - AE(T - T_S)\alpha \tag{4}$$

or

$$P_S = P_O - \frac{AE(L_n - L_S)}{L_n} + AE(T - T_S)\alpha$$
 (5)

Substituting equation (5) for  $P_S$  in the general equation (1), we have

$$L = L_n + \frac{L_n (P - P_O)}{AE} - \frac{L_n (W * L_n / N * P_C)^2}{24}$$
 (6)

The distance, L, of the tape interval again can be set to the designated nominal length,  $L_n$ , for determining the tension of accuracy,  $P_C$ , while the tape is supported in catenary suspensions, by writing this equation as follows

$$L_n = L_n + \frac{L_n (P_C - P_O)}{AE} - \frac{L_n (W * L_n / N * P_C)^2}{24}$$
 (7)

from which:

$$P_C^2(P_C - P_O) = \frac{AE(W * L_n/N)^2}{24}$$
 (8)

or

$$P_{C}^{2} \left[ P_{C} - P_{S} - \frac{AE(L_{n} - L_{S})}{L_{n}} + AE(T - T_{S})\alpha \right] = \frac{AE(W * L_{n}/N)^{2}}{24}$$
(9)

The value of  $P_C$  can be solved by first determining the right side of the equal signs in equations (8) or (9), then substituting various values for  $P_C$  until the left side approaches the right side within the desired limits. If the value is greater than the right side, reduce the value of  $P_C$ .

# **Standard Operating Procedures**



#### SOP No. 10

#### Recommended Standard Operating Procedure for Calibration of Rigid Rules

#### 1. Introduction

#### 1.1. Purpose

This SOP describes the procedure to be followed for the calibration of rigid rules by comparison to the 18 inch metal rule issued to each State as the State reference standard which is calibrated in the interval from 1 inch to 13 inches. Equations provided here assume that the rigid rules being compared have the same coefficient of expansions.

The maximum length of rule that can be directly compared to the standard rule is 12 inches. However, longer lengths can be calibrated in segments of 12 inches with reference to the standard rule.

#### 1.2. Prerequisites

- 1.2.1. Valid calibration certificates with appropriate values and uncertainties must be available for all of the standards used in the calibration. All standards must have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST.
- 1.2.2. The ocular microscope used in measuring differences in lengths must be in good operating condition and must be equipped with a graduated reticle having established traceability.
- 1.2.3. The operator must be trained and experienced in precision measuring techniques with specific training in GMP 2, GMP 8, GMP 9, SOP 10, and SOP 29.
- 1.2.4. Laboratory facilities must comply with following minimum conditions to meet the expected uncertainty possible with this procedure. Equilibration of supporting surface, standard rule and rule to be calibrated requires environmental stability of the laboratory within the stated limits for a minimum of 24 hours before a calibration.

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Table 1. Environmental conditions.

Temperature Requirements During a Calibration	Relative Humidity (%)
Lower and upper limits: 18 °C to 22 °C Maximum changes: $< \pm 1$ °C $/$ 24 h and $\pm 0.5$ °C $/$ 1 h	$40 \text{ to } 60 \pm 10 / 4 \text{ h}$

#### 2. Methodology

#### 2.1. Scope, Precision, Accuracy

The accuracy of calibration of standard rules is possible to within 0.000 1 inch, provided suitable calibration of standards are obtained. The precision of intercomparison and the accuracy of the standard limit the uncertainty of calibration to 0.002 inch under optimum conditions.

#### 2.2. Summary

A rigid rule (test rule) is calibrated by comparing intervals on it with intervals of the standard rule. A reticle eye piece (ocular micrometer) is used for this purpose. Test rules longer than the standard rule may be calibrated in segments, using the last calibrated graduation as the zero graduation mark for the succeeding segments. Deviations from nominal that are calculated for each successive interval are cumulative.

#### 2.3. Apparatus/Equipment Required

- 2.3.1. Length bench or similar flat surface on which to lay the test rule and the standard rule.
- 2.3.2. Calibrated standard rule.
- 2.3.3. Microscope with calibrated graduated reticle having graduations spaced at intervals no greater than 0.002 inch.

#### 2.4. Symbols

Table 2. Symbols used in this procedure.

Symbol	Description
A, B, C, D	Calculated centers of graduations
S	Standard
X	Unknown
$X_{OL}$	Left edge of zero on unknown
$X_{OR}$	Right edge of zero on unknown
$S_{OL}$	Left edge of zero on standard
$S_{OR}$	Right edge of zero on standard
$X_m$	Center of graduation of unknown
$S_m$	Center of graduation of standard
$d_i$	Difference between <i>X</i> and <i>S</i> . The subscript <i>i</i> , designates the trial number, 1 or 2.
$L_X$	Calibrated length of test rule

#### 2.5. Procedure

- 2.5.1. Both the test rule and the standard rule must be in temperature equilibrium with the environment of the length laboratory.
- 2.5.2. Place the test rule and the standard rule on the length bench or similar flat surface, parallel to one another with the reading edges adjacent. It is not necessary to have the "zero" graduations in coincidence.

Ordinarily, this will require that one rule reads left-to-right (increasing units) while the other reads right-to-left (decreasing units). In this case, for convenience of calculation, the standard rule is placed in the right-to-left position. A worksheet to reverse the calibration on the standard rule is provided at the end of this SOP. Place shims under the rules as necessary so that the upper surfaces of both are in the same plane.

- 2.5.3. Place the microscope on the bench in the vicinity of the zero position and align it so that its reticle scale is parallel to the scales under test. (See GMP No. 2 for instructions on reading graduations with the microscope.)
  - 2.5.3.1. Observe readings of left and right sides of the corresponding graduation of the standard rule and record to the nearest 0.001 inch. The average of these readings will give a value for A.
  - 2.5.3.2. Observe readings of left and right sides of the corresponding graduation of the test rule and record to the nearest 0.001 inch. The average of these readings will give a value for B. If the

"zero" graduation is the end of the rule, only the reading for the end of the rule is taken.

- 2.5.4. Move the microscope successively to each graduation to be calibrated and record readings as described in 2.5.3.1 and 2.5.3.2 identifying the readings for the standard rule as C and for the unknown as D. CAUTION! Be certain that the rules are not disturbed during movement of the microscope.
- 2.5.5. Return the microscope to the zero graduation and repeat readings (2.5.3.1. and 2.5.3.2.) to verify that the rules have not been disturbed. Accept all previous data if the initial zero readings do not disagree with the final zero readings by more than 0.002 inch; otherwise, discard all previous data and repeat entire sequence of readings until a satisfactory set of zero readings are obtained.
- 2.5.6. Repeat 2.5.3. thru 2.5.5. for every additional segment of the test rule requiring calibration. This will require repositioning the rules, aligning the last measured interval graduation on the test rule with the initial graduation of the standard rule. Deviations from nominal for successive segments are cumulative.
- 2.5.7. Move the rules and reposition, making a second set of measurements as directed in 2.5.3, 2.5.4, and 2.5.5 for Trial 2.

#### 3. Calculations

3.1. Calculate the difference between the Initial (A, B) and Final (A<sub>1</sub>, B<sub>1</sub>) zero measurements to ensure that the initial and final readings agree within 0.002 inch. (2.5.5.)

$$|A - B - A_1 + B_1| \tag{1}$$

3.2. Calculate the center of graduation for unknown,  $X_m$ , and standard,  $S_m$ , for each set of measurements and each scale interval. The center of the "starting" graduation on the standard rule is A; for the test rule it is B. The subsequent centers of graduations for the standard and the test rule are recorded as C and D, respectively, for each interval measured. For each trial, the values for A and B will remain constant for all measured intervals on that trial, and will be used to compute the measured differences between the test rule and the standard.

$$S_m = \frac{(S_L + S_R)}{2} = A, C \tag{2}$$

$$X_{m} = \frac{(X_{L} + X_{R})}{2} = B, D \tag{3}$$

3.3. Calculate the differences,  $d_1$ , and  $d_2$  between X and S for each scale interval and each trial. (The subscript i is associated with the trial number.)

$$d_i = A - B - C_i + D_i \tag{4}$$

- 3.4. Obtain the calibrated length of the standard,  $L_S$ , for the measured interval from the calibration certificate for the standard rule.
- 3.5. Calculate the length of the test rule,  $L_X$ , for each interval measured using the mean of the measured differences.

$$L_X = \frac{(d_1 + d_2)}{2} + L_S \tag{5}$$

- 4. Measurement Assurance
  - 4.1. Duplicate the process with a suitable check standard or have a suitable range of check standards for the laboratory. See NISTIR 7383 SOP 17, SOP 20 and NISTIR 6969 SOP 30. Plot the check standard length and verify it is within established limits OR a *t*-test may be incorporated to check the observed value against an accepted value. The mean of the check standard observations is used to evaluate bias and drift over time. Check standard observations are used to calculate the standard deviation of the measurement process which contributes to the Type A uncertainty components.
  - 4.2 If a standard deviation chart is used for measurement assurance, the standard deviation of each combination of Trial 1 and Trial 2 is calculated and the pooled (or average) standard deviation is used as the estimate of variability in the measurement process. Note: the pooled or average standard deviation over time will reflect varying conditions of test items that are submitted to the laboratory. A standard deviation chart will be needed for each interval calibrated so that the variability resulting from transfers will be measured (the number of charts may be adjusted through analysis using F-tests).
- 5. Assignment of Uncertainty
  - 5.1. The limits of expanded uncertainty, U, include estimates of the standard uncertainty of the length standards used,  $u_s$ , estimates of the standard deviation of the measurement process,  $s_p$ , and estimates of the effect of other components associated with this procedure,  $u_o$ . These estimates should be combined using the root-sum-squared method (RSS), and the expanded uncertainty, U, reported with

a coverage factor to be determined based on degrees of freedom, which if large enough will be 2, (k = 2), to give an approximate 95 percent level of confidence. See NISTIR 6969, SOP 29 (Standard Operating Procedure for the Assignment of Uncertainty) for the complete standard operating procedure for calculating the uncertainty.

- 5.1.1 The expanded uncertainty for the standard, U, is obtained from the calibration report. The combined standard uncertainty,  $u_c$ , is used and not the expanded uncertainty, U, therefore the reported uncertainty for the standard will usually need to be divided by the coverage factor k. When transfers are used,  $u_s$  for values after the transfer are dependent and cumulative. See NISTIR 6969, SOP 29 for handling of dependent uncertainties.
- 5.1.2. The standard deviation of the measurement process,  $s_p$ , is taken from a control chart for a check standard or standard deviation charts. See NISTIR 7383, SOP 17, SOP 20, and NISTIR 6969, SOP 30.
- 5.1.3. Uncertainty associated with bias,  $u_d$ . Any noted bias that has been determined through analysis of control charts and round robin data must be less than limits provided in SOP 29 and included if corrective action is not taken. See SOP 29 for additional details
- 5.1.4. Other standard uncertainties usually included at this calibration level include uncertainties associated with the ability to read the graduated reticle, only part of which is included in the process variability due to parallax and visual capabilities, and uncertainties associated with the graduations of the reticle.

Table 3. Example uncertainty budget table.

Component	Description	Reference
$u_s$	Standard uncertainty for standards	Calibration report, divide by k
$S_p$	Standard uncertainty for the process	Measurement assurance process; range charts
$u_{gr}$	Standard uncertainty for graduated reticle	Must be assessed experimentally or from a calibration certificate
$u_d$	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u <sub>res</sub>	Standard uncertainty due to resetting of the rules	Must be assessed experimentally; if an interval based standard deviation chart is used u <sub>res</sub> will be included in the control chart standard deviation value.
$u_{o}$	Standard uncertainty for other factors	

#### 6. Report

Report results as described in SOP No. 1 Preparation of Calibration Certificates.

# Appendix A

Rigid Rule Calibration Data Sheet

Date			E	Environmental parameters						
Metro	ologist			Before After Unc/ability t		ility to me	asure			
Test l	No.		Temperature			°C		°C		
$S_p$		in	Pressure			mmHg		mmHg		
df			Humidity			%		%		
	Based on NISTIR 6969, SOP 29, Appendix A at 95.45 % probability distribution: k factor									

	ID	Range	
S			
X			

			Tri	al 1				Trial 2	
Rule	Graduation	Left	Right		Center of	Left	Right		Center of
					graduation				graduation
S				A =				A =	
X				B =				B =	
- C				<u> </u>					
$S_1$				$C_1 =$				$C_1 =$	
$X_1$				$D_1 =$				$D_1 =$	
		$d_1 = 1$	$A-B-C_1+$	$D_1 =$		$d_2 = A$	$-B-C_1$	$+ D_1 =$	
			$d = (d_1 + d_2)$				•	$L_S =$	
					average $d + L_S =$			5	
$S_2$				$C_2 =$				$C_2 =$	
$X_2$				$D_2 =$				$D_2 =$	
		$d_1 = d_2$	$A-B-C_2+$	$D_2 =$		$d_2 = A$	$-B-C_2$	$+ D_2 =$	
			$\frac{1}{d} = (d_1 + d_2)$			W <sub>2</sub> 11	<u> </u>	$L_S =$	
		Tiverage			average $d + L_S =$			23	
			zungur orr	1, Z <sub>A</sub>	areinge a : 25				
$S_3$				$C_3 =$				$C_3 =$	
$X_3$				$D_3 =$				$D_3 =$	
		d. –	$A-B-C_3+$	. D. –		$d_2 - \Delta$	_ R _ C .	+ D <sub>2</sub> -	
			$\frac{d - B - C_3}{d = (d_1 + d_2)}$			$u_2 - n$	$-B-C_3$	$L_S =$	
		Tiverage			average $d + L_S =$			L <sub>S</sub> –	
			Length of 2	$L_X =$	average a + Ly =				
S				$A_1 =$				$A_1 =$	
X				$B_1 =$				$\mathbf{B}_1 =$	
				Trial	1: Absolute differ	ence betwe	en initial		
	and final zero measurements = $ A - B - A_1 + B_1 $ =								
						0.002 inch		If YES	, accept all data.
				Trial	2: Absolute differ				, - T
			and		ero measurements				
						0.002 inch		If YES	, accept all data.

Form A-2. Worksheet to Reverse the Calibration of the Rigid Rule Standard

1 01111 71 2. V	VOIRSHEEL TO INC VEISO	tiit emileimileii	01 1110	11810 11010 2 10110			
Nominal							
Length of	Calc	Calculations to be performed					
Interval	Carc	ulations to be pe		u	(inches)		
(inches)							
1	$L_1 = C_{13} - C_{12} =$		ı				
2	$L_2 = C_{13} - C_{11} =$		1				
3	$L_3 = C_{13} - C_{10} =$		1				
4	$L_4 = C_{13} - C_9 =$		ı				
5	$L_5 = C_{13} - C_8 =$		1				
6	$L_6 = C_{13} - C_7 =$		1				
7	$L_7 = C_{13} - C_6 =$		1				
8	$L_8 = C_{13} - C_5 =$		1				
9	$L_9 = C_{13} - C_4 =$		1				
10	$L_{10} = C_{13} - C_3 =$	-	1	_			
11	$L_{11} = C_{13} - C_2 =$		1				
12	$L_{12} = C_{13} - C_1 =$		-				

To reverse the calibration on the State standard rigid rule, subtract the lengths for each calibrated interval from the length from the 1 inch to the 13 inch graduation.

#### Example:

Length from 13 to 12 = Length from 1 to 13 – Length from 1 to 12 Length from 13 to 12 = 11.999 8 – 10.999 7 = 1.000 1 inch

#### **SOP No. 11**

#### Recommended Standard Operating Procedure for Calibration of Metal Tapes Bench Method

#### 1. Introduction

#### 1.1. Purpose

Metal tapes are used by contractors, surveyors, and others for building layouts, measurement of land areas, establishment of land boundaries, and similar purposes. Inaccuracies in such measurements can cause structural misalignments, boundary controversies, and other problems. The test method described here provides a procedure to calibrate such tapes to four decimal places in the case of measurements in customary units (inches), and to 6 decimal places for measurements expressed in SI units (meters). The calibrated length bench is used as the standard.

#### 1.2. Prerequisites

- 1.2.1. Valid calibration certificates with appropriate values and uncertainties must be available for all of the standards used in the calibration. All standards must have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST.
- 1.2.2. The ocular microscope used in measuring differences in lengths must be in good operating condition and must be equipped with a calibrated graduated reticle having established traceability.
- 1.2.3. The operator must be trained and experienced in precision measuring techniques with specific training in GMP 2, GMP 8, GMP 9, SOP 11, and SOP 29.
- 1.2.4. Laboratory facilities must comply with following minimum conditions to meet the expected uncertainty possible with this procedure. Equilibration of length bench and tapes requires environmental stability of the laboratory within the stated limits for a minimum of 24 hours before a calibration

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Table 1. Environmental conditions.

Temperature Requirements During a Calibration	Relative Humidity (%)
Lower and upper limits: 18 °C to 22 °C Maximum changes: $< \pm 1$ °C / 24 h and $\pm 0.5$ °C / 1 h	40 to 60 ± 10 / 4 h

#### 2. Methodology

#### 2.1. Scope, Precision, Accuracy

The precision and accuracy attainable depend upon the care exercised in aligning the tape on the length bench, and the skill acquired in the use of an optical micrometer to measure scale differences.

#### 2.2. Summary

The tape to be calibrated is laid over the bench scale and sufficient tension is applied to insure that it lies flat on the bench. Differences between the graduation on the tape and that of the bench scale are measured using an ocular micrometer. The temperature of the tape is observed, and corrections applied, to calculate the length at the reference temperature of 20 °C using formulae or tables which are in Appendix B. Tapes longer than the length bench may be calibrated in segments, using the last calibrated graduation as the zero graduation mark for the succeeding segments. Typical lengths to be tested on a 100 foot tape when using the 16 foot length bench are: every foot through 10 feet and then 15, 20, 30, 40, 45, 50, 60, 70, 75, 80, 90, and 100 foot lengths.

#### 2.3. Apparatus/Equipment Required

- 2.3.1. Calibrated thermometers with sufficiently small resolution, stability, and uncertainty, capable of indicating temperatures in the range of 15 °C to 30 °C, and accurate to  $\pm$  0.5 °C.
- 2.3.2. Calibrated 5 meter (16 foot) length bench.
- 2.3.3. Microscope with calibrated graduated reticle having graduations spaced at intervals no greater than 0.002 inch.
- 2.3.4. Clamps, fabric tape in lengths required to connect tension weights to the tape being calibrated, and weights to apply an appropriate tension to the tape under calibration. (See C.3.)

#### 2.4. Symbols

Table 2. Symbols used in this procedure.

Symbol	Description
S	Standard
X	Unknown
$d_{0i}$	Initial zero difference
$d_{0f}$	Final zero difference
$X_{0L}$	Left edge of zero on unknown
$X_{0R}$	Right edge of zero on unknown
$S_{0L}$	Left edge of zero on standard
$S_{\mathit{OR}}$	Right edge of zero on standard
$X_m$	Center of graduation of unknown
$S_m$	Center of graduation of standard
d	Difference of interval between <i>X</i> and <i>S</i>
t	Average of initial and final temperatures
$t_{corr}$	Temperature correction
$L_n$	Nominal length of tape interval under test in inches
$L_X$	Calibrated length of tape interval under test in inches
$\alpha$	Linear coefficient of thermal expansion for the standard bench (0.00001063 / °C)
β	Linear coefficient of thermal expansion for the tape (0.00001160 / °C)
AE	Cross-sectional area times Young's modulus of elasticity
$Q_0$	Lower load (tension) applied to the tape
$Q_I$	Higher load (tension) applied to the tape
$L_0$	Length of tape under load $Q_0$
$L_1$	Length of tape under load $Q_I$

#### 2.5. Procedure

- 2.5.1. Inspect the tape to ensure that no kinks, dents, or other damage are present that will affect the accuracy of the tape
- 2.5.2. Clean the tape and bench by first wiping with a soft cloth, and then with a soft cloth saturated with alcohol to remove protective oil film. (See Appendix C, Section C.1.)
- 2.5.3. Place the tape, under the clamp, at the zero end of the bench, and adjust so the starting mark on the tape is near the zero graduation on the length bench. When the ring is part of the measuring tape, a special clamp must be used. See Appendix C.5 for further information on this matter.
- 2.5.4. Lay the tape flat on the bench. The tape should extend well beyond the end roller of the bench to permit tension to be applied. Slide one end of a fabric strap onto another tape clamp. Fasten this tape clamp to the tape on the portion that extends below the end roller. Hang the tension weight from the

bottom of the fabric strap. Check to see that the tape is lying straight on the bench and parallel to the bench scale. Adjust, if necessary. Apply tension using a weight of 10 pounds, unless other tension is required (See Appendix C.3.) When a tape is shorter than the 5 m length bench, a fabric strap is used to effectively lengthen the tape so that the tension weight is properly suspended below the rollers of the length bench.

- 2.5.5. Lay two thermometers (See 2.3.1.) on the bench and tape at intervals of 1/3 and 2/3 of the length of the bench to determine its initial and final temperatures at the time of the test.
- 2.5.6. Adjust the tape clamp on the zero end of the bench so that the starting graduation on the tape coincides with the center of the zero graduation of the bench. Note: Some tapes are calibrated using the one foot mark as the reference point, rather than the zero graduation. In this case, align the 1 foot mark of the tape with the zero graduation mark of the bench.
- 2.5.7. Check all alignments and coincidence of the graduations down the length of the bench before proceeding with calibration. Use the lateral adjustments at the left end of the bench to facilitate alignment. Caution! Take care that tape is not touched or disturbed during the following sequence of measurements. Record all observations on a suitable data sheet.
- 2.5.8. Record the temperatures indicated by the two thermometers.
- 2.5.9. Place the microscope on the bench in the vicinity of the zero position and align it so that its reticle scale is parallel to the tape under test. (See GMP No. 2 for instructions on reading graduations with the microscope.)
  - 2.5.9.1. Observe readings of left and right sides of zero graduation of tape and record to the nearest 0.001 inch.
  - 2.5.9.2. Observe readings of left and right sides of zero graduation of bench and record to the nearest 0.001 inch.
- 2.5.10. Move microscope successively to each graduation that needs to be calibrated and record readings similarly as in 2.5.9.1 and 2.5.9.2.
- 2.5.11. Return microscope to the zero graduation and repeat readings (2.5.9.1. and 2.5.9.2.) to verify that the tape has not moved. Accept all previous data if the zero readings have not changed by more than 0.001 inch; otherwise, discard all previous data and repeat entire sequence of readings until a satisfactory set of zero readings are obtained.
- 2.5.12. Loosen the tape by removing the tension weights, move the tape, hang the

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tension weights back on the fabric strap, and realign the zero marks on the tape and the bench to coincidence.

- 2.5.13. Repeat 2.5.8 thru 2.5.12 for every 15 foot (5 meter) section of the tape requiring calibration. This will require repositioning the tape, aligning the last measured interval graduation on the tape with the zero graduation of the bench.
- 2.5.14. Make a second set of measurements as directed in 2.5.8 to 2.5.13.
- 2.5.15. After the final measurement is taken and accepted, record the temperatures indicated by the two thermometers.
- 2.5.16. After all measurements are completed; apply a thin film of oil to the tape.
- 3. Calculations
  - 3.1. Calculate the Initial Zero Difference,  $d_{0i}$  (2.5.8.)

$$d_{0i} = \frac{(X_{0L} + X_{0R} - S_{0L} - S_{0R})}{2} \tag{1}$$

3.2. Calculate the Final Zero Difference,  $d_{0f}(2.5.10.)$ 

$$d_{0f} = \frac{(X_{0L} + X_{0R} - S_{0L} - S_{0R})}{2} \tag{2}$$

3.3. Calculate the center of graduation for unknown,  $X_m$ , and standard,  $S_m$ , for each set of measurements and each scale interval.

$$X_m = \frac{(X_L + X_R)}{2} \tag{3}$$

$$S_m = \frac{(S_L + S_R)}{2} \tag{4}$$

3.4. Calculate the difference, d, between X and S for each scale interval.

$$d = X_m - S_m \tag{5}$$

3.5. Obtain the correction to the standard,  $C_S$ , for the measured interval from the calibration certificate for the length bench scale.

3.6. Calculate the temperature correction,  $t_{corr}$ .

$$t_{corr} = L_n[(t-20)(\alpha-\beta)] \tag{6}$$

3.7. Calculate a correction,  $C_X$ , for each trial and each scale interval.

$$C_X = d + C_S + t_{corr} (7A)$$

Be sure to include cumulative errors from previous intervals when transfers are made. (For example, corrections for two prior intervals added for the case when a third interval is calibrated.)

$$C_X = d + C_S + t_{corr} + C_{x'} + C_{x''}$$
 (7B)

- 3.8. Calculate and report the mean,  $\overline{C_X}$  of the two corrections for each interval.
- 3.9. Calculate the length of tape under the 10 pound load.

$$L_X = L_n + \overline{C_X} \tag{8}$$

- 4. Measurement Assurance
  - 4.1. Duplicate the process with a suitable check standard or have a suitable range of check standards for the laboratory. See NISTIR 7383 SOP 17, SOP 20 and NISTIR 6969 SOP 30. Plot the check standard length and verify it is within established limits OR a *t*-test may be incorporated to check the observed value against an accepted value. The mean of the check standard observations is used to evaluate bias and drift over time. Check standard observations are used to calculate the standard deviation of the measurement process which contributes to the Type A uncertainty components.
  - 4.2 If a standard deviation chart is used for measurement assurance, the standard deviation of each combination of Trial 1 and Trial 2 is calculated and the pooled (or average) standard deviation is used as the estimate of variability in the measurement process. Note: the pooled or average standard deviation over time will reflect varying conditions of test items that are submitted to the laboratory. A standard deviation chart will be needed for each interval calibrated (at least initially) so that the variability resulting from transfers will be measured.
- 5. Assignment of Uncertainty
  - 5.1. The limits of expanded uncertainty, U, include estimates of the standard uncertainty of the length standards used,  $u_s$ , estimates of the standard deviation of the measurement process,  $s_p$ , and estimates of the effect of other components associated with this procedure,  $u_o$ . These estimates should be combined using the

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root-sum-squared method (RSS), and the expanded uncertainty, U, reported with a coverage factor to be determined based on degrees of freedom, which if large enough will be 2, (k = 2), to give an approximate 95 percent level of confidence. See NISTIR 6969, SOP 29 (Standard Operating Procedure for the Assignment of Uncertainty) for the complete standard operating procedure for calculating the uncertainty.

- 5.1.1 The expanded uncertainty for the standard, U, is obtained from the calibration report. The combined standard uncertainty,  $u_c$ , is used and not the expanded uncertainty, U, therefore the reported uncertainty for the standard will usually need to be divided by the coverage factor k. When transfers are used,  $u_s$  for values after the transfer are dependent and cumulative. See NISTIR 6969, SOP 29 for handling of dependent uncertainties.
- 5.1.2. The standard deviation of the measurement process,  $s_p$ , is taken from a control chart for a check standard or standard deviation charts. (See SOP 17, SOP 20, and SOP 30)
- 5.1.3. Uncertainty associated with bias,  $u_d$ . Any noted bias that has been determined through analysis of control charts and round robin data must be less than limits provided in SOP 29 and included if corrective action is not taken. See SOP 29 for additional details
- 5.1.4. Uncertainty associated with temperature correction includes values for the linear coefficients of expansion for the Standard and Unknown, the accuracy of temperature measurements, and factors associated with potential gradients in measuring the temperature on the length bench and tape.
- 5.1.5. Other standard uncertainties usually included at this calibration level include uncertainties associated with the ability to read the graduated reticle, only part of which is included in the process variability due to parallax and visual capabilities, and uncertainties associated with the graduations of the reticle.

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Table 3. Example uncertainty budget table.

Component	Description	Reference
$u_s$	Standard uncertainty for standards	Calibration report, divide by k
Sp	Standard uncertainty for the process	Measurement assurance process; range charts
$u_{gr}$	Standard uncertainty for graduated reticle	Must be assessed experimentally or from a calibration certificate
$u_{tc}$	Standard uncertainty for temperature correction	Calibration certificate
$u_t$	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.1 °C
$u_{lce}(\alpha,\beta)$	Standard uncertainty for linear coefficient of expansion	5 % to 10 % of the coefficient of expansion value
$u_{tw}$	Standard uncertainty for tension weights	Calibration certificate
$u_d$	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u <sub>res</sub>	Standard uncertainty due to resetting of the tape	Must be assessed experimentally; may be included in the control chart standard deviation where present
$u_o$	Standard uncertainty for other factors	

# 6. Report

Report results as described in SOP No. 1 Preparation of Calibration Certificates.

# Appendix A

# Bench Method Data Sheet

Date			Environmental parameters					
Metro	ologist			Before	After		Unc/ability to mea	asure
Test 1	No.		Temperature			°C	°C	
$S_p$		in	Pressure			mmHg	mmHg	
df			Humidity			%	%	
Based on NISTIR 6969, SOP 29, Appendix A at 95.45 % probability distribution: k factor							r	

		ID	Range		Linear co	efficient of t	hermal exp	ansion
S	Bench		16 ft (5 m)		α	0.000 010	63	/°C
X	Tape				β	0.000 011	60	/°C
Tension		lb		Support				

Initial temperatures			Final temper	ratures		
$t_I$	°C	$t_3$		°C	Average temperature	°C
$t_2$	°C	$t_4$		°C		

Interv	al, ft	Twick		Initial Zero Graduations									
Bench	Tape	Trial		Tape			Bench						
		1	$X_{0L}$	$X_{0R}$		$S_{0L}$	$S_{0R}$						
		1									<u> </u>		
0	0	Initial z	ero, diff	erence b	etween.	X and $S$ ,	$d_{0i}$			in			
U	U	2	$X_{0L}$	$X_{0R}$		$S_{0L}$	$S_{OR}$						
			Init	ial zero,	differen	ce betw	een X an	$dS, d_{0i}$		in			
	Intervals after zero												
				Tape			Bench						
			$X_{IL}$	$X_{IR}$	$X_m$	$S_{IL}$	$S_{IR}$	$S_m$	d	$C_S$	$t_{corr}$	$C_X$	Range
		1											<u> </u>
		2											
										Ave	rage $C_X$		in
			$X_{IL}$	$X_{IR}$	$X_m$	$S_{IL}$	$S_{IR}$	$S_m$	d	$C_S$	$t_{corr}$	$C_X$	Range
		1											
		2											
										Avei	rage $C_X$		in
				Fina	l Zero (	<u>Gradua</u>	tions						
				Tape			Bench						
			$X_{0L}$	$X_{0R}$		$S_{0L}$	$S_{0R}$						
		1								1	T		
							een X an			in			
0	0	Differen	nce betw	een Init	ial Zero		al Zero,		101150	in			
					ı		$ d_{0f}  \leq 0.0$	)01 in?	If YES	s, accept	previou	s data.	
			$X_{0L}$	$X_{\theta R}$		$S_{0L}$	$S_{0R}$		 				
		2	E.	1	1: 00	1 .	17	1.6.1		1 -	I		
		D:cc					een X an			in			
		Differen	nce betw	een mit	iai Zero		al Zero,		IfVE	in	proviou	a data	
						$ a_{0i} $	$ d_{0f}  \le 0.0$	JUL III!		s, accept	previou	s uata.	

$$t_{corr} = L_n[(t-20)(\alpha-\beta)]$$

$$C_X = d + C_S + t_{corr}$$

$$C_X = d + C_{S'} + C_{X''}$$

$$C_X = d + C_S + t_{corr} + C_{X''} + C_{X''}$$

# Appendix B

Table B-1. Temperature Correction Factors for Calibration of Steel Tapes graduated in feet when linear coefficient of thermal expansion for the tape is 0.00001160 / °C. All values in inches x  $10^{-4}$ 

L <sub>n</sub>						Tem	perature	e, °C				
Feet	19	20	21	22	23	24	25	26	27	28	29	30
1	0.1	0.0	-0.1	-0.2	-0.3	-0.5	-0.6	-0.7	-0.8	-0.9	-1.0	-1.2
2	0.2	0.0	-0.2	-0.5	-0.7	-0.9	-1.2	-1.4	-1.6	-1.9	-2.1	-2.3
3	0.3	0.0	-0.3	-0.7	-1.0	-1.4	-1.7	-2.1	-2.4	-2.8	-3.1	-3.5
4	0.5	0.0	-0.5	-0.9	-1.4	-1.9	-2.3	-2.8	-3.3	-3.7	-4.2	-4.7
5	0.6	0.0	-0.6	-1.2	-1.7	-2.3	-2.9	-3.5	<b>-4</b> .1	-4.7	-5.2	-5.8
6	0.7	0.0	-0.7	-1.4	-2.1	-2.8	-3.5	-4.2	-4.9	-5.6	-6.3	-7.0
7	0.8	0.0	-0.8	-1.6	-2.4	-3.3	-4.1	-4.9	-5.7	-6.5	-7.3	-8.1
8	0.9	0.0	-0.9	-1.9	-2.8	-3.7	<b>-</b> 4.7	-5.6	-6.5	-7.4	-8.4	-9.3
9	1.0	0.0	-1.0	-2.1	-3.1	-4.2	-5.2	-6.3	-7.3	-8.4	<b>-</b> 9.4	-10.5
10	1.2	0.0	-1.2	-2.3	-3.5	-4.7	-5.8	-7.0	-8.1	-9.3	-10.5	-11.6
15	1.7	0.0	-1.7	-3.5	-5.2	-7.0	-8.7	-10.5	-12.2	-14.0	-15.7	-17.5
20	2.3	0.0	-2.3	<b>-</b> 4.7	-7.0	-9.3	-11.6	-14.0	-16.3	-18.6	-21.0	-23.3
30	3.5	0.0	-3.5	-7.0	-10.5	-14.0	-17.5	-21.0	-24.4	-27.9	-31.4	-34.9
40	4.7	0.0	-4.7	-9.3	-14.0	-18.6	-23.3	-27.9	-32.6	-37.2	-41.9	-46.6
45	5.2	0.0	-5.2	-10.5	-15.7	-21.0	-26.2	-31.4	-36.7	<b>-</b> 41.9	-47.1	-52.4
50	5.8	0.0	-5.8	-11.6	-17.5	-23.3	-29.1	-34.9	-40.7	-46.6	-52.4	-58.2
60	7.0	0.0	-7.0	-14.0	-21.0	-27.9	-34.9	<b>-</b> 41.9	-48.9	-55.9	-62.9	-69.8
70	8.1	0.0	-8.1	-16.3	-24.4	-32.6	<b>-</b> 40.7	-48.9	-57.0	-65.2	-73.3	-81.5
75	8.7	0.0	-8.7	-17.5	-26.2	-34.9	-43.7	-52.4	-61.1	-69.8	-78.6	-87.3
80	9.3	0.0	-9.3	-18.6	-27.9	-37.2	-46.6	-55.9	-65.2	-74.5	-83.8	-93.1
90	10.5	0.0	-10.5	-21.0	-31.4	-41.9	-52.4	-62.9	-73.3	-83.8	-94.3	-104.8
100	11.6	0.0	-11.6	-23.3	-34.9	-46.6	-58.2	-69.8	-81.5	-93.1	-104.8	-116.4

For example: For  $L_n = 20$  feet at 25 °C, the temperature correction is -0.001 16 inches.

Table B-2. Temperature Correction Factors for Calibration of Steel Tapes graduated in meters when linear coefficient of thermal expansion for the tape is 0.00001160 /  $^{\circ}$ C. All values in meters x  $10^{-6}$ 

L <sub>n</sub>						Т	emperatur	e, °C				
Meters	19	20	21	22	23	24	25	26	27	28	29	30
0.1	0.1	0.0	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-1.0
0.2	0.2	0.0	-0.2	-0.4	-0.6	-0.8	-1.0	-1.2	-1.4	-1.6	-1.7	-1.9
0.3	0.3	0.0	-0.3	-0.6	-0.9	-1.2	-1.5	-1.7	-2.0	-2.3	-2.6	-2.9
0.4	0.4	0.0	-0.4	-0.8	-1.2	-1.6	-1.9	-2.3	-2.7	-3.1	-3.5	-3.9
0.5	0.5	0.0	-0.5	-1.0	-1.5	-1.9	-2.4	-2.9	-3.4	-3.9	-4.4	-4.8
0.6	0.6	0.0	-0.6	-1.2	-1.7	-2.3	-2.9	-3.5	-4.1	-4.7	-5.2	-5.8
0.7	0.7	0.0	-0.7	-1.4	-2.0	-2.7	-3.4	<b>-4</b> .1	-4.8	-5.4	<b>-6</b> .1	-6.8
0.8	0.8	0.0	-0.8	-1.6	-2.3	-3.1	-3.9	<b>-</b> 4.7	-5.4	-6.2	-7.0	-7.8
0.9	0.9	0.0	-0.9	-1.7	-2.6	-3.5	-4.4	-5.2	-6.1	-7.0	-7.9	-8.7
1	1.0	0.0	-1.0	-1.9	-2.9	-3.9	-4.8	-5.8	-6.8	-7.8	-8.7	-9.7
2	1.9	0.0	-1.9	-3.9	-5.8	-7.8	-9.7	-11.6	-13.6	-15.5	-17.5	-19.4
3	2.9	0.0	-2.9	-5.8	-8.7	-11.6	-14.6	-17.5	-20.4	-23.3	-26.2	-29.1
4	3.9	0.0	-3.9	-7.8	-11.6	-15.5	-19.4	-23.3	-27.2	-31.0	-34.9	-38.8
5	4.8	0.0	-4.8	<b>-</b> 9.7	-14.6	-19.4	-24.3	-29.1	-34.0	-38.8	-43.6	-48.5
10	9.7	0.0	-9.7	-19.4	-29.1	-38.8	-48.5	-58.2	-67.9	-77.6	-87.3	-97.0
15	14.6	0.0	-14.6	-29.1	-43.6	-58.2	-72.7	-87.3	-101.9	-116.4	-131.0	-145.5
20	19.4	0.0	-19.4	-38.8	-58.2	-77.6	-97.0	-116.4	-135.8	-155.2	-174.6	-194.0
25	24.3	0.0	-24.3	-48.5	-72.7	-97.0	-121.3	-145.5	-169.8	-194.0	-218.3	-242.5
30	29.1	0.0	-29.1	-58.2	-87.3	-116.4	-145.5	-174.6	-203.7	-232.8	-261.9	-291.0

For example: For  $L_n$  = 4 meters at 25 °C, the temperature correction is -0.000 019 4 meters.

## Appendix C

# Supplemental Information

## C.1. Cleaning

To clean a steel tape before calibration, first wipe the tape with a soft cloth; then with a soft cloth saturated with alcohol to remove the film of oil used to protect the tape.

After calibration, a thin film of light oil, such as sewing machine oil, should be applied to the tape for protection.

#### C.2. Tolerances

The tolerances for measuring tapes are those stated below.

Table C-1. Tolerances for a 30 meter tape.

Length Interval	Tolerances
0 through 15 meters	1.27 mm (0.050 inch)
15 through 22 meters	1.91 mm (0.075 inch)
22 through 30 meters	2.54 mm (0.100 inch)

C.2.1. The inaccuracy in the length of the ribbon, when supported on a horizontal surface with a tension of 10 pounds at a temperature of 20 °C (68 °F) shall not exceed 0.050 inch for the 75 foot length, and 0.100 inch for the 100 foot length.

# C.3. Tension Specifications

The length of a tape will be affected by the temperature of the tape, the tension applied to the tape, and the manner in which the tape is supported. The tape will stretch when tension is applied and will return to its normal length when the tension is removed, provided the tape has not been permanently deformed when it was stretched. The tensions, at which steel tapes are to be calibrated, expressed in terms of the load in kilograms (or pounds) to be applied to obtain the tension, are stated below. The loads should be accurate within 45 g (0.01 lb).

Table C-2. Tension to be applied, in terms of load.

Length Interval	Tension
Less than 10 m (25 ft)	2 kg (3.5 lb)
10 m through 30 m (25 ft through 100 ft)	5 kg (10 lb)
Greater than 30 m (100 ft)	10 kg (20 lb)

#### C.4. Methods of Support and Tension Considerations

Tapes calibrated in a State laboratory are normally supported on a horizontal surface

throughout the entire length of the tape. Also, tapes may be calibrated and used when supported in catenary types of suspension. In these cases, the tape is supported at equidistant points because the weight of the tape affects its length. The weight of the tape increases the tension and the "sag" causes the horizontal length to be shorter than when the tape is supported throughout its length. Equations are given in GMP No. 10 to compute the horizontal straight-line distance of a tape supported at N number of equidistant catenary suspensions and for computing the tension of accuracy, defined as the tension that must be applied to the tape interval to produce its designated nominal length at the observed temperature of the tape.

It is sufficient to provide the user of a steel tape with the calibrated length of the tape under standard temperature and tension conditions, the weight per unit-length of the tape, and the AE value for the tape, as might be requested. This information will enable the user to compute the values desired using the equations cited above.

#### C.5. Zero Reference Point

Metal measuring tapes submitted to a State laboratory for calibration normally will be made of steel. Generally, these tapes will have a ring on the end of the tape. For maximum calibration and measurement accuracy, a tape should have a blank end between the ring and the zero graduation. The zero graduation is then more precisely defined and more easily referenced for calibration and use.

Tapes that have the ring as part of the measuring portion of the tape are more difficult to calibrate than a tape with a blank end. When the ring is part of the measuring portion of the tape, the zero reference point shall be the outside end of the ring unless otherwise specified. It is more difficult to obtain a good zero reference setting on the ring due to its curvature and to parallax in reading the edge of the ring against a reference mark. Additionally, the ring may become permanently deformed in use and change the length of the tape. For these tapes the NIST normally calibrates from the 1-foot mark over the length of the tape, and then calibrates from the ring to the 1-foot mark. These values are reported separately so the user can obtain maximum measurement accuracy by using the 1-foot graduation as the zero reference point.

When the ring is part of the measuring range of the tape, a special holder for the ring is needed to clamp the tape to the length bench. A strap with an open area in the middle is needed to permit the end of the ring to be seen. The strap is slipped through the ring and the strap is clamped to the length bench. See example below.

Figure C-1. Strap, holder, and tape ring.



The edge of the tape to be calibrated (the reading edge) is the edge nearest the observer

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when the zero graduation is to the observer's left. When viewed through a microscope, some graduations will appear to have irregular edges. The portion of the graduation to be used for calibration is the portion of the graduation at the bottom of the reading edge of the tape. This provides a reference point that can be repeated and referenced by others. Do not attempt to estimate the 'best overall' edge of a graduation because this is not easily repeatable and cannot be accurately reproduced by other. If the graduations to be calibrated do not reach to the edge of the tape, the tape should not be calibrated.

# C.6. Temperature Considerations

The reference temperature for length calibrations is 20 °C. The length of the tape can be determined at any other temperature T by using the following equation.

$$L_T = L_{20} [1 + \alpha (T - 20)] \tag{C-1}$$

Since  $\alpha$  is always positive, it can be seen that for temperatures above 20 °C, the tape is longer than it is at 20 °C. For temperatures below 20 °C, the tape is shorter than it is at 20 °C.

If two length standards have different coefficients of expansion because they are made of different materials, the lengths of the tapes will change at different rates as the temperature changes. If two tapes are being compared at a temperature other than 20 °C, these lengths must be corrected back to 20 °C for calibration.

If two length standards are being compared and they have the same coefficient of expansion, then as the temperature changes the lengths will change by the same amount. Hence, if the standards are compared at a temperature other than 20 °C, the relationship between the two standards will be the same as if they were being compared at 20 °C; thus, no temperature correction is needed.

Table C-3. Coefficient of expansion of various length standards.

Type	/°C	/° <b>F</b>
Steel tape	11.60 x 10 <sup>-6</sup>	$6.45 \times 10^{-6}$
Length bench	10.63 x 10 <sup>-6</sup>	5.91 x 10 <sup>-6</sup>
Invar tape	$4.0 \times 10^{-7}$	$2.2 \times 10^{-7}$

## C.7. Invar tapes

Invar is an alloy of nickel and steel. Invar tapes are used to obtain measurements of greater accuracy than can be made with steel tapes, because invar has a very low coefficient of expansion. It has the added benefit of being very slow to tarnish from exposure to the atmosphere. However, invar tapes require very careful handling to prevent twists and kinks.

The load to be applied to an invar tape to maintain the desired tension is normally 20 lb. A load of 40 lb is used for the higher tension to determine the AE value (see C.8.). For

metric tapes, the normal load is 5 kg. A load of 10 kg is used to determine the AE value.

#### C.8. AE Value

The AE Value (area elongation value) for a tape is determined by first calibrating the tape under its normal tension. The load is then increased by 10 lb or 20 lb and one length interval is recalibrated to determine the length of the tape under the increased tension. The AE factor is computed with the following equation.

$$AE = \frac{Q_1 - Q_0}{L_1 - L_0} L_n \tag{C-2}$$

For example, a 100 foot tape is calibrated from 0 feet to 100 feet with a load of 10 lb applied to the tape with a resulting length of 99.992 feet. The load is increased to 20 lb and the new length is found to be 100.004 feet. The AE value is:

$$AE = \frac{(20 lb - 10 lb)}{(100.004 - 99.992)} 100 feet = \frac{1000 lb feet}{0.012 feet} = 83333 lb$$

It is recommended that the AE value be determined over the longest interval that is convenient to measure. This minimizes the error in the AE value because of the better readability of the change in length

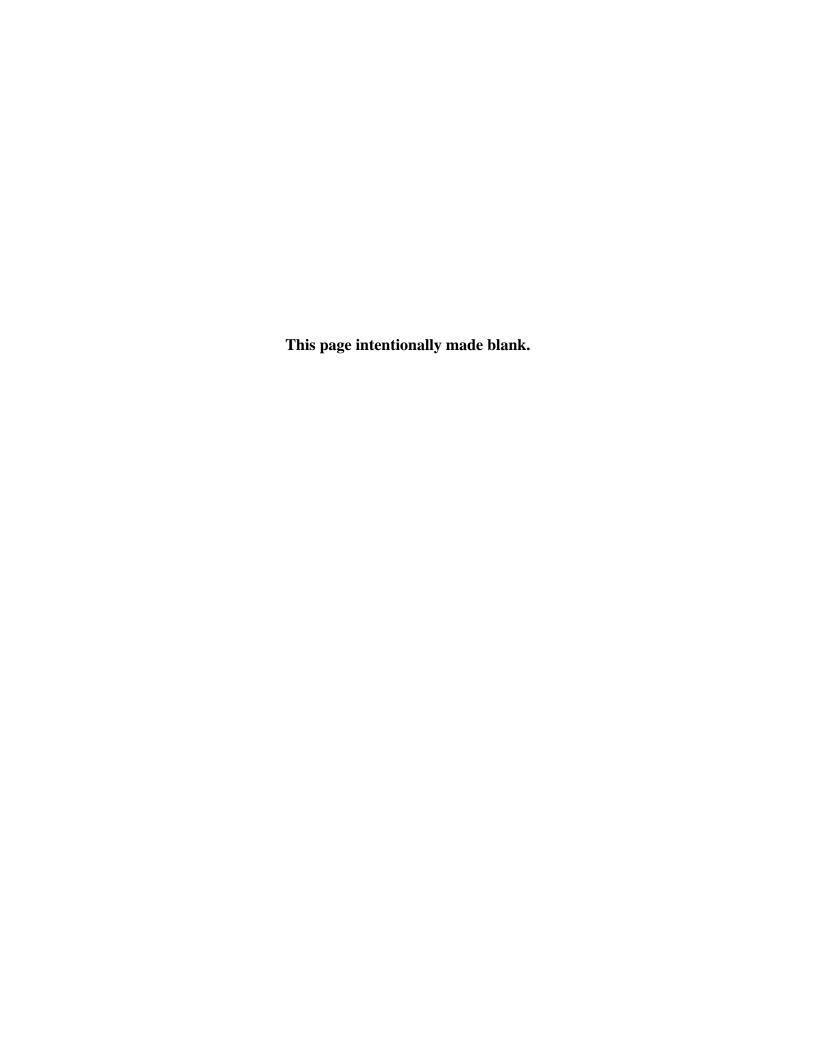
## C.9 Weight per Unit Length

The weight per unit length of a tape can be determined as follows:

- C.9.1. Weigh the tape and reel (or case).
- C.9.2. Remove the tape from the reel or case and weigh the empty reel (or case).
- C.9.3. Measure the length of any blank ends on the tape and add this to the measuring length.
- C.9.4. Correct for the weight of the loop on the tape. The weight of the loop that is normally used on steel tapes is approximately 2.5 grams.

The weight per unit length is the computed as follows.

$$Weight \ per \ Unit \ Length = \frac{Weight \ of \ loaded \ reel - weight \ of \ empty \ reel - weight \ of \ loop}{length \ of \ tape + length \ of \ blank \ ends}$$



#### **SOP No. 12**

# Recommended Standard Operating Procedure for Calibration of Metal Tapes<sup>1</sup> Tape-to-Tape Method

#### I. Introduction

# 1.1. Purpose

Metal tapes are used by contractors, surveyors, and others for building layouts, measurement of land areas, establishment of land boundaries, and similar purposes. The accuracy of the measurement often must be provable to 0.000 1 meter on a 100 meter tape (0.000 1 foot on a 100 foot tape), sometimes in a court of law. A significant feature of such proof is the knowledge of the accurate length of the tape used. Inaccuracies in such measurements can cause structural misalignments, boundary controversies, and other problems. The test method described here provides a procedure to calibrate such tapes by comparison to a calibrated standard tape.

### 1.2. Prerequisites

- 1.2.1. Valid calibration certificates with appropriate values and uncertainties must be available for all of the standards used in the calibration. All standards must have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST.
- 1.2.2. The operator must be trained and experienced in precision measuring techniques with specific training in GMP 8, GMP 9, SOP 12, and SOP 29.
- 1.2.3. Laboratory facilities must comply with following minimum conditions to meet the expected uncertainty possible with this procedure. Equilibration of tapes requires environmental stability of the laboratory within the stated limits for a minimum of 24 hours before a calibration.

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<sup>&</sup>lt;sup>1</sup> Many references are provided in in-pound units due to common use in U.S. weights and measures requirements. The International System of Units, SI, is the official system of units for metrological traceability.

Table 1. Environmental conditions.

Temperature Requirements During a Calibration	Relative Humidity (%)
Lower and upper limits: 18 °C to 22 °C Maximum changes: $< \pm 1$ °C / 24 h and $\pm 0.5$ °C / 1 h	40 to 60 ± 10 / 4 h

### 2. Methodology

#### 2.1. Scope, Precision, Accuracy

The method is applicable to the calibration of metal tapes such as used by surveyors, builders, and contractors. The overall length and specified intermediate lengths may be checked by the technique. The accuracy is limited by the accuracy of the calibration of the standard tape and by the precision of intercomparison. The latter should be within  $\pm 0.001$  foot, corresponding to 10 parts per million in a 100 foot tape. The method is limited to calibration of steel tapes (because the tension is specified as the result of a 10 pound load).

# 2.2. Summary

This procedure is based upon the method developed by C. Leon Carroll Jr., National Bureau of Standards, NBSIR 74-451, "Field Comparisons of Steel Surveyors' Tapes."

The tape to be calibrated is stretched out parallel to a standard tape on a reasonably flat surface. Paper scales (graph paper), graduated in millimeters are used at the zero and at each specified interval of calibration to measure any difference between the two tapes. The length of the tape undergoing calibration is computed from the known length of the standard tape and the observed differences between the test tape and the standard.

Calibrations are usually made at each 1 foot interval for the first 10 feet, and at each 10 foot interval to the full length of the tape.

## 2.3. Apparatus/Equipment Required

- 2.3.1. Standard tape, calibrated to within  $\pm$  0.001 foot.
- 2.3.2. Pieces of graph paper (10 x 10 to the centimeter, i.e., millimeter graduations), approximately 5 cm in width by 15 cm in height. Number the horizontal centimeter graduations, 0, 1, 2, etc.. Evaluate and document the range of use to within 0.1 mm using a calibrated rigid rule or a microscope with calibrated reticles to ensure that the graph paper has sufficient accuracy.

- 2.3.3. Equipment, see Figure 1, to apply tension to the tapes under test, consisting of the following.
  - 2.3.3.1. Spring scales (two); one capable of indicating a load of 10 pounds and the other capable of indicating a load of 20 pounds. The scales should be calibrated with an accuracy of  $\pm$  0.1 lb. This may be done by the arrangement shown in Figure 2 with results internally documented.
  - 2.3.3.2. Turnbuckles, suitable for adjusting tension on the tapes.
  - 2.3.3.3. Swivel connectors to prevent axial twisting of the tapes.
  - 2.3.3.4. Magnifying glass to aid in reading the graph paper values.
  - 2.3.3.5. Small weights (< 5 lb) to hold the tapes down flat to the paper.

Figure 1. Experimental Arrangement.

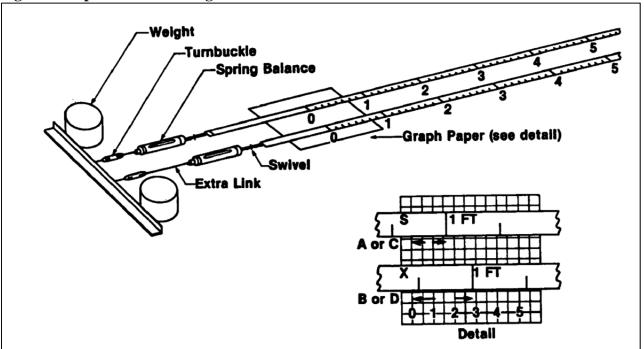
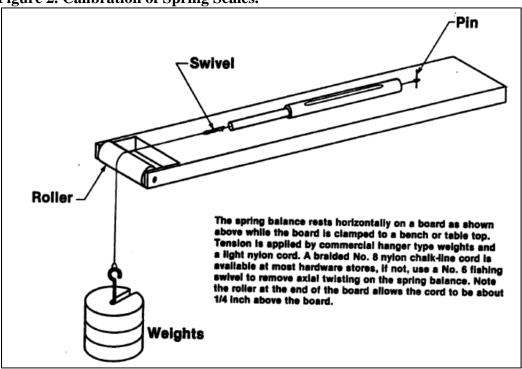


Figure 2. Calibration of Spring Scales.



## 2.4. Symbols

Table 2. Symbols used in this procedure.

Symbol	Description				
S	Length of the Standard Tape at the calibration interval				
$L_X$	Length of the Test Tape at the calibration interval				
cf	cf Conversion factor,				
	0.032 808 ft/cm for tapes graduated in feet				
	0.010 000 m/cm for tapes graduated in meters				
$L_n$	Nominal length of tape interval under test				
AE	Cross-sectional area times Young's modulus of elasticity				
$Q_0$	Lower load (tension) applied to the tape				
$Q_I$	Higher load (tension) applied to the tape				
$L_0$	Length of tape under load $Q_0$				
$L_{I}$	Length of tape under load $Q_1$				

#### 2.5. Procedure

- 2.5.1. Inspect the tapes to ensure that no kinks, dents, or other damage are present which will affect the accuracy of the calibration.
- 2.5.2. Clean the tapes by first wiping with a soft cloth, and then with a soft cloth saturated with alcohol to remove protective oil film.

- 2.5.3. Stretch the test tape and standard tape parallel to each other on a reasonably flat surface. The evenness of the surface is less important than the parallelism of the tapes. The two tapes should be separated by a constant distance of about 1 to 3 centimeters. The zero and test intervals of the two tapes should not be in coincidence but rather displaced by one or two centimeters, as indicated in the detail of Figure 1.
- 2.5.4. Use the turnbuckles to apply equal loads of 10 pounds to the two tapes as indicated by the spring scales. (Note the use of swivels to prevent axial twisting.)
- 2.5.5. Place a piece of graph paper under the zero interval and each interval to be calibrated as shown in the detail in Figure 1. Adjust the tapes and the paper so that the former are precisely aligned with the lateral rulings of the paper. It is convenient but not necessary for these to be the bold centimeter rulings of the paper. Note the amount of separation of the tapes at the zero interval and make corresponding adjustments at each calibration interval of interest. In this way, parallelism of the two tapes is easily verified.
- 2.5.6. Make final adjustment of tensions on the tapes and recheck for parallelism at all test points before taking the readings described in 2.5.7. Do not disturb during the measurement sequence.
- 2.5.7 Read the distances A, B, C, and D as indicated in the detail of Figure 1. Note that A and B are for the zero (or first) interval and are the same for all test intervals. C and D have subscripts 1, 2, etc. corresponding to the interval, *i*, calibrated. Make all readings to the center of the graduation mark tested and estimate to the closest 0.1 mm. Record all readings in centimeters.
- 2.5.8. Record all measurements as Trial 1.
- 2.5.9. Release the tension to the tapes and reapply it.
- 2.5.10. Displace each piece of graph paper a few millimeters, then readjust the load, check for parallelism, and record a second series of measurements as Trial 2.
- 2.5.11. Release the tension to the tapes and reapply it.
- 2.5.12. Displace each piece of graph paper a few millimeters, then readjust the load, check for parallelism, and record a second series of measurements as Trial 3.
- 2.5.13. After all measurements are completed; apply a thin film of oil to the tapes.

#### 3. Calculations

- 3.1. Calculate and record A B C + D for each trial, then record the value of R, the range of these values (difference of highest and lowest) for each scale interval. The range should not exceed 0.15 cm. Sum the values for A, B, C, D for the three trials to use when calculating the length, L, of each interval.
- 3.2. The value obtained from  $\Sigma A \Sigma B \Sigma C + \Sigma D$  must equal the sum of the column A B C + D, otherwise an error has been made in the calculations.
- 3.3. Calculate the length of the test tape at each calibration interval according to the following equation.

$$L_X = S + \frac{cf}{3} \Sigma (A - B - C + D) \tag{1}$$

#### 4. Measurement Assurance

- 4.1. Duplicate the process with a suitable check standard or have a suitable range of check standards for the laboratory. See NISTIR 7383 SOP 17, SOP 20 and NISTIR 6969 SOP 30. Plot the check standard length and verify it is within established limits OR a *t*-test may be incorporated to check the observed value against an accepted value. The mean of the check standard observations is used to evaluate bias and drift over time. Check standard observations are used to calculate the standard deviation of the measurement process which contributes to the Type A uncertainty components.
- 4.2 If a standard deviation chart is used for measurement assurance, the standard deviation of each combination of 3 Trials is calculated and the pooled (or average) standard deviation is used as the estimate of variability in the measurement process. Note: the pooled or average standard deviation over time will reflect varying conditions of test items that are submitted to the laboratory. A standard deviation chart will be needed for each interval calibrated (at least initially) so that the variability resulting from transfers will be measured.

# 5. Assignment of Uncertainty

5.1. The limits of expanded uncertainty, U, include estimates of the standard uncertainty of the length standards used,  $u_s$ , estimates of the standard deviation of the measurement process,  $s_p$ , and estimates of the effect of other components associated with this procedure,  $u_o$ . These estimates should be combined using the root-sum-squared method (RSS), and the expanded uncertainty, U, reported with a coverage factor to be determined based on degrees of freedom, which if large enough will be 2, (k = 2), to give an approximate 95 percent level of confidence. See NISTIR 6969, SOP 29 (Standard Operating Procedure for the Assignment of

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Uncertainty) for the complete standard operating procedure for calculating the uncertainty.

- 5.1.1 The expanded uncertainty for the standard, U, is obtained from the calibration report. The combined standard uncertainty,  $u_c$ , is used and not the expanded uncertainty, U, therefore the reported uncertainty for the standard will usually need to be divided by the coverage factor k.
- 5.1.2. The standard deviation of the measurement process,  $s_p$ , is taken from a control chart for a check standard or standard deviation charts. (See SOP 17, SOP 20, and SOP 30)
- 5.1.3. Uncertainty associated with bias,  $u_d$ . Any noted bias that has been determined through analysis of control charts and round robin data must be less than limits provided in SOP 29 and included if corrective action is not taken. See SOP 29 for additional details
- 5.1.4. Other standard uncertainties usually included at this calibration level include uncertainties associated with the ability to read the graph paper, only part of which is included in the process variability due to parallax and visual capabilities, and uncertainties associated with the graduations of the graph paper.

Table 3. Example uncertainty budget table.

Component	Description	Reference
$u_s$	Standard uncertainty for standards	Calibration report, divide by k
Sp	Standard uncertainty for the process	Measurement assurance process; range charts
$u_{gp}$	Standard uncertainty for graph paper	Must be assessed experimentally
$u_{tw}$	Standard uncertainty for the spring scales	Must be assessed experimentally or from a calibration certificate
$u_d$	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u <sub>res</sub>	Standard uncertainty due to resetting of the tapes	Must be assessed experimentally
$u_o$	Standard uncertainty for other factors	

## 6. Report

Report results as described in SOP No. 1 Preparation of Calibration Certificates.

# Appendix A

Tape-to-Tape Method Data Sheet

Date				E	Environmental parameters					
Metrologist			Before	After		Unc/ability to measure		asure		
Test	Test No.		Temperature			°C		°C		
$S_p$			in	Pressure			mmHg		mmHg	
df				Humidity			%		%	
	Based on NISTIR 6969, SOP 29, Appendix A at 95.45 % probability distribution: k factor									r

	ID		Range
S		·	
X			
Tension	lb	•	

Interval	Trial	A	В	С	D	A-B-C+D			
	1								
	2						$\frac{cf}{3}\Sigma(A-B-C+D)$	S	L
	2 3						3		
	Σ								
	, – ,			l	Range				
	1 1			1					
	1						$\frac{cf}{3}\Sigma(A-B-C+D)$	S	т.
	3						$\frac{1}{3}$ $\sum (A-B-C+D)$	5	L
	$\Sigma$								
	2				Range				
			•		Range				
	1						cf		
	3						$\frac{cf}{3}\Sigma(A-B-C+D)$	S	L
	3								
	Σ								
					Range				
	1						_		
	2						$\frac{cf}{3}\Sigma(A-B-C+D)$	S	L
	3						3		
	Σ								
			•	•	Range			•	•
	1								
	2						$\frac{cf}{3}\Sigma(A-B-C+D)$	S	L
	3			<del> </del>			3 2(11 15 0 1 15)	3	
	$\frac{\sigma}{\Sigma}$								
			1	1	Range			<u>I</u>	<u>I</u>
	1 1				1				
	1			-			$cf_{\Sigma(A-B-C+B)}$	C	,
	2						$\frac{cf}{3}\Sigma(A-B-C+D)$	S	L
	3								
	Σ				D				
					Range				

### Appendix B

# Supplemental Information

## B.1. Cleaning

To clean a steel tape before calibration, first wipe the tape with a soft cloth; then with a soft cloth saturated with alcohol to remove the film of oil used to protect the tape.

After calibration, a thin film of light oil, such as sewing machine oil, should be applied to the tape for protection.

#### B.2. Tolerances

The tolerances for measuring tapes are those stated below.

Table B-1. Tolerances for a 30 meter tape.

Length Interval	Tolerances		
0 through 15 meter	1.27 mm (0.050 inch)		
15 through 22 meter	1.91 mm (0.075 inch)		
22 through 30 meter	2.54 mm (0.100 inch)		

B.2.1. The inaccuracy in the length of the ribbon, when supported on a horizontal surface with a tension of 10 pounds at a temperature of 20 °C (68 °F) shall not exceed 0.050 inch for the 75 foot length, and 0.100 inch for the 100 foot length.

#### B.3. Tension Specifications

The length of a tape will be affected by the temperature of the tape, the tension applied to the tape, and the manner in which the tape is supported. The tape will stretch when tension is applied and will return to its normal length when the tension is removed, provided the tape has not been permanently deformed when it was stretched. The tensions, at which steel tapes are to be calibrated, expressed in terms of the load in kilograms (or pounds) to be applied to obtain the tension, are stated below. The loads should be accurate within 45 g (0.01 lb).

Table B-2. Tension to be applied, in terms of load.

Length Interval	Tension
Less than 10 m (25 ft)	2 kg (3.5 lb)
10 m through 30 m (25 ft through 100 ft)	5 kg (10 lb)
Greater than 30 m (100 ft)	10 kg (20 lb)

#### B.4. Methods of Support and Tension Considerations

Tapes calibrated in a State laboratory are normally supported on a horizontal surface

throughout the entire length of the tape. Also, tapes may be calibrated and used when supported in catenary types of suspension. In these cases, the tape is supported at equidistant points because the weight of the tape affects its length. The weight of the tape increases the tension and the "sag" causes the horizontal length to be shorter than when the tape is supported throughout its length. Equations are given in GMP No. 10 to compute the horizontal straight-line distance of a tape supported at N number of equidistant catenary suspensions and for computing the tension of accuracy, defined as the tension that must be applied to the tape interval to produce its designated nominal length at the observed temperature of the tape.

It is sufficient to provide the user of a steel tape with the calibrated length of the tape under standard temperature and tension conditions, the weight per unit-length of the tape, and the AE value for the tape, as might be requested. This information will enable the user to compute the values desired using the equations cited above.

#### B.5. Zero Reference Point

Metal measuring tapes submitted to a State laboratory for calibration normally will be made of steel. Generally, these tapes will have a ring on the end of the tape. For maximum calibration and measurement accuracy, a tape should have a blank end between the ring and the zero graduation. The zero graduation is then more precisely defined and more easily referenced for calibration and use.

Tapes that have the ring as part of the measuring portion of the tape are more difficult to calibrate than a tape with a blank end. When the ring is part of the measuring portion of the tape, the zero reference point shall be the outside end of the ring unless otherwise specified. It is more difficult to obtain a good zero reference setting on the ring due to its curvature and to parallax in reading the edge of the ring against a reference mark. Additionally, the ring may become permanently deformed in use and change the length of the tape. For these tapes the NIST normally calibrates from the 1 foot mark over the length of the tape, and then calibrates from the ring to the 1 foot mark. These values are reported separately so the user can obtain maximum measurement accuracy by using the 1 foot graduation as the zero reference point.

When the ring is part of the measuring range of the tape, a special holder for the ring is needed to clamp the tape to the length bench. A strap with an open area in the middle is needed to permit the end of the ring to be seen. The strap is slipped through the ring and the strap is clamped to the length bench. See example below.

Figure B-1. Strap, holder, and tape ring.



The edge of the tape to be calibrated (the reading edge) is the edge nearest the observer

when the zero graduation is to the observer's left. When viewed through a microscope, some graduations will appear to have irregular edges. The portion of the graduation to be used for calibration is the portion of the graduation at the bottom of the reading edge of the tape. This provides a reference point that can be repeated and referenced by others. Do not attempt to estimate the 'best overall' edge of a graduation because this is not easily repeatable and cannot be accurately reproduced by other. If the graduations to be calibrated do not reach to the edge of the tape, the tape should not be calibrated.

#### B.6. Temperature Considerations

No temperature correction is required, provided the test tape and the standard tape are at the same temperature and of the same material. This will be the case when the measurements are made inside a building. Tapes of the same color would be expected to attain the same temperature, even in sunlight. However, black and white tapes have shown temperature differences of as much as 8 °C when exposed to direct sunlight. In such cases, the temperature differences, even when measured, would be uncertain due to variability of exposure along the length of the tape. Accordingly, calibrations in the laboratory are preferred, when possible.

## B.7. Invar tapes

Invar is an alloy of nickel and steel. Invar tapes are used to obtain measurements of greater accuracy than can be made with steel tapes, because invar has a very low coefficient of expansion. It has the added benefit of being very slow to tarnish from exposure to the atmosphere. However, invar tapes require very careful handling to prevent twists and kinks.

The load to be applied to an invar tape to maintain the desired tension is normally 20 lb. A load of 40 lb is used for the higher tension to determine the AE value (see C.8.). For metric tapes, the normal load is 5 kg. A load of 10 kg is used to determine the AE value.

#### B.8. AE Value

The AE Value (area elongation value) for a tape is determined by first calibrating the tape under its normal tension. The load is then increased by 10 lb or 20 lb and one length interval is recalibrated to determine the length of the tape under the increased tension. The AE factor is computed with the following equation.

$$AE = \frac{Q_1 - Q_0}{L_1 - L_0} L_n \tag{B-1}$$

For example, a 100 foot tape is calibrated from 0 feet to 100 feet with a load of 10 lb applied to the tape with a resulting length of 99.992 feet. The load is increased to 20 lb and the new length is found to be 100.004 feet. The AE value is:

$$AE = \frac{(20 lb - 10 lb)}{(100.004 - 99.992)} 100 feet = \frac{1000 lb feet}{0.012 feet} = 83333 lb$$

It is recommended that the AE value be determined over the longest interval that is convenient to measure. This minimizes the error in the AE value because of the better readability of the change in length

B.9 Weight per Unit Length

The weight per unit length of a tape can be determined as follows:

- B.9.1. Weigh the tape and reel (or case).
- B.9.2. Remove the tape from the reel or case and weigh the empty reel (or case).
- B.9.3. Measure the length of any blank ends on the tape and add this to the measuring length.
- B.9.4. Correct for the weight of the loop on the tape. The weight of the loop that is normally used on steel tapes is approximately 2.5 grams.

The weight per unit length is the computed as follows.

$$Weight \ per \ Unit \ Length \ = \frac{Weight \ of \ loaded \ reel - weight \ of \ empty \ reel - weight \ of \ loop}{length \ of \ tape + length \ of \ blank \ ends}$$

#### **SOP 23**

# Recommended Standard Operations Procedure for Calibrations of PI Tapes Bench Method

#### 1. Introduction

## 1.1 Purpose

PI tapes are circumferential measuring tapes that are wrapped around a circular or cylindrical object and when read, indicate the diameter of the object. The tapes are manufactured for various diameter ranges and for either outside diameters (OD) or inside diameters (ID). The test method described here provides a procedure to calibrate such tapes to four decimal places in inches (diameter). A calibrated length bench (transfer standard, calibrated using SOP 11) is used as the standard.

## 1.2 Prerequisites

- 1.2.1 Valid calibration certificates with appropriate values and uncertainties must be available for all of the standards used in the calibration. All standards must have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST.
- 1.2.2. The ocular microscope used in measuring differences in lengths must be in good operating condition and must be equipped with a graduated reticle having established traceability.
- 1.2.3. The operator must be trained and experienced in precision measuring techniques with specific training in GMP 2, GMP 8, GMP 9, SOP 11, SOP 23, and SOP 29.
- 1.2.4. Laboratory facilities must comply with following minimum conditions to meet the expected uncertainty possible with this procedure. Equilibration of length bench and tapes requires environmental stability of the laboratory within the stated limits for a minimum of 24 hours before a calibration.

Table 1. Environmental conditions.

Temperature Requirements During a Calibration	Relative Humidity (%)
Lower and upper limits: 18 °C to 22 °C Maximum changes: < ± 1 °C / 24 h and ± 0.5 °C / 1 h	$40 \text{ to } 60 \pm 10 / 4 \text{ h}$

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# 2. Methodology

### 2.1. Scope, Precision, Accuracy

The precision and accuracy attainable depend upon the care exercised in aligning the tape on the length bench, and the skill acquired in the use of an optical micrometer to measure scale differences.

#### 2.2 Physical Characteristics

2.2.1. PI tapes are manufactured with corrections built into the tape where they will indicate the proper diameter without using extensive mathematical corrections. These corrections will have to be taken into consideration when calibrating a PI tape on a flat horizontal surface.

OD PI tapes are referenced to 20 °C with 2.25 kg (5 lb) tension (within  $\pm$  0.5 percent of the load, i.e., Class F mass standards are more than adequate.) applied. There is concern about the proper tension to apply to an ID PI tape since they are normally used under some degree of compression. Most tapes are made of 1095 spring steel, which has a coefficient of thermal expansion of 0.00000633 /°F.

Figure 1. Schematic drawing of Pi tape

The main body of the tape is graduated in inches and tenths of an inch with four subdivisions, each of which are equal to 0.025 inches.

The vernier (on the left end of the tape) has 25 graduations and each graduation equals 0.001 inch. Metric graduations are 0.01 mm on the vernier.

PI tapes are normally manufactured for specific ranges of diameters, such as 2" to 24", 24" to 48", 48" to 60", etc.
PI tapes are also available in metric units, such as 100 mm to 300 mm, 300 mm to 900 mm.

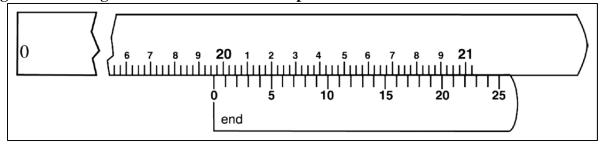
The vernier scale and the reading scale are off-set to allow proper alignment of both scales without causing unnatural stresses on the tape when it is wrapped around the circumference of an object.

As the tape is wrapped around the circumference of an object, the reading portion of the tape and the vernier come into alignment. The smallest graduation on the main body equals 0.5 mm (0.025 in). The line on the

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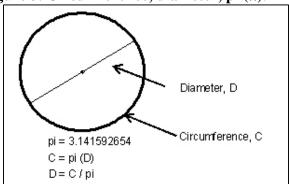
vernier scale which comes closest to lining up with a graduation on the main body is the value to be added to the reading. The vernier allows readings to be made to the nearest 0.001 in, as shown in Figure 2.

Figure 2. Reading the Vernier scale on a Pi tape.



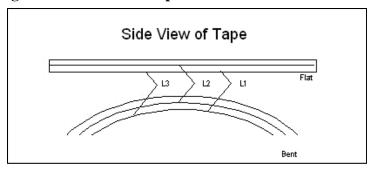
- 2.3. Fundamental Considerations
  - 2.3.1. The relationship of circumference, C, diameter, D, and pi  $(\pi)$  is  $C = \pi(D)$ ,  $D = C/\pi$ . See Figure 3 where  $\pi = 3.141592654$

Figure 3. Circumference, diameter, pi  $(\pi)$ .



2.3.2. The thickness of the tape will affect the measurements in two locations, as shown in Figure 4. L1 is the outside portion of the tape, which becomes longer when the tape is bent. L2 is the center of the tape (neutral axis). It remains the same length when the tape is bent. L3 is the inside portion of the tape and it compresses and becomes shorter when the tape is bent.

Figure 4. Side view of tape.



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2.3.3 Another point that must be taken into consideration is that the tape was tested for accuracy on a flat horizontal surface but now it is curved around an object. The tape becomes temporarily deformed but the amount of distortion is predictable using the modulus of elasticity. As shown in the diagram (Figure 4.), the outside portion of the tape stretches and the inside compresses but there is a point midway through the thickness of the tape which does not change in length and is called the neutral axis.

For all homogeneous beams with the same modulus of elasticity in tension and compression, the neutral axis passes through the center of the section. If the tape has a thickness of 0.010 inch, the distance from the neutral axis to the surface of the object is 0.005 inch. Therefore, the reading we obtained is not the distance on the outside surface of the tape but is actually the distance along the neutral axis of the tape. When we calculate a diameter, this is actually the diameter of the neutral axis of the tape. Since this thickness affects the diameter in two locations, the amount of correction that must be taken into account is 0.010 inches.

#### 2.4. Summary

The tape to be calibrated is laid over the bench scale and tension is applied to ensure that it lies flat on the bench. Differences between the graduation on the tape and that of the bench scale are measured using an ocular micrometer. The temperature of the tape is observed and corrections applied to the reference temperature of 20 °C. Tapes longer than the length bench may be calibrated in segments, using the last calibrated graduation as the zero graduation mark for the succeeding segments. PI tape graduations represent object diameters and may not directly coincide with length bench graduations.

### 2.5. Apparatus/Equipment Required

- 2.5.1. Calibrated thermometers with sufficiently small resolution, stability, and uncertainty, capable of indicating temperatures in the range of 15 °C to 30 °C, and accurate to  $\pm$  0.5 °C.
- 2.5.2. Calibrated 5 meter (16 foot) length bench.
- 2.5.3. Microscope with graduated reticle having graduations spaced at intervals no greater than 0.002 inch.
- 2.5.4. Clamps, fabric tape in lengths required to connect tension weights to the tape being calibrated, and weights to apply an appropriate tension to the tape under calibration. (See "Tension Specifications", in Appendix C.3 in SOP 11).

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### 2.6. Symbols

Table 2. Symbols used in this procedure.

Symbol	Description
S	Standard
X	Unknown
$d_{0i}$	Initial zero difference
$d_{0f}$	Final zero difference
$X_{0L}$	Left edge of zero on unknown
$X_{0R}$	Right edge of zero on unknown
$S_{0L}$	Left edge of zero on standard
$S_{0R}$	Right edge of zero on standard
$X_m$	Center of graduation of unknown
$S_m$	Center of graduation of standard
d	Difference of interval between <i>X</i> and <i>S</i>
t	Average of initial and final temperatures
$t_{corr}$	Temperature correction
$L_n$	Nominal length of tape interval under test in inches
$L_X$	Calibrated length of tape interval under test in inches
$\alpha$	Linear coefficient of thermal expansion for the standard bench (0.00001063 / °C)
β	Linear coefficient of thermal expansion for the tape (0.00001160 / °C)
$L_d$	Diameter length of interval
$\pi$	3.141592654

#### 2.7. Procedure

- 2.7.1. Inspect the tape to ensure that no kinks, dents, or other damage are present which will affect the accuracy of the tape.
- 2.7.2. Clean the tape by first wiping with a soft cloth, and then with a soft cloth saturated with alcohol to remove the protective oil film. (See Appendix C, Section C.1 for SOP 11).
- 2.7.3. Place the tape, under the clamp, at the zero end of the bench, and adjust so the starting mark on the tape is near the zero graduation on the length bench.
- 2.7.4. Lay the tape flat on the bench. The tape should extend well beyond the end roller of the bench to permit tension to be applied. Slide one end of a fabric strap onto another tape clamp. Fasten this tape clamp to the tape on the portion that extends below the end roller. Hang the tension weight from the bottom of the fabric strap. Check to see that the tape is lying straight on the bench and parallel to the bench scale. Adjust, if necessary. Apply tension using a weight of 2.25 kg (5 lb) unless other tension is specified. (See Appendix C.3 in SOP 11.) When a tape is shorter than the

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- 5 m length bench, fabric strap will be be used to effectively lengthen the tape so that the tension weight is properly suspended below the rollers of the length bench.
- 2.7.5. Lay two thermometers (see 2.4.1.) on the bench at intervals of one-third and two-thirds of the length of the bench to determine its temperature at the time of test
- 2.7.6. Adjust the tape clamp on the zero end of the bench so that the tape zero graduation coincides with the center of the zero graduation of the bench.
- 2.7.7. Check all alignments and coincidence of zero graduations before proceeding with the calibration. Use the lateral adjustments of the left end of the bench to facilitate alignment. Caution! Take care that the tape is not touched or disturbed during the following sequence of measurements. Record all observations on a suitable data sheet. (See SOP 11, Appendix A.)
- 2.7.8. Record the temperatures indicated by the two thermometers.
- 2.7.9. Place the ocular microscope on the bench in the vicinity of the zero position and align it so that its scale is parallel to the tape under test. (See GMP No. 2 for instructions on how to make readings.)
  - 2.7.9.1.Observe readings of left and right sides of zero graduation of tape and record to the nearest 0.001 inch. The mean of these values will be used to determine the center of the tape graduation.
  - 2.7.9.2.Observe readings of left and right sides of zero graduation of bench and record to the nearest 0.001 inch. The mean of these values will be used to determine the center of the bench graduation.
- 2.7.10. Move the ocular microscope successively to each graduation that is to be calibrated and record readings similarly as in 2.7.9.1 and 2.7.9.2.
- 2.7.11. Return the ocular microscope to the zero graduation and repeat readings to verify that the tape has not moved. Accept all previous data if the initial zero reading does not disagree with the final zero reading by more than 0.001 inch; otherwise, discard previous data and repeat entire sequence of readings until a satisfactory set is obtained.
- 2.7.12. Loosen the tape by removing the tension weights, move the tape, hang the tension weights back on the fabric strap, and realign the zero marks on the tape and bench to coincidence.

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- 2.7.13. Repeat 2.7.9. thru 2.7.12. for every 15 foot (5 meter) section of the tape requiring calibration. This will require repositioning the tape, aligning the last measured interval graduation on the tape with the zero graduation of the bench.
- 2.7.14. Make a second set of measurements as directed in 2.7.9 through 2.7.13.
- 2.7.15. After the final measurement is taken and accepted, record the temperatures indicated by the two thermometers.
- 2.7.16. After all measurements are completed, apply a thin film of oil to the tape.

#### 3. Calculations

3.1. Calculate the Initial Zero Difference,  $d_{0i}$  (2.6.9.)

$$d_{0i} = \frac{(X_{0L} + X_{0R} - S_{0L} - S_{0R})}{2} \tag{1}$$

3.2. Calculate the Final Zero Difference,  $d_{0f}(2.6.11.)$ 

$$d_{0f} = \frac{(X_{0L} + X_{0R} - S_{0L} - S_{0R})}{2} \tag{2}$$

3.3. Calculate the center of graduation for unknown,  $X_m$ , and standard,  $S_m$ , for each set of measurements and each scale interval.

$$X_m = \frac{(X_L + X_R)}{2} \tag{3}$$

$$S_m = \frac{(S_L + S_R)}{2} \tag{4}$$

3.4. Calculate the difference, *d*, between *X* and *S* for each scale interval.

$$d = X_m - S_m \tag{5}$$

- 3.5. Obtain the correction to the standard,  $C_S$ , for the measured interval from the calibration certificate for the length bench scale.
- 3.6. Calculate the temperature correction,  $t_{corr}$ .

$$t_{corr} = L_n[(t-20)(\alpha-\beta)]$$
(6)

3.7. Calculate a correction,  $C_X$ , for each trial and each scale interval.

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$$C_X = d + C_S + t_{corr} \tag{7}$$

- 3.8. Calculate and report the mean,  $\overline{C_X}$ , of the two corrections for each interval.
- 3.9. Calculate the length of tape under the 10 pound load.

$$L_X = L_n + \overline{C_X} \tag{8}$$

3.10. As appropriate, convert the flat length of the tape to an outside diameter (OD) measurement using the following formula:

$$L_{OD} = \frac{L_X}{\pi} - (tape\ thickness\ in\ inches) \tag{9}$$

3.11. As appropriate, convert the flat length of the tape to an inside diameter (ID) measurement using the following formula:

$$L_{ID} = \frac{L_X}{\pi} + (tape\ thickness\ in\ inches) \tag{9}$$

- 4. Measurement Assurance
  - 4.1. Duplicate the process with a suitable check standard or have a suitable range of check standards for the laboratory. See NISTIR 7383 SOP 17, SOP 20 and NISTIR 6969 SOP 30. Plot the check standard length and verify it is within established limits OR a *t*-test may be incorporated to check the observed value against an accepted value. The mean of the check standard observations is used to evaluate bias and drift over time. Check standard observations are used to calculate the standard deviation of the measurement process which contributes to the Type A uncertainty components.
  - 4.2 If a standard deviation chart is used for measurement assurance, the standard deviation of each combination of Trial 1 and Trial 2 is calculated and the pooled (or average) standard deviation is used as the estimate of variability in the measurement process. Note: the pooled or average standard deviation over time will reflect varying conditions of test items that are submitted to the laboratory. A standard deviation chart will be needed for each interval calibrated (at least initially) so that the variability resulting from transfers will be measured.
- 5. Assignment of Uncertainty
  - 5.1. The limits of expanded uncertainty, U, include estimates of the standard uncertainty of the length standards used,  $u_s$ , estimates of the standard deviation of the measurement process,  $s_p$ , and estimates of the effect of other components associated with this procedure,  $u_o$ . These estimates should be combined using the root-sum-squared method (RSS), and the expanded uncertainty, U, reported with a coverage factor to be determined based on degrees of freedom, which if large

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enough will be 2, (k = 2), to give an approximate 95 percent level of confidence. See NISTIR 6969, SOP 29 (Standard Operating Procedure for the Assignment of Uncertainty) for the complete standard operating procedure for calculating the uncertainty.

- 5.1.1 The expanded uncertainty for the standard, U, is obtained from the calibration report. The combined standard uncertainty,  $u_c$ , is used and not the expanded uncertainty, U, therefore the reported uncertainty for the standard will usually need to be divided by the coverage factor k.
- 5.1.2. The standard deviation of the measurement process,  $s_p$ , is taken from a control chart for a check standard or standard deviation charts. (See SOP 17, SOP 20, and SOP 30)
- 5.1.3. Uncertainty associated with bias,  $u_d$ . Any noted bias that has been determined through analysis of control charts and round robin data must be less than limits provided in SOP 29 and included if corrective action is not taken. See SOP 29 for additional details
- 5.1.4. Uncertainty associated with temperature correction includes values for the linear coefficients of expansion for the Standard and Unknown, the accuracy of temperature measurements, and factors associated with potential gradients in measuring the temperature on the length bench and tape.
- 5.1.5. Other standard uncertainties usually included at this calibration level include uncertainties associated with the ability to read the graduated reticle, only part of which is included in the process variability due to parallax and visual capabilities, and uncertainties associated with the graduations of the reticle.

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Table 3. Example uncertainty budget table.

Component	Description	Reference
$u_s$	Standard uncertainty for standards	Calibration report, divide by k
$S_p$	Standard uncertainty for the process	Measurement assurance process; range charts
$u_{gr}$	Standard uncertainty for graduated reticle	Must be assessed experimentally or from a calibration certificate
$u_{tc}$	Standard uncertainty for temperature correction	Calibration certificate
u <sub>t</sub>	Standard uncertainty for temperature	Handbook 143 accuracy guideline, 0.1 °C
$u_{lce}(\alpha,\beta)$	Standard uncertainty for linear coefficient of expansion	5 % to 10 % of the coefficient of expansion value
$u_{tw}$	Standard uncertainty for tension weights	Calibration certificate
$u_d$	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u <sub>res</sub>	Standard uncertainty due to resetting of the tape	Must be assessed experimentally
$u_o$	Standard uncertainty for other factors	

# 6. Report

Report results as described in SOP No. 1 Preparation of Calibration Certificates.

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# Appendix A

#### Bench Method Data Sheet

Date				Environmental parameters						
Metro	Metrologist			Before	After		Unc/abil	c/ability to measure		
Test 1	Test No.			Temperature			°C		°C	
$S_p$			in	Pressure			mmHg		mmHg	
df				Humidity			%		%	
Based on NISTIR 6969, SOP 29, Appendix A at 95.45 % probability distribution: k factor									•	

		ID		Range		Linear coefficient of thermal expansion			
S	Bench			16 ft (5 m)		α	0.000 010	63	/°C
X	Tape					β	0.000 011	60	/°C
Tension		lb			Support				

Initial temperatures			Final temper	ratures		
$t_I$	°C	$t_3$		°C	Average temperature	°C
$t_2$	°C	$t_4$		°C		

Interval, ft		T!-1	Initial Zero Graduations										
Bench	Tape	Trial	Tape Bench										
0	0	1	$X_{0L}$	$X_{0R}$		$S_{0L}$	$S_{0R}$						
		Initial 2	Initial zero, difference between $X$ and $S$ , $d_{0i}$							in	1		
		IIIItiai Z								111	+		
		2	$X_{0L}$	$\Lambda_{0R}$		$\mathcal{S}_{0L}$	$\mathcal{S}_{0R}$				1		
			Initial zero, difference between $X$ and $S$ , $d_{0i}$							in			
			Intervals after zero										
			Tape Bench										
			$X_{IL}$	$X_{IR}$	$X_m$	$S_{IL}$	$S_{IR}$	$S_m$	d	$C_S$	$t_{corr}$	$C_X$	Range
		1											
		2											
									Average $C_X$				in
			$X_{IL}$	$X_{IR}$	$X_m$	$S_{IL}$	$S_{IR}$	$S_m$	d	$C_S$	$t_{corr}$	$C_X$	Range
		1											
		2											
	Einel Zone Cue des dieses								1	Ave	rage $C_X$		in
		Final Zero Graduations						1					
			$X_{0L}$	Tape $X_{\partial R}$		$S_{0L}$	Bench $S_{0R}$	1					
0	0	1	$\Lambda_{0L}$	$\Lambda_{0R}$		$S_{0L}$	$\mathcal{S}_{0R}$						
		1	Fir	nal zero	differer	nce betw	een <i>X</i> ar	$dS d_{oc}$		in			
		Final zero, difference between $X$ and $S$ , $d_{0f}$ Difference between Initial Zero and Final Zero, $d_{0i}$ - $d_{0f}$								in	-		
		$ d_{0i} - d_{0f}  \le 0.001$ in?							If YES, accept previous data.				
		2	$X_{0L}$	$X_{0R}$		$S_{0L}$	$S_{0R}$						
			VI			Ü.	011		1				
			Final zero, difference between <i>X</i> and <i>S</i> , $d_{0f}$							in			
	•	Difference between Initial Zero and Final Zero, $d_{0i}$ - $d_{0f}$								in	1		
		$ d_{0i} - d_{0f}  \le 0.001 \text{ in}$ ?							If YE	S, accep	t previou	s data.	

$$t_{corr} = L_n [(t - 20)(\alpha - \beta)] \qquad C_X = d + C_S + t_{corr} \qquad L_X = L_n + \overline{C_X}$$

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