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COMMENTS ON UNITS IN MAGNETISM

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ABSTRACT

Suggestions are given on how to express magnetic quantities in SI units.

INTRODUCTION

Perusing the 1974 M^3 Conference Proceedings indicates that, at the present time, Systeme Internationale (SI) units are avoided by most leading scientists and engineers in the field of magnetism. Throughout the Proceedings, almost universal preference is displayed for the cgs electromagnetic system (or for the Gaussian system, which gives an equivalent description of magnetic quantities). However, usage of SI units in the field of magnetism will undoubtedly increase with time. One barrier to increased usage is the present lack of standardized and agreed upon relationships between magnetic quantities within the SI. In this paper we will tentatively propose notation and definitions for those relationships most frequently used by experimentalists, with the hope that this will help stimulate the magnetism community to make their views known on preferred definitions.

SOME CONSIDERATIONS ON THE TWO SYSTEMS

One major property of the Gaussian (and the cgs emu) system, considered an advantage by some and a disadvantage by others, is that B and H have the same numerical value in empty space. Changing to the SI, where not only do B and H have different units in empty space, but also different numerical magnitudes, puts one somewhat in the position of Casimir's[1] mythical tangenometrists who decided that, "The volumetric displacement of empty space - although equal to unity - had the dimension Archimedes per Euclid".

The SI is a "rationalized" system, whereas the Gaussian is unrationalized. Thus, when magnetic susceptibilities are converted between the two systems a factor of 4π is involved. Further factors of 10 are involved depending on whether volume, mass, or molar susceptibility is in question. This gives considerable latitude for errors and ambiguities in data compilations, handbooks, and treatises which attempt to convert existing numerical values to SI units, and numerous examples of such errors can be found. For example, in the recent treatise on magnetic materials by Heck[2], who endeavors to use SI units as much as possible, a table of paramagnetic susceptibilities apparently gives the rationalized mass susceptibility for Pt in cm^3/g, the unrationalized mass susceptibility for γ-Fe in cm^3/g, and the rationalized volume susceptibility for Li (dimensionless). Since these differences in units are not listed in the table, an unsuspecting user could easily be misled. As most commonly used with SI, the relation between B, H, and M is defined as $B=\mu_0(H+M)$, $\chi=M/H$. Some authors[3] exhibit the μ_0 associated with the SI explicitly by replacing H by B/μ_0, giving $\chi=\mu_0 M/B$. This is, of course, approximately correct for the small susceptibilities found in most diamagnetic and paramagnetic materials, but could be misapplied to superparamagnetic or ferromagnetic materials.

Table 1

Symbols and names for magnetic qnantities in SI and cgs, Gaussian (or cgs emu).

Symbol	Name: cgs emu	Name: SI
B	flux density magnetic induction	flux density (magnetic induction)
H	magnetic field strength	magnetic field strength
M	magnetization	magnetization
J	---	magnetic polarization
χ	volume susceptibility	---
κ	---	rationalized volume susceptibility
χ_ρ	mass susceptibility	---
κ_ρ	---	rationalized mass susceptibility
χ_{mole}	molar susceptibility	---
κ_{mole}	---	rationalized molar susceptibility
m	magnetic moment	magnetic moment
μ_B	Bohr magneton	Bohr magneton

RECOMMENDATIONS

In order to ease conversion from Gaussian (and cgs emu) to SI units, the *names*, definitions, and symbols for magnetic quantities should be standardized. This requires agreement within the magnetism community. Our current recommendations are summarized in the Tables.

Table 1 lists recommended symbols and names for magnetic quantities in SI and cgs emu. When using SI units to express susceptibility, we believe it would be useful to label it 'rationalized' and give it the symbol κ, reserving χ for the nonrationalized cgs emu system. What we have labeled the "volume susceptibility" in Table 1 is often referred to simply as just "susceptihility". The

TABLE 2

Corresponding equations in SI and cgs Gaussian (or cgs emu). In this table, F refers to force, W refers to the energy of a magnetic dipole in a field, w refers to the volume energy density. Other symbols are defined in Table 1.

Gaussian (or cgs emu)	SI	
$B = H + 4\pi M$	$B = \mu_0 (H + M)$	(1)
	$B = \mu_0 H + J$	(2)
$\chi = M/H$	$\kappa = M/H$	(3)
$F = \chi V H \frac{\partial H}{\partial x}$	$F = \mu_0 \kappa V H \, \partial H/\partial x$	(4)
$W = -mB\cos\theta$	$W = -mB\cos\theta$	(5)
$w = \frac{BH}{8\pi}$	$w = \frac{1}{2}BH$	(6)

Table 3. Conversion from Gaussian to S.I. Units

Multiply the Number for: Gaussian Quantity	Unit	by	To Obtain the Number for: SI Quantity	Unit
flux density, B	G	10^{-4}	flux density, B	T($\equiv$Wb/m^2$\equiv$Vs/m^2)
magnetic field strength, H	Oe	$10^3/4\pi$	magnetic field strength, H	A/m
volume susceptibility, χ	emu/cm^3(dimensionless)	4π	rationalized volume susceptibility, κ	dimensionless
mass susceptibility, χ_ρ	emu/g ($\equiv$cm^3/g)	$4\pi\cdot 10^{-3}$	rationalized mass susceptibility, κ_ρ	m^3/kg
molar susceptibility,* χ_{mole}	emu/mol ($\equiv$cm^3/mol)	$4\pi\cdot 10^{-6}$	rationalized molar susceptibility, κ_{mole}	m^3/mol
magnetization, M	G or Oe	10^3	magnetization, M	A/m
		$4\pi\cdot 10^{-4}$	magnetic polarization, J	T
magnetization, 4πM	G or Oe	$10^3/4\pi$	magnetization, M	A/m
		10^{-4}	magnetic polarization, J	T
magnetization, M	μ_B/atom or μ_B/form. unit, etc.**	1	magnetization, M	μ_B/atom or μ_B/form. unit, etc.**
magnetic moment of a dipole, m	erg/G	10^{-3}	magnetic moment of a dipole, m	J/T ($\equiv$Am^2)
demagnetizing factor, N	dimensionless	$1/4\pi$	rationalized demagnetizing factor, N	dimensionless

* Also called atomic susceptibility. Molar susceptibility is preferred since atomic susceptibility has also been used to refer to the susceptibility per atom.

** "Natural" units, independent of unit system. However, the numerical value of the Bohr magneton does depend on the unit system.

introduction of the symbol J (where $J = \mu_0 M$) in the SI is useful due to the controversy[4] over whether one should define $B=\mu_0(H+M)$ or $B=\mu_0 H+M$. Further, the symbol J and the associated name 'magnetic polarization', are in current use[5].

Table 2 compares several of the more important equations in the field of magnetism. Eqs. (1) and (2) define the recommended usage of the symbols M and J in SI, as mentioned above. In both Gaussian and SI units, the volume susceptibility, defined by Eq. (3), is dimensionless and is the ratio of M to H, (both with magnitudes which will change by a factor of 4π upon rationalization). Eq. (4) gives the force on a material placed in a magnetic field gradient. (This equation involves certain assumptions and is most useful for small samples with small susceptibilities.) Eq. (5) gives the energy of a (point) magnetic moment in a magnetic field, and Eq. (6) gives the volume energy density associated with a magnetostatic field.

Table 3 gives numerical factors for converting between the two unit systems. The conversions for flux density, B, and ~~susceptibility, χ and κ,~~ are independent of the conventions adopted, i.e. whether $B=H+M$, $B=\mu_0 H+M$, etc. Other conversions will depend on these conventions. One problem for those not thoroughly familar with current magnetic unit usage is that 'emu' is not really a unit but rather a flag to describe the unit system being used. Often, though not always, a dimensional anaylsis on susceptibility units may be performed if 'emu' is replaced by cm^3. Another problem which undoubtedly gives further difficulty to the uninitiated is the variety of units used for the same quantity in the Gaussian system. For example, in the 1974 M^3 conference proceedings we find the following units used for "magnetization": G, Oe, emu/g, μ_B/atom, B.M./FORMULA UNIT, μ_B/impurity, G cm^3/g, emu/cm^3, and emu: and for "susceptibility" we find the following variety of units: emu/g, emu/cm^3, emu/mole, emu/g kOe, emu/gm-At. V, and emu/Oe mole.

To convert an equation given in the Gaussian system to the corresponding equations in the SI, Table 4 can often be useful. For example, in the Gaussian system the magnetization can be considered as the magnetic moment per unit volume,

TABLE 4

Substitutional Symbols for Equations

To convert an equation in Gaussian units to a corresponding equation in SI, replace the symbols in the column labled Gaussian by the combination of symbols in the column labeled SI. Symbols representing quantities with units involving only volume, force, energy, and length transform directly.

Gaussian Quality	Gaussian symbol	SI symbol
flux density	B	$\sqrt{4\pi/\mu_0}\ B$
magnetic field	H	$\sqrt{4\pi\mu_0}\ H$
magnetization	M	$\sqrt{\mu_0/4\pi}\ M$, or $\sqrt{1/4\pi\mu_0}\ J$
volume susceptibility	χ	$(1/4\pi)\kappa$
magnetic moment	m	$\sqrt{\mu_0/4\pi}\ m$

Table 5

Important Fundamental Constants

Quantity	Gaussian	SI
μ_o, permeability of free space	1 (dimensionless)	$4\pi \times 10^{-7}$ H/m $\left(\equiv \frac{Tm}{A} \equiv \frac{Vs}{Am}\right)$
μ_B, Bohr magneton	$9.274078(36) \times 10^{-21}$ $\frac{erg}{G}$	$9.274078(36) \times 10^{-24}$ $\frac{J}{T}$ $\left(\equiv Am^2\right)$
μ_N, Nuclear magneton	$5.050824(20) \times 10^{-24}$ erg/G	$5.050824(20) \times 10^{-27}$ J/T

$$M = \frac{m}{V} \tag{1}$$

where M is the magnetization in G, m is an appropriate magnetic moment in erg/G, and V is an appropriate volume in cm^3. Using the substitutions of Table 4 we have

$$\sqrt{\frac{\mu_o}{4\pi}}\ M = \frac{\sqrt{\mu_o/4\pi}\ m}{V} \tag{2}$$

which reduces to

$$M = \frac{m}{V} \tag{3}$$

Thus the magnetization in our suggested SI system can also be considered as the magnetic moment per unit volume, with magnetization in A/m, dipole moment in J/T, and volume in m^3. Table 5 gives the numerical value of three important fundamental magnetic constants in the two unit systems, and Table 6 compares demagnetizing coefficients, N, for several familiar shapes, where the defining equation for N for both systems is

$$H = H_o - NM \tag{4}$$

with H the magnetic field strength within the magnetized body and H_o the applied magnetic field strength.

DISCUSSION

There are currently several systems of electromagnetic equations that may be used with SI units[4,6]. In order to apply SI units in the field of magnetism with a minimum of confusion, agreement and uniformity in symbols and definitions would be extremely helpful. Here we have suggested such a set of symbols and definitions which covers most of the quantities of current interest to those who publish in the M^3 proceedings. We would emphasize that this set is possibly not the one most desirable to a majority of magneticians. It was selected as one which appeared to us to be most in conformity with current international usage. An example of an alternative system would be the SI analog of a rationalized 'Gaussian' system. In such a system B, H, and M would be given the relation B=H+M, and H and M would also have units of 'tesla'. This would overcome the problem, troublesome to some, of giving B and H different numerical values in a vacuum. Another possibility, favored by Coleman[7], is the "SI electric" in which one defines $B=H+\mu_o M$ as the general relationship between B, H and M. In this system the unit for B and H is tesla and the unit for M is Am^{-1}, again giving B and H the same numerical value in empty space. However, both of these systems have the advantage (or disadvantage) found in the Gaussian system that B and H have the same numerical value in empty space.

Many of the details listed in the Tables given here depend on the particular SI relationship adopted for magnetic quantities. However, whichever relationships are adopted, the conversions for magnetic induction and susceptibility listed in Table 3 will remain valid, and the use of the proper unit and of the term 'rationalized' whenever susceptibility values are given in SI units would do much to reduce the possibility for errors and misinterpretation.

TABLE 6

Demagnetizing Coefficients, N, for homogeneous isotropic bodies of various shapes.

Shape	N Gaussian (unrationalized)	N SI (rationalized)
‖ to axis of long needle	0	0
⊥ to axis of long needle	2π	1/2
sphere	$4\pi/3$	1/3
⊥ to plane of a thin disc	4π	1

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