

# Detecting Anomalies in Time and Frequency Data

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# Outline

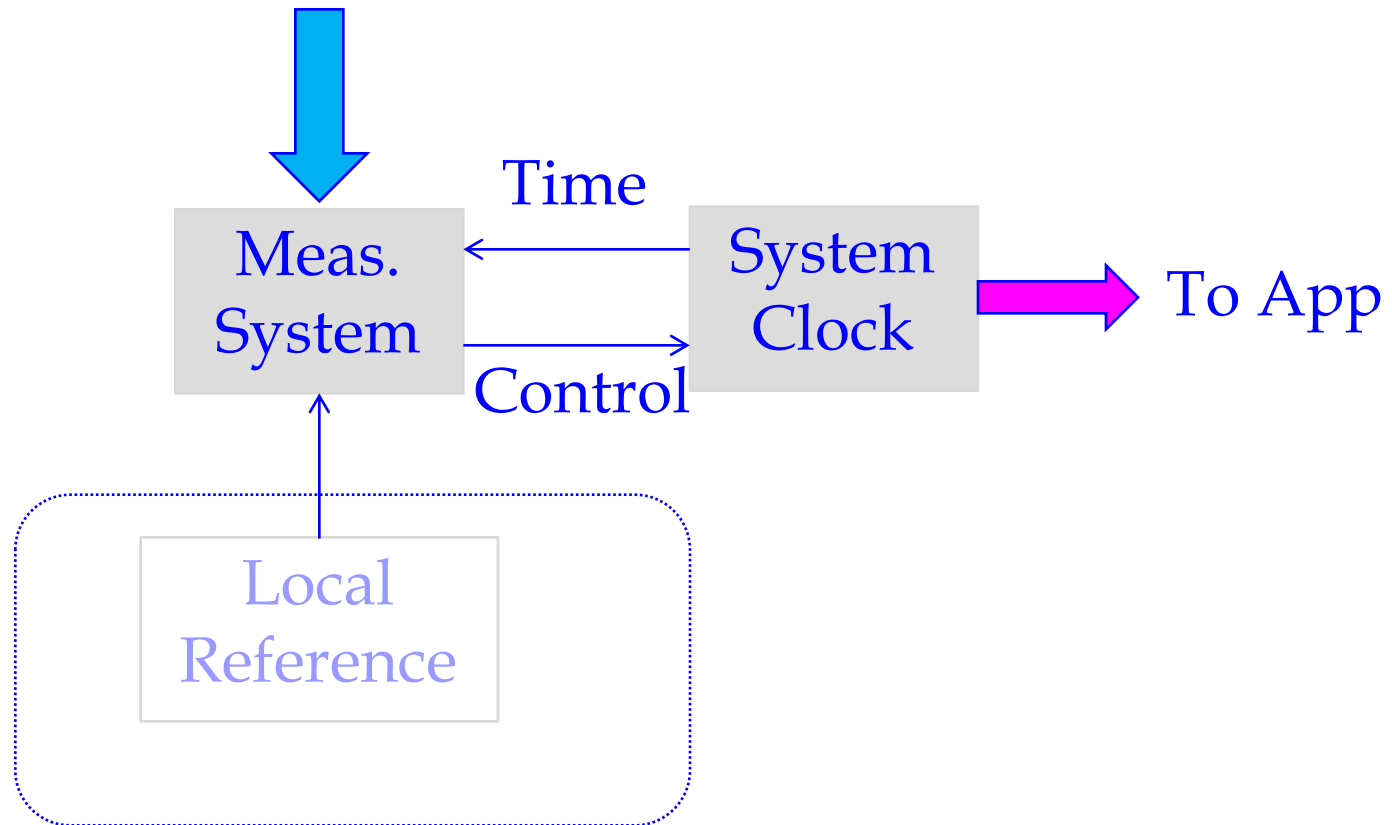
- Types of Anomalies
- Measurement configurations
- Analysis methods
- Examples
- Summary

# Types of anomalies

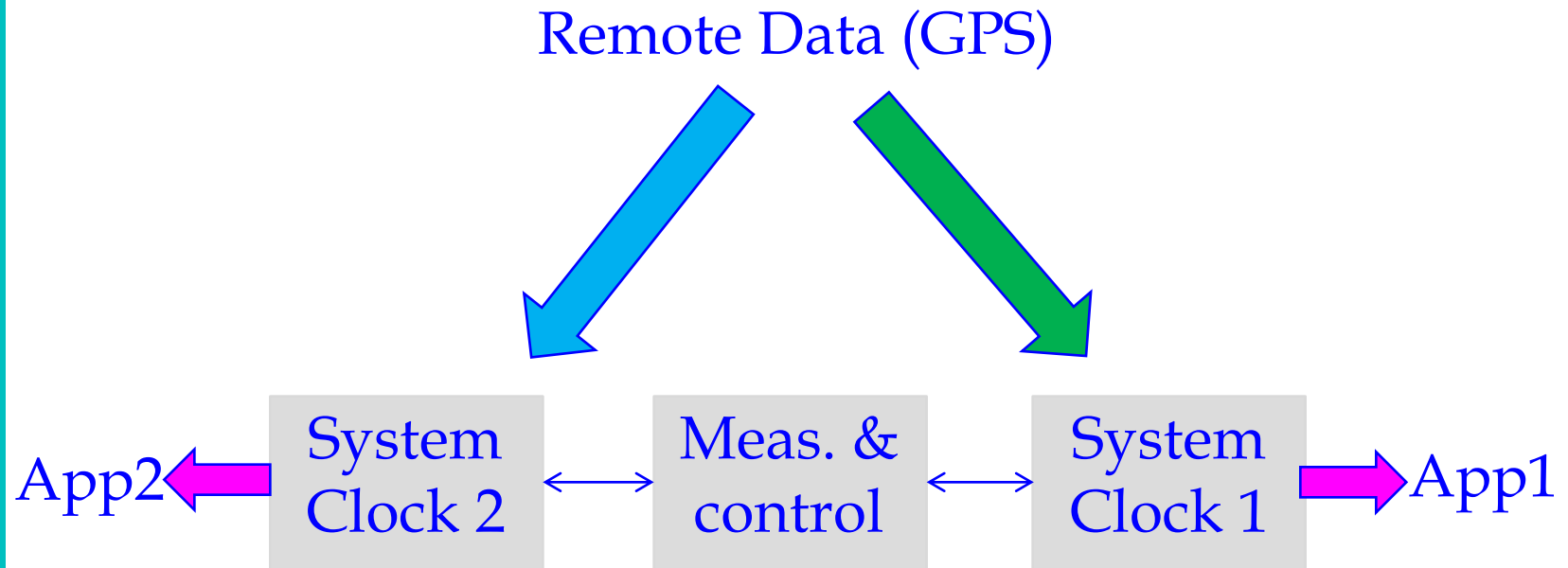
- Time Step
- Frequency Step
- Missing Data

# Measurement Configuration - 1

Remote Data (GPS)



# Measurement Configuration - 2



The Common-View Method

# Data Analysis - 1

Recursive model for measurement channel  $j$ :

$$\hat{x}_j(t_k) = x_j(t_{k-1}) + y_j(t_{k-1})\tau$$

$$\varepsilon_j(t_k) = X_j(t_k) - \hat{x}_j(t_k)$$

$X_j$  is measured time difference

$x, y$  are model time and frequency

$\tau$  is time interval between measurements

*Parameters are constant between measurements*

# Data Analysis - 2

Time variance for each measurement:  $\xi_j$  and  $\eta_j\tau^s$

Choice of  $\tau$  important!

Case 1: The good news:

$$\varepsilon_j(t_k) \ll \text{All noise terms}$$

# Error Models - 1

## Time Step:

For all channels:

$$\varepsilon_j(t_k) > \text{noise terms}$$

$$\varepsilon_j(t_{k-1}) < \text{noise terms}$$

$$\text{Magnitude} = \langle \varepsilon_j(t_k) \rangle$$

## Frequency Step:

For all channels:

$$\varepsilon_j(t_k) > \text{noise terms}$$

$$\varepsilon_j(t_{k-1}) > \text{noise terms}$$

$$\text{Magnitude} = \langle \varepsilon_j(t_k) / \tau \rangle$$



# Error Models - 2

Local clock broken:

For all channels:

$$\varepsilon_j(t_k) \gg \text{noise terms}$$

$$\varepsilon_j(t_{k-1}) \gg \text{noise terms}$$

$$\varepsilon_j(t_{k-2}) \gg \text{noise terms}$$

Reference Channel  $m$  is broken:

For channel  $m$ :

$$\varepsilon_m(t_k) > \text{noise terms}$$

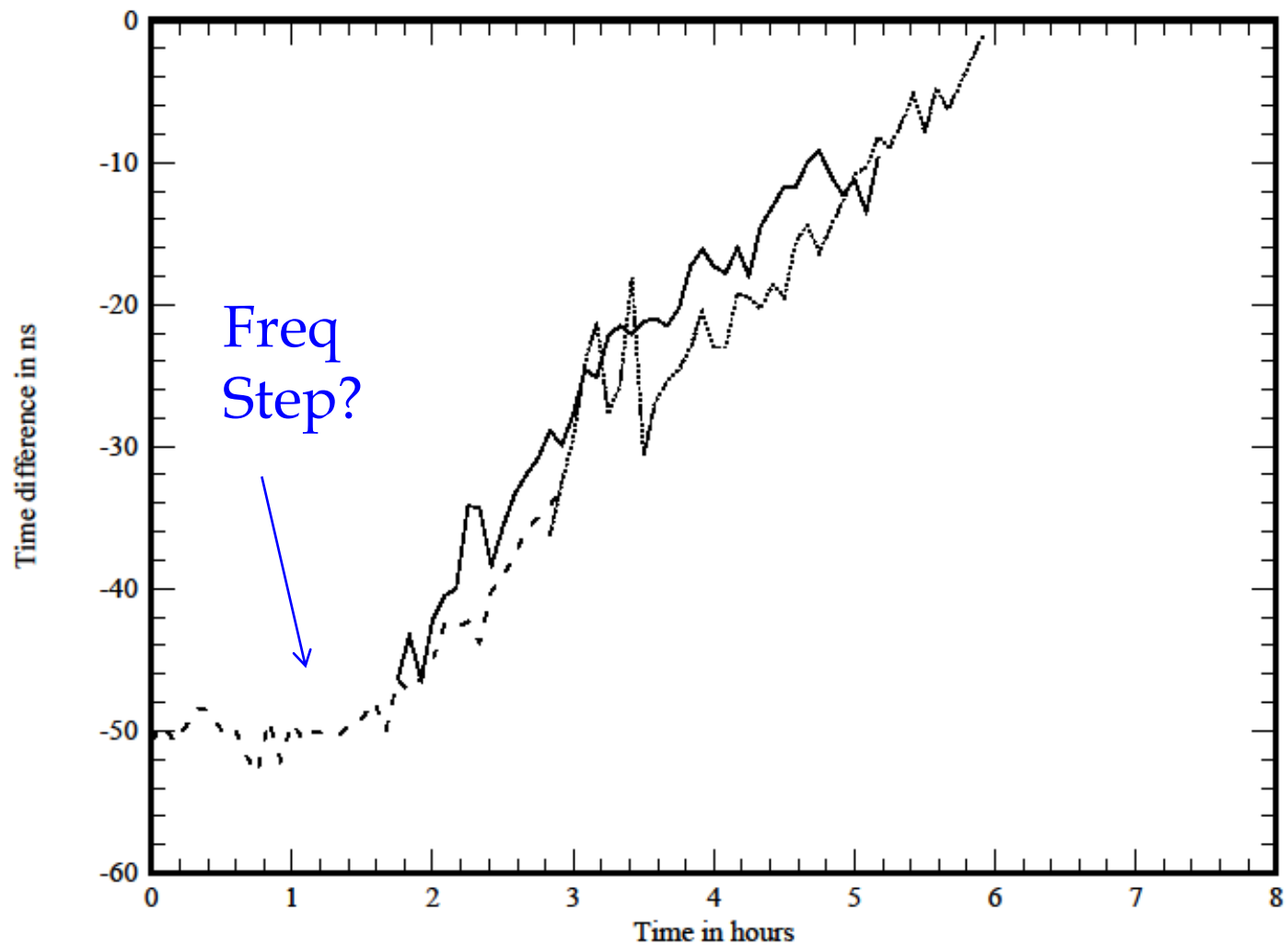
For other channels:

$$\varepsilon_n(t_k) < \text{noise terms}$$

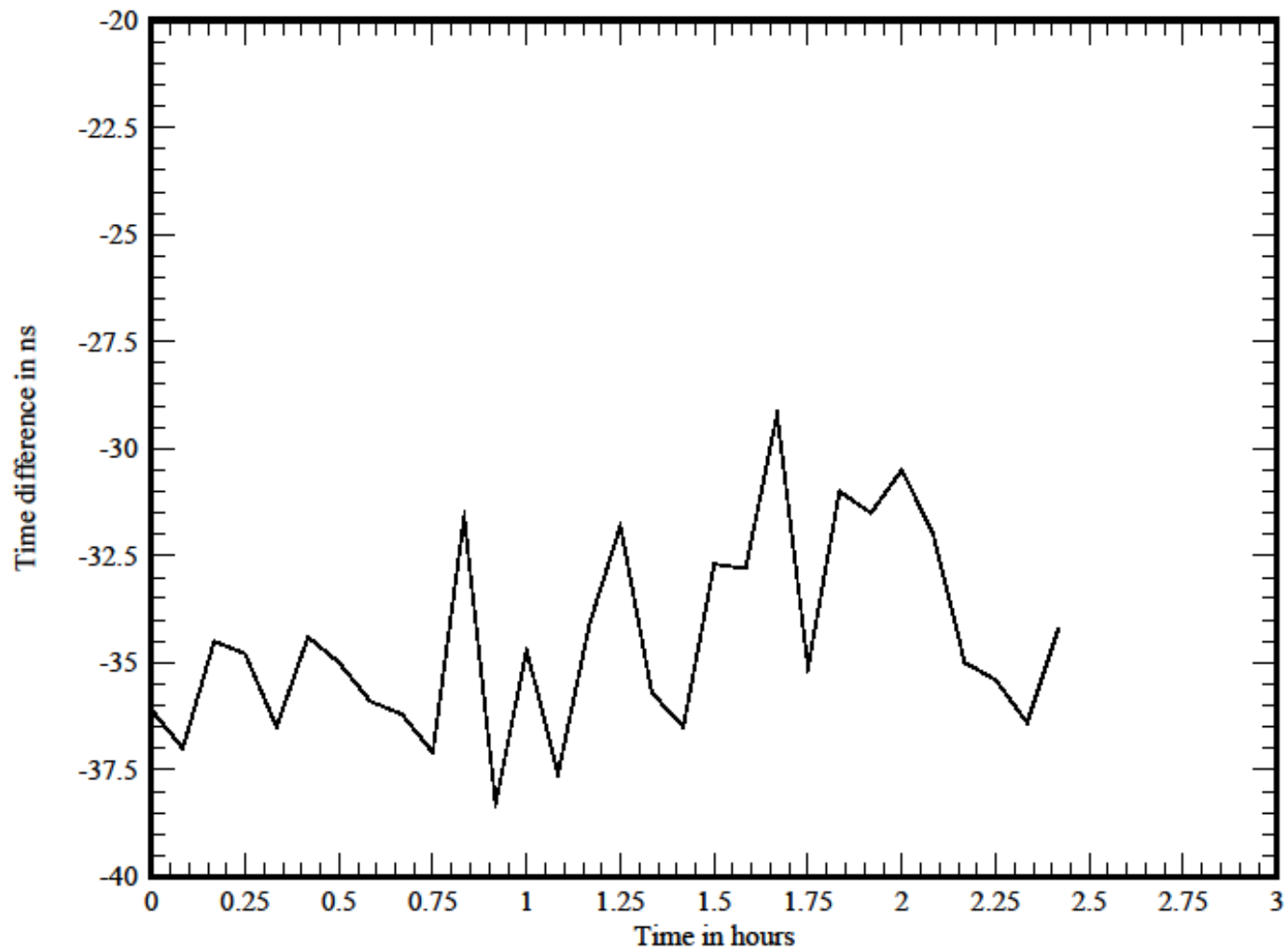
# Error Model - 3

- No data at time  $t_k$ :
  - Extrapolate by using model equation
$$x_j(t_k) = x_j(t_{k-1}) + y_j(t_{k-1})\tau$$
  - Extrapolation error:  $\sqrt{\xi^2 + (\eta\tau)^2}$

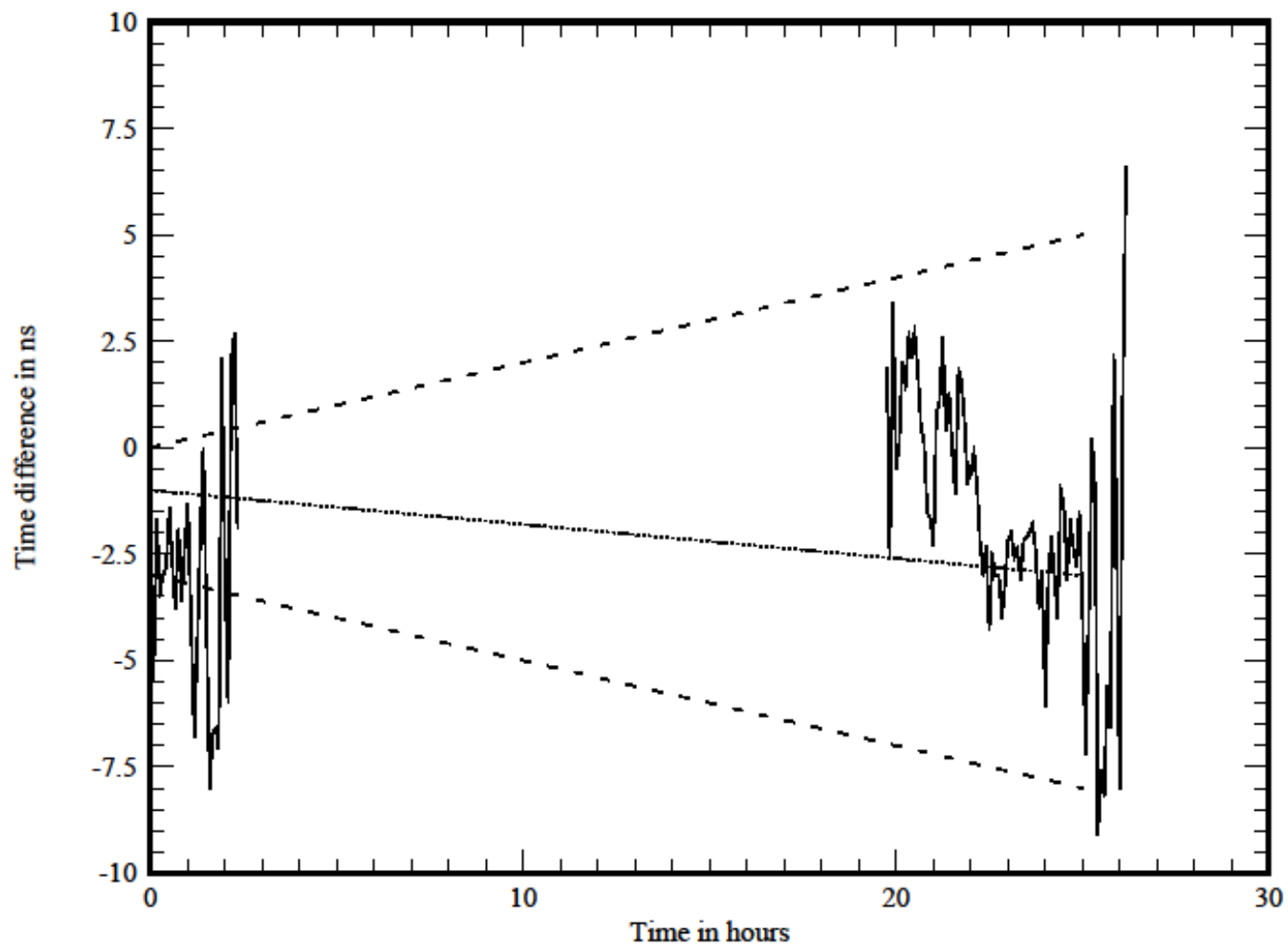
Local clock - GPS, s18, s1, s3



### Common View via s18



### Local clock - GPS via s18



# Summary

- Anomaly Detection compares data with model of measurement channel(s)
  - Optimum strategy based on redundant measurements
    - Multiple satellite GPS common application
    - Local reference clock can be very useful
- Missing data holdover uses model for extrapolation
- Performance depends on statistical parameters