

Robustness of the Sheet Metal Springback Cup Test

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The robustness of a proposed test for elastic springback characterization of sheet metal has been examined using a matrix of defined experimental errors. A series of flat bottom deep drawn cups made from AISI 1010 steel sheet were examined. It was found that misalignment of the blank over the forming tool and error in the vertical location where the springback ring was cut from the cup sidewall had the largest effect on the resulting springback opening. Other experimental errors involving cup height and ring width were found to be less important. The effect of in-plane anisotropy of mechanical properties on springback was negligible. The results are examined in terms of measured through thickness residual stresses and elastic bending of beams with circumferential thickness gradients.

I. INTRODUCTION

SPRINGBACK is the elastic shape change of a sheet metal part after forming and removal from the die. In the automotive industry, the poor predictability of springback has always been a problem, but it was somewhat alleviated by the use of rather thick walled, low strength steels and by the accumulation of empirical methods of compensation during die design and rework. Due to the need for weight reduction, there is a major shift underway to use thinner high-strength steels and aluminum alloys. In both cases, these materials exhibit larger and unpredictable springback magnitudes, thus making an accurate prediction all the more necessary in order to avoid a costly redesign of the stamping tool.

Modeling efforts in die design are beginning to treat springback in the same way that other phenomena such as forming limits and friction have been handled, through the modeling of a standard test geometry to verify theoretical predictions. There have been attempts to measure springback deflections in near-production parts,^[1] but often these measurements are difficult to perform because the displacements can be small and body forces (gravity) often distort the part, so that fixturing becomes very important and is difficult to do in a similar manner for each differently shaped part. There is a need for a standard test that will characterize a material's springback properties in a quick and consistent way.

For a standard test, it is desirable that it be easy to perform, repeatable and in the case of this particular need, have a large springback component that can be easily measured. A test that has received considerable attention^[2-6] consists of a ring sample taken from the sidewall of a flat bottom, deep drawn cup. The test geometry is presented graphically below in Figure 1. This geometry provides an axisymmetric sample with a large residual hoop stress that, when the ring is split, provides a large opening displacement. Residual stresses in the cup exist because different

locations in the cup have accumulated different magnitudes of plastic strain during the draw-bend-unbend process. These stresses are key for the modeling of the springback because, integrated over the thickness, they create a bending moment in the split ring, and thus the shape change.

In preparation for submitting this test to the standardization process, a variational analysis has been performed to study the robustness of the springback measurement to experimental errors. A number of different experimental errors that might be expected during forming and slicing the samples were introduced systematically, and the resulting change in springback opening was measured. Likely causes for any changes in observed springback opening in the rings are examined and analyzed in terms of measured residual stress profiles and ring thickness variations.

II. EXPERIMENTAL PROCEDURE

The cups were made in the Forming Laboratory of the NIST Metallurgy Division using a hydraulic deep drawing machine with 500 kN capacity.^[7,8] The AISI 1010 steel blanks, with mechanical properties listed in Table I, were circular disks of 200-mm diameter and 0.88-mm nominal thickness. The diameter of the die bore was 110 mm with a rim radius of 10 mm. The forming tool had a diameter of 100 mm with an edge radius of 10 mm and a recessed center. The depth of the cups was nominally 56 mm. Vegetable oil was used as a lubricant. The forming rate was constant at 5 mm/s, and the hold-down load was constant at 90 kN with no draw or lock bead in use. The forming rate and hold-down load were not varied as one of the sources of possible error, as it is possible to accurately calibrate modern servohydraulic equipment for both of these inputs and apply them accurately. The current study was designed to measure the effects of likely errors introduced by experimenters and technicians in performing the test.

The errors in forming the cups that were examined were thought to be among the more common that someone performing the test might commit. These can be divided broadly into sectioning variables, or how the ring is isolated from the cup and sliced open, and forming variables, which are errors in how the cup is formed. The former include the method used to cut the ring open, any errors in the vertical location or width of the ring taken from the sidewall, and where along the ring's circumference (with respect to the rolling direction of the sheet) the ring is cut open. The latter

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Manuscript submitted March 21, 2006.

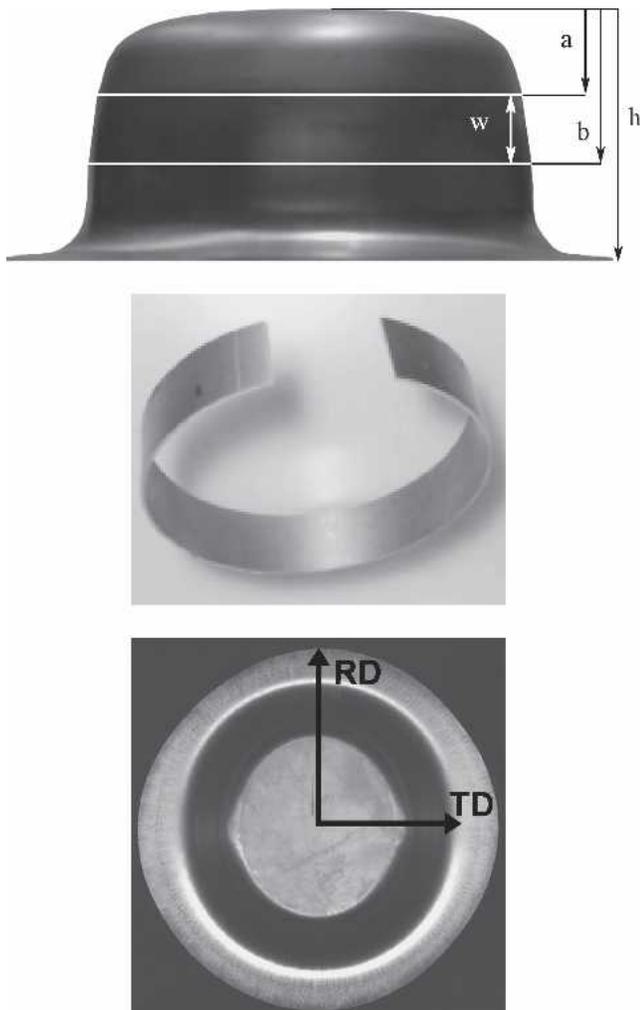


Fig. 1—Schematic of the sheet metal springback cup test. The top image shows the important geometric variable for the sidewall ring, and the bottom image denotes the rolling direction (RD) and transverse direction (TD).

include forming the cup too deeply or shallowly and forming an asymmetric cup because the circular blank was not properly aligned with the punch. These variables were tested by systematically changing each independently on each test. As an example, when examining the effect of a misaligned blank, the sheet sample was displaced by a number of millimeters from a centered circle scribed on the tooling using a micrometer, in either the rolling or transverse direction, before forming the cup.

After forming, each cup was cleaned and placed on a flatbed scanner so that an image of the earing profile could be taken. Subsequent to this, the thickness profiles were measured along the centerline of the ring to be cut from the sidewall using a coordinate measuring machine. The inner and outer profiles of the cups were measured at a number of points, fit with a spline, and the spline values were calculated every 10 deg and subtracted, giving a circumferential plot of the thickness. Prior to sectioning, the total cup height was measured. The rings were then cut from the sidewalls using wire electrical discharge machining (EDM), ensuring excellent location of the cuts and

Table I. Mechanical Properties of 1010 Steel Used in this Study; Values are Averages and Standard Deviations over Five Tests

Mechanical Property	Rolling Direction	Transverse Direction
Yield strength (MPa)	217.0 ± 6.7	220.8 ± 7.8
Ultimate tensile strength (MPa)	331.3 ± 4.1	328.3 ± 2.8
Tensile strain (1-in.gage) (pct)	41.7 ± 1.9	41.2 ± 1.1
Strain hardening Exponent	0.24 ± 0.008	0.25 ± 0.005
Strain ratio (R)	1.74 ± 0.02	1.79 ± 0.04

parallel edges of the rings. The rings were given fiducial marks along the edge on both sides of the intended slice location, approximately 1 cm apart. The rings were imaged on the flatbed scanner, and initial fiducial mark and diameter measurements were made. Finally, the rings were split and imaged again, with measurements made of the final fiducial mark spacing and ring diameter. Calculations were made of ring opening and diametral strain. In total, 202 cups with various intentional errors were produced and analyzed. Standard deviations reported subsequently were calculated using

$$s = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

and the correlation coefficients reported in the figure captions of the data plots were calculated using

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$$

III. DISCUSSION

Drawing of flat-bottomed cups is considered to be at one end of the forming spectrum in terms of strain state,^[10] where one principal strain is positive and the other negative. During the cupping operation, the change in thickness is small. The Swift cup test^[11] was devised to determine the limiting draw ratio (LDR) that a material can withstand, whereby the diameter of the cup tooling was decreased to a point where a given material would fail when partially drawn. The parameters and geometry in this test protocol were chosen such that materials with low LDR, and thus low formability in general, will not produce useful test specimens, but that materials with higher LDR and formability, and thus usefulness for automotive forming applications, could be characterized using this test. This was primarily done through the proper choice of punch edge radius and tooling gap between the punch and hold-down ring. The protocol proposed that gave input to the present investigation dictates the forming of a flat bottomed cup 56-mm tall from a 200-mm round blank. The ring is then

cut from the sidewall, having a width of 15 mm centered on a circumference located 37.5 mm down the cup from the outside flat bottom. Following this procedure, a springback opening of 48.2 ± 2.1 mm was found for these “perfect” cups of 1010 steel sheet. More cups were made for which deliberate systematic experimental errors were introduced into how the cup was formed or how the ring was cut from the sidewall, and the effect on the resulting springback opening was observed. One error was introduced at a time, and their effects are discussed in the following sections. In the cases of geometric parameter errors, the reader is referred to Figure 1 for visualization purposes.

For samples of all types to be discussed in this study, there was a consistent scatter of about 10 pct about the mean value of the ring opening (with forming/slicing parameters constant). A possible cause of this persistent error was found during an investigation of how the thickness profile of the sidewall might be affected by the in-plane anisotropy of the mechanical properties of the sheet. If the sheet has significant mechanical anisotropy, finite element modeling predicts a sinusoidal thickness variation around the circumference of the sidewall of the cup. The cup would thin (draw more) in regions where the material is softer, and *vice versa* where stronger. If the anisotropy showed typical fourfold symmetry in the plane of the sheet, the thickness variation measured should be sinusoidal with a wavelength of pi radians with no phase shifts. What was found was various samples exhibited roughly sinusoidal thickness profiles with a wavelength of 2 pi radians, with random phase shifts from one sample to another, as seen in Figure 3.

The observed thickness variations are likely an artifact of how the test is performed even under “perfect” conditions. Forming a circular blank into a flat bottomed cup requires the material to undergo a draw/bend/unbend operation. As this is occurring, the diameter of the starting blank is reduced as the material is drawn into the die. If the blankholder force is sufficiently large, the formation of wrinkles will be suppressed and the frictional holding force will be uniform. However, during actual operation, depending on material, blankholder forces sufficient to completely suppress wrinkling are often too large to allow the cup to form without splitting. In practice, small wrinkles form around the periphery of the blank and are ironed out as they are drawn. These effects translate into an unpredictable variation in the thickness of the sidewall from point to point, as the material in the wrinkle is thicker than the neighboring material, and results in a slightly thicker section of the sidewall. In this study of 1010 steel, the thickness variations due to hold-down inhomogeneities overwhelm any variations due to in plane mechanical anisotropy. This may not be the case for a material exhibiting more mechanical anisotropy. In addition, neither the blank nor the tooling are perfectly smooth, and local variations in frictional force and physical contact exist, allowing more gross variations in thickness as random locations within the blankholder stick and release.

A. Model Considerations

The driving force for ring opening is the residual hoop stress induced in the sidewall of the cup during the forming

process. This stress has been investigated^[9] and found to differ significantly from formalisms that model through thickness stresses for work hardening materials.^[10] It was also found that there are variations in the hoop stress both around the circumference and in the vertical direction, both of which affect the springback opening. A plot of the average hoop stress measured in a thick-walled AISI-1010 cup is seen in Figure 2. Plots are presented showing both intact and cut rings. The component of the hoop stress that drives opening is actually the amount that is released after slicing, or the difference between the intact and sliced through thickness stress profiles. This is also plotted in Figure 2. This linear stress change allows the use of a simple model that relates the hoop stress, the thickness, the ring radius, and the springback opening.

In this model, we can express the stress on the outside of the ring (which determines the stress change completely) as^[10,13]

$$\sigma_t = E \frac{t}{2} (\rho_0 - \rho) \quad [1]$$

(ρ_0 is the curvature of the intact ring, ρ is the curvature of the split ring, t is the thickness, E is the Young’s modulus, and σ_t is the change of hoop stress on the outside of the ring; this is identical to the intersection of the linear stress change with the stress axis in Figure 2). A simple geometric consideration shows that the curvature of the split ring is related to the gap g of the split ring by

$$g = \frac{2}{\rho} \sin \left(\pi - \frac{\pi\rho}{\rho_0} \right) \quad [2]$$

Using the series expansion for the sine-function for $\rho/\rho_0 \leq 1$, we obtain for g

$$g \cong 2\pi \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) = 2\pi \left(\frac{1}{\rho_0 - 2\sigma_t/Et} - \frac{1}{\rho_0} \right) \quad [3]$$

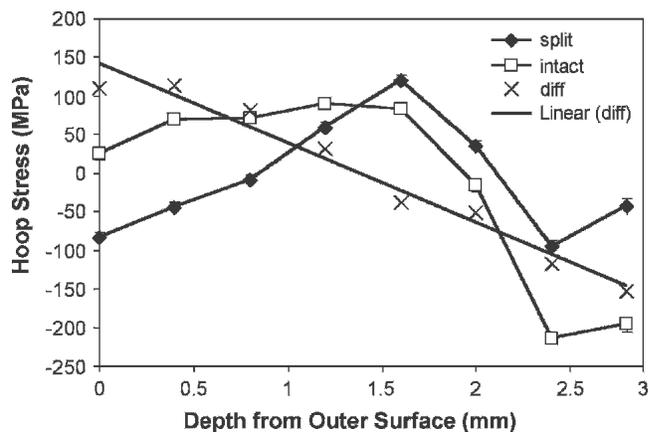


Fig. 2—Plot of residual stresses through the thickness of a ring taken from a 1010 steel cup, showing loaded and unloaded states, and the difference. Some error bars are not visible because they are smaller than the symbol size. The straight line is the linear regression of the stress difference split intact (Eq. [1]).

It is conceivable that preferred orientation due to rolling and deep drawing produces circumferential variations of the elastic modulus E , thus affecting the ring opening g . However, as shown in Reference 12, for rolling textures, the in-plane variations of the elastic moduli are usually small and insufficient to produce significant stress changes. Thus, the two variables determining the magnitude of the ring opening are the hoop stress and the wall thickness. Evidence for stress variations was found in Reference 9, and Eq. [1] shows that local stress variations can affect the local curvature of the split rings. Thus, the curvature ρ of the split ring may not be uniform and the variations are given by the ratio of the local hoop stress and the local thickness.

B. Effect of Thickness Variations

The spread of local thickness around the average value reaches 12 pct, therefore suggesting variations in the local curvature due to the thickness effect. What was found is that the local diameters of selected split rings (with thickness variations shown in Figure 3) are not correlated with the thickness but rather that the split rings are near elliptical in their shape with the largest diameters always close to the transverse direction (thus implying a lower curvature in the regions of the original rolling direction), as shown in Figure 4. Relative diameter variations were <2 pct for split rings, while for intact rings, the variations were <0.1 pct.

These findings suggest that local variations in thickness are accompanied by variations in the hoop stresses such that the curvature stays relatively unaffected. A lower thickness by itself would lower the curvature and increase the ring opening, but this also means higher levels of plastic strain at this location. The latter implies deformations farther beyond the yield stress, thus reducing locally the hoop stress, which, in turn, increases the local curvature. This is supported by the findings in Reference 9, which show lower-than-average hoop stresses in the region of the ring around the transverse direction and higher-than-average hoop stresses in the region of the rolling direction.

Our findings regarding the thickness effect also provide an answer to the question of whether the location of the cut affects the springback opening. The material immediately adjacent to the slice does not contribute to the bending moment of the beam, because it is not stressed. Thus, when considering the thickness not locally but as an effective

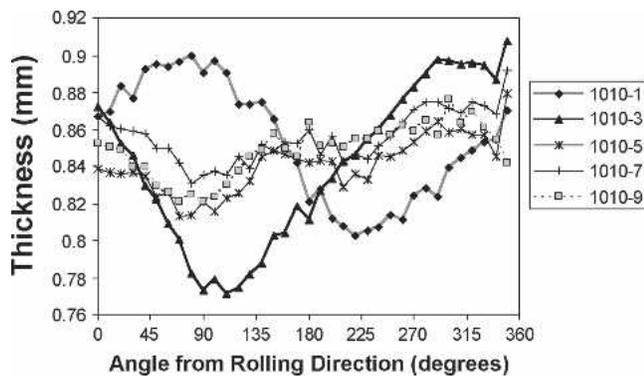


Fig. 3—Plot of collection of circumferential thickness profiles. The uncertainty of the thickness measurement is ± 0.005 mm

thickness for the entire ring, there could be an effect whether the cut is located in a thin region or a thick region. This is not the case because, first, there is no indication for a thickness effect, and, second, according to Saint Venant's principle, these edge effects subside at a distance from the slice comparable to the thickness (which is a small quantity).

C. Effect of Sample Sectioning Parameters

The EDM was chosen to isolate the ring from the cup sidewall for the reasons stated in the previous section: precision and parallelism of the cuts. However, slitting open the rings to produce the springback component can be accomplished in a number of ways, and several of these were compared to determine their effects on the final measurement. Slitting methods were selected based on two factors: the ease of use and the minimization of mechanical or thermal effect on the sample. The techniques chosen and the measured values of springback opening for each technique are presented in Table II. As seen, there was no observable influence of the chosen slitting method on the resultant opening. It was found that as long as the slitting method selected localized the deformation to a small section of the ring, whether by shear or material removal, and did not introduce bending moments on the rest of the ring, the resulting opening was unaffected.

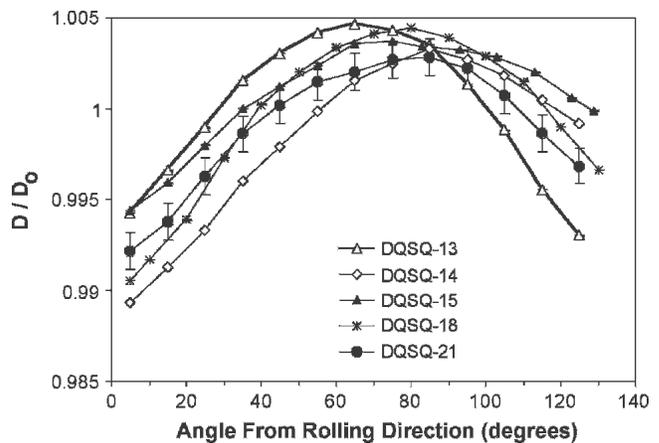


Fig. 4—Relative diameter variations of selected split rings. Slitting of the rings was performed using EDM. The uncertainty of the relative diameter is 4×10^{-4} .

Table II. Springback Openings Resulting from Various Slicing Methods; Values are Averages and Standard Deviations over Five Tests

Slicing Method	Ring Opening \pm Standard Deviation (mm)
Jeweler's saw (80 teeth per inch)	47.8 \pm 1.7
Shear	48.2 \pm 2.2
High-speed cutoff saw	47.6 \pm 2.4
Low-speed diamond saw	48.4 \pm 1.8
EDM	48.3 \pm 1.8

D. Effect of Geometric Parameters

Effect of error in vertical location of ring and ring width

The largest variation in springback opening displacement is seen when the location where the ring is cut from the sidewall is displaced vertically. This can be visualized in Figure 1 as a change of the same magnitude and sign of the distances a and b , and the resulting data are presented in Figure 5. Again, the considerable vertical decrease of the ring opening confirms the findings in Reference 9, where a corresponding vertical decrease of the hoop stresses toward the cup bottom was measured. This was attributed to a rising level of accumulated plastic strain as the rings are closer to the bottom. The forming parameters in Reference 9 were essentially the same as here; however, a major discrepancy exists in the vertical thickness distribution. While the wall thickness for cups made of 3-mm-thick blanks decreases toward the bottom, here, it increases slightly (but is still lower than the blank thickness), as shown in Figure 6. This is most likely a result of the tooling gap-to-thickness ratio (5 to 3 mm vs 5 to 0.88 mm). The different behavior of vertical thickness variations represents a problem insofar as, for the thick walled cup, a decrease in thick-

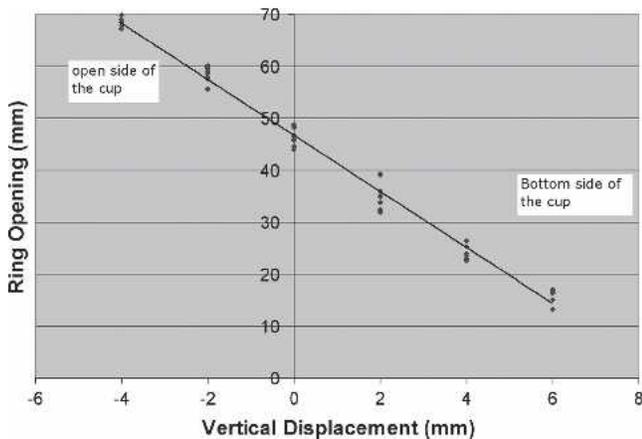


Fig. 5—Plot of effect of vertical ring location displacement on springback opening. Each value for the ring opening represents the average of nine rings. The correlation coefficient is -0.99 .

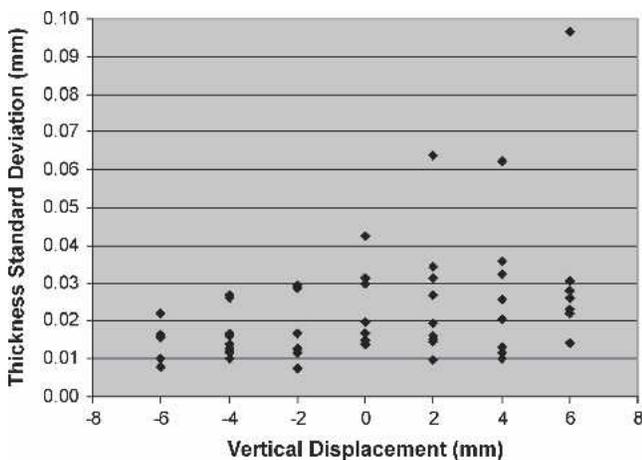


Fig. 6—Plot of thickness standard deviations over the vertical position. The correlation coefficient is 0.97 .

ness is a clear indication of uniform vertical stretching (rising plastic strain), and it explains the lower springback at the cup bottom. The slight increase in thickness toward the bottom found for the thin walled cup seemingly contradicts that, because with no rise in vertical stretching, the springback is still lower at the bottom. The explanation lies in the different circumferential plastic flow of material during the deep drawing process. For example, the thickness variations of the thick walled cup are approximately 3 pct, while the thin walled cups exhibit variations up to 12 pct, which indicates that the level of tangential plastic strain is generally higher in the thin walled cups as compared to thick walled cups where higher strains are predominantly caused by vertical stretching. More evidence for higher circumferential strains in thin walled cups is apparent in the standard deviations of the thickness profiles, which increase toward the cup bottom, as shown in Figure 7. The thickness standard deviations are a measure for how much the wall thickness varies at a given vertical position. Higher standard deviations mean larger thickness differences, which implies an increased circumferential material flow and higher strain levels. The latter have the effect of reducing springback as more material is deformed beyond the yield stress.

In the same manner as the vertical ring position, an error in the width of the ring can also affect the resulting springback, but to a much smaller degree. In this case, the ring cut from the sidewall is still centered on 37.5 mm down from the flat outside of the cup, but the width is varied (in Figure 1, change a and b the same amount, with opposite sign). The effect is plotted in Figure 8. The slight decrease in opening for increasing ring width is a result of the more pronounced conical shape of the ring. Both the wall thickness and the initial curvature increase toward the cup bottom with the result that the ring opening is not simply the average of values from Figure 5, but it is somewhat dominated by the near-bottom portions of the ring.

E. Effect of Blank Misplacement

Due to the overall configuration of the tooling needed to perform the test, one of the more likely expected errors might be the misplacement of the blank with respect to a

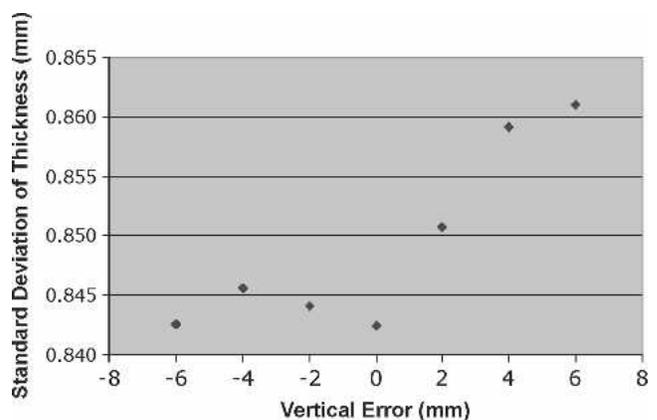


Fig. 7—Thicknesses at different vertical locations on cup sidewall. Each data point represents the average of thicknesses of eight rings sectioned at the same locations. The correlation coefficient is 0.88 .

position centered on the punch. This would result in several effects on the formed cups. The earing pattern from a misplaced blank shows the fourfold symmetry seen in the correctly formed cups described previously (Figure 9(a)), but superimposed upon this is a twofold symmetry from the misalignment (Figure 9(b)). In cases of severe misalignment (in excess of 5 mm), one side of the blank escapes the hold-down platen before the other side, and the cup is severely misformed. A series of cups were produced with prescribed misalignments over the center of the forming ram of up to 4 mm, displaced at 0, 45, and 90 deg with respect to the rolling direction of the blank. The misalignment was limited to 4 mm, as beyond this the shape of the cup is so asymmetric as to not produce a useable ring from the sidewall.

The resulting springback measurements are presented in Figure 10. There appears to be a small drop-off in springback within the first 2 mm of misalignment, and then the value plateaus. Also, there is seemingly no effect of changing the direction of misalignment with respect to the sheet rolling direction. The thickness distributions and their standard deviations in Figure 11 show no clear correlation with the misalignment. Thus, the springback as shown in Figure 10 appears to be unaffected by the misalignment.

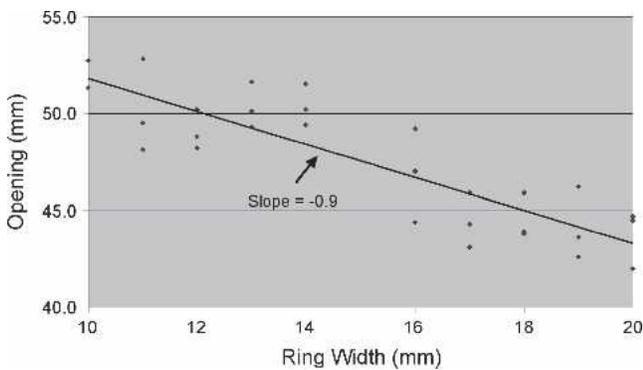


Fig. 8—Plot of effect of error in the width of the ring on springback opening. The correlation coefficient is 0.95.

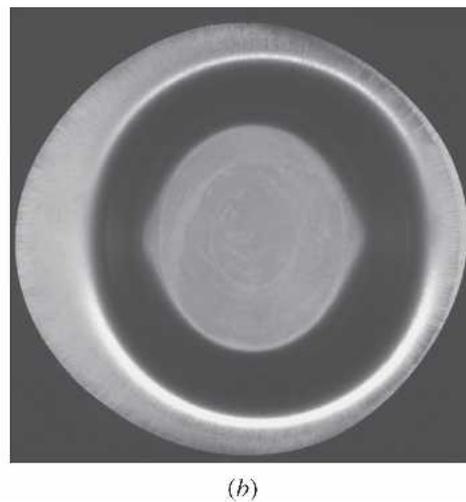
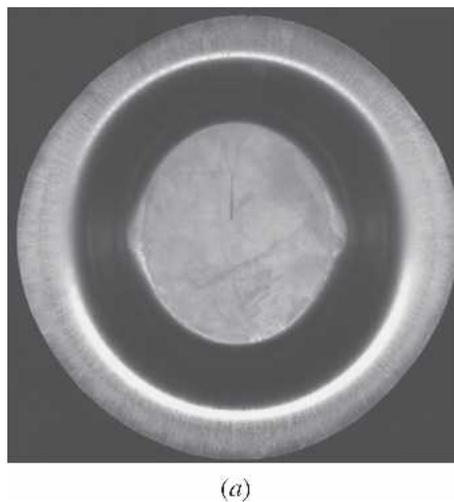


Fig. 9—Earing profiles imaged on (a) correctly formed and (b) blank misaligned cups.

F. Effect of Error in Cup Height

Another possible source of error in forming the cups is an error in the final height of the sample. For this proposed test, the target value of the total height of the formed cup is 56 mm. A series of cups were made with final heights as much as 8 mm shorter and 8 mm taller than this, and the resulting springback values are seen in Figure 12. Note that again in each case the ring cut from the sidewall of the cup was still 15-mm wide and centered on 37.5 mm from the outside of the flat end of the cup. The effect of the error in total cup height is approximately 1 mm less springback opening for each millimeter of additional height.

This can be partly analyzed if one considers the vertical change in residual hoop stresses seen in a previous study.^[9] As the cup is drawn deeper, the fraction of uniform plastic strain in the cup wall increases. As a result, the material in all physical locations in the cup wall is deformed significantly past the yield stress, and the magnitude of the through-thickness strain inhomogeneities (difference in plastic strain from one location to another) introduced by the bending and unbending of the blank decreases. As the uniform portion of the plastic strain increases, the material at the neutral plane is deformed above the yield point. In order to produce this effect, the strains involved can be small, and the latter are accompanied by equally small thickness changes (Figure 13). Thus, in terms of the stress-strain curve of the material, the deformation of the entire cup wall proceeds into the strain region where large strain differences correspond to relatively small stress differences. The latter translates into reduced residual stresses and springback.

G. Effect of Direction of Slicing with Respect to Rolling Direction

Due to in-plane anisotropy of the mechanical properties of the sheet, it might be expected that slitting the springback ring open at various angles with respect to the rolling direction might produce different amounts of opening. A series of samples were prepared that were identical except that they were slit open at 15 deg increments with respect to the rolling direction. The results can be seen in Figure 14,

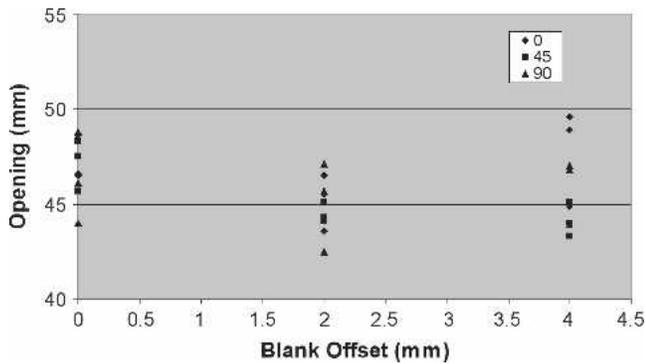


Fig. 10—Plot of effect of error in aligning the blank over the center of the forming ram for three misalignment directions: along the rolling direction (0 deg), 45 deg between the rolling direction and the transverse direction, and along the transverse direction (90 deg). The correlation coefficients are 0.12 (0 deg), -0.79 (45 deg), and -0.08 (90 deg).

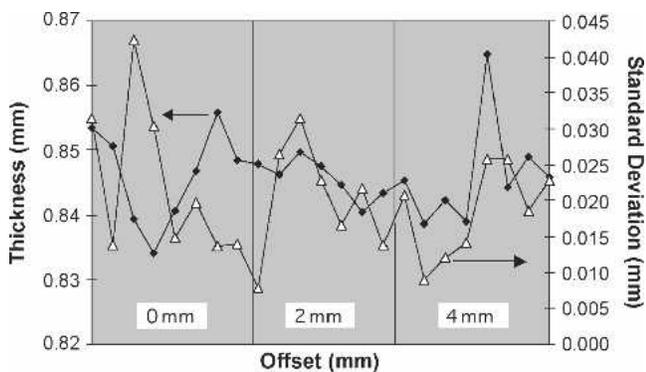


Fig. 11—Plot of thicknesses and standard deviations with blank misalignment.

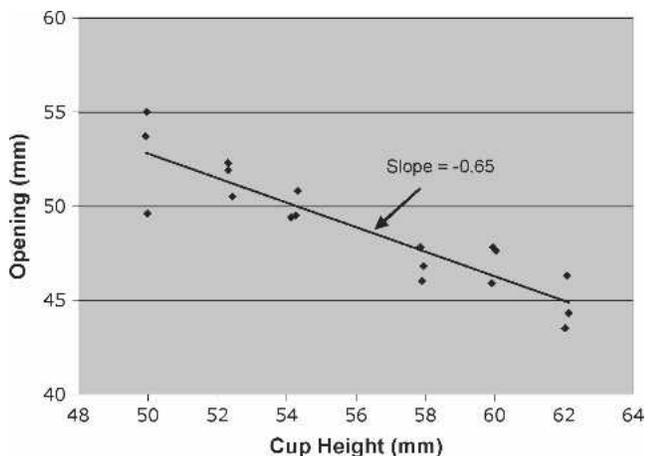


Fig. 12—Plot of effect of error in final cup height on springback opening. The correlation coefficient is -0.94 .

and the effect was found to be negligible in this material. However, in materials that exhibit much more mechanical anisotropy, this might become a more important factor.

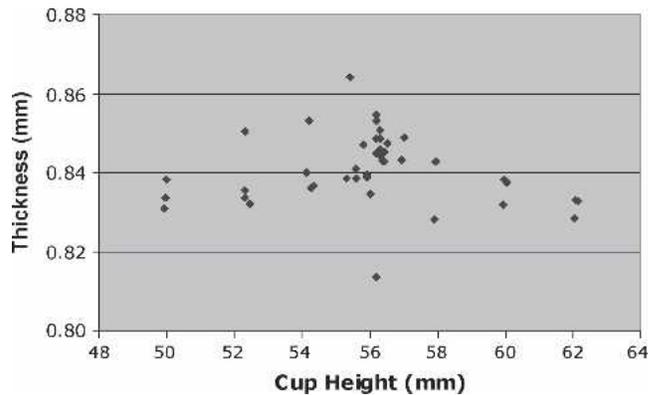


Fig. 13—Plot of ring thickness with cup height.

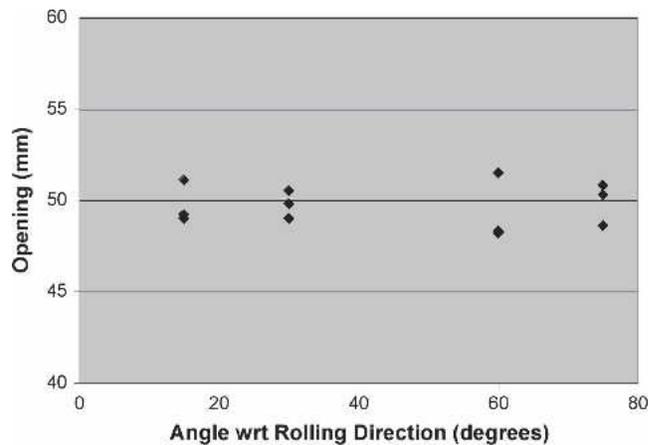


Fig. 14—Plot of the effect of orientation of slicing direction with respect to the rolling direction on springback opening. The correlation coefficient is -0.12 .

IV. CONCLUSIONS

The systematic introduction of errors in running the springback cup test produced systematic errors in ring opening in the sample. The largest effects were found due to errors in initial blank alignment over the punch and errors in the vertical location of the sidewall ring. Other errors produced small or negligible effects on measured springback. The observations can be rationalized by considering how the error affects the magnitude of plastic strain in the ring, and how this subsequently changes the springback opening through the residual stresses in the wall.

ACKNOWLEDGMENTS

The authors thank R. Fields, NIST, M. Demeri, Ford and M. Shi, US Steel, for helpful discussions.

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