Arizona State University

DESIGN AUTOMATION LAB

Tolerance Map Models for Design and Manufacturing

MBE Summit @ NIST

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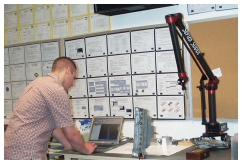
$$A = \frac{\partial f}{\partial d_1}\bar{d}_1 + \frac{\partial f}{\partial d_2}\bar{d}_2 + \ldots \frac{\partial f}{\partial d_n}\bar{d}_n$$

$$\sigma_A = \sqrt{\left(\frac{\partial f}{\partial d_1}\right)^2 \sigma_{d1}^2 + \left(\frac{\partial f}{\partial d_2}\right)^2 \sigma_{d2}^2 + \ldots \left(\frac{\partial f}{\partial d_n}\right)^2 \sigma_{dn}^2}$$

Motivation

- Geometric dimensions & tolerances are of concern in all aspects of product development.

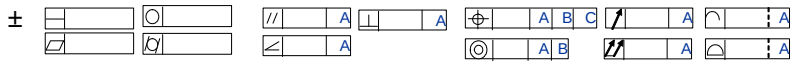


  - Designers are concerned with assemblability and function.
  - Process planners are concerned with selection of set-ups, fixturing, machines and operation tolerances to minimize manufacturing cost and time.
  - QA must verify that manufactured parts comply with design specifications.
- Miscommunication and misinterpretation between these groups can result in low acceptance rates or expensive rework.
- While 3D Computer aided Tolerance Analysis tools are available to designers, the same is not true for manufacturing process planning
- A fundamental understanding of geometric variations, their accumulation, and their implications in design, manufacturing and inspection is needed.
- These are the motivations for developing mathematical models for GD&T

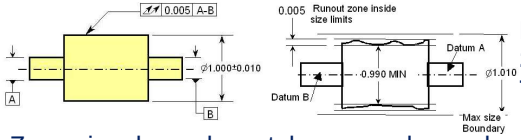
The Challenge

- In engineering practice, tolerances are specified using national and international GD&T standards, such as ASME Y14.5M and ISO 1101.
- This standards are not based on any math foundation; they are a set of symbols, conventions and practices
- If a model is to gain acceptance, it must be consistent with the standards
- Many methods have been proposed for tolerance representation & analysis, based on elegant math models but failed to gain acceptance because they were not compatible with the standards
  - PARAMETRIC MODELS [Hillyard & Braid 78, Light & Gossard 82]
  - OFFSET ZONES [Requicha 83, Requicha & Chan 84]
  - VARIATIONAL SURFACES [Martinsen 93, Turner 90]
  - VECTOR SPACES [Turner & Wozny 90]
  - KINEMATIC MODELS [Chase & Magelby 98, Rivest 94, Kramer 92]
  - DEGREE OF FREEDOM (DOF) MODELS [Bernstein 89, Clement 91, Zhang 92, Solomons 95, Kandikjian 98]; TTRS MODELS [Clement 91, Desroschers 99]
- Develop math models for GD&T consistent with the standard, i.e. retroactively fit a math model to the conventions in ASME Y14.5M
- *Therefore, it is important to understand the key concepts in GD&T standards before discussing tolerance analysis*

ASME Y14.5 Conventions: A Quick Look

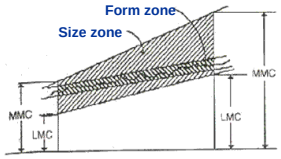


Geometric variations have been decomposed into specific types because they affect function & assembly in different ways

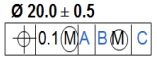


Each variation is represented by zones whose shape depends on the toleranced feature type;

Zone size depends on tolerance value and modifiers;
Zone location depends on tolerance type and datums

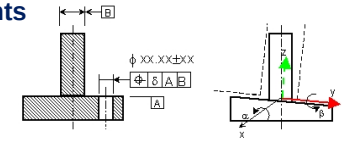


Certain tolerances are refinements of others; some zones float within other zones (Rule#1)

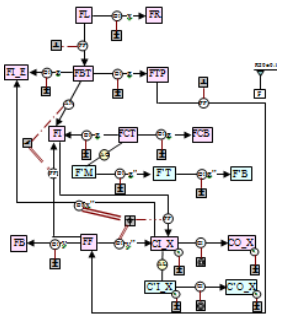


Bonus Tolerance & Shift : Material conditions (MMC, LMC) can enlarge position tolerance zones by the difference between MMC (or LMC) and actual size

Datum order influences directions of measurements

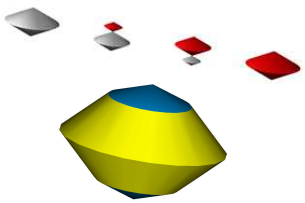


ASU Bi-Level Math Model*



TOPOLOGICAL MODEL (DoF algebra; CTF graph):

- Similar to topology or control schema
- Models relationships between all feature control frames, datum reference frames (DRF) and their precedence (datum flow chain)
- provides basis for geometric validation of D&T scheme, loop detection for analysis and DoFs
- Supported by DoF algebra



METRIC MODEL (T-Maps):

- models the composite quantitative effect of all tolerances on a given feature
- interaction of size, form, orientation, position is clearly identified
- Rule #1 is embedded in the formulation
- relative volumes of regions can be used to study trade-offs in tolerance allocation (size vs form vs orientation..)

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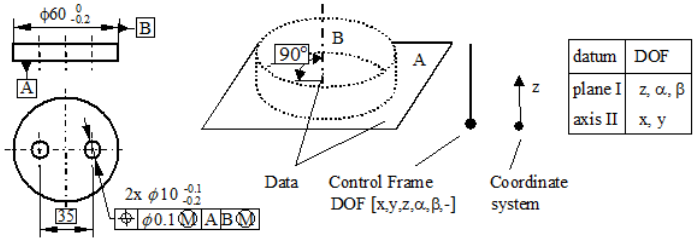
* US Patent No. 6,963,824

Topological Model: Basic Concepts



Surface mapping: The degrees of freedom of all types of surfaces can be represented by combinations of points, lines, and planes establishing a mapping between surfaces and control frames.

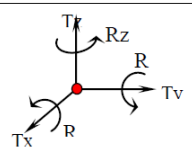
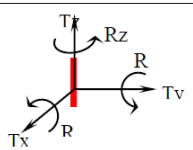
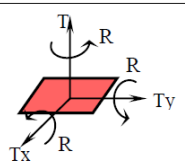
Control frames, {D,T,R}: Directed geometric relations R between datum D and target T rigid sets.

Degrees of freedom (DoFs) of an entity or rigid set: Translations (x,y,z) or rotations (α,β,γ) not constrained by geometric relations minus the invariant directions.

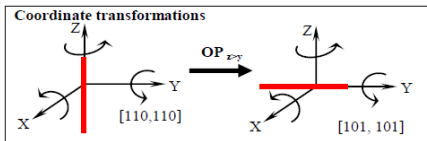
Invariant DoFs: An entity or rigid set is invariant in those transformations that have no effect on its location or orientation.

Topological Model: DoF Algebra

- DRFs and TRFs are clusters of points, lines and planes with different geometric relations to each other (coincident, //, ⊥, …)
- DoF Algebra includes symbolic ops to determine free and invariant DoFs of entity clusters.
- This algebra was validated by applying it to all cases in the Y14.5.1.

	Point	Line	Plane
			
X_{fdof}	$(T_x\ T_y\ T_z, - - -) \rightarrow (111,000)$	$(T_x\ T_y - , R_x R_y\ -) \rightarrow (110,110)$	$(- -\ T_z, R_x R_y -) \rightarrow (001,110)$
X_{inv}	$(- - -, R_x R_y R_z) \rightarrow (000,111)$	$(- - T_z \ldots R_z) \rightarrow (001,001)$	$(T_x\ T_y - , - Rz) \rightarrow (110,001)$

Coordinate transformations



We define the CS Transform operator **OP** $_{i>j}$ as swapping of the i and j element values for both TDof and RDoF.
Thus, **OP** $_{z>y}$ { [110,110]} = [101,101].
This is valid for any entity or cluster and for active or invariant DoF vector.

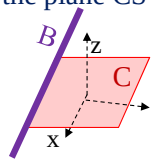
Algebraic Operators

Union Operation ∪			Intersection Op ∩		
a_i	b_i	ai ∪ bi	a_i	b_i	ai ∩ bi
1	1	1	1	1	1
1	0	1	1	0	0
0	1	1	0	1	0
0	0	0	0	0	0

Combining DoFs for clusters

$[X_{fdof}] = [A_{fdof}] \cup [B_{fdof}]$;
$[X_{inv}] = [A_{inv}] \cap [B_{inv}]$

Example: Line-Plane (coincident):
the plane CS will be used as the cluster CS; the line CS needs to be transformed.



Plane C: C_{dof} = [001,110] and C_{inv} = [110,001]
Line B: B_{dof} = [110,110] and B_{inv} = [001,001]
$[(BC)_{dof}] = [\mathbf{OP}_{z>x}\{B_{dof}\}] \cup [C_{dof}]$
= [011,011] ∪ [001,110] = [011,111]
$[(BC)_{inv}] = [B_{inv}] \cap [C_{inv}]$ = [100,000]

Algebraic Relations

- $[A] \cup [B] = [B] \cup [A]$ ……Commutative relation
- $[A_{fdof}] \cap [A_{inv}] = [\varnothing]$=[000,000] …….Null set
- $[A_{fdof}] \cup [A_{inv}] = [\mathbf{I}]$=[111,111]…… "Identity" vector
- $[A_{inv}]$ = RCP $\{[A_{fdof}]\}$ …. …… Reciprocal relation (or $\bar{A}$)
- +Standard Associative, Distributive and Idempotence relations

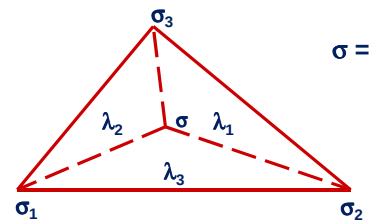
DOF representation of Tolerances

- DoF algebra models datum flow chains, proper DRF combinations and tolerance classes
- The constrained DOFs are the intersection of the DOFs of the three tolerance elements.
- The target, DRF and tolerance classes are completely represented in terms of DOF vector.
- No matter what the target cluster is, the DOF vector of target entity is one of six combinations.
- The DRF candidates for a tolerance specification should have common DOFs with target entity.

No.	Target	DRFs	Tol. Class	Constrained DOFs
1	(111,000)	(111,000) (111,110) (111,111)	(111,111)	(111,000)
2	(110,110)	(110,110) (001,110)	(000,111)	(000,110)
		(111,110) (110,111) (111,111)	(111,111)	(110,110)
3	(001,110)	(110,110) (001,110)	(000,111)	(000,110)
		(111,110) (111,111)	(111,111)	(001,110)
4	(111,110)	(110,110) (001,110)	(000,111)	(000,110)
		(111,110) (111,111)	(111,111)	(111,110)
5	(110,111)	(110,111) (111,111)	(111,111)	(110,111)
6	(111,111)	(110,110) (001,110)	(000,111)	(000,110)
		(111,111)	(111,111)	(111,111)

Metric Model For Planar Faces

- Areal (barycentric) coordinates ➞ A point in 2-D space is represented by 3 homogeneous coordinates



$\sigma = \lambda_1\sigma_1 + \lambda_2\sigma_2 + \lambda_3\sigma_3 \qquad \lambda_1 + \lambda_2 + \lambda_3 = 1$

$\sigma_1 \equiv \{\lambda_1, \lambda_2, \lambda_3\} = \{1,0,0\}$
$\sigma_2 \equiv \{\lambda_1, \lambda_2, \lambda_3\} = \{0,1,0\}$
$\sigma_3 \equiv \{\lambda_1, \lambda_2, \lambda_3\} = \{0,0,1\}$

By appropriate choice for σ_1, σ_2, σ_3, p, q, & s are proportional to the scale for Cartesian frame placed on the E-space.

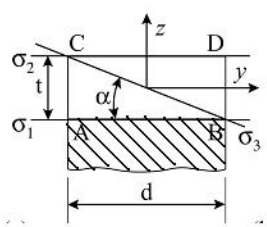
- Duality of space of points and planes:

$$px + qy + rz + sw = 0$$

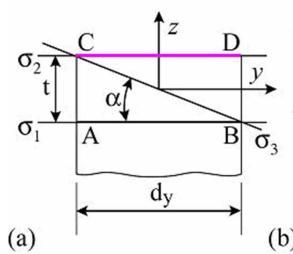
Points (x, y, z, w) lie on plane (p, q, r, s)
All planes (p, q, r, s) passing through the point (x, y, z, w)

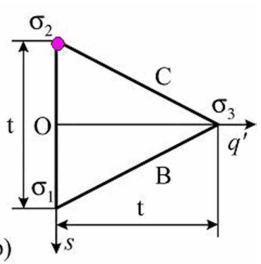
Tolerance Maps for size: planar feature

Cylindrical bar cross-sections



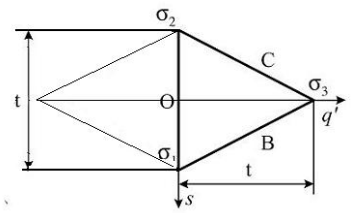
Basis Planes





Any plane (point) $\sigma = \lambda_1\sigma_1 + \lambda_2\sigma_2 + \lambda_3\sigma_3$

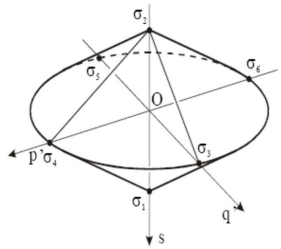
Cross section of planar T-map©



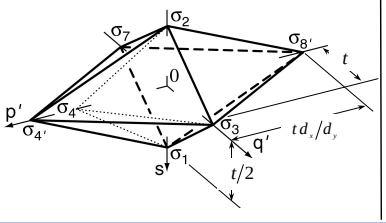
T-Maps for size: other planar sections

Any plane (point) $\sigma = \lambda_1\sigma_1 + \lambda_2\sigma_2 + \lambda_3\sigma_3 + \lambda_4\sigma_4$

Round Bar

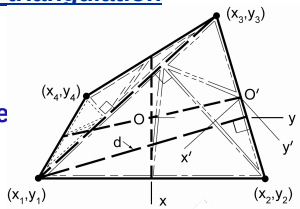


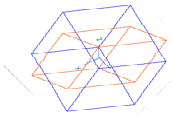
Rectangular Bar

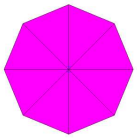


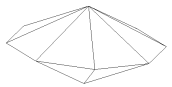
Arbitrary X-sec by triangulation

- isosceles triangle used as a primitive element
- Only 2 params neede
- any convex shape produced by iso-triangulation
- T-Map obtained as the ∩ of the Δ T-Maps, appropriately juxtaposed









Form & Orientation Tolerances: Planar Features

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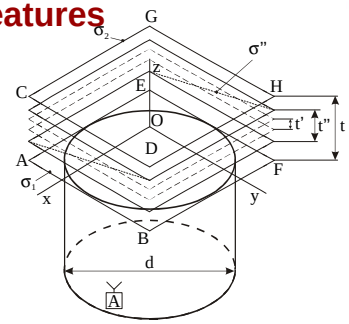
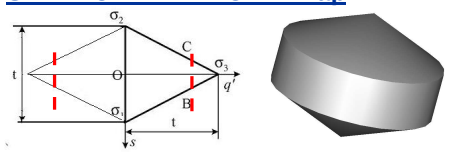
FLOATING ZONES

ORIENTATION zone (t'') translates ↑↓ can rotate about *x*- or *y-axes*

FORM zone (t') translates ↑↓ and rotates about *x*- or *y-axes*

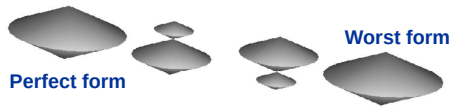


SIZE + ORIENTATION T-map



Addition of orientation tol t'' to size reduces the allowable tilt
Orientation T-map can be obtained from size by truncating the σ_3 axis

SIZE + FORM T-map



As per Y14.5 Rule#1
- **Worst form occupies the entire zone**
- **Perfect form occupies none**

Therefore, size + form is modeled by splitting into two planar T-maps that together must conform to size map

Tolerance Maps For Lines: 4-D Solid of Points

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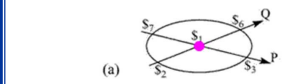
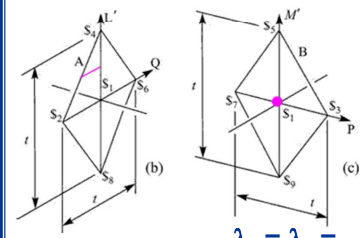
2D cross-sections of the T-Map

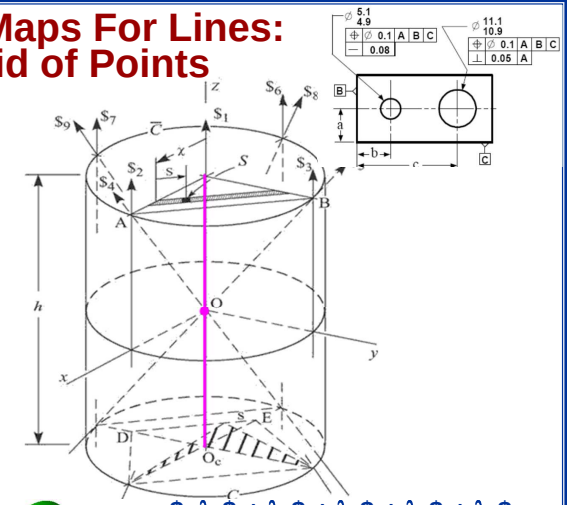


$\lambda_4 = \lambda_5 = 0$

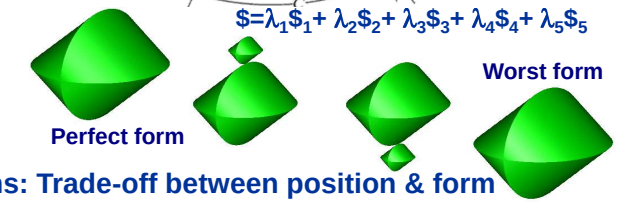


$\lambda_3 = \lambda_5 = 0$

$\lambda_2 = \lambda_4 = 0$



$\$ = \lambda_1\$_1 + \lambda_2\$_2 + \lambda_3\$_3 + \lambda_4\$_4 + \lambda_5\$_5$



3D cross-sections: Trade-off between position & form

Material Modifiers in T-map models

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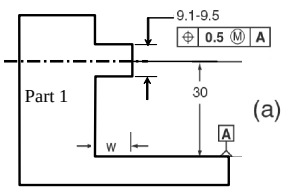
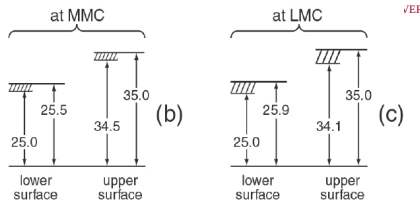
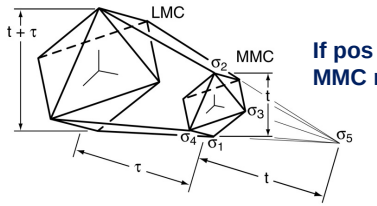
t = pos. tol
τ = size tol.

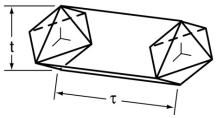




4-D T-Maps: size is the 4th dimension
The dipyramid now is the T-map for position of the medial plane.



If pos tol uses MMC modifier



If pos tol uses RFS modifier

Hyper-Volume computation
Hyperpyramid of dimension n

$$ {}_n\mathcal{V} = \frac{1}{n}\left({}_{n-1}\mathcal{C}\right)h $$

$$ {}_4\mathcal{V}_f = \frac{1}{4}\left\{{}_3\mathcal{C}_{t+\tau}(t+\tau) - {}_3\mathcal{C}_t\, t\right\} = \frac{1}{6}\left\{(t+\tau)^4 - t^4\right\}. $$

Hyperprism of dimension n

$$ {}_n\mathcal{V} = {}_{n-1}\mathcal{C}\,h $$

$$ {}_4\mathcal{V}_p = {}_3\mathcal{C}_t\,\tau = \tfrac{2}{3}t^3\tau $$

Insight: if t = τ $\mathcal{V}_{\mathrm{MMC}} = 3.75\ \mathcal{V}_{\mathrm{RFS}}$

Tolerance Analysis

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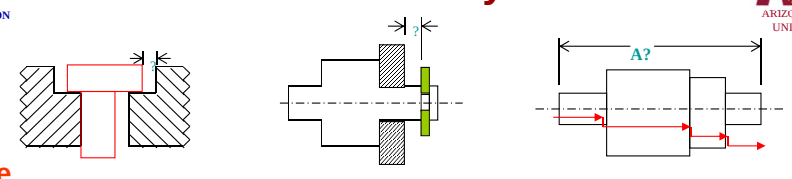

Purpose

Determine accumulation of geometric variations caused by all contributing elements (dimension, location, orientation, etc)
In general, the analyzed dimension *A* is a non-linear function of independent dimensions & geometric variations

In parts variations are controlled by datum flow chains
In assemblies tolerances accumulate (stack-up)

Non-linear problem; hard to do with both dimensional & geometric tolerances

Types of analysis

Worst case analysis – 100% interchange-ability
Statistical analysis – selective assembly

Tolerance Analysis with T-maps: Minkowski Sums

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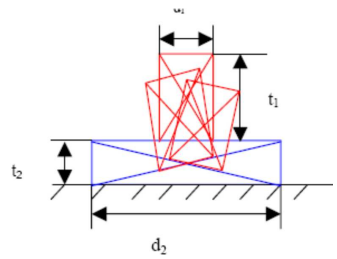
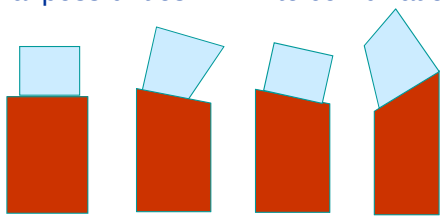
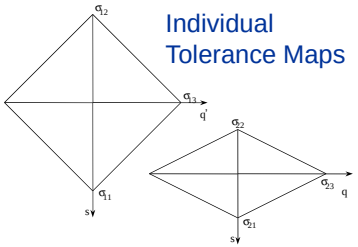


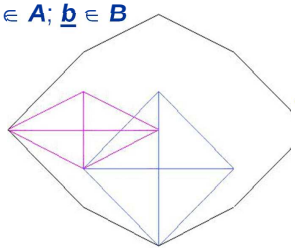
Variational possibilities – infinite combinations



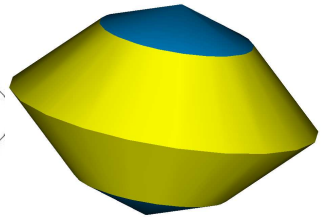
Individual Tolerance Maps



Minkowski sum: C = U*c*, where ***c*** = ***a*** + ***b*** and ***a*** ∈ ***A***; ***b*** ∈ ***B***



Accumulation map



Worst case analysis with T-maps: Functional & Accumulation Maps

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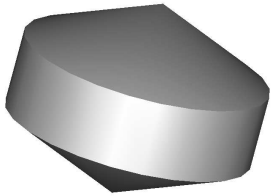
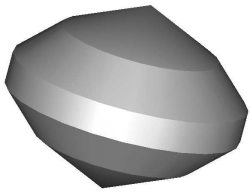
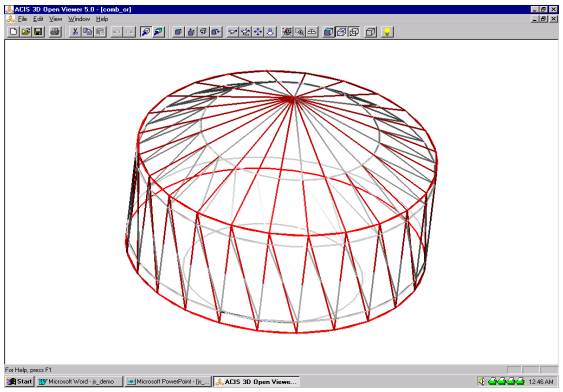
Functional Map



Accumulation Map



Fit accumulation map inside functional map



$$t_f = t_2 + t_1$$

$$t''_f = t_2 + d_2 t''_1 / d_1$$

Clearance Distribution due to Position & Size of Mating Features

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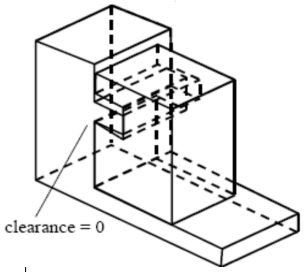

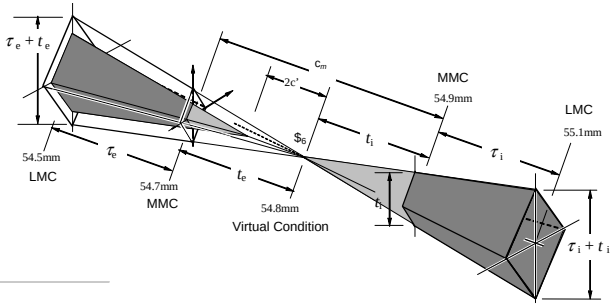

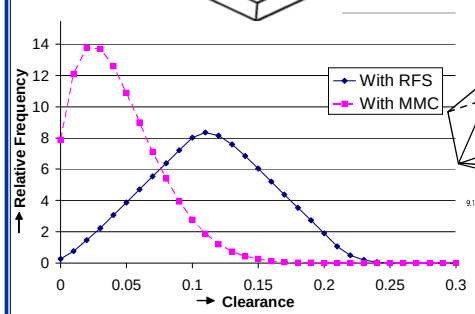

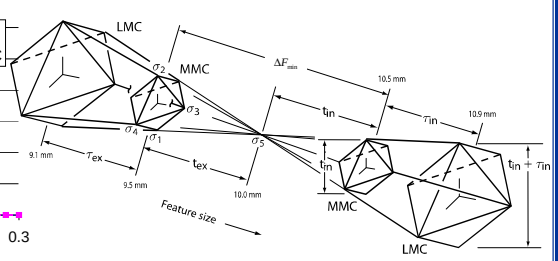

Circular Runout Model

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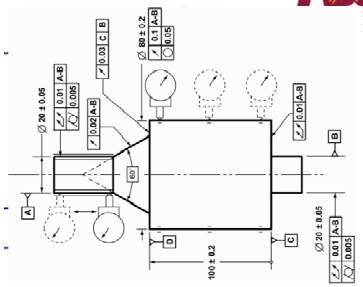
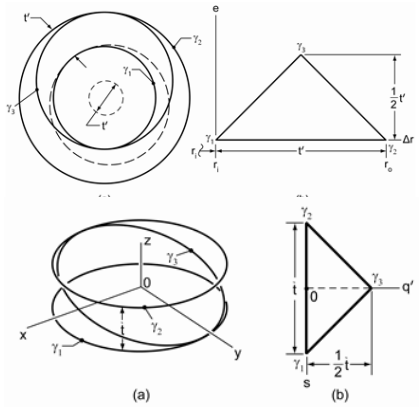
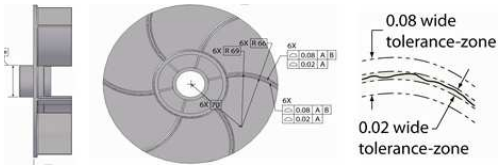
- Circular runout is a composite tolerance that controls both circularity and concentricity (position), independent of size
- Applied to any axisymmetric X-sec



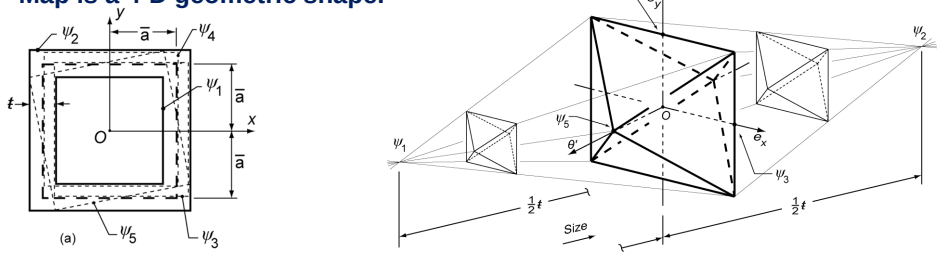


- circular X-sec, involves two variables: circularity (annular zone) + eccentricity
  (a) An annular tolerance-zone of amount t′ which lies between the inner and outer boundaries γ_1 and γ_2 of radii r_i and r_o, respectively. (b) Its 2D T-Map
- planar (end) involves two variables: linear offset + angle
  (a) A cylindrical tolerance-zone of height t̀ which lies between the upper and lower boundaries of y1 and y2. (b) Its 2D T-Map.

Line Profile: parametric model

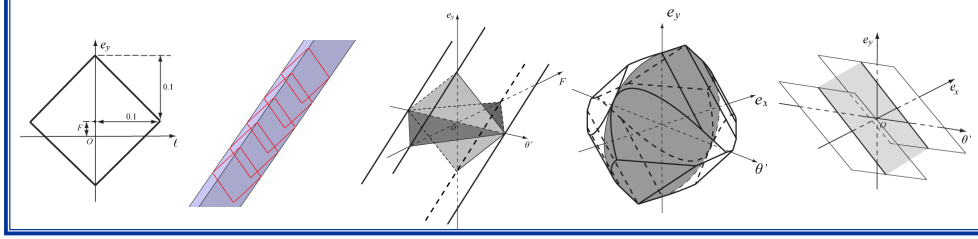


- Profile tolerances control the shape, size, and position of complex features, e.g. turbine blades and pump vanes.
- For line profiles, four variables are required to identify a variation of the theoretical shape within its tolerance-zone.
- Example: A square line-profile
- For line profiles, each point in the T-Map represents one square with a given size and x-, y-, and θ-position in the tolerance-zone. Consequently, the T-Map is a 4-D geometric shape.



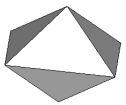
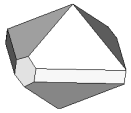
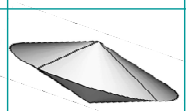
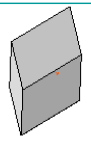
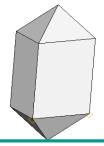
Line Profile: Decomposition model

1. Decompose the entire line-profile into its line, circular-arc, and/or free-form segments.
2. Define a local reference system for each of the segments, and create the primitive T-Map for each one.
3. Arbitrarily set a temporary reference frame for the entire profile and represent each primitive T-Map in this reference system.
4. Intersect the transformed primitive T-Maps in the temporary reference frame to get a tentative T-Map for the entire profile.
5. Find the maximum rotation center (pole) of the profile. Reset the origin of the reference frame to the pole and transform the tentative T-Map to its representation in this new frame.



T-map Catalog: Sample page

More than 50 T-map models have been developed so far based on combinations of target feature, tolerance type and datum type

T-map	Geometry, tolerance, datum	T-map	Geometry, tolerance, datum
	Geom: Rect bar; plane Tol class: size Datum: none		Geom: Rect bar; plane Tol class: size + orient Datum: planar face
	Geom: Round bar; plane Tol class: size Datum: none		Geom: Round bar; plane Tol class: size + orient Datum: offset axis
	Geom: Round bar; plane Tol class: size + orient Datum: planar face		Geom: Planar circular face Tol class: circular runout Datum: axis
	Geom: traing bar; plane Tol class: size Datum: none		Geom: Rect bar; plane Tol class: size + orient Datum: two datums

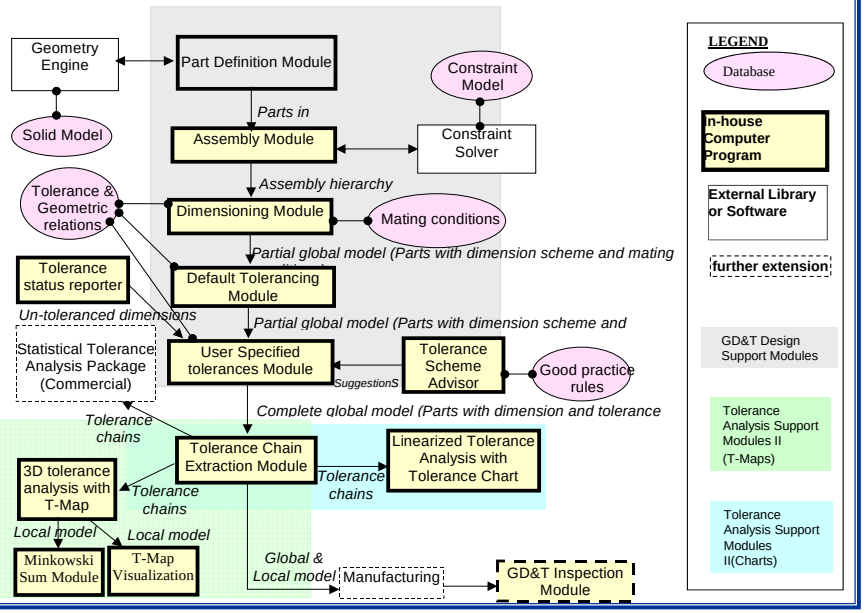
The Tolerance Analysis Maze

Many variations of tolerance analysis approaches exist in practice

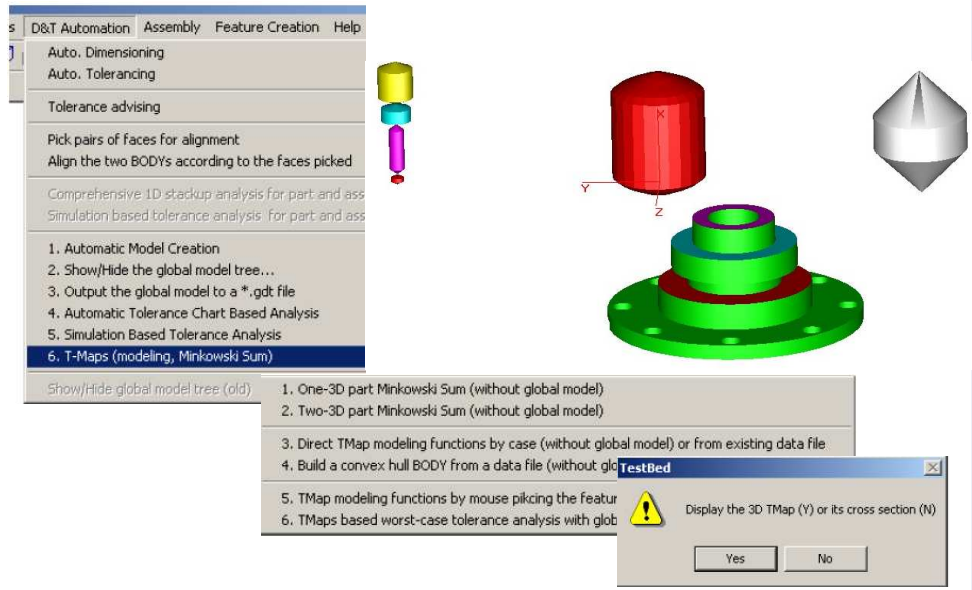
Min/Max charts **I-DEAS** **VSA, eTolmate**

Dimensionality	Analysis	Tolerance classes	GDT standards	Level	Linear-ization	Auto-mation
1-D	worst case	dimensional	compatible	part	Linear	manual
2-D	Statistical: Gaussian	geometric	Partially compatible	assembly	Linearized	Interactive
3-D	Statistical: Any dist.	all	Not compatible	Parts + assembly	Non-linear	automated
	Worst case & statistical	Dimensional + orientation				

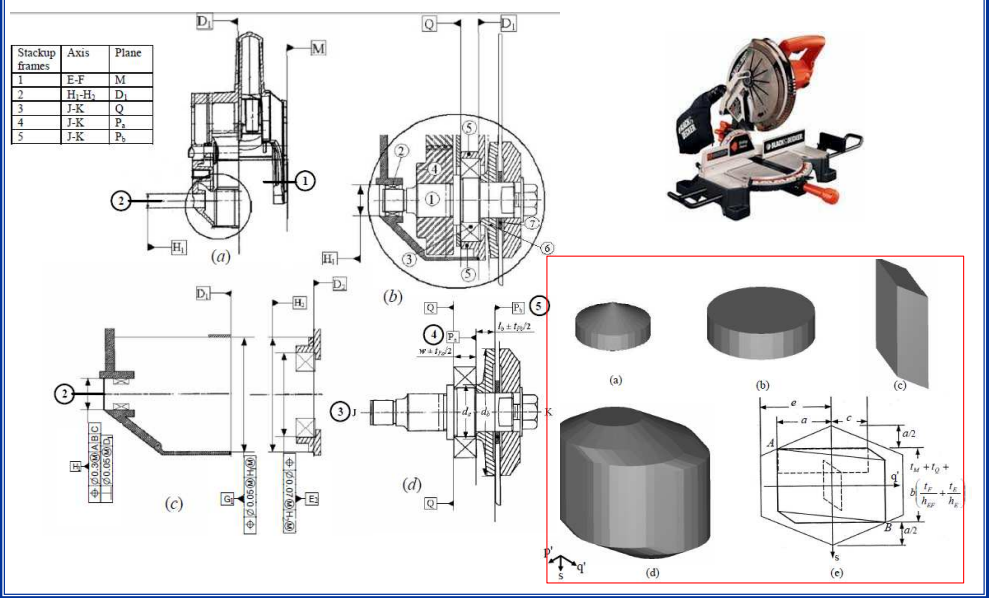
Integrated GD&T: System Architecture



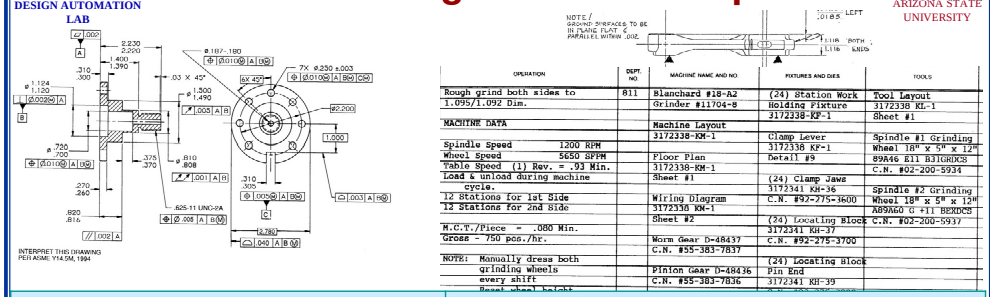
ASU GDT Testbed



B&D miter saw



GD&T in Design vs. Process plans



Designs (formal GD&T)	Process plan (implied GD&T)
DRFs explicitly shown	DRFs are implicit in setups, fixtures
Formal GD&T frames	At most, +/- for dimensions, No GD&T
Datum flow chain directly extracted	Datum, and flow chain implicit , distributed
Consolidated info, in single Drg.	Distributed info (in multiple steps/pages)
Drawings represent final parts	Plans represent many transitions
Many tolerance analysis methods used (1D/2D/3D)	Mostly 1-D tolerance charts are used by process planners

GD&T Mapping

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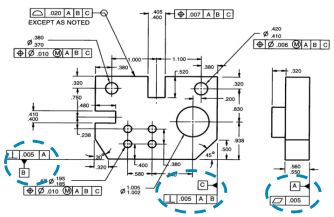
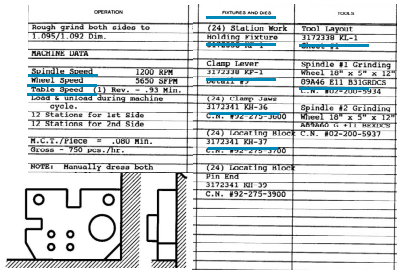
DESIGN



Tolerance Conversion

Tolerance Explication

PROCESS PLAN



- Process planners use their knowledge of machine accuracy, operation variability and fixturing elements to develop mfg plans
- Process planners must <u>convert</u> the GD&T schema to their setups, operation sequence and fixture plans (different datums)
- Stack analyses is typically done with 1D charts and plan documentation only contains conventional ± tolerances
- What if want to independently verify/audit process plan GD&T?
- That would require tolerance <u>explication</u> from process plans
- The T-map model cann be used for both objectives

Tolerance Explication

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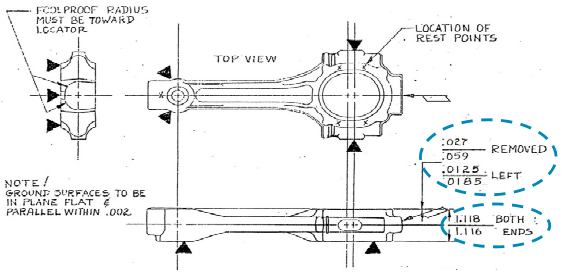

- Geometric & Dimensional tolerance values and type Extraction
- Datum reference frame (DRF) Extraction
- Datum flow chain Extraction, including Transient features
- Convert the +/- dimensional sizes and locations to basic dims, sizes, position tolerances
- Take into account the following errors/deviations in each stage:

I. Locating/positioning errors, coming from:
Fixture errors, Datum errors, Raw stock errors

II. Machining errors, coming from: Machine tool errors, Cutting tool errors

From the dimensional sizes and tolerances we can extract some information regarding position and size tolerances:

Tolerance Conversion

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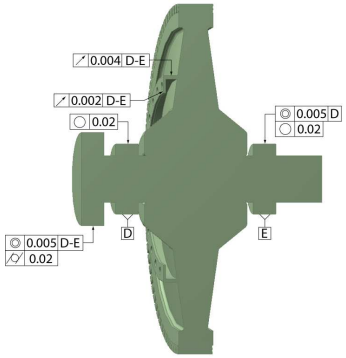
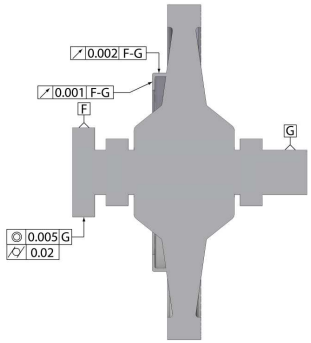
- Process plans typically call for multiple setups
- the datum flow chain used by manufacturing is different from design.
- This requires tolerance conversion and datum transfer.
- Example: design runout tolerances with bearing surfaces D, E; process plan for turning may call for the part to use surfaces E,G instead





- The Tool & Mfg Engineers Handbook documents the manufacturing charts procedure for verifying design tolerances in process sequences
- This is just a 1D stack involving dimensional tolerances only. Trig functions are used to convert angular feature

Datum transformation

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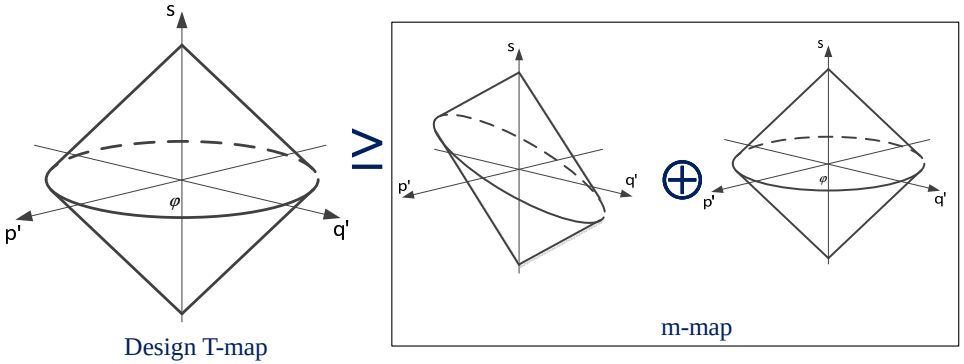
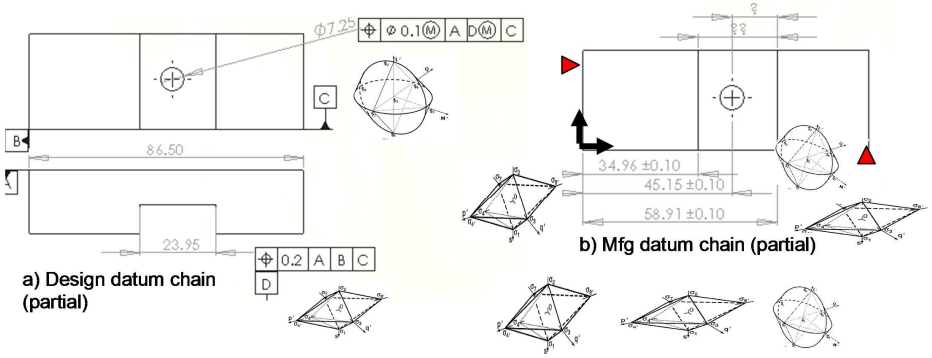
- establish relation between design tolerances and machining tolerance in transferring of the datum
- enables 3D tolerance analysis consistent with Y14.5 standard



- Minkowski Sum of the Manufacturing T-map and the Datum transformation T-map should fit into the Design T-map.

Tolerance Conversion: m-maps



a) Design datum chain (partial)

b) Mfg datum chain (partial)

c) Worst case m-Map for hole feature

- **Procedure**
  - determine relationships between original datum flow and machining ops
  - generate T-Maps corresponding to variations that were controlled directly in design but have become indirect in manufacturing (m-maps)
  - chains can include transient features, as well
  - m-map will depend on all the contributors in the stack and will need to be determined by a Minkowski sum,

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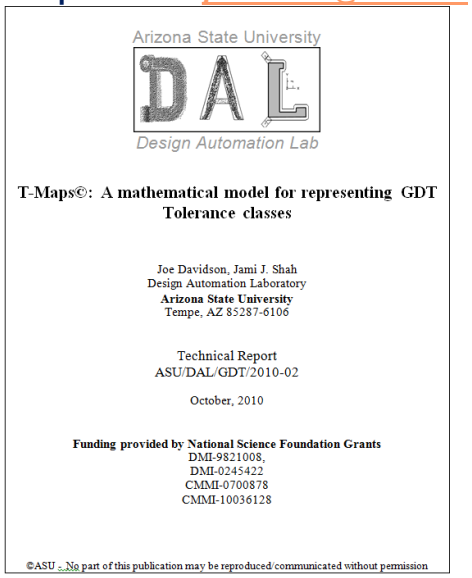
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T-Maps©: A mathematical model for representing GDT Tolerance classes

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Tempe, AZ 85287-6106

Technical Report
ASU/DAL/GDT/2010-02

October, 2010

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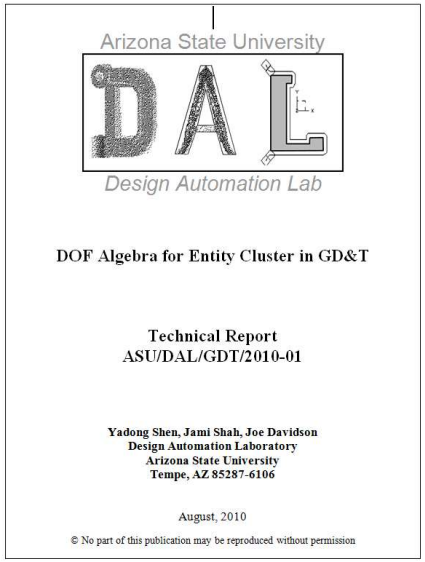
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DOF Algebra for Entity Cluster in GD&T

Technical Report
ASU/DAL/GDT/2010-01

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