A TECHNIQUE FOR EXTRAPOLATING THE 1 KC VALUES OF SECONDARY CAPACITANCE STANDARDS TO HIGHER FREQUENCIES

R. N. Jones
Radio Standards Laboratory
National Bureau of Standards
Boulder, Colorado
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A TECHNIQUE FOR EXTRAPOLATING THE 1 KC VALUES OF SECONDARY CAPACITANCE STANDARDS TO HIGHER FREQUENCIES

A simple technique is described for extrapolating the 1 kc values of certain two-terminal capacitors to higher frequencies without incurring serious losses in accuracy. The method is intended for use with air capacitors having binding post or banana plug type connectors. Because of inherent errors, such connectors are not appropriate for measurement where the highest accuracy is required. For this reason, it is recommended that calibration of this type be performed by secondary laboratories and not be submitted to the National Bureau of Standards.

1. INTRODUCTION

With the establishment of requirements that military contractors show traceability to the National Bureau of Standards in all areas of measurement, increasing demands for new and improved types of calibration services have appeared. Included in these demands is the entire area of high frequency impedance measurements. Prior to the increased demand for traceability there was little requirement for impedance standards and calibration services at frequencies much above 1 kc, and as a result there existed almost no commercially available standards which were designed for high frequency applications. Consequently, NBS has been requested to perform high frequency calibrations of standards which were primarily intended for use at dc and low frequencies. Such requests have presented difficulties because NBS does not possess appropriate standards to use for comparison, nor is it practical to attempt to develop them.

A limiting factor in the accuracy of impedance measurements is associated with the connector systems which are employed as a part
of the instrumentation set-up. This is true to a varying degree at all frequencies, but in general, as the frequency of measurement is increased, the importance of connectors must be given added consideration. There are certain minimum requirements which should be satisfied by any connector system in order to assure accurate results in impedance measurements. First, the connector should be coaxial for purposes of self-shielding so as to avoid any influence of surrounding objects upon the measurement result. This requirement is an important one even at the very low frequencies, especially when small values of capacitance are to be determined. Two other requirements are a well-defined plane of electrical reference and a voltage standing wave ratio (VSWR) as nearly 1.0 as possible. These are most important at the higher frequencies where short wavelengths are encountered and measurement accuracy depends upon the degree of impedance matching within the connector.

Many of the impedance standards submitted to NBS for calibration at frequencies above 30 kc employ banana plug connectors and are, therefore, subject to connection errors which render them inappropriate for use where highest accuracy is desired. Of particular interest here are capacitance standards which are equipped with banana plug connectors of either the plug or jack type.

Because the calibration methods employed at NBS for standards of this type do not depend upon any national reference standard, it is possible for a laboratory to perform its own high-frequency calibrations of such capacitors and avoid the necessity of submitting them to NBS. By employing accepted techniques to calibrate such secondary standards, mutual benefits can accrue both to NBS and to the laboratories requiring calibration services. NBS, by limiting services to only the
highest quality standards, can devote increased effort to research and, as a result, provide better services. On the other hand, other standards laboratories can save money and time and avoid the perils of transporting standards by utilizing as many such self-calibration techniques as possible.

The measurement technique to be described is not new, nor is it complicated. It is presented as an aid to overall efficiency, and while it does contain several sources of error, they are not out of reason when compared with the general accuracy to which standards to this type may be considered reliable.

2. PROCEDURE FOR DETERMINING HIGH FREQUENCY VALUE

Let us assume that we have a two-terminal air capacitor, with binding post or banana plug connectors, whose known low frequency (1 kc) value in farads is \( C_0 \), and that we have an application requiring that we know the capacitance value, \( C_e \), at some higher angular frequency, \( \omega = 2\pi f \), where \( f \) is in cycles per second. Inherent in any capacitor is a certain amount of conductance, series resistance, and series inductance. All of these contribute to the effective value of the capacitor at any given frequency, but, in this case, it is only the series inductance which is considered significant. This is especially true for high \( Q \) capacitors. Consider the simplified equivalent circuit for the capacitor as shown on the right in figure 1.

![Simplified Equivalent Circuit](image)

**Figure 1**  
Simplified Equivalent Circuit
Here \( L \) is the residual series inductance of the capacitor in henries and \( C_0 \) is the low frequency (1 kc) value of the capacitor in farads. At some angular frequency, \( \omega \), we wish to find the effective value \( C_e \). Equating the reactances of these two circuits gives:

\[
\frac{1}{\omega C_e} = \frac{1}{\omega C_0} - \omega L
\]

and

\[
C_e = \frac{C_0}{1 - \omega^2 L C_0}
\]

(1)

To arrive at a value for \( C_e \), it is necessary to evaluate the residual series inductance, \( L \). This is easily done by short circuiting the terminals of the capacitor and employing a grid-dip meter to ascertain the angular frequency, \( \omega_r = 2\pi f_r \), at which the resulting circuit resonates. The residual series inductance of the capacitor is then given by the expression:

\[
L = \frac{1}{\omega_r^2 C_0} - L_s
\]

(2)

where \( C_0 \) is again the low frequency value of the capacitor and \( L_s \) is the inductance of the device used to short circuit the terminals. The effective capacitance is then obtained by substituting the value for \( L \) into (1). A typical set-up for determinations of this type is shown in figure 2.

The solid line in figure 3 indicates the percentage correction, as a function of frequency, for a 1000 picofarad capacitor having a residual series inductance of 0.050 microhenry. This is a typical value for the inductance of a good capacitance standard and may be expected to be nearly the same for all capacitors of the same general type of construction and comparable physical size. With this in mind,
Figure 2

Grid-Dip Meter Arrangement Used to Determine the Resonant Frequency of a Short-Circuited Capacitor.
PERCENT CORRECTION VERSUS FREQUENCY FOR A 1000 pf CAPACITOR HAVING A RESIDUAL INDUCTANCE OF 0.050 MICROHENRY

Figure 3
it should be noted that the larger the value of the capacitor being considered, the more the effective value will rise for a given frequency.

3. SOURCES OF ERROR

Four principal sources of error associated with this measurement procedure are as follows: (a) inaccuracy in the determination of the resonant frequency of the short-circuited capacitor, (b) mutual coupling between the shorting device and the grid-dip meter coil, (c) the inductance of the shorting device, and (d) the equivalent circuit used to represent the capacitor. In examining these sources of error it is necessary to generalize in a number of instances because of the absence of a convenient geometrical arrangement upon which to base a rigorous analysis. This is especially true in the latter three instances where current distributions in conductors and the resulting fields are extremely complex and true representations of actual circuit conditions are almost impossible to realize.

In measuring the resonant frequency of the short circuited capacitor, the grid-dip meter indication is very distinct because the capacitor and short constitute a high Q circuit. This condition makes it possible to achieve repeatability which is beyond the resolution of the frequency scale on the grid-dip meter oscillator. Either of two methods may be employed to improve this resolution. The frequency of the radiated signal of the oscillator may be measured with a heterodyne frequency meter, or the signal may be amplified and fed into a frequency counter. In this manner, it is possible to achieve repeatability well within one percent. Assuming that the accuracy is also of this order, the effect on the accuracy of the total circuit inductance would be approximately two percent. For frequency accuracies within a few
percent, the accuracy in inductance is given by the expression:

\[ \frac{\Delta L}{L} \approx \frac{\Delta \omega}{\omega} \]

where \( \Delta L \) and \( \Delta \omega \) represent the errors in inductance and angular frequency respectively.

The error due to mutual coupling between the short and the grid-dip meter coil is evidently very small because no distinct change in the resonant frequency is caused by varying the distance between them. Any change in the resonant frequency resulting from an increase or decrease in the separation is probably of the same order as the stability and stability of the grid-dip meter oscillator. In any event, the coupling error may be minimized by maintaining a separation of an inch or more between them.

Probably the largest source of error is in the value used to represent the self-inductance of the short. It is possible to improve the situation by making a judicious choice of the size and configuration of the device used so as to permit the use of available formulas [Terman, 1943] to calculate the self-inductance. Even so, there remains a margin of uncertainty due to the geometry of the current path, and such formulas may not be expected to yield accuracies better than ten to fifty percent. However, since the inductance of the short is usually only a small portion of the total inductance of the resonant circuit, this does not represent a serious degradation of the value obtained for \( L \) in equation (2). Figure 4 is a photograph showing two shorting bars which are convenient to use. These are brass bars fitted with banana plugs or jacks appropriately spaced to fit the capacitor terminals.
Finally, we must consider the equivalent circuit used to represent the capacitor. The circuit of figure 1 may be criticized because, in actuality, the inductance and capacitance probably do not exist as shown. Depending upon the physical details of the construction, the capacitor may be more accurately represented by other equivalent circuits. As an example, consider the equivalent circuit of figure 5.

![Equivalent Circuit Diagram]

**Figure 5**

**High Frequency Equivalent Circuit**

Here the inductance and capacitance, instead of being lumped together at one point, are separated into fractional parts of the total and arranged in a manner which more nearly approximates the actual situation. \( C_t \) is the capacitance between the case and the high potential lead passing through it, and \( C_p \) is the capacitance in the stack of plates. \( L_t \) and \( L_p \) are the lead inductances arranged as they may appear in relation to \( C_t \) and \( C_p \). Problems arise, however, because it is difficult to estimate the values to assign these components, and also because complex circuits are cumbersome to handle mathematically. To illustrate the importance of the equivalent circuit, consider a capacitor having a 1 kc value of 1000 picofarads and a residual series inductance of 0.050 microhenry. Using the equivalent circuit of figure 1, the value at 1 Mc is 1001.98 picofarads. The circuit of figure 5 gives a value of 1001.96 picofarads if we let \( L_t = L_p = 0.025 \) microhenry, \( C_t = 10 \) picofarads, and \( C_p = 990 \) picofarads. At this frequency the difference is essentially negligible,
but at 15 Mc the values are 1799.00 picofarads for the circuit of figure 1 and 1786.32 picofarads for the circuit of figure 5, using the same values for \( L_t \), \( L_p \), \( C_t \), and \( C_p \). Thus it is seen that the equivalent circuit assumes increasing importance at the higher frequencies.

Because of the sources of error, it is believed that the reliability of the value obtained for the residual series inductance by the method described is not better than ten percent. Again referring to figure 3, the broken lines indicate the upper and lower limits of the corrections which result from this ten percent uncertainty in the inductance value. In general, it may be said that the effective capacitance may be determined by this method to accuracies of one percent or better up to frequencies where the correction amounts to ten percent of the 1 kc value.

4. CONCLUSION

With the advent of improved rf connector systems, many standards and instruments using the older type connectors are no longer appropriate for use at the level of primary standards. NBS has developed and does maintain standards which are only appropriate for use with the improved type of coaxial connectors [Jones and Nelson, 1962]. Because it is impractical for NBS to attempt to maintain standards for a wide variety of connector configurations, secondary standards laboratories are urged to employ this calibration procedure where possible and when compatible with traceability requirements.
5. REFERENCES
