EVALUATION OF THREE-TERMINAL AND FOUR-TERMINAL PAIR CAPACITORS AT HIGH FREQUENCIES

R.N. Jones

Electromagnetic Fields Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

U.S. DEPARTMENT OF COMMERCE, Philip M. Klutznick, Secretary

Luther H. Hodges, Jr., Deputy Secretary
Jordan J. Baruch, Assistant Secretary for Productivity, Technology and Innovation

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

Issued September 1980

CONTENTS

LIST OF FIGURES AND TABLES

Evaluation of Three-Terminal and Four-Terminal Pair Capacitors at High Frequencies

R. N. Jones

Electromagnetic Fields Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

The low frequency (1 kHz) capacitance values of three-terminal and four-terminal pair air dielectric capacitors can be extrapolated to higher frequencies if the residual series inductance is known. A resonance method for evaluating the residual series inductance of these capacitor types, together with the extrapolation procedure, is described. For the region where the product of capacitance in farads and frequency in hertz is 10^{-2} or less, uncertainties of one percent or less may be obtained.

Key words: Calibration; capacitance; immittance standards; inductance; four-terminal pair capacitance; residual series inductance; three-terminal capacitance; resonance techniques.

1. Introduction

Manufacturers of capacitance standards usually provide the buyer with the capacitance value at 1 kHz. However, many such standards are useful for calibrating immittance measuring instruments well into the high frequency range, provided that appropriate allowance is made for changes resulting from residual series inductance in the capacitor. The techniques described are used at the National Bureau of Standards to establish standards for high frequency use. The procedure was previously described [1][1] for use with two-terminal capacitors fitted with unshielded connectors. It is described again here, but with modifications for use with three-terminal and four-terminal pair (4TP) standards. The method is useful for both fixed and variable air capacitors.

2. Background

A resonance technique for evaluating residual series inductance, together with an accompanying procedure for mathematically extrapolating low frequency (1 kHz) capacitance values to higher frequencies is described in reference [1]. This procedure was specifically intended for use with two-terminal air-dielectric capacitors equipped with unshielded connectors of the binding post or banana plug type. In the ensuing years, three-terminal

---

[1] Figures in brackets indicate the literature references at the end of this paper.

capacitance measuring instruments have appeared which cover the range from 0.001 to 1000 pF at frequencies up to 1 MHz and capacitance standards have been established at NBS to provide supporting calibration services. More recently, immittance measuring instruments have been introduced which utilize an active measurement principle [2] and a four-terminal pair arrangement for connecting the device under test to the instrument. These instruments, often called LCR meters,[2] operate at frequencies well above 1 MHz and utilize 4TP capacitors as calibration standards.

To establish the high frequency values for both the three-terminal capacitance standards as well as the 4TP capacitance standards, NBS has again employed resonance methods combined with a mathematical extrapolation similar to that used for two-terminal types. Thus the procedures to be described are essentially only adaptions in the procedure in reference [1] modified for use with three-terminal capacitors and 4TP capacitors.

It is to be emphasized that the procedures described in this technical note are applicable only to air dielectric capacitors or capacitors having dielectrics whose dielectric coefficient does not change with frequency. Due to interfacial polarization, the dielectric coefficient of some materials, such as silvered mica, decreases as frequency increases. This occurs in the audio region and above. Hence, a simple linear extrapolation from 1 kHz based solely on the residual series inductance, does not yield an accurate result. A second limitation to this procedure is that it should not be employed at frequencies approaching self-resonance of the capacitor where the reactance of the residual series inductance approaches that of the capacitance. In this region the effective capacitance increases rapidly and approaches infinity, and the uncertainty becomes unacceptably large due to inaccuracies in the residual series inductance as determined by the resonance technique.

3. Determination of High Frequency Effective Capacitance

The procedure for using the residual series inductance of a capacitor to calculate its effective capacitance value at higher frequencies is basically the same for two-terminal, three-terminal and 4TP capacitors. It is not an exact method because it includes a number of approximations and assumptions, but if its use is confined to a reasonable range of frequencies and capacitance values, it will yield accuracies that are adequate to provide excellent calibration support for instruments in the 0.1 to 1.0 percent uncertainty range. Because most of the capacitors used as high frequency immittance standards are of comparable physical size and configuration, they are usually found to have residual series inductances in the range of 10 to 50 nanohenries. This being true, it may be assumed that as long as the product of the nominal (1 kHz) value of capacitance in farads and the frequency in hertz is equal to or less than 10^{-2}, the uncertainty of the extrapolated value of effective

[2] Inductance, capacitance and resistance

capacitance will be one percent or less. This criteria roughly defines the frequency and capacitance ranges where the technique is used by NBS. Briefly, the extrapolation principle is illustrated by the following diagram and derivation:

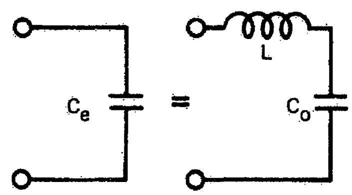


Figure 1. Simplified high frequency equivalent circuit for a capacitor

For any two-terminal, three-terminal or 4TP capacitor, the effective capacitance, C_e, at any frequency above 1 kHz is obtained by equating the reactances:

$$\frac{1}{\omega C_e} = \frac{1}{\omega C_o} - \omega L_r$$

and

$$C_e = \frac{C_o}{1 - \omega^2 L_r C_o}, \qquad (1)$$

where C_e is the capacitance in farads, L_r is the residual series inductance in henries, C_o is the low frequency (1 kHz) value of the capacitance in farads, and, ω, is the angular frequency in radians per second. This simple derivation assumes that any contribution from series resistance or shunt conductance in the equivalent circuit of figure 1 is negligible, which is true of high quality capacitors having dissipation factors of 10^{-3} or less.

4. The Resonance Method for Three-Terminal Capacitors

The diagram in figure 2 is a representation of the circuit elements which are of significance in the determination of residual series inductance, L_r. The main capacitance element, represented by C_o, is enclosed in a conducting shield. The two coaxial connectors, "H" and "L", provide the means of connection to an external circuit. C_H and C_L are the stray capacitances from each side of C_o to ground. The residual inductance, L_r, is a property of the internal leads to the capacitance element and, to some extent, the capacitance structure. To indicate the distributed nature of L_r, it is represented as two equal halves, $L_r/2$, shown on each side of C_o. It is this residual inductance that causes a significant change in the effective value of the capacitance standard as the frequency is increased. To evaluate L_r, The capacitor is fitted with a shorting element having a self-inductance L_s, and is shown connected between the center conductors of "H" and "L", but not contacting the outer conductors of the two coaxial connectors. A grid-dip meter is shown coupled inductively to L_s. By varying the frequency of the grid-dip meter oscillator, a sharp dip will be observed when the series resonant frequency of the circuit is reached. This circuit includes the capacitances C_o, C_H and C_L, plus the inductances L_r and L_s.

Assuming that the mutual inductance between the shorting element, L_s, and the grid-dip meter coil is negligible, the series resonance relationship may be written as:

$$X_L = X_C$$

or

$$\omega_r (L_r + L_s) = \frac{1}{\omega_r C_T} \qquad (2)$$

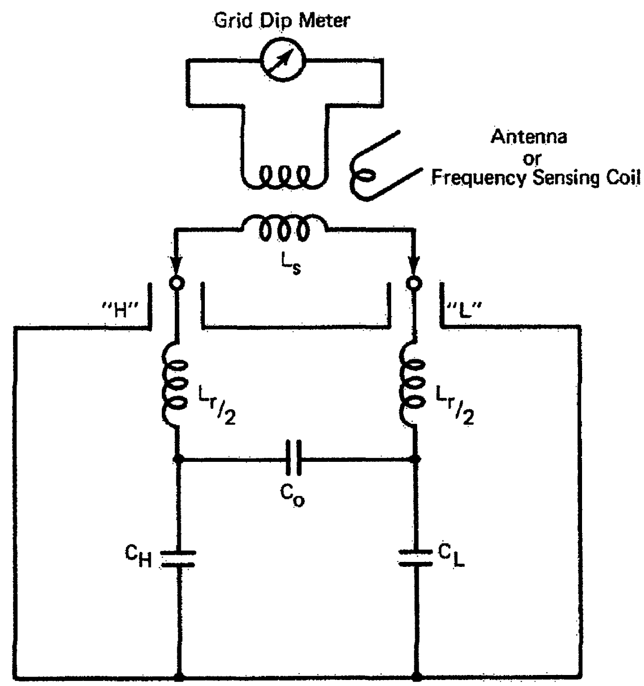


Figure 2. Circuit arrangement for measuring the residual series inductance of a three-terminal capacitor.

where ω_r is the angular resonant frequency in radians per second, $(L_r + L_s)$ is the sum of the inductances in henries, and the total circuit capacitance, C_T, is in farads. The circuit configuration of figure 3 shows the arrangement of the capacitance elements, C_o, C_H and C_L which comprise C_T. The value of C_T is given by the expression

$$C_T = C_o + \frac{C_H C_L}{C_H + C_L} \qquad (3)$$

The value of C_o is usually given to satisfactory accuracy by the manufacturer or may be determined by a three-terminal capacitance measurement at 1 kHz. The values of C_H and C_L are obtained by two-terminal capacitance measurements at 1 kHz. Placing a coaxial short on terminal "L" and measuring the two-terminal capacitance of "H" (see figure 2) yields the sum of C_o and C_H. Then, by similarly shorting terminal "H" and measuring the two-terminal capacitance at "L", the sum of C_o and C_L is obtained. With C_o previously determined, the values of C_H and C_L may be found and the value of C_T calculated by eq (3). Rearranging eq (2), the residual series inductance, L_r, is given by:

$$L_r = \frac{1}{\omega_r^2 \; C_T} - L_s . \qquad (4)$$

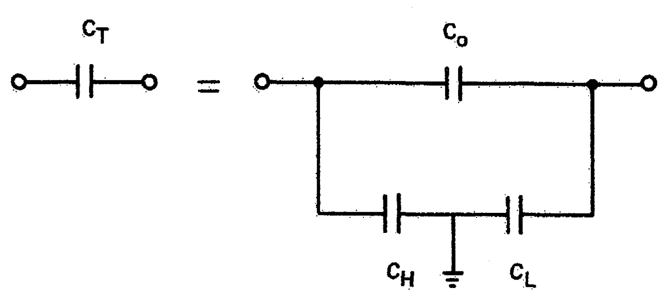


Figure 3. The capacitances comprising the total resonating capacitance, C_T.

Figure 4 is a block diagram showing an arrangement of instrumentation that has proven convenient for measuring the self-resonant frequency of the short circuited capacitor. The resonant frequency may be read directly from the grid-dip meter to about three significant figures, but the use of an rf amplifier and a frequency counter yields better resolution, thereby allowing a much better determination of the precision of the resonant frequency. A typical measurement setup is illustrated in the photograph of figure 5.

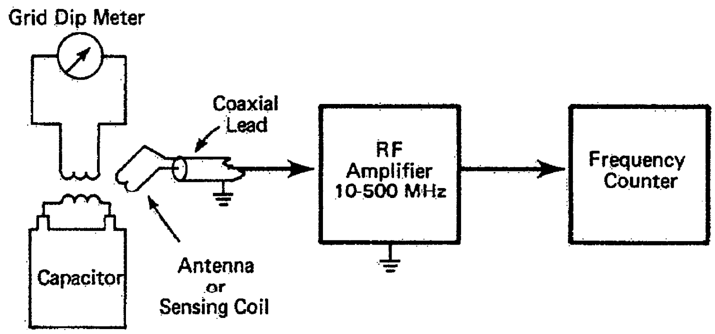


Figure 4. A block diagram showing a typical arran ment for measuring the resonant frequency of a short circuited capacitc

5. The Resonance Method for Four-Terminal Pair Capacitors

Although the same basic resonance approach is used for 4TP capacitors as that just described for three-terminal capacitors, there are some necessary modifications which arise because of differences in capacitor construction. Also, the 4TP measurement principle is different from the three-terminal method. A complete explanation of 4TP LCR meter operation is provided in ref [2], but the principle will be briefly discussed here in order that the modifications referred to above may be better understood. The diagram of figure 6 illustrates the 4TP measurement principle and figure 7 is a diagram of a 4TP capacitance standard of the type used to calibrate 4TP immittance (LCR) meters. In figure 6, a current is passed through the device under test (DUT) and also through a reference standard resistor, R_S. The complex impedance of the DUT is then determined by taking the product of the complex ratio of the voltages V_X and V_S, and the value of R_S. This measurement principle makes it necessary to use separate pairs of coaxial connectors on the DUT as shown in figure 7. One pair is used for current injection and is labeled H_{cur} and L_{cur}. The other pair is used to detect the voltage (or potential), V_X, across the DUT and these are labeled H_{pot} and L_{pot}. The use of H and L designates the input (High) and output (Low) sides of the DUT. It is this external connector arrangement together with the inaccessability of the points "H" and "L" (fig. 7) for connecting the shorting element that require the modification of the procedure used for three-terminal capacitors.

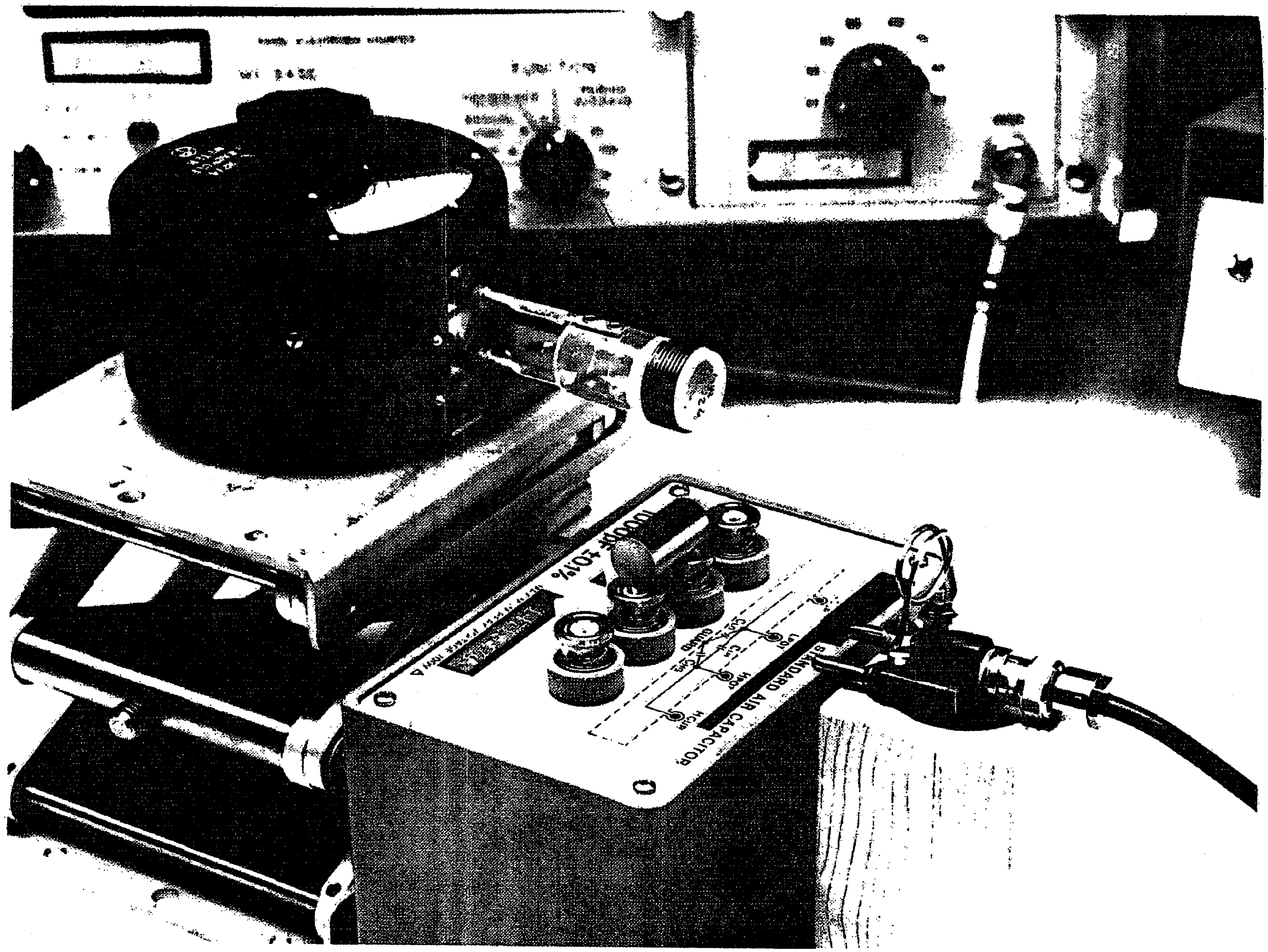


Figure 5. Photograph of instrumentation setup for determining capacitor self-resonance.

Referring to figure 7 the main capacitor, C_0, is usually an air dielectric parallel plate capacitor, and the other elements are the residuals shown as lumped inductors and capacitors. Loss components are omitted because they are not of first order significance in this discussion, assuming the capacitance standard is of good quality. A shorting element for use in determining the residual series inductance is shown as L_s. Examination of figure 7 will reveal that below the points "H" and "L" the equivalent circuit is identical to the three-terminal equivalent circuit of figure 2. In fact, it is only the capacitance between points "H" and "L" that is measured by a 4TP instrument. The inductances L_1, L_2, L_3 and L_4

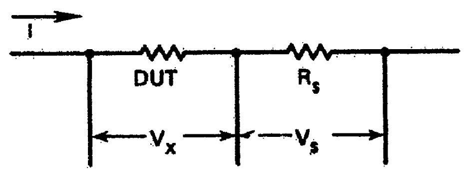


Figure 6. The four-tei .inal pair concept.

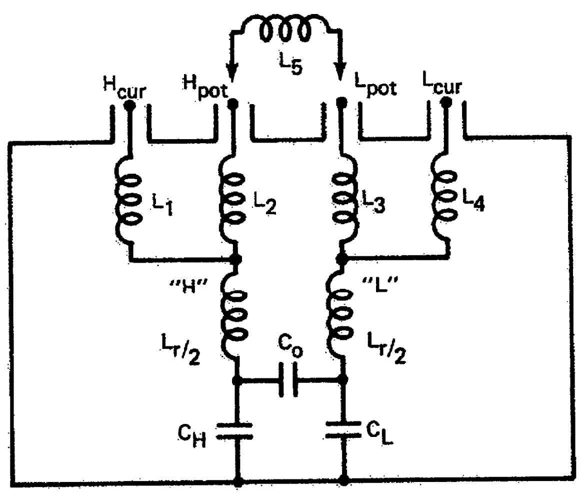


Figure 7. Circuit model for four-terminal pair capacitor.

are the residuals associated with the internal leads connecting the points "H" and "L" to the external coaxial connectors, and do not effect the high frequency effective value of the main capacitor, C_o. Because the internal points "H" and "L" are not readily accessible the shorting element must be connected externally. As a result, two of the four lead inductances (usually L_2 and L_3) will be part of the circuit whose resonant frequency is determined by the grid-dip meter. Otherwise, the procedure for determining the residual inductance of a 4TP capacitor by the resonance method is essentially the same as that already described for three-terminal capacitors. Thus the residual inductance of the 4TP capacitor is given by:

$$L_r = \frac{1}{\omega_r^2 C_T} - (L_s + L_2 + L_3), \qquad (5)$$

if the shorting element is connected between the H_{pot} and L_{pot} terminals as shown in figures 5 and 7. In the case of one particular type of 4TP capacitor, L_2 and L_3 amounted to the inductance of two BNC type panel feed through connectors, or approximately 14 nH. This value was derived by measuring panel feed through connectors of an identical type on an inductance bridge. In most instances, it will probably be found that the internal portions of the inductance paths between "H" and "L" and the external connectors are not directly accessible and some indirect method for determining the inductances L_2 and L_3 is required. However, with care it should be possible to achieve accuracies to within one or two nH, thereby preserving the accuracy of L_r to within ±10 percent. Of course, if other terminal combinations are used such as H_{cur} and L_{pot}, or H_{pot} and L_{cur}, or H_{cur} and L_{cur}, then other inductances would replace L_2 and L_3 in eq (5). Having found the residual inductance, L_r, for the 4TP capacitor, the extrapolation represented by eq (1) is used to determine the effective capacitance at a given frequency.

6. The Shorting Elements

Of utmost importance to the accuracy of this procedure is the value used for the inductance, L_s, of the shorting element. Because it may be nearly as large as the residual series inductance to be measured, it is necessary to determine L_s as accurately as possible. Figure 8 is a close-up photograph of several shorting elements in use with three-terminal and 4TP capacitors. Using a high frequency impedance bridge, some attempts at measuring the inductances of these shorting devices have been made; however, more accurate results are probably realized through computation using various inductance formulas for conductors of round or rectangular cross section. These formulas were originally derived by NBS [3] and have been more recently published by Terman [4]. For convenience the expressions for the self-inductances of a straight round rod and a straight rectangular bar of materials whose permeability, $\mu = 1$ are given here.

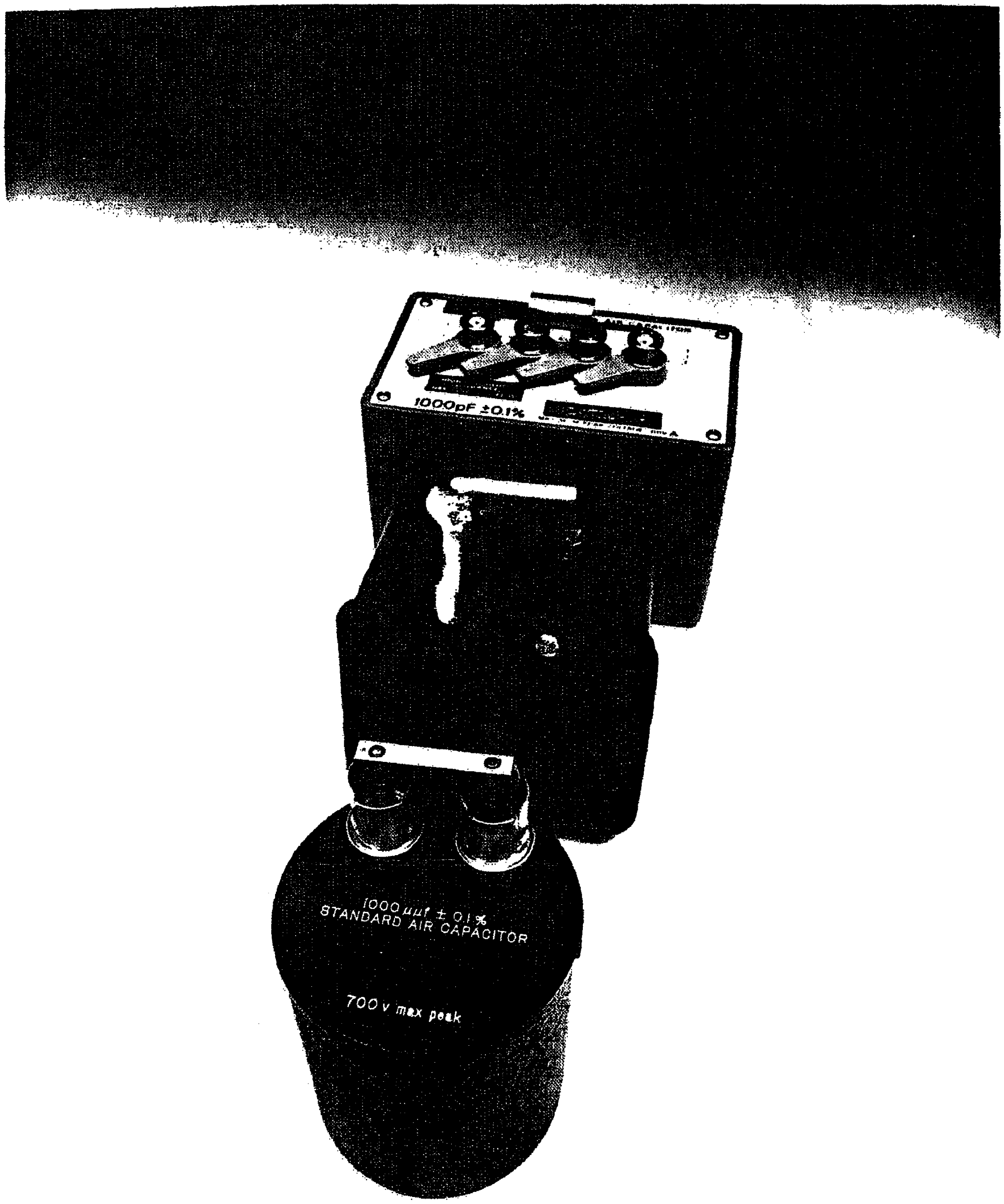


Figure 8. From top to bottom: One 4TP capacitor and two three-terminal capacitors fitted with shorting elements.

For a straight round rod of length, $\ell < 100d$, where d is the diameter, the self inductance at high frequencies is:

$$L_s = 0.00200\ell \; (2.303 \log_{10} \frac{4\ell}{d} - 0.75 + \frac{d}{2\ell}) \text{ microhenry.} \qquad (6)$$

For a straight rectangular bar, the expression for self-inductance is:

$$L_s = 0.00200\ell \; (2.303 \log_{10} \frac{2\ell}{b+c} + 0.5 + 0.2235 \frac{b+c}{\ell}) \text{ microhenry} \qquad (7)$$

where b, c and ℓ are the width, thickness and length in cm respectively. If the dimensions are in inches as in ref [4], the constant, 0.00508, must be used in place of, 0.00200, in eqs (6) and (7).

In order to minimize the uncertainty in the inductance value calculated for the shorting element, L_s, it is helpful if it is configured so that the inductance is as s ll as possible. This will make the error in the calculated value of L_s less significant in relation to L_r. Three shorting elements are pictured in figure 9. Note especially the one made of a cylindrical rod, and the one made of A.W.G.#18 copper wire. Each is 2.25 cm long. For least uncertainty in the calculated value, the larger diameter rod is better because its inductance as calculated from eq (6) is only 6.6 x 10^{-9} henry as compared to 14.6 x 10^{-9} henry for the wire element. Thus the final result will be more accurate where the larger diameter shorting element is used.

7. Sources of Error

Three sources of error associated with this measurement procedure are as follows: (a) inaccuracy in the determination of the resonant frequency of the short-circuited capacitor, (b) mutual coupling between the shorting element and the grid-dip meter coil, and (c) the inductance of the shorting element. In examining these sources of error it is necessary to generalize in a number of instances because of the absence of a suitable geometrical arrangement upon which to base a rigorous analysis.

In measuring the resonant frequency of the short circuited capacitor, the grid-dip meter indication is very distinct because the capacitor and shorting element constitute a high Q circuit. This condition makes it possible to achieve repeatability which is beyond the resolution of the frequency scale on the grid-dip meter oscillator. Either of two methods may be employed to improve this resolution. The frequency of the radiated signal of the oscillator may be measured with a heterodyne frequency meter, or the signal may be amplified and fed into a frequency counter. In this manner, it is possible to achieve

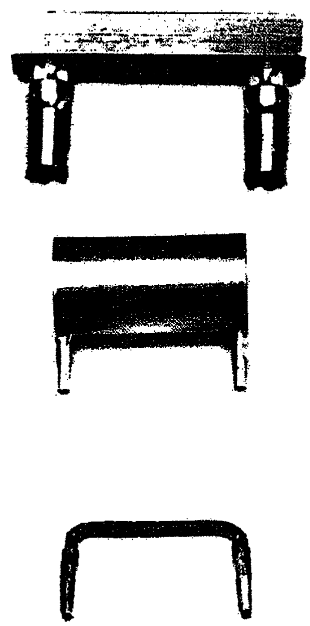


Figure 9. Various types of shorting elements for use with three- erminal and four- terminal pair capacitors.

repeatability well within one percent. Assuming that the accuracy is also of this order, the effect on the value of the total circuit inductance would be approximately two percent. For frequency accuracies within a few percent, the accuracy in inductance is given by the expression:

$$\frac{\Delta L_r}{L_r} \approx \frac{\Delta\omega}{\omega} \qquad (8)$$

where ΔL_r and $\Delta\omega$ represent the errors in inductance and angular frequency respectively.

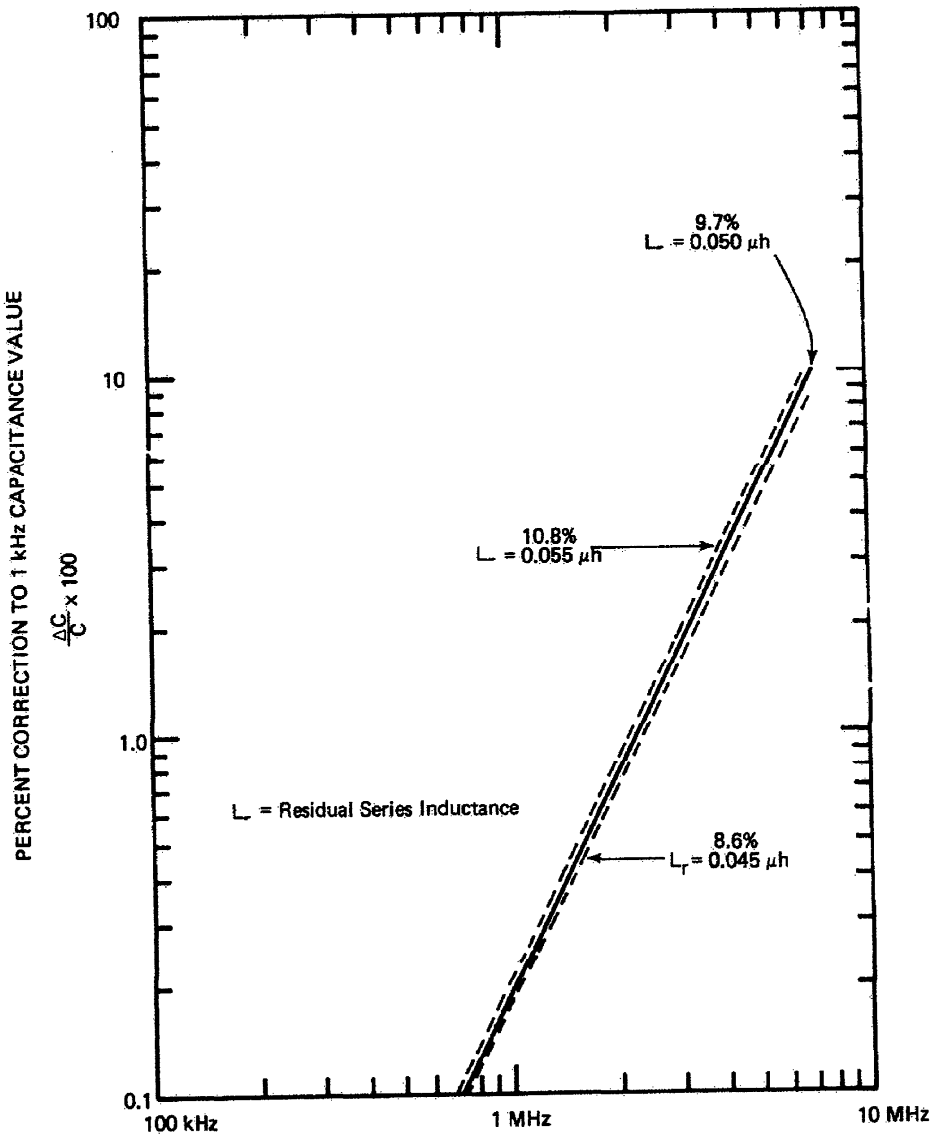
The error due to mutual coupling between the shorting element and the grid-dip meter coil is evidently very small because no distinct change in the resonant frequency is caused by varying the distance between them. Any change in the resonant frequency resulting from an increase or decrease in the separation is probably of the same order as the stability and setability of the grid-dip meter oscillator. In any event, the coupling error may be minimized by maintaining a separation of two or three centimeters between them.

Probably the largest source of error is in the value used to represent the self-inductance of the shorting element. As already mentioned, it is possible to improve the overall accuracy of the procedure by optimizing the configuration of the shorting element to obtain the minimum self-inductance. At best, however, eqs (6) and (7) provide only approximations of the inductance of shorting elements because of the assumptions made in the course of their derivations. These assumptions include a free space environment, a perfectly straight current path and the absence of end effects, all of which are violated to some degreee in this application.

Because of the sources of error, it is estimated that the reliability of the value obtained for the residual series inductance by the method described is of the order of ten percent. Referring to figure 10, the broken lines indicate the upper and lower limits of the corrections which result from this ten percent uncertainty in the inductance value. In general, the effective capacitance may be determined by this method to accuracies of one percent or better up to frequencies where the correction amounts to ten percent of the 1 kHz value.

8. Summary

Procedures have been described for evaluating the residual series inductance of air-dielectric three-terminal and 4TP capacitors and for extrapolating the low frequency (1 kHz) values of these capacitors to higher frequencies. The procedures require only what is considered to be commonly available equipment. A high degree of skill or experience is not required, and the measurements and calculations can be performed in a short time. Uncertainties in effective capacitance of one percent or less are obtainable in the region where the product of capacitance in farads and frequency in hertz is 10^{-2} or less.



PERCENT CORRECTION FOR C_0 VERSUS FREQUENCY FOR A 1000 pf CAPACITOR HAVING A RESIDUAL INDUCTANCE OF 0.050 MICROHENRY

Figure 10. Percent correction to C_0 versus frequency for a 1000 pF capacitor having a residual series inductance of 0.050 microhenry.

9. References

[1] Jones, R. N., A technique for extrapolating the 1 KC values of secondary capacitance standards to higher frequencies, Nat. Bur. Stand. (U. S.) Tech. Note 201, 16 pages (Nov. 5, 1963).

[2] Maeda, K. and Narimatsu, Y., Multifrequency LCR Meters Test Components Under Realistic Conditions, Hewlett-Packard Journal, 24-31 (Feb. 1979).

[3] Rosa, E. B., and Grover, F. W., Formulas and tables for the calculation of mutual and self-inductances, Nat. Bur. Stand. (U.S.) Bull. 8, [1], 1-237 (Jan. 1912).

[4] Terman, F. E., Radio Engineers Handbook, 1st ed., 48-51, (McGraw-Hill Book Co., Inc., New York, 1943).