

# Capacitance and Dissipation Factor Measurements<sup>†</sup> from 1 kHz to 10 MHz

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**Abstract:** A measurement technique developed by K. Yokoi et al. at Hewlett-Packard Japan, Ltd. has been duplicated and evaluated at the National Institute of Standards and Technology (NIST) to characterize four-terminal pair capacitors. The technique is based on an accurate three-terminal measurement made at 1 kHz using a capacitance bridge and wideband single-port measurements made between 30 MHz and 200 MHz using a network analyzer. The measurement data are fitted to the four-terminal pair admittance model defined by R. Cutkosky to compute capacitance and dissipation factor at any frequency up to 10 MHz. Capacitors characterized using this technique will be used as impedance reference standards for a general-purpose digital impedance bridge recently developed at NIST to calibrate inductors and ac resistors. The technique could also lead to a future NIST Special Test for dissipation factor.

## INTRODUCTION

Recent efforts by the Electricity Division at the National Institute of Standards and Technology (NIST) to develop improved impedance comparison methods from 20 Hz to 1 MHz<sup>(1)</sup> have made it necessary to find a means to evaluate the reference impedances used in such comparisons. The impedance-characterization method discussed in this paper is described by Suzuki, et al.<sup>(2)</sup> and is based on the four-terminal pair impedance work of Cutkosky<sup>(3)</sup> and Jones<sup>(4,5)</sup>.

The four-terminal pair impedance,  $Z_{4tp}$ , of a device is defined as a combination of its real component,  $R$ , and its imaginary component,  $X$ :

$$Z_{4tp} = R + jX.$$

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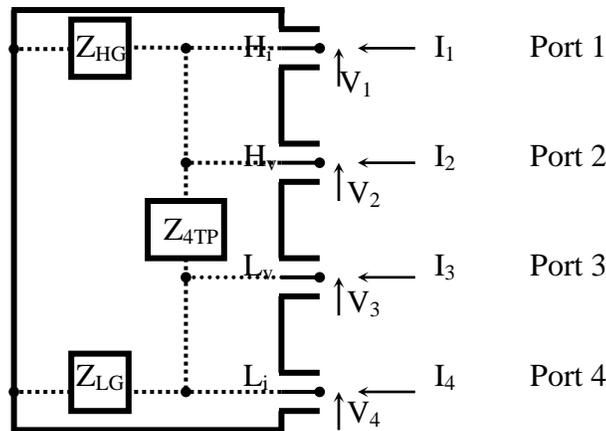
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For a capacitor, this is often expressed in terms of dissipation factor,  $D$ , and capacitance,  $C$ . The frequency response of a capacitor can be expressed as the capacitance change relative to a reference frequency (1 kHz, for this measurement method).

$$D = \arctan \frac{R}{X} \quad C = \frac{1}{2\pi f X}, \quad \text{and} \quad \frac{C}{C_{1\text{kHz}}} = \frac{C(f)}{C_{1\text{kHz}}}$$

where  $C$  is the capacitance at frequency  $f$ . In general, four-terminal pair capacitors can be described with  $z$ -parameters in matrix form. Using Cutkosky's definition of four-terminal pair impedance, and referring to Fig. 1,

$$Z_{4TP} = \frac{V_2}{I_2}, \quad V_3 = 0, \quad I_3 = 0, \quad \text{and} \quad I_2 = 0.$$



**Figure 1. Four-Terminal Pair Capacitor**

It is possible to derive an expression showing that the four-terminal pair impedance is equal to

$$Z_{4TP} = \frac{Z_{21}Z_{34} - Z_{31}Z_{24}}{Z_{31}}; \quad \text{where} \quad Z_{ij} = Z_{ji} = \sqrt{Z_{jj}(Z_{ii} - Z_{iisj})}.$$

$Z_{ii}$  is the driving-point impedance at port  $i$  with all other ports left open and  $Z_{iisj}$  is the driving-point impedance at port  $i$  with port  $j$  shorted and all other ports left open.  $H_1$  (port 1) and  $L_i$  (port 4) refer to the high- and low-current inputs of the capacitor.  $H_v$  (port 2) and  $L_v$  (port 3) refer to the high- and low-voltage terminals.

## MEASUREMENT PROCEDURE

Capacitance measurements are taken at the reference frequency of 1 kHz using a precision capacitance meter. The high-frequency measurements are made at frequencies from 40 MHz to 170 MHz and are used to extrapolate inductance and resistance behavior at frequencies between 1 kHz and 10 MHz. The extrapolations are then used to compute capacitance for the device under test for the same frequencies. The measurement steps are outlined in the following paragraphs. We have implemented the inductance and resistance extrapolations and the capacitance computations in a commercially available mathematical software environment. The measurements are taken using a graphical programming package for controlling instrumentation.

## Measurements at 1 kHz Using a Precision Capacitance Bridge

A precision capacitance meter is used at 1 kHz to make measurements between the low and high terminals,  $C_{lh}$ , the high and ground terminals,  $C_{hg}$ , and the low and ground terminals,  $C_{lg}$ , of the 4TP capacitor under test (see Fig. 2). These measurements are used as a reference with which to compare higher frequency measurements using a network analyzer.

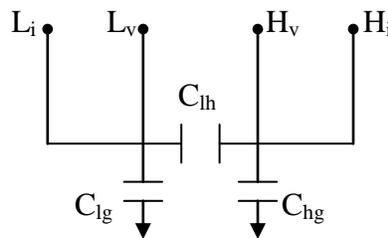


Figure 2. Four-Terminal Pair Capacitor: Simple Model

## High-Frequency Measurements Using a Network Analyzer

A network analyzer is used to measure a series of driving-point impedances at the capacitor terminals. The measurements are automated; however, the user is prompted to manually connect the proper driving-point impedance terminals to the network analyzer via a connector adapter. The delay, loss frequency, and fringing capacitance of the adapter are pre-programmed into the network analyzer via the controlling program and are measured and updated periodically. Additionally, the network analyzer is calibrated periodically using a reference air line.

## Open/Short/Load Calibration of the Network Analyzer

Prior to each measurement or set of measurements, an open/short/load (O/S/L) calibration of the measurement port of the network analyzer is performed using known standards. The controlling program allows for the measurement data and sweep parameter data to be saved in user-specified files. During the O/S/L calibration, the standard open, short, and load terminations are connected using a torque wrench to ensure a consistent measurement plane. Averaging is used in taking the network analyzer measurements to reduce measurement noise.

## High-Frequency Driving-Point Impedance Measurements

A software program has been developed to automate the data retrieval; however, the physical connections for the individual driving-point impedance measurements must be performed interactively with the software program. A driving-point impedance is the impedance across any one pair of terminals, for example,  $Z_{11}$  refers to the driving-point impedance at port 1,  $Z_{22}$  to port 2, etc. As mentioned, a four-terminal pair capacitor can be described using z-parameters in matrix form as developed by Cutkosky<sup>(3)</sup> and refined by Suzuki, et al.<sup>(2)</sup> This approach requires driving-point impedance difference measurements in which one of the driving-point impedances is measured with one of the other terminals shorted to ground. For example,  $Z_{11s2}$  is the driving-point impedance at port 1 with port 2 shorted to ground.

## MEASUREMENT RESULTS

Commercial four-terminal pair standard air capacitors were used for these measurements. The values of the capacitors were 1 pF, 10 pF, 100 pF, and 1000 pF.

### Measurements at 1 kHz

Each capacitance measurement result is an average of 20 measurements taken in one day. The voltage levels for the 1 pF, 10 pF, 100 pF, and 1000 pF capacitor measurements are 15 V, 15 V, 7.5 V, and 0.75 V, respectively. The 1 kHz measurement results along with type A uncertainties are shown in Table 1.

Nominal Capacitance (pF)	Measured Capacitance (pF)	Type A Uncertainty (pF)
1	0.999951	0.000004
10	10.00020	0.000003
100	99.9923	0.00009
1000	1000.046	0.0000

**Table 1. 1 kHz Measurement Results**

### High-Frequency Measurements

Frequency	Capacitance (pF)	Type A Unc. (pF)	Dissipation Factor (x1e-6)	Type A Unc. (x1e-6)
1 pF Standard Capacitor				
100 kHz	0.99995	0.000000	-1	0.2
1 MHz	0.99983	0.000001	-18	7.4
10 MHz	0.98766	0.000101	-593	240
10 pF Standard Capacitor				
100 kHz	10.0002	0.00000	0	0.0
1 MHz	10.0002	0.00000	0	0.2
10 MHz	10.0032	0.00006	-11	5.7
100 pF Standard Capacitor				
100 kHz	99.992	0.0000	0	0.0
1 MHz	99.995	0.0001	1	0.6
10 MHz	100.311	0.0063	19	19.4
1000 pF Standard Capacitor				
100 kHz	1000.05	0.000	1	0.1
1 MHz	1000.30	0.004	20	2.4
10 MHz	1026.42	0.403	646	73.6

**Table 2. Capacitance and Dissipation Factor Results at Selected Frequencies**

A commercial network analyzer was used for these measurements. Driving-point impedance measurements were performed at frequencies from 40 MHz to 170 MHz and extrapolations and computations were made to establish capacitance and dissipation factor characterization over frequencies from up to 10 MHz. Table 2 presents the capacitance and dissipation factor results

for six repeat measurements for each of the four standard capacitors for frequencies of 100 kHz, 1 MHz, and 10 MHz. All of the measurements were performed using a single O/S/L calibration of the network analyzer. Note that only type A uncertainties are reported. A detailed error analysis of the measurement system (including influences of the network analyzer) is still underway.

## **FACTORS INVOLVED IN AN ERROR ANALYSIS**

The measurement process consists of taking a set of measurements using a capacitance meter at 1 kHz and a network analyzer at around 100 MHz, and then processing the data using a mathematical model. The techniques employed at the 1 kHz reference frequency are well established and their influences on the overall measurement uncertainty are known. The factors to be determined for a complete error analysis of the measurement process consist mainly of the errors of the network analyzer. Just prior to taking a measurement set using the network analyzer, an O/S/L calibration is performed on the network analyzer so that updated corrections are used when measuring a standard capacitor. Periodically the network analyzer is calibrated using a standard air line to verify measurement results. Further study is underway to precisely establish the influence of the network analyzer calibrations on the overall measurement uncertainty.

The mathematical model used to estimate a standard capacitor's frequency characteristics is based upon an assumption that it is possible to use mathematical regressions on driving-point impedance measurements at high frequencies (40 MHz to 170 MHz) in order to estimate the capacitor's behavior at lower frequencies (up to 10 MHz). The regression must be performed for resistive and inductive components of the impedance. Capacitance is assumed to be constant with frequency. These inductance and resistance regressions must also be examined to determine their effects on the overall measurement uncertainty.

## **CONCLUSIONS AND FUTURE WORK**

Recent work at NIST to develop a digital impedance bridge to compare generalized multi-terminal impedances has led to the requirement of impedance reference standards in the 1 Hz to 1 MHz range. The four-terminal pair measurement system, developed at Hewlett-Packard Japan and described in this paper, is being evaluated to provide reference impedances in this frequency range. In duplicating the system at NIST, our intent has been to confirm the theory and modeling approach, code the data collection and data analysis software, learn the test procedures and conduct tests, and estimate the system errors. All but the last step has now been completed. Once the error analysis is done, we will begin using a set of calibrated four-terminal pair capacitors as wideband impedance standards for the digital impedance bridge. If there is sufficient interest, we also plan to offer a NIST Special Test for four-terminal pair capacitors.

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