EVALUATION OF A CAPACITANCE SCALING SYSTEM

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Abstract

An improved error analysis of an existing capacitance scaling system to support measurements of higher valued (10 nF to 100 \textmu F), ceramic dielectric, 4-terminal-pair (4TP) capacitance standards over the 100 Hz to 100 kHz frequency range is described. The capacitance scaling system uses a commercial impedance (LCR) meter and a single-decade inductive voltage divider (IVD) as an impedance comparator. 4TP capacitors in decade (10:1) steps from 10 nF to 100 \textmu F are measured. The system’s 10:1 scaling error is determined using 100 pF and 1 nF air-dielectric 4TP capacitance standards with known capacitance and loss characteristics over frequency. This paper discusses the significant reductions in measurement uncertainty that were attained through the use of improved calibration standards and measurement method refinements. Details of the uncertainty analysis for a 10 nF capacitor and verification data are presented.

Index terms: Inductive Voltage Dividers, IVD, capacitance, calibration.

Introduction

A capacitance scaling system developed to support the measurement of a specific manufacturer’s particular type of higher valued (10 nF to 100 \textmu F), ceramic-dielectric, 4TP
capacitance standards over the 100 Hz to 100 kHz frequency range has been described previously [1]. Two desirable aspects of this system are that it is based mostly on commercially available instrumentation, and the calibration procedure is largely automated. An investigation was conducted at the National Institute of Standards and Technology (NIST) to determine whether the measurement uncertainty of this system could be reduced to support the routine measurement of improved 4TP capacitance standards, either as they become available commercially, or as recently developed at NIST to support generalized ac impedance metrology [2]. The chief goal of this investigation was to separate the relative contributions of measurement uncertainty due to the impedance comparator itself from those of the 100 pF and 1 nF 4TP air-dielectric capacitance standards used to calibrate it. A manual four-terminal-pair (4TP) capacitance bridge designed at NIST by Cutkosky in 1970 [3] was used to perform the investigation. This bridge can compare standards (with short-term stability better than a part in $10^7$) over the frequency range of 100 Hz to 20 kHz. The stability of the capacitor under test is crucial because of the complexity of the manual balancing process. Thus, it is not practical to calibrate commercial 4TP air and ceramic dielectric capacitors, which have temperature coefficients on the order of 10 to 30 parts in $10^6$. As a result, commercially available, nitrogen-dielectric capacitance standards were selected for use in the investigation.

**4TP scaling system based on an LCR meter**

In 1997, Aoki and Yokoi [1] described a method of calibrating higher value 4TP capacitors (10 nF to 100 μF) using a 4TP impedance (LCR) meter, a decade inductive voltage divider (IVD), and two reference capacitors. The LCR meter is not only a stable impedance detector, but also a variable-frequency and variable-amplitude (current or voltage) source. The decade IVD is used to make a capacitor “appear” to be ten times larger by scaling the voltage applied to the sense terminals of the LCR meter. This allows the LCR meter to be used as a transfer
device at approximately the same operating point (all but eliminating its linearity errors [4] and influence of short term stability) for capacitors that differ in value by a factor of 10.

A simplified diagram of the NIST capacitance scaling system is shown in Fig. 1. The scaling fixture controls the circuitry used to adjust signal inputs, and levels and connections of the 10:1 IVD used to scale from lower to higher values of capacitance. The LCR meter and scaling fixture (interface and IVD) is self-calibrated by comparing two calibrated 4TP air capacitors with known frequency characteristics [5,6]. This essentially establishes the scaling ratio of the fixture as a 1:1 and 10:1 interface. The scaling ratio self-calibration procedure and the uncertainty analysis are given in detail in the following section.
Four measurements are required to calibrate the 10:1 scaling ratio. First, a 100 pF capacitor is measured using a 3T capacitance bridge, $C_{100\,\text{pF}}$. The impedance of this standard is then measured using the LCR meter, $Z_{\text{100\,pF,LCR}}$, with the 10:1 IVD switched into the circuit (this makes the 100 pF capacitor appear to have the impedance of a 1 nF capacitor). The impedance of a 1 nF standard is measured using the LCR meter without the IVD, $Z_{\text{1\,nF,LCR}}$. And finally the capacitance of the 1 nF standard is measured using the capacitance bridge, $C_{\text{1\,nF}}$. The scaling ratio $K$ is determined by:

$$K = \frac{Z_{\text{100\,pF,LCR}}}{Z_{\text{1\,nF,LCR}}} \cdot \frac{C_{\text{1\,nF}}}{C_{\text{100\,pF}}}$$  \hspace{1cm} (1)$$

The capacitance scaling method described in [1] uses 4TP air capacitors. While the 100 pF and 1 nF air capacitors have insignificant dissipation factors over the frequency range of interest, their capacitance is not particularly stable (typically 30 parts in $10^6$ per degree C). Over the course of several months of measurements using the capacitance scaling system, we observed fluctuations in the 1 kHz capacitance of standard air capacitors on the order of 7 parts in $10^6$. In order to discriminate between the uncertainty of the scaling system and the instability of the 4TP air standards, 3T nitrogen capacitors were introduced. At 1 kHz, the nitrogen capacitors
varied less than 0.1 parts in $10^6$. The nitrogen capacitors are connected to the 4TP connector at the scaling fixture using a coaxial adaptor.

The relative uncertainty of the scaling ratio, $u_r(K)$, is given by

$$u_r(K) = \left( \frac{\sigma\left(\frac{Z_{100pLCR}}{Z_{inLCP}}\right)}{\frac{Z_{100pLCR}}{Z_{inLCP}}} + \frac{\sigma\left(\frac{C_{100pCB}}{C_{inCB}}\right)}{\frac{C_{100pCB}}{C_{inCB}}} \right)^2 + (NL_{LCR})^2 + (NL_{CB})^2,$$  

where $\sigma(x)$ represents the standard deviation of measurement $x$, $NL_{LCR}$ is the uncertainty contribution from the nonlinearity of the LCR meter and scaling fixture, and $NL_{CB}$ is the uncertainty contribution from the nonlinearity of the capacitance bridge. Measurements were performed to determine the nonlinearity of the LCR meter. Using capacitors that varied from nominal as much as 100 parts in $10^6$, the LCR readings were in error by less than 1 part in $10^6$, supporting previous studies [4]. Similarly, the nonlinearity of the capacitance bridge was found to be less than 1 part in $10^6$. The major contribution to the uncertainty of the scaling ratio, $K$, comes from the type of reference capacitors used to determine $K$.

Based on equation (2), $u_r(K)$ is typically 2 parts in $10^6$ using nitrogen capacitors and 10 parts in $10^6$ using air capacitors.

The IVD used in the scaling fixture should be very stable. To separate the influence of the interface from the IVD, we replaced it with a commercial IVD and observed the same level of variations in $K$ (2 parts in $10^6$ using nitrogen capacitors). This indicates that the stabilities of the interface, the LCR meter, and the capacitors are the main contributors to the uncertainty of the scaling ratio calibration. To determine whether this instability is time dependent, we shortened the measurement process from 30 minutes to 5 minutes. The standard deviation of the scaling ratio was reduced by nearly an order of magnitude to approximately 0.3 parts in $10^6$. 
Scaling ratio uncertainty tests were performed between 100 Hz to 100 kHz using 10 nF air and nitrogen capacitors and the results are presented in Table 1.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$u_r$($K$) using air capacitors</th>
<th>$u_r$($K$) using nitrogen capacitors</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>1 kHz</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>10 kHz</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>100 kHz</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1. Frequency dependence of scaling ratio uncertainty, $u_r(K)$, in parts in $10^6$ for 10 nF capacitors.

10 nF Capacitor Measurement

Once the scaling ratio is determined, the system can be used to calibrate capacitors. A 1 nF reference capacitor is measured using the capacitance bridge, $C_{inCB}$, and using the LCR meter with the IVD, $Z_{inLCR}$. Then, a 10 nF device under test $Z_{DUT10n}$, is measured using the LCR meter without the IVD, $Z_{10nLCR}$. These measurements are used to find the impedance of the device under the test.

$$Z_{DUT10n} = K \frac{1}{\sigma C_{inCB}} \frac{Z_{10nLCR}}{Z_{inLCR}}$$

(3)

Note that the frequency characteristic of the 1 nF reference capacitor is assumed flat.

The relative uncertainty $u_r(Z_{DUT10n})$ is given by:

$$u_r(Z_{DUT10n}) = \sqrt{\left(\frac{\sigma K}{K}\right)^2 + \left(\frac{\sigma Z_{10nLCR}}{Z_{inLCR}}\right)^2 + \left(\frac{\sigma Z_{10nLCR}}{Z_{inLCR}}\right)^2 + \left(\frac{\sigma C_{inCB}}{C_{inCB}}\right)^2 + (u_{inCB})^2}$$

(4)
The same notation as in equation (2) is followed. Note that the measurement of $C_{\text{InCB}}$ has both Type-A (standard deviation of the measurements) and Type-B ($u_{\text{InCB}}$) error components. Numerical estimation of the 10 nF impedance uncertainty is shown in equation (5).

$$\frac{\partial Z_{DUT10_n}}{Z_{DUT10_n}} \approx \frac{\partial C_{DUT10_n}}{C_{DUT10_n}} = 10^{-6} \sqrt{2^2 + 6^2 + 1^2 + 0.1^2 + 1^2} = 6.5 \times 10^{-6}$$ \hspace{1cm} (5)

The actual fluctuations of the measured 10 nF nitrogen capacitance standard produced a standard deviation of 4 parts in $10^6$, in good agreement with Equation (5).

A 10 nF nitrogen dielectric capacitor with short-term stability of better than 0.1 part in $10^6$ was measured using the NIST 4TP bridge and the capacitance scaling system. A summary of the results is shown in Table 2.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Deviation from NIST 4TP Bridge Measurement</th>
<th>Measurement Uncertainty Coverage Factor ($k = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>NIST 4TP Bridge</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Capacitance Scaling System</td>
<td>19</td>
<td>1.9</td>
</tr>
</tbody>
</table>

* measurement standard deviation

Table 2. 10 nF capacitor measurements and the uncertainty levels (in parts in $10^6$) at several frequencies.
The results in Table 2 show excellent agreement between the NIST manual bridge and the capacitance scaling system at 1 kHz and 10 kHz using stable 1 nF and 10 nF 3T nitrogen capacitors. The agreement at 100 Hz is not as good but is still within 20 parts in $10^6$. The results are considerably better than those obtained using 4TP air capacitors, due to their inherent instability.

The capacitance scaling system was originally designed [1] to use 4TP air standards as references due to their known frequency characteristics [5]. To determine the influence of the frequency dependence of the nitrogen capacitors on determination of the 10 nF capacitance, a 1 nF nitrogen capacitor was measured to vary less than 2 parts in $10^6$ in the range from 100 Hz to 10 kHz. This variation influenced the results for the 10 nF capacitor less than 1 part in $10^6$. This result indicates that nitrogen dielectric capacitors should be used as references at frequencies up to 10 kHz, whenever possible. However, air capacitors should be used as references for the 100 kHz range because the uncertainty of the frequency dependence of nitrogen dielectric capacitors is larger than the temperature coefficient instability of the 4TP air standards.

**Conclusions**

This evaluation of an LCR-based capacitance scaling system shows its potential for calibrating stable capacitors in the 100 pF to 10 nF range. Preliminary results indicate that the system is capable of measuring capacitance and dissipation factor for higher valued 4TP solid dielectric standards (up to 100 μF) over a wide frequency range (100 Hz to 100 kHz). The stability of these measurements is limited by the stability of the device under test. However, further tests are required to determine the uncertainties of the system over this range. For further information on this work see reference [8].
Acknowledgements

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References


