### I. How to Entangle Remote Emitters by Projection

#### Example for three trapped ions

![Detector](image)

**Detection**

$$D_0 = \sum_{\alpha=0}^2 e^{i\alpha\phi} (a_0|\psi\rangle \langle a_0| + a_1|\phi\rangle \langle a_1|)$$

**Atomic state after detection of the first photon with polarization \(\sigma^+\)**

$$|\psi_{(1)}\rangle = D_1 |e, e, e\rangle = |+, +, +\rangle + |+, -, +\rangle + |-, +, +\rangle$$

**Detection operator**

$$D_0$$

**Polarizer**

$$\sigma^+$$

**Detector**

$$\sigma^-$$

**Optical fibers**


### II. Generation of long-living W-state in Emitters

#### Detection of three photons

**Initial atomic state:**

$$|\psi_{(0)}\rangle = |e, e, e\rangle$$

**Detection of a \(\sigma^-\)-polarized photon**

$$|\psi_{(1)}\rangle = D_1 |e, e, e\rangle = |+, +, +\rangle + |+, -, +\rangle + |-, +, +\rangle$$

**Detection of a \(\sigma^-\)-polarized photon**

$$|\psi_{(2)}\rangle = D_2 D_1 |e, e, e\rangle = |+, +, +\rangle + |+, -, +\rangle + |-, +, +\rangle$$

**Detection of a \(\sigma^-\)-polarized photon**

$$|\psi_{(3)}\rangle = D_3 D_2 D_1 |e, e, e\rangle = |+, +, +\rangle + |+, -, +\rangle + |-, +, +\rangle + |-, -, +\rangle + |-, +, -\rangle + |-, -, +\rangle$$

#### Explanation of the algorithm: mimicking the coupling of angular momenta

Total angular momentum (TAM) eigenstates \([3, m]\) are defined as the eigenstates of the total angular momentum operator \(I_3\) and its z-component \(I_3_z\).

Generally, N qubits can be coupled to \(2^N\) different quantum states.

1. **Example: 2-qubit system**

   **Spin-1 triplet:**
   
   $$(|+1\rangle + |10\rangle + |01\rangle)/\sqrt{3}$$

   **Spin-1 singlet:**
   
   $$(|11\rangle - |00\rangle)/\sqrt{2}$$

2. **Example: 3-qubit system**

   **Spin-2 triplet:**
   
   $$(|+2\rangle + |11\rangle + |00\rangle)/\sqrt{3}$$

   **Spin-2 singlet:**
   
   $$(|11\rangle - |00\rangle)/\sqrt{2}$$

#### This approach corresponds to a successive coupling of angular momenta.

#### We have found an algorithm that transforms the coupling of angular momenta into explicit experimental setups.

### III. Generation of any symmetric State in Emitters

#### Usage of elliptical polarizers allows the generation of any symmetric N-qubit state

Any symmetric N-qubit state \(|\phi\rangle\) can be expressed as a sum of symmetric Dicke states \(|D_k\rangle\), with \(|D_k\rangle\) being the symmetric Dicke state with \(k|+\rangle\) excitations.

$$|\phi\rangle = \sum_{k=0}^N n_k |D_k\rangle(0)$$

**Example: 3-qubit Dicke states**

$$|D_0\rangle = |1\rangle$$

$$|D_1\rangle = (|2\rangle + |1\rangle)/\sqrt{2}$$

$$|D_2\rangle = (|3\rangle + |1, 1\rangle + |0, 2\rangle)/\sqrt{3}$$

**The coefficients \(n_k\) are used to construct the polynomial \(P(\frac{1}{2})\).**

$$P(\frac{1}{2}) = \sum_{m=0}^N n_m |D_m\rangle(0)$$

**The roots \(z_2^n\) of this polynomial define the polarizer orientations \(e_i = a_i|\sigma^+\rangle + b_i|\sigma^-\rangle\).**

In case of a 3-qubit state there is furthermore a simple correspondence between polarizer orientation and the entanglement class of the generated state [1,5].

### IV. Generation of all Total Angular Momentum Eigenstates in Emitters

#### Moving the polarizers from the detectors to the single photon sources allows to generate the equivalent states encoded in Photonic qubits.

**Photonic qubits**

$$|\psi_{\text{PHOT}}\rangle = \sum_{k=0}^N n_k |D_k\rangle(0)$$

**Atomic qubits**

$$|\psi_{\text{ATOM}}\rangle = \sum_{k=0}^N n_k |D_k\rangle(0)$$

**State \(|\psi_{\text{PHOT}}\rangle\) compatible with a successful detection event:**

$$|\psi_{\text{PHOT}}\rangle = \prod_{k=0}^N P_{k} |0\rangle |0\rangle$$

**State \(|\psi_{\text{ATOM}}\rangle\) compatible with a successful detection event:**

$$|\psi_{\text{ATOM}}\rangle = \prod_{k=0}^N D_{k} |0\rangle |0\rangle$$

**Direct transfer of results obtained for the generation of entangled states in matter qubits is possible.**

### V. Generation of all above States in Photonic Qubits

Moving the polarizers from the detectors to the single photon sources allows to generate the equivalent states encoded in the polarization degrees of freedom of N photons [4].