Efficient sympathetic cooling of an ion crystal in a double-well trap potential

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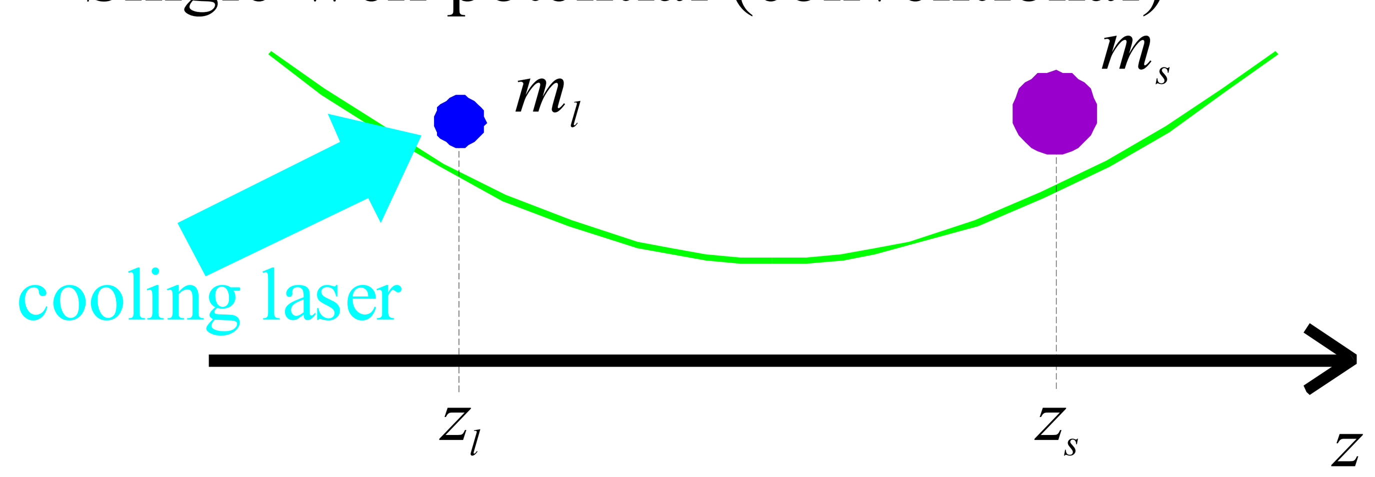
Abstract

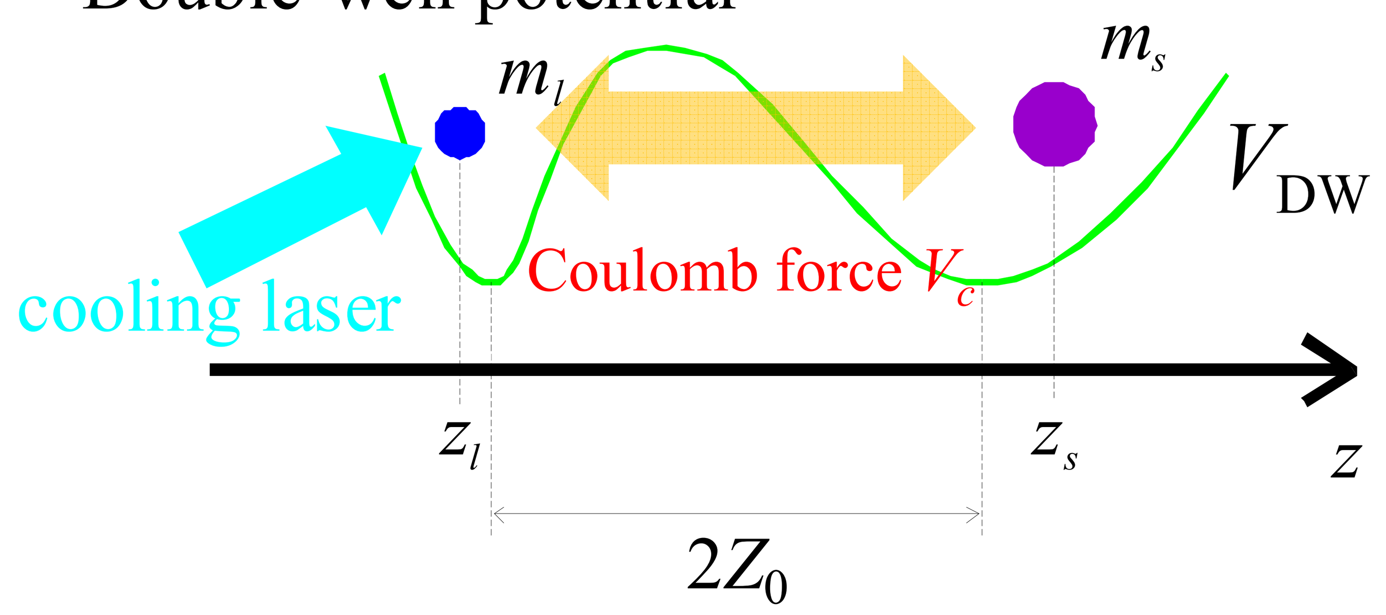
Efficient sympathetic cooling of two-ion system, in which one is laser-cooled and the other is sympathetically cooled, in a linear rf trap with a double-well potential is proposed. The double-well potential consists of two parabolic wells, and there is one ion in each well. By theoretical analysis, the normal modes of the small oscillations around the equilibrium are derived, and a measure of the sympathetic cooling rate is obtained. As a result, it is found that the sympathetic cooling is efficient when the resonant frequency of the small oscillation of the sympathetically cooled ion is close to that of the laser-cooled ion. In the double-well potential, therefore, the sympathetic cooling of the ion species whose mass is much heavier or lighter than that of the laser-cooled ion can be efficient. According to the estimation, the double-well potential may be made by the microfabricated electrode configuration or by the optical dipole force trap.

Double-well potential

Single-well potential (conventional)



Double-well potential



Radial confinement by the pseudopotential is supposed to be tight

Motions along *z* direction are considered

In case of m_s>>m_l, the sympathetic cooling is not so efficient in the single-well potential

Hamiltonian of small oscillation around the equilibrium

$$V = V_C + V_{\mathrm{DW}}$$

$$V_{\mathrm{DW}} = \frac{1}{2} m_l \Omega_l^2 (z_l + Z_0)^2 + \frac{1}{2} m_s \Omega_s^2 (z_s + Z_0)^2$$

$$V_C = \frac{q^2}{4\pi\varepsilon_0 (z_s - z_l)}$$

- **Equilibrium Z_l Z_s**

$$\left.\frac{\partial V}{\partial z_l}\right|_{\substack{z_l = Z_l \\ z_s = Z_s}} = 0, \quad \left.\frac{\partial V}{\partial z_s}\right|_{\substack{z_l = Z_l \\ z_s = Z_s}} = 0$$

- **Mass-normalized small displacements**

$$\delta_l = \sqrt{m_l}(z_l - Z_l), \quad \delta_s = \sqrt{m_s}(z_s - Z_s)$$

Approximation up to the second order of δ_l and δ_s

$$H = \frac{1}{2}\left(p_l^2 + p_s^2\right) + \frac{1}{2}(\delta_l, \delta_s) \left.\begin{pmatrix} \partial^2 V/\partial \delta_l^2 & \partial^2 V/\partial \delta_l \partial \delta_s \\ \partial^2 V/\partial \delta_l \partial \delta_s & \partial^2 V/\partial \delta_s^2 \end{pmatrix}\right|_{\delta_l = \delta_s = 0} \begin{pmatrix} \delta_l \\ \delta_s \end{pmatrix}$$

Explicit expressions are available, although they are very complicated.

Normal modes

By diagonalizing the matrix:

$$U = \left.\begin{pmatrix} \partial^2 V/\partial \delta_l^2 & \partial^2 V/\partial \delta_l \partial \delta_s \\ \partial^2 V/\partial \delta_l \partial \delta_s & \partial^2 V/\partial \delta_s^2 \end{pmatrix}\right|_{\delta_l = \delta_s = 0}$$

- Eigenfrequencies ω_+ and ω_-
- Eigenvectors →Transfer matrix T

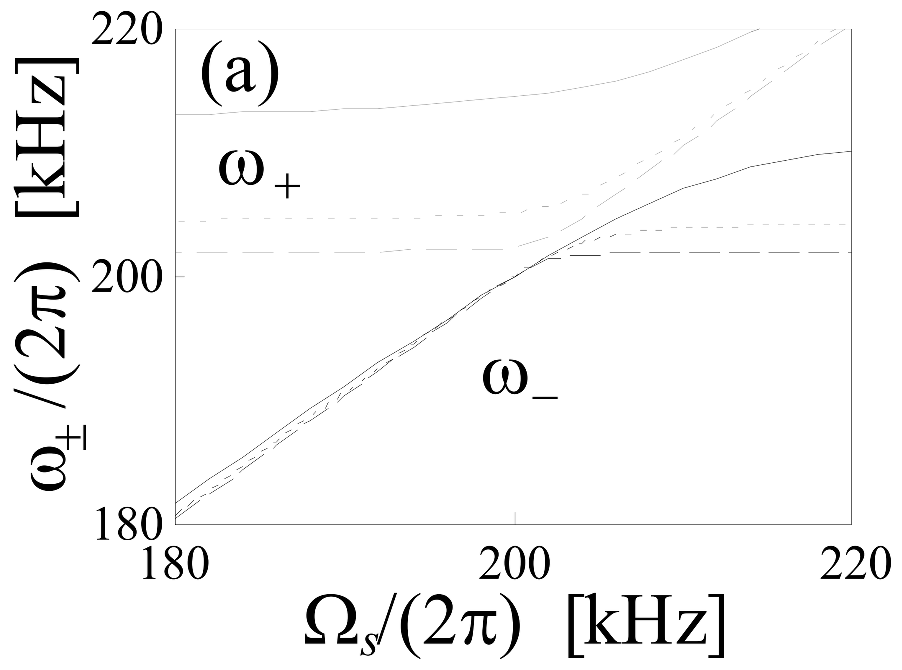
$$\begin{pmatrix} \delta_+ \\ \delta_- \end{pmatrix} = T \begin{pmatrix} \delta_l \\ \delta_s \end{pmatrix} \qquad T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$H = \frac{1}{2}\left(p_+^2 + p_-^2\right) + \frac{1}{2}\omega_+^2 \delta_+^2 + \frac{1}{2}\omega_-^2 \delta_-^2$$

Example of eigenfrequencies

Laser-cooled ion: Ba^+
Sympathetically cooled ion: C_{60}^+

$\Omega_l = 2\pi \times 200\text{kHz}$



Solid lines $Z_0 = 20\mu\text{m}$

Dotted lines $Z_0 = 30\mu\text{m}$

Dashed lines $Z_0 = 40\mu\text{m}$

Cooling rate

Classical analysis

Linear damping (1-D optical molasses)

Equation of motion

$$\frac{d^2}{dt^2}\begin{pmatrix}\delta_l\\ \delta_s\end{pmatrix} + \begin{pmatrix}\kappa & 0\\ 0 & 0\end{pmatrix}\frac{d}{dt}\begin{pmatrix}\delta_l\\ \delta_s\end{pmatrix} + U\begin{pmatrix}\delta_l\\ \delta_s\end{pmatrix} = 0$$

In the normal-mode expression:

$$\frac{d^2}{dt^2}\begin{pmatrix}\delta_+\\ \delta_-\end{pmatrix} + \kappa\begin{pmatrix}c^2 & cs\\ cs & s^2\end{pmatrix}\frac{d}{dt}\begin{pmatrix}\delta_+\\ \delta_-\end{pmatrix} + \begin{pmatrix}\omega_+^2 & 0\\ 0 & \omega_-^2\end{pmatrix}\begin{pmatrix}\delta_l\\ \delta_s\end{pmatrix} = 0$$

$c = \cos\theta, s = \sin\theta$

Quantum-mechanical analysis

Sideband cooling

Transition rate that reduces the phonon number

$$P_+ = \alpha I_+ \eta_+^2 c^2 n_+$$

$$P_- = \alpha I_- \eta_-^2 s^2 n_-$$

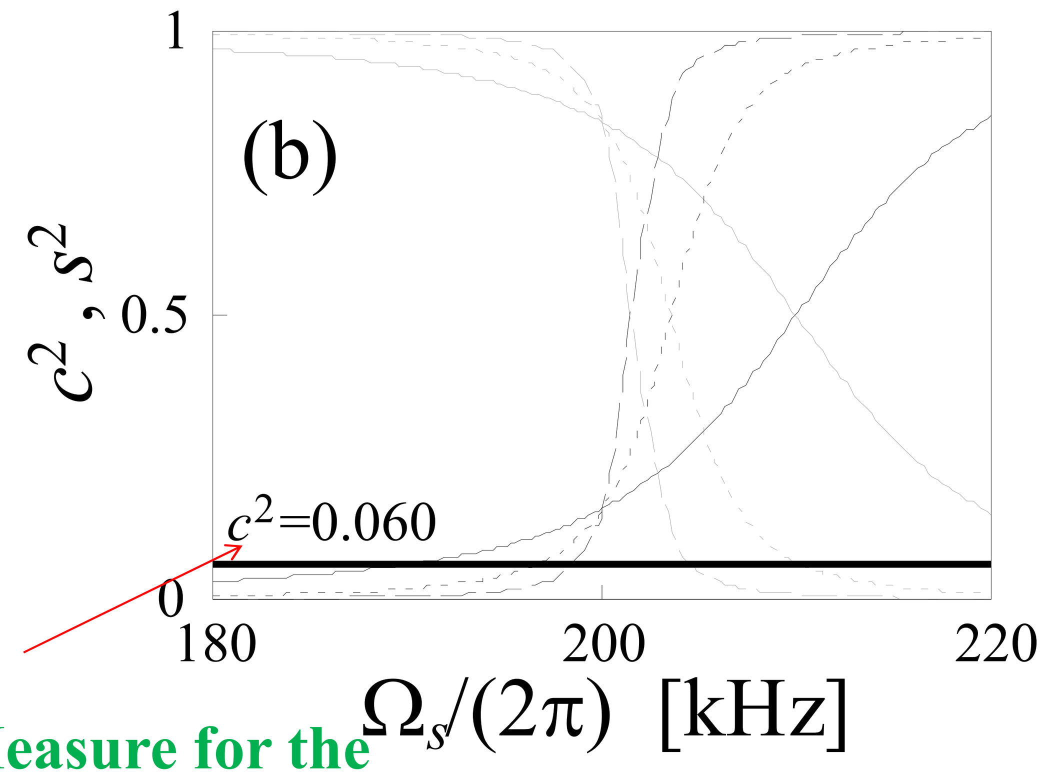
α: Constant that represents the transition strength
$I_\pm$: Laser intensity tuned to the sideband
c: Speed of light
$n_\pm$: Vibrational quantum number

$$\eta_\pm = \sqrt{2\hbar k^2 / m_l \omega_\pm}$$

c^2 and s^2 are measures of the sympathetic cooling

Examples

Laser-cooled ion: Ba^+
Sympathetically cooled ion: C_{60}^+



Measure for the single-well potential

$\Omega_l = 2\pi \times 200\text{kHz}$

Solid lines $Z_0 = 20\mu\text{m}$

Dotted lines $Z_0 = 30\mu\text{m}$

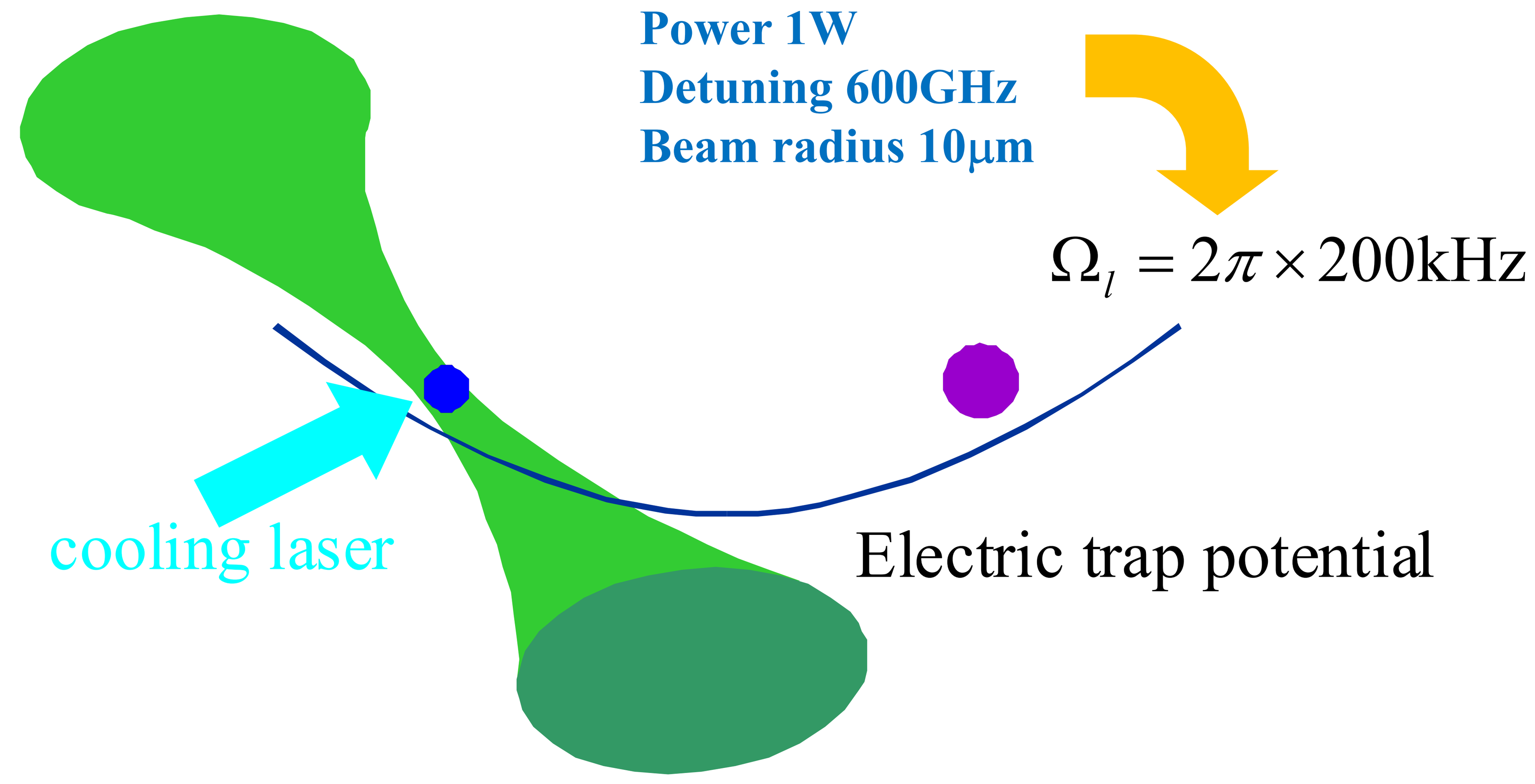
Dashed lines $Z_0 = 40\mu\text{m}$

- **The sympathetic cooling is efficient when $\Omega_l \sim \Omega_s$**
- **Z_0 determines the bandwidth of the resonance.**

How to make the double-well potential

- **Surface trap**
- **Optical dipole force trap**

Superimposed optical dipole force trap



Conclusions

- In the double-well potential, the sympathetic cooling is efficient when the resonant frequency of the sympathetically cooled ion motion is close to that of the laser-cooled ion motion.
- The ion motions are expressed by the normal modes, and the measure of the sympathetic cooling rate is obtained.
- According to the estimation, it is possible to satisfy the condition for the efficient sympathetic cooling experimentally in the case of a laser-cooled Ba^+ and a sympathetically cooled C_{60}^+ with the use of surface traps or optical dipole force traps.