Harmonic oscillators coupled at the single quantum level

K. R. Brown, C. Ospelkaus, Y. Colombe, A. C. Wilson, D. Leibfried, and D. J. Wineland
National Institute of Standards and Technology, Boulder, Colorado, U.S.A.

Introduction

The harmonic oscillator is one of the simplest physical systems but also one of the most fundamental. Realizations of harmonic oscillators in the quantum regime include electromagnetic fields in a cavity and the mechanical modes of a trapped atom or macroscopic solid. Quantized interaction between two motional modes of an individual trapped ion has been achieved by coupling through optical fields, and entangled motion of two ions in separate locations has been accomplished indirectly through their internal states. However, direct controllable coupling between quantized mechanical oscillators held in separate locations has not been realized previously. Here we implement such coupling through the mutual Coulomb interaction of two ions held in trapping potentials separated by 40 µm. By tuning the confining wells into resonance, energy is exchanged between the ions at the quantum level. Such coupling is a key feature of proposals to implement quantum simulation, and it could allow logic operations to be performed in a multi-zone quantum information processor without the requirement of bringing the ion qubits into the same trapping potential. It could also extend the capabilities of quantum logic spectroscopy to ions that cannot be trapped within the same potential well as the measurement ion, such as oppositely charged ions or even antimatter particles.

Apparatus

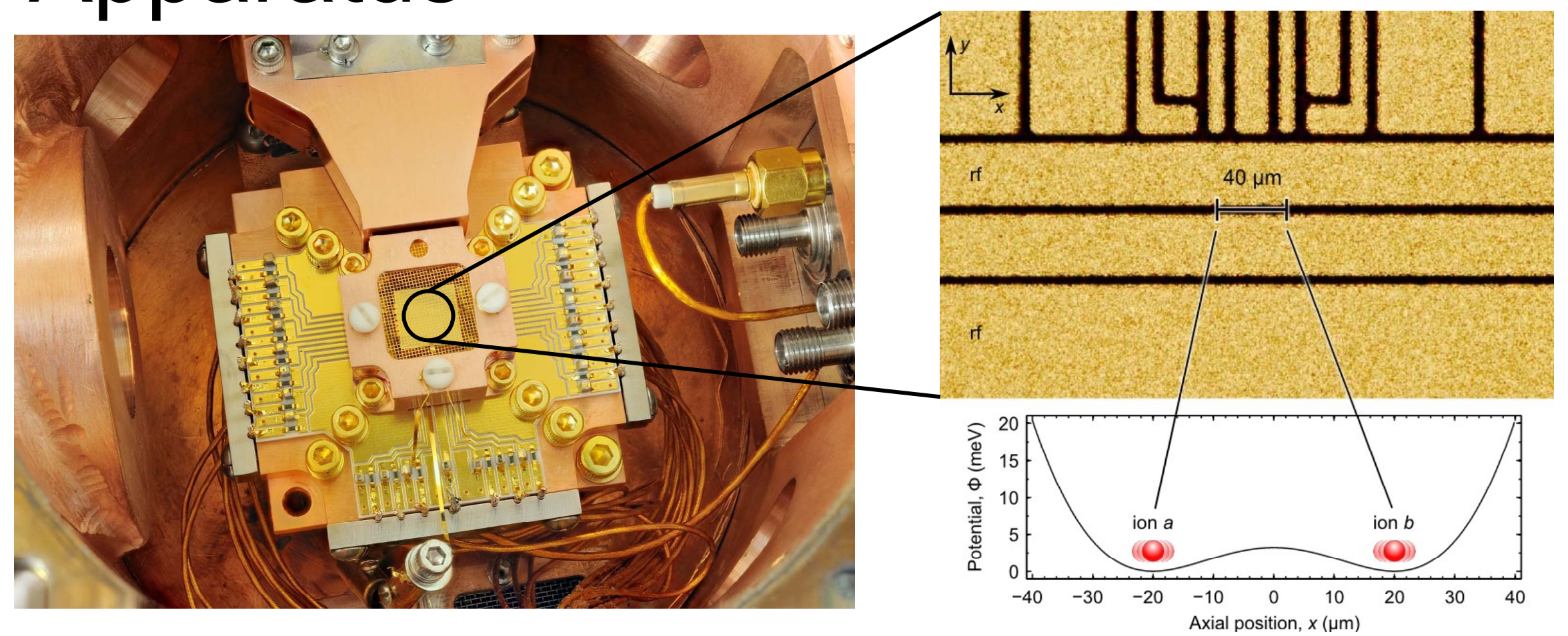

Figure 1. Photograph of the ion trap, showing r.f. and d.c. electrodes, and gaps between electrodes (darker areas). Electrodes are 8 µm thick gold electroplated onto a single-crystal quartz substrate. The graph indicates the simulated potential along the trap *x* axis. Two trapping wells are separated by s_0 = 40 µm at a height d_0 = 40 µm above the surface. The d.c. electrodes are sufficient to control the axial frequency and the position of each ion independently. Here both frequencies are ~4 MHz and the potential barrier between the two ions is ~3 meV.

Coulomb coupling

The Coulomb interaction between two ions of charges q_a and q_b is

$$U(x_a,x_b)=\frac{1}{4\pi\varepsilon_0}\frac{q_aq_b}{s_0-x_a+x_b}$$

$$\approx\frac{q_aq_b}{4\pi\varepsilon_0 s_0}\left(1+\frac{x_a-x_b}{s_0}+\frac{x_a^2+x_b^2}{s_0^2}\boxed{-\frac{2x_ax_b}{s_0^2}}\right)$$

"exchange" coupling

Quantum mechanically, this exchange coupling is

$$\frac{-q_aq_b}{2\pi\varepsilon_0 s_0^3}x_ax_b\approx-\hbar\Omega_{\mathrm{ex}}(ab^\dagger+a^\dagger b)$$

$$\Omega_{\mathrm{ex}}=\frac{q_aq_b}{4\pi\varepsilon_0 s_0^3\sqrt{m_am_b}\sqrt{\omega_a\omega_b}}$$

- Periodically swaps energy between two oscillators
- For $\omega_a=\omega_b$ ("resonance"), full swaps with period $2\tau_{\mathrm{ex}}=\pi/\Omega_{\mathrm{ex}}$
- For ^{9}Be$^+$ at $\omega_0/2\pi$ = 4.04 MHz, predict τ_{ex} = 162 µs

Heating rates

- Ion heating is the primary impediment to observing exchange coupling at the quantum level
- Cooling trap to 4.2 K suppresses heating rates
- We have observed a heating rate as low as 70 quanta/s at $\omega_0/2\pi$ = 2.3 MHz (see below). However, for the coupling experiments reported here, we observed a heating rate ~500 – 2,000 quanta/s that varied between the two wells.

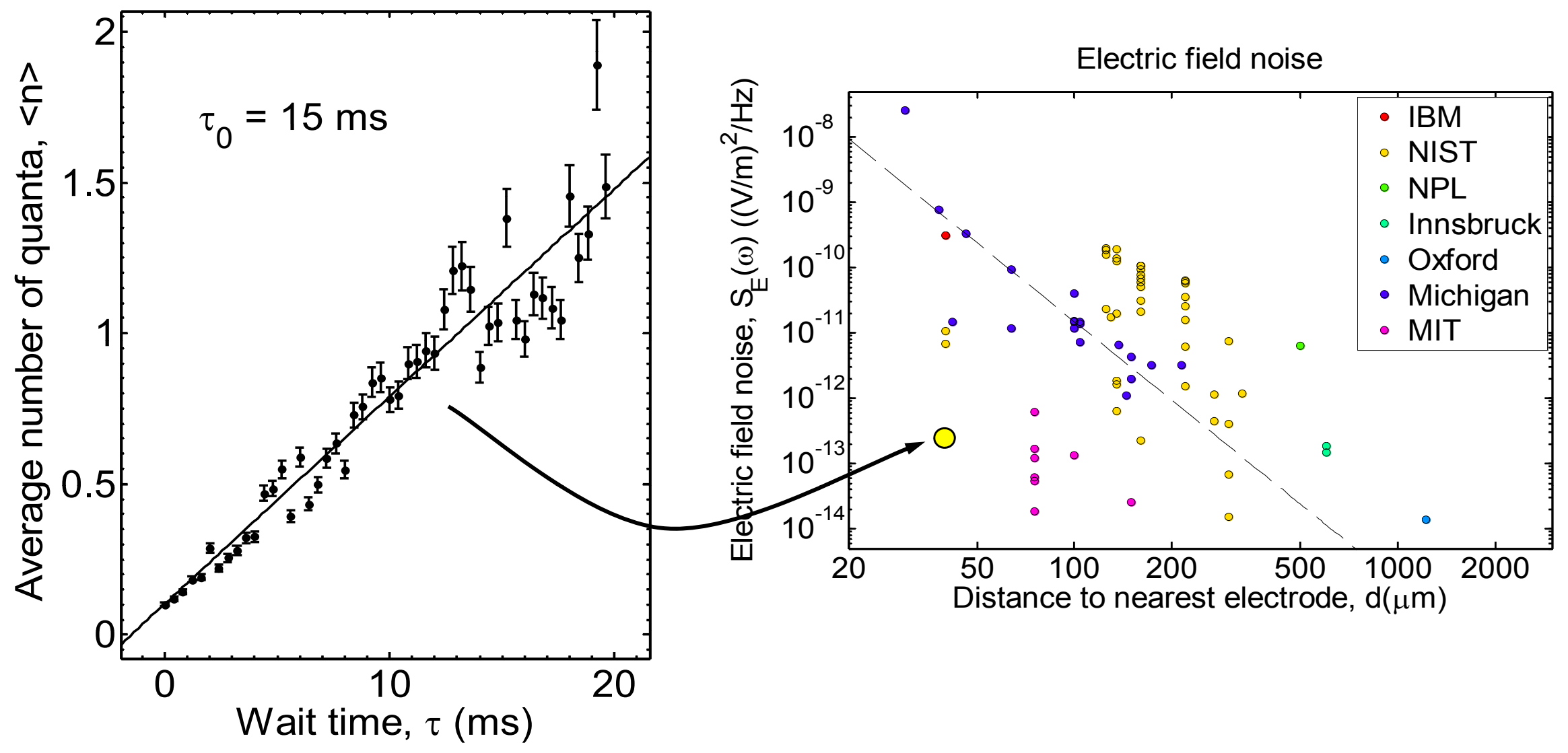

Figure 2. Heating rate measured in the ^{9}Be$^+$ cryogenic trap. The ion was cooled to the ground state with hyperfine Raman sideband transitions and then allowed to heat up for a time τ. (left) The average number of quanta ‹*n*› was then measured by observing asymmetry between the red and blue motional sidebands. (right) The observed heating rate is compared with other heating rates reported in the literature.

Avoided crossing

- A signature of coupling between ions is the splitting between the two axial normal mode frequencies
- Splitting reaches a theoretical minimum $\delta f=\Omega_{\mathrm{ex}}/\pi$ = 3.1 kHz
- We measure the mode frequencies by applying a nearly resonant oscillating potential pulse ("tickle") to one of the trap electrodes. A decrease in ion fluorescence indicates that a mode of the ions' motion has been resonantly excited.

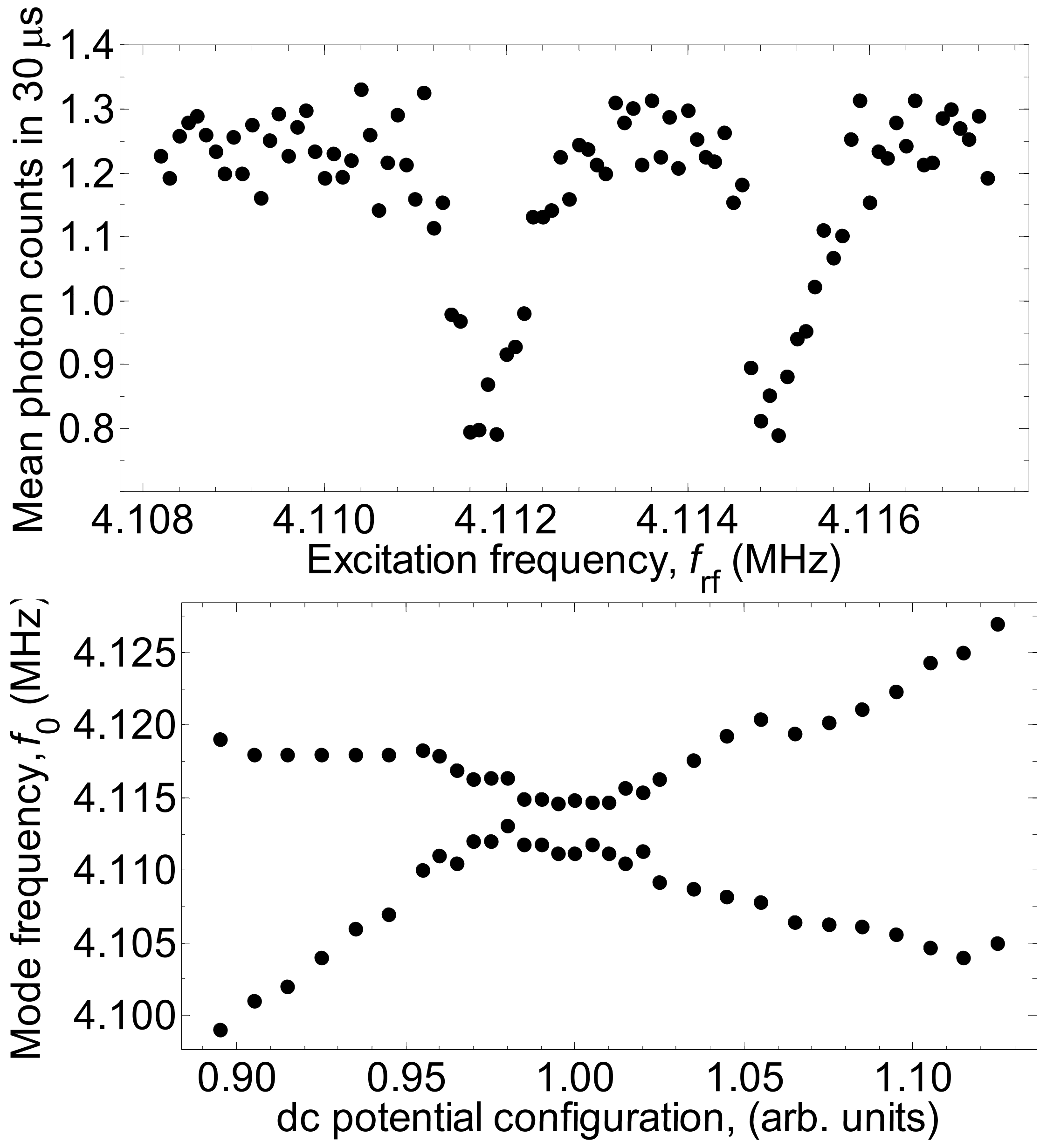

Figure 3. (top) Dips in fluorescence for two ions appear as their motion is excited with an rf electric field. (bottom) Mode frequencies for varying d.c. potentials. The 3.1 kHz minimum splitting between the two modes is in good agreement with theory.

Energy exchange

Energy is exchanged between the two ions when their frequencies are brought into resonance.

Thermal case Off resonance, ions *a* and *b* are both Doppler cooled, and ion *a* is further cooled to the ground state. The ions are then brought into resonance for an interval τ, after which the motional quantum number of ion *a* is measured.

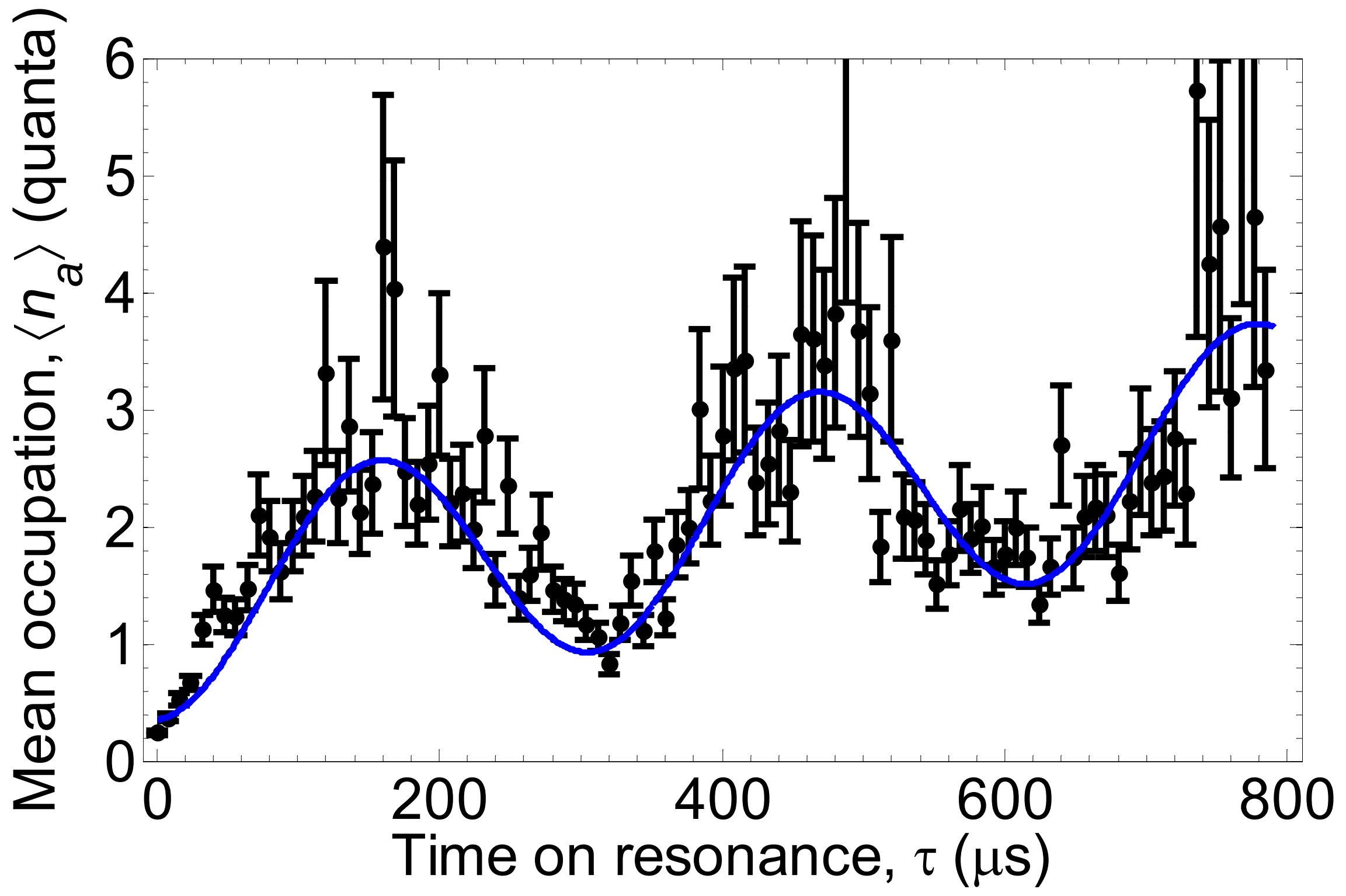

Figure 4. Energy swapping between two ions at the Doppler limit. The blue curve represents a fit to theory with four free parameters: the two initial mean quantum numbers, the exchange time and the heating rate. Energy exchanges between the ions at 155(1) µs intervals, in reasonable agreement with the theoretical prediction of 162 µs. The linearly increasing trend in n_a is due to ion heating at a rate of 1,885(10) quanta/s.

Single quantum case While the ions are on resonance, ion *a* is cooled to the ground state during a time much longer than the exchange period, sympathetically cooling ion *b*. A single quantum is then injected into ion *a*, and after interval τ the probability that this quantum remains in ion *a* is measured.

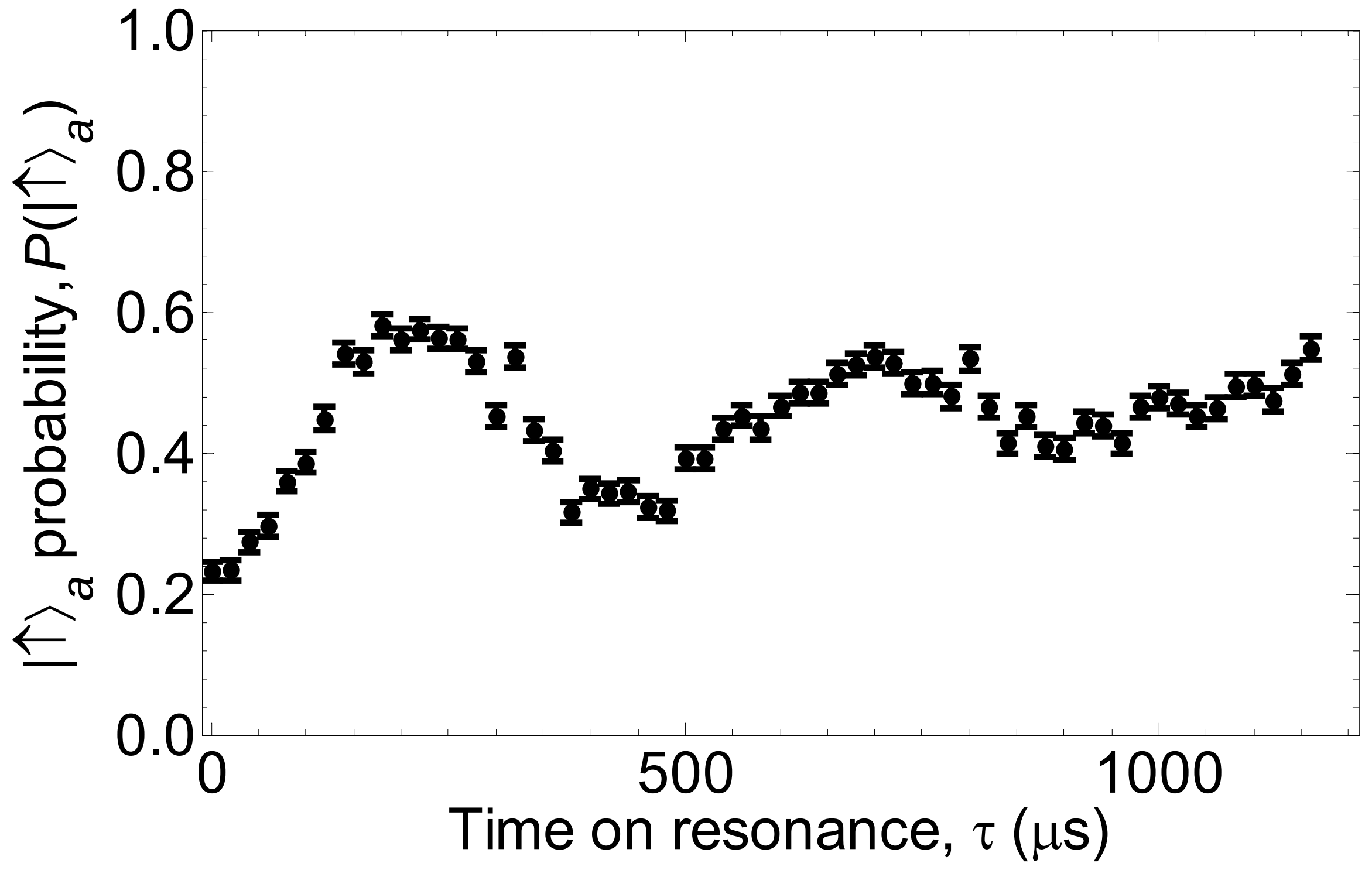

Figure 5. Energy swapping between two ions at approximately the single quantum level. $P(|\uparrow\rangle_a)$ oscillates with period $2\tau_{\mathrm{ex}}$ = 437(4) µs as a quantum exchanges between the ions. The longer period here compared with the thermal case arises from the higher trap frequency ($\omega_0/2\pi$ = 5.56 MHz).

Further information

A preprint describing the coupling experiment in greater detail (to appear in *Nature*) is at http://http://arxiv.org/abs/1011.0473.

Future improvements

The fidelity of the coupling demonstrated here is limited primarily by ion heating, as well as by beam pointing instability, cryostat vibrations, and laser intensity fluctuations. We plan to

- Shield the laser beams from air currents
- Firmly secure the inside of cryostat with Kevlar straps
- Actively stabilize the laser intensity
- Fabricate a new trap to try to recreate our lowest heating rates

If the fidelity can be sufficiently improved, it should be possible to use the coupling to create a distant entangled pair.

Other experiments

Randomized benchmarking of single qubit operations

The cryogenic copper enclosure surrounding the trap shields the ion from magnetic field fluctuations, the primary decoherence mechanism for single qubit rotations. We quantify the single qubit gate fidelity with randomized benchmarking:

- Prepare the ion in a known state
- Apply random sequence of gates with pre-determined outcome
- Fidelity of the sequence outcome falls for longer sequences

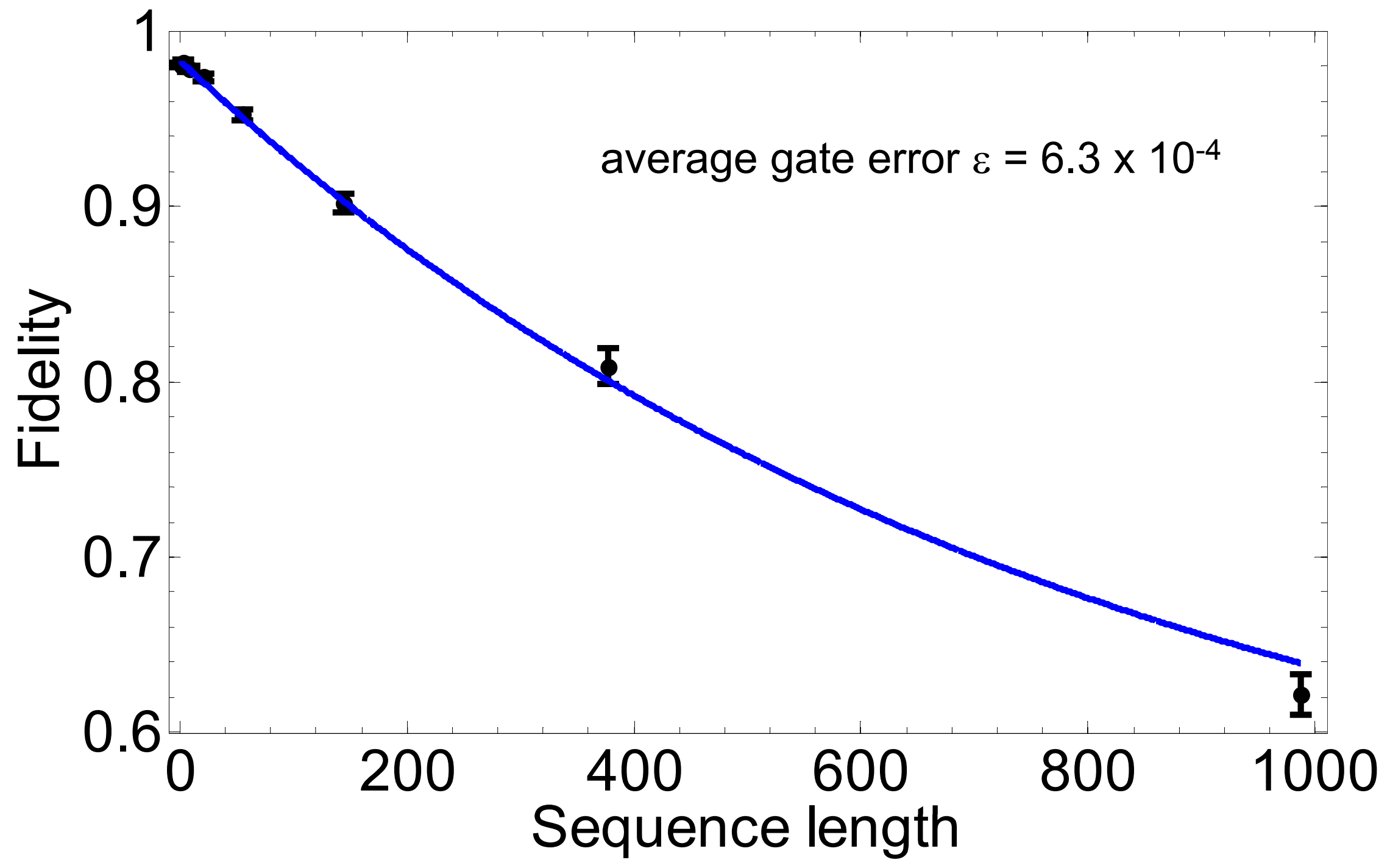

Figure 6. Randomized sequence fidelity for sequences of increasing length. The blue line is an exponential fit to the observed decay.

Ground-state cooling with the optical dipole transition

The light mass of ^{9}Be$^+$ means that very high trapping frequencies (>30 MHz) are possible. High frequencies combined with relatively narrow optical linewidth (20 MHz) should enable ground-state cooling (resolved-sideband regime).

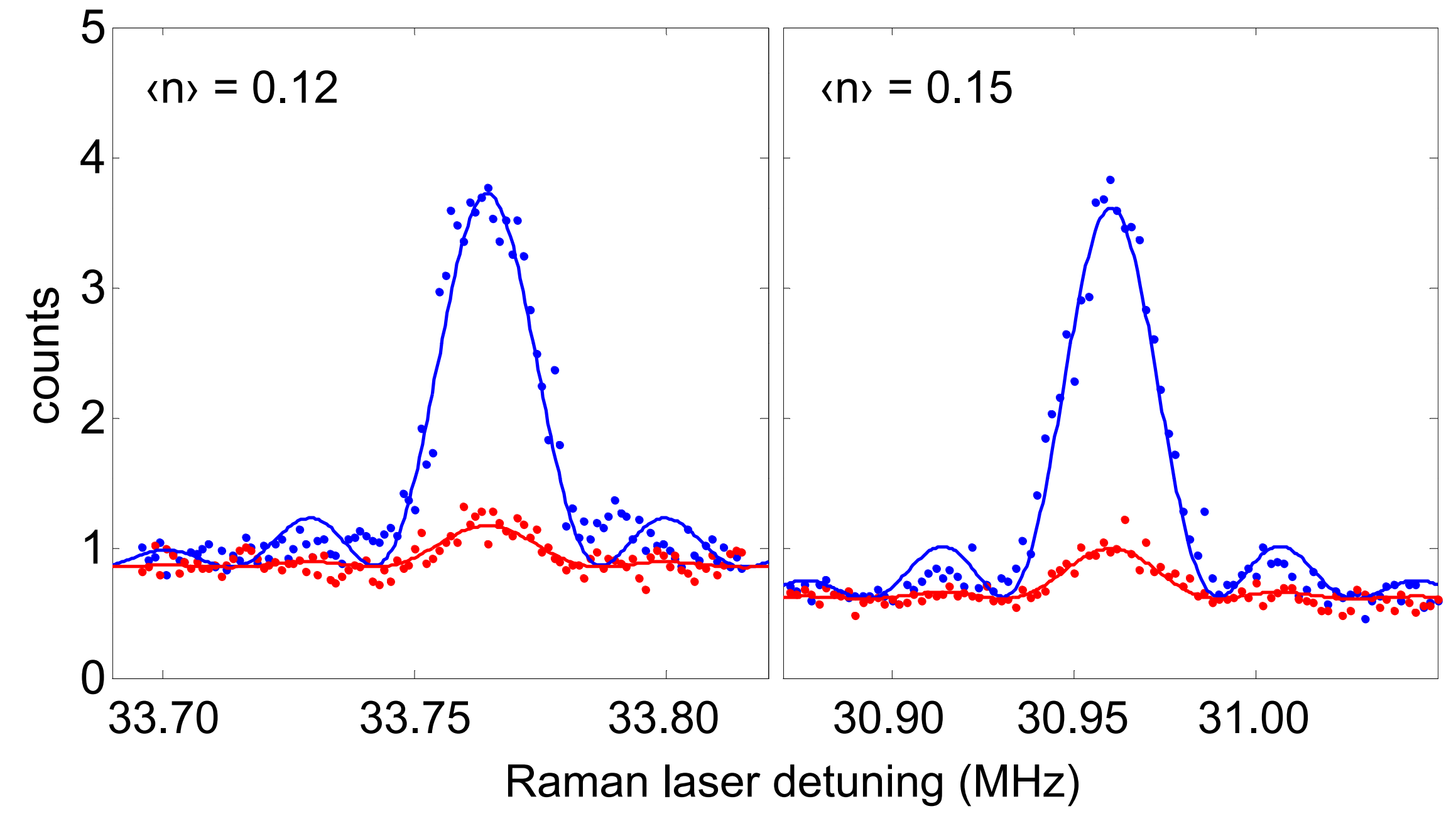

Figure 7. Raman hyperfine blue and red motional sidebands for the radial modes of a single ^{9}Be$^+$ ion after cooling with the Doppler laser. Pronounced asymmetry between the sidebands indicates that the ion is nearly in the ground state.