Uncertainty: some concepts and a firearms example

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NIST, the capital of uncertainty?

- Uncertainty has a special meaning at NIST
- It is NIST policy that each measurement result be accompanied by an uncertainty
- Prioritizing uncertainty estimation has enhanced NIST's scientific and measurement reputation
- Uncertainty characterizes one's imperfect state of knowledge

Uncertainty conceptually

- In a typical metrology situation, there is measurement of a **numeric** quantity
- **Uncertainty** (of measurement) is a quantity that "reflects the lack of exact knowledge of the value of the measurand" (GUM)

  – GUM is the *Guide to the Expression of Uncertainty in Measurement*

Uncertainty Operationally

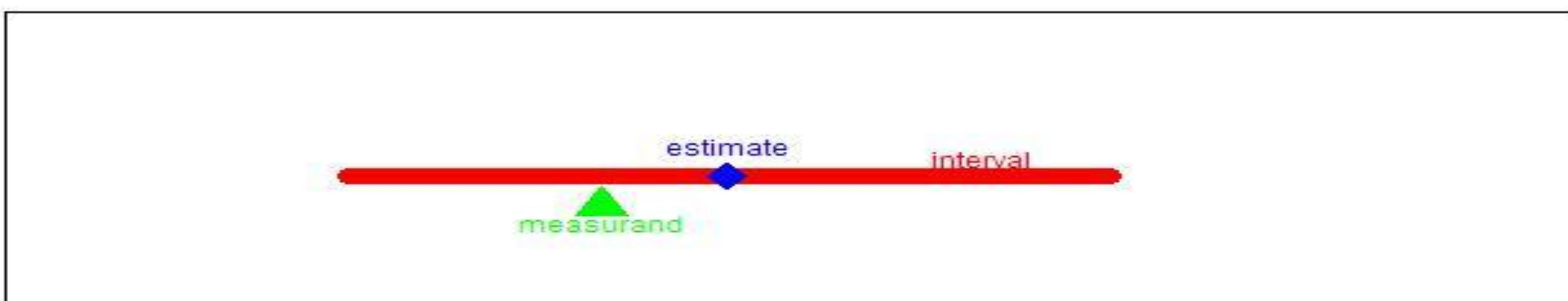
- The **uncertainty** “characterizes the dispersion of the values that could be reasonably attributed to the measurand” (GUM)
- **Standard** uncertainty: Uncertainty expressed as a a standard deviation
- **Combined** standard uncertainty incorporates components of uncertainty from various sources: variability of measurements, uncertainty due calibration standards or moisture correction, etc.

Expanded Uncertainty

- **Expanded** uncertainty "provides an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could be reasonably attributed to the measurand" (GUM)
- Classic example of expanded uncertainty is (2 x Standard Uncertainty) for an interval with 95% confidence

Intervals and Probability

- What is an interval with 95% confidence?
- One interpretation: It is a procedure that covers the true value 95 % of the time
- Alternatively, the odds are 19-1 in favor of containing the true value
- 80%, 90%, and 99% are also commonly used



Non-typical uncertainties at NIST

- Some uncertainties don't take the "*+/- k* x *uncertainty* " form
- SRM 2461 has a "ACCF > xx.x" uncertainty statement
- Uncertainties could be in the form of a probability
- Uncertainties have been stated qualitatively (with ongoing efforts for a more quantitative approach)

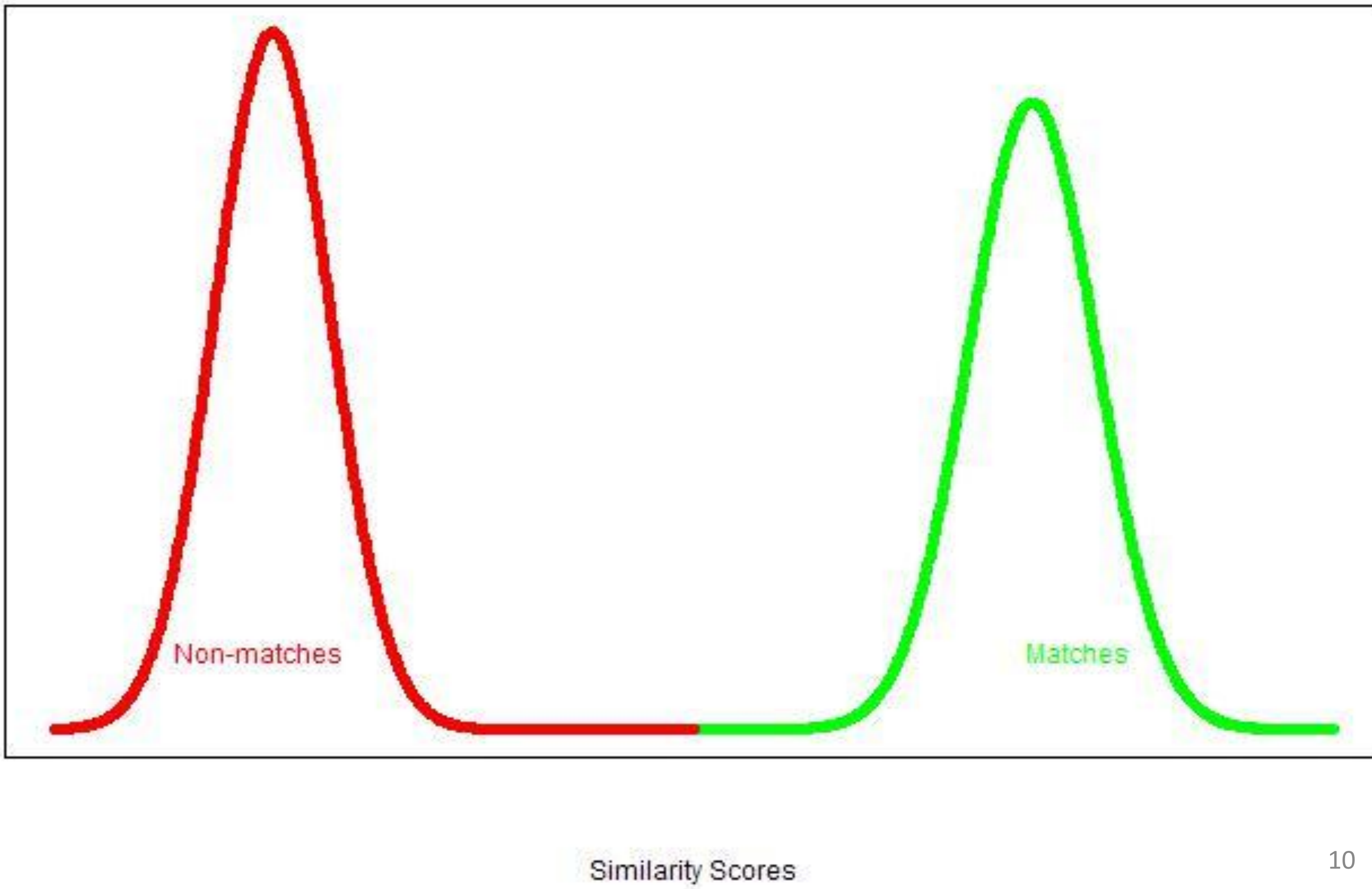
False positives and negatives

- 95% intervals contain those values not rejected by a hypothesis test with "95% significance" (p-value < 0.05)
- Null Hypothesis : If it were *really* random, the chances of getting a result this extreme (i.e. this good of a match) is……
  - (false positive rate from the 'background [non-match distribution')
- High False negative rates indicates a not very powerful procedure
- Tradeoff between the two error rates

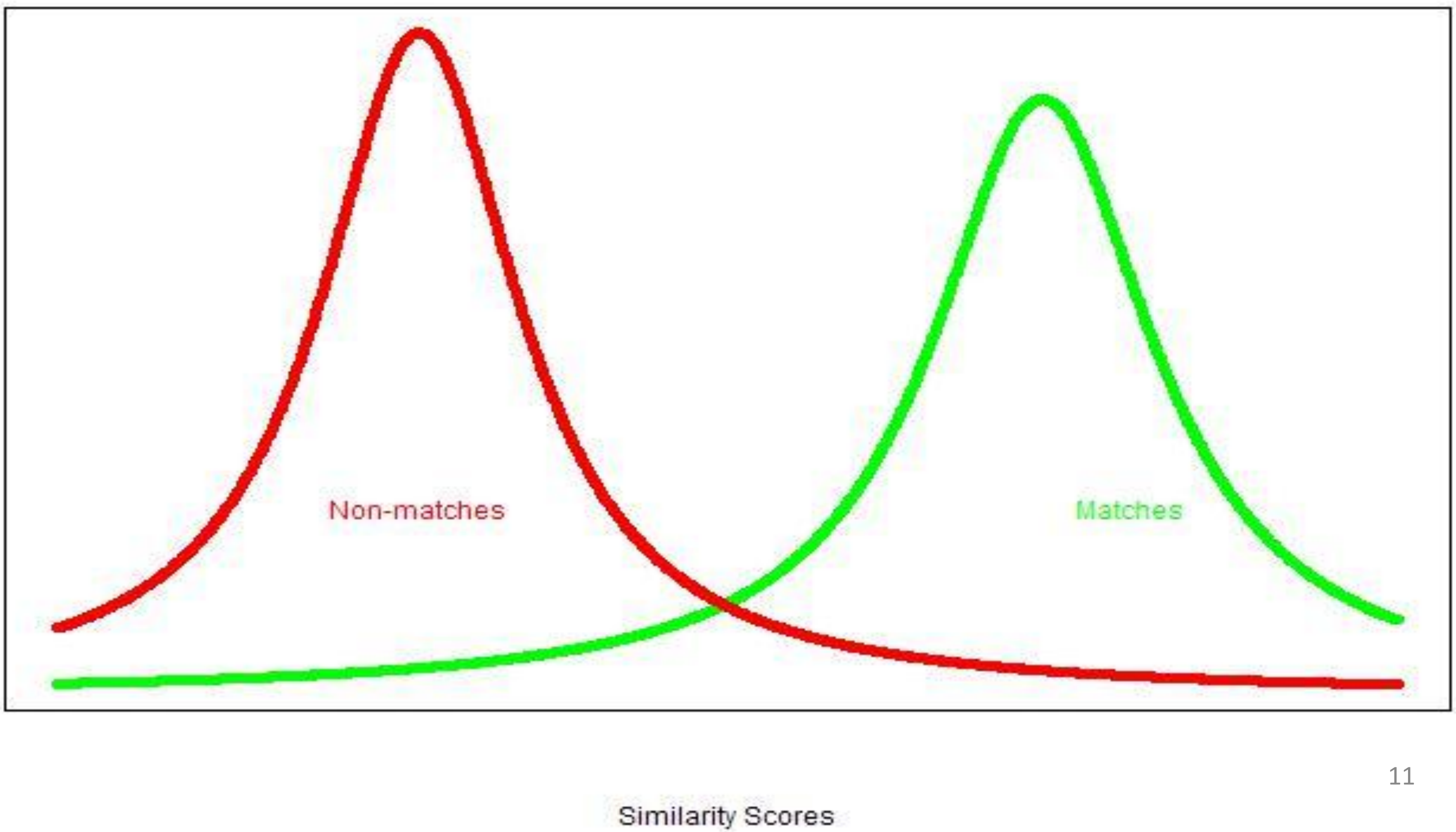
Similarity scores

- Quantitative measures of similarity between casings and bullets are being actively researched: Correlation, ACCF, CMS, CMC, number of feature points, etc.
- High=Match, Low=Non-match

A possible Matching vs. Non-matching distributions of similarity measures



Overlapping distributions of similarity scores



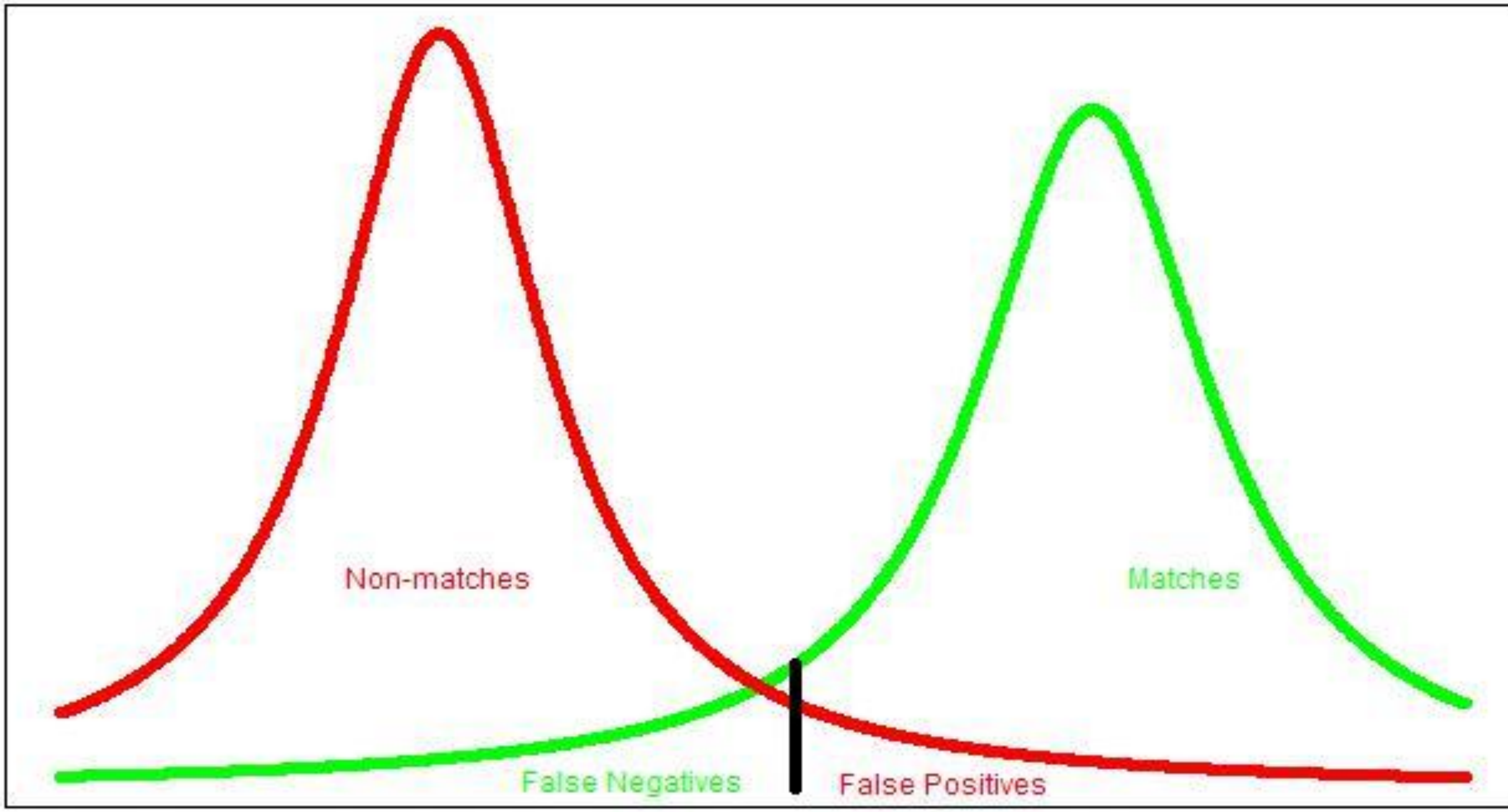
Matching and non-matching distributions

- Different pair of distributions for each specific scenario (gun, ammo, etc.)
- We look at the degree of separation or overlap between match and non-match distributions
- There may be scores with uncertain conclusion (inconclusive?)
- What is the threshold(s)?
- Trade off between false positives and false negatives

Example Table of actual vs. predicted for a particular threshold

Threshold= 0.321	Actual Match	Actual Non-Match
Predicted Match	700	20
Predicted non-match	250	6900

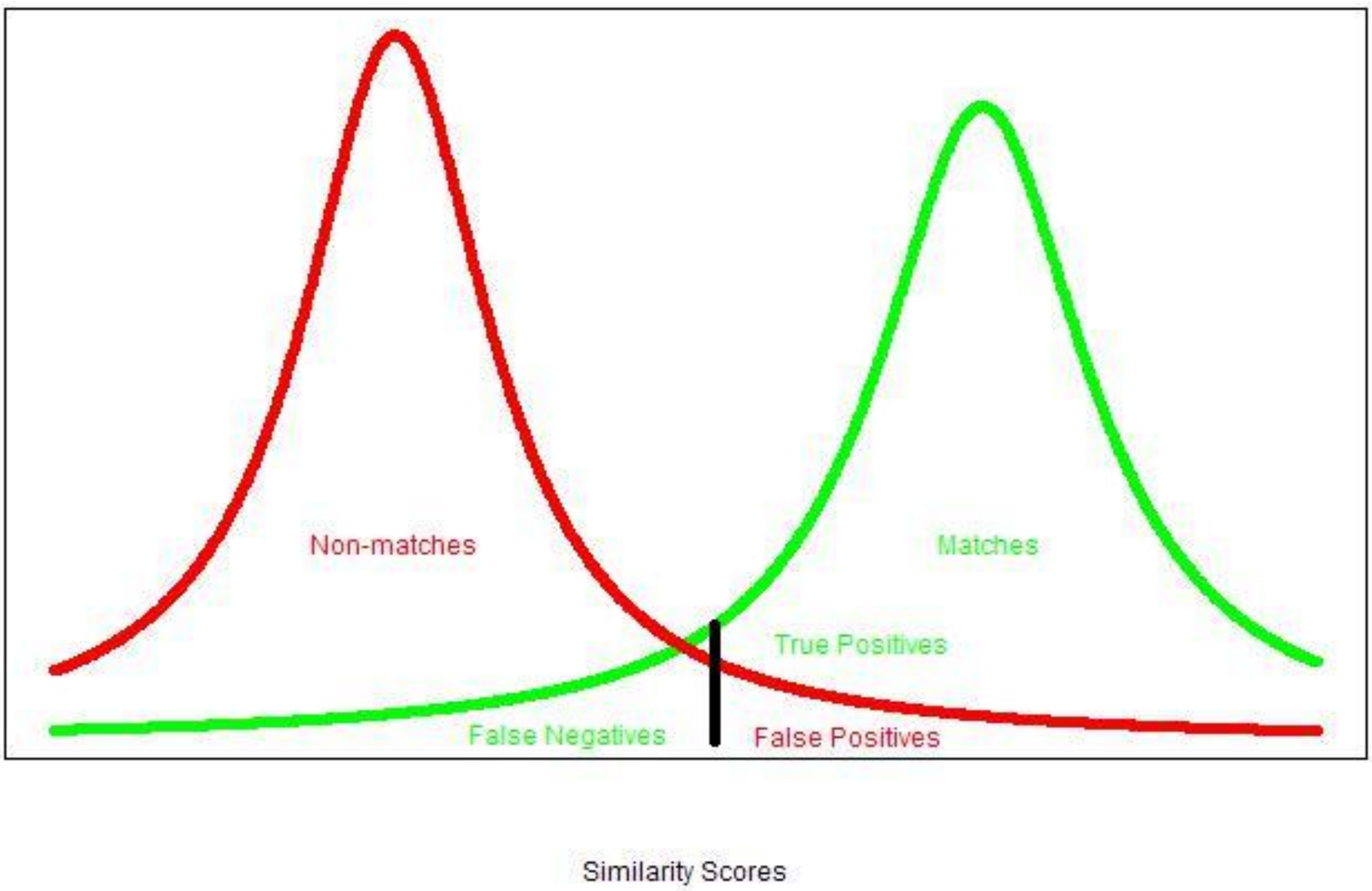
Threshold determines error rates



ROC Curves

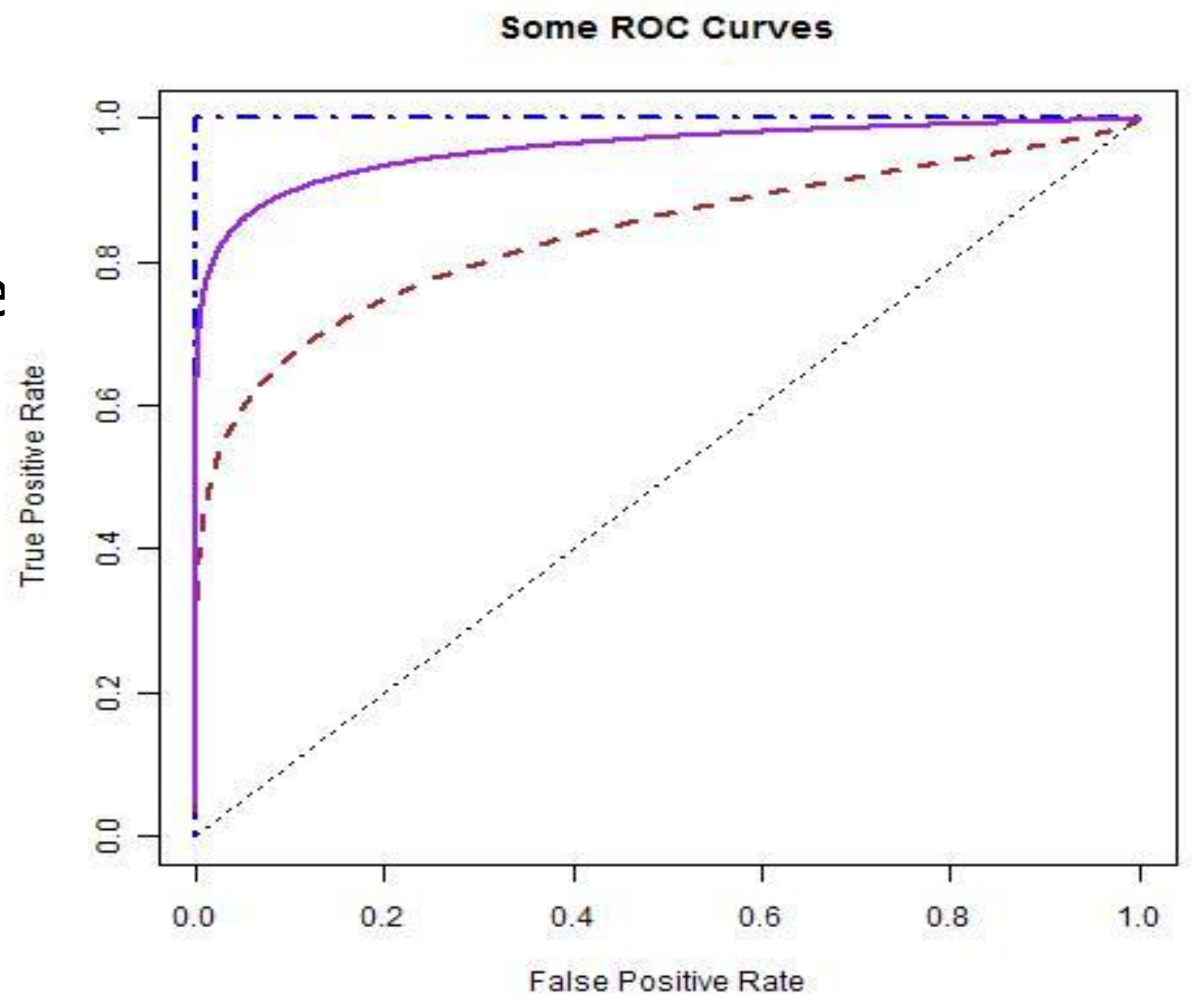
- ROC (receiver operating characteristic) curves depict tradeoff between False Positive and False Negative rates
- ROC curves can show the degree of separation between various match vs. non-match distributions for various scenarios: different classifiers/metrics, different gun brands, etc.

Threshold determines error rates

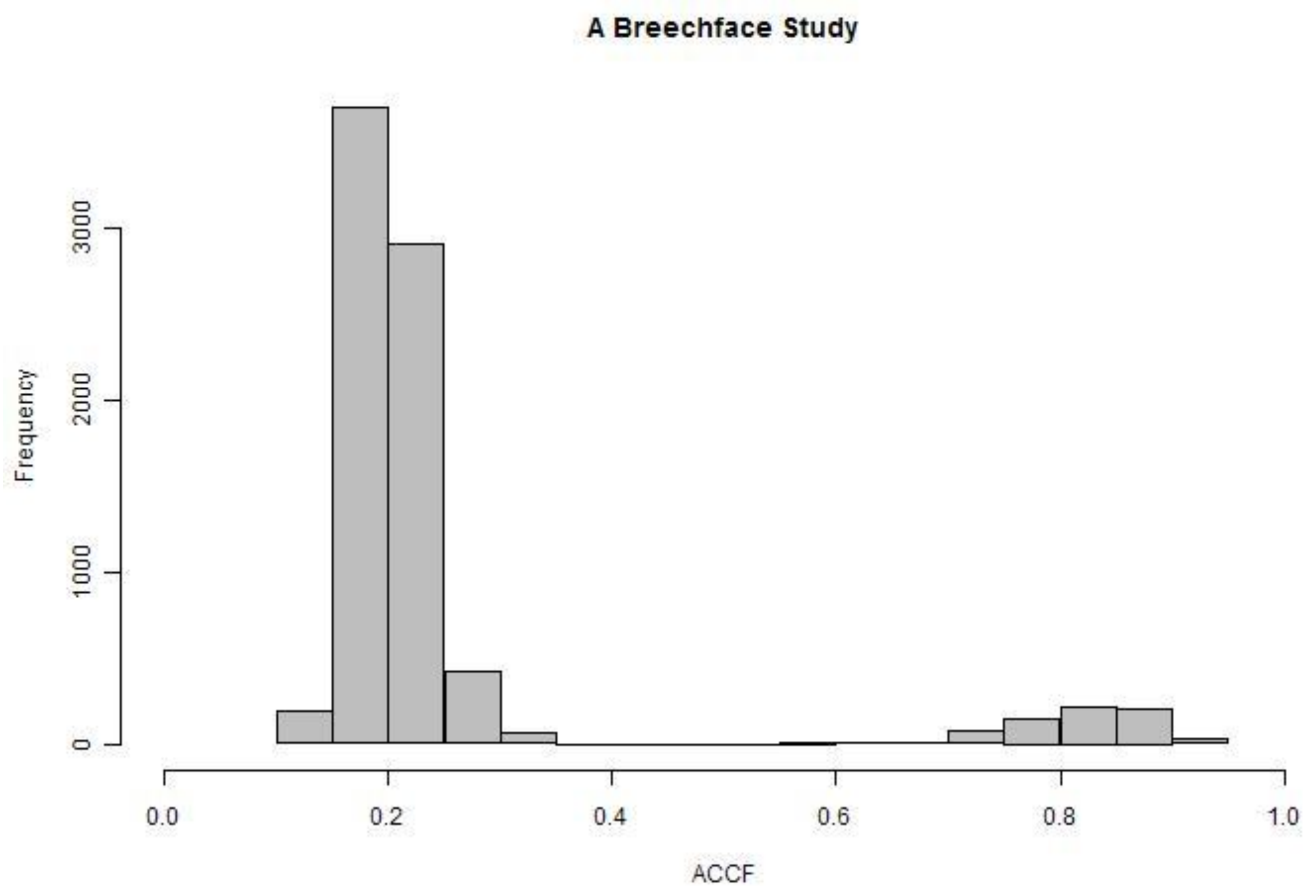


ROC Curve examples

- Different ROC curves demonstrate different degrees of separation



A real data example (Zheng, Weller)



AUC of ROC

- An overlap metric: what is the probability that a randomly chosen non-match score is smaller than a random match score?
- Metric related to Area Under ROC curves (AUC)
- AUC can be estimated various ways:
  - Fit distributions if they are known
  - Use Mann-Whitney procedure (compare every match score to every non-match score and keep score): Equivalent to using empirical distributions
  - Integration of area under curve

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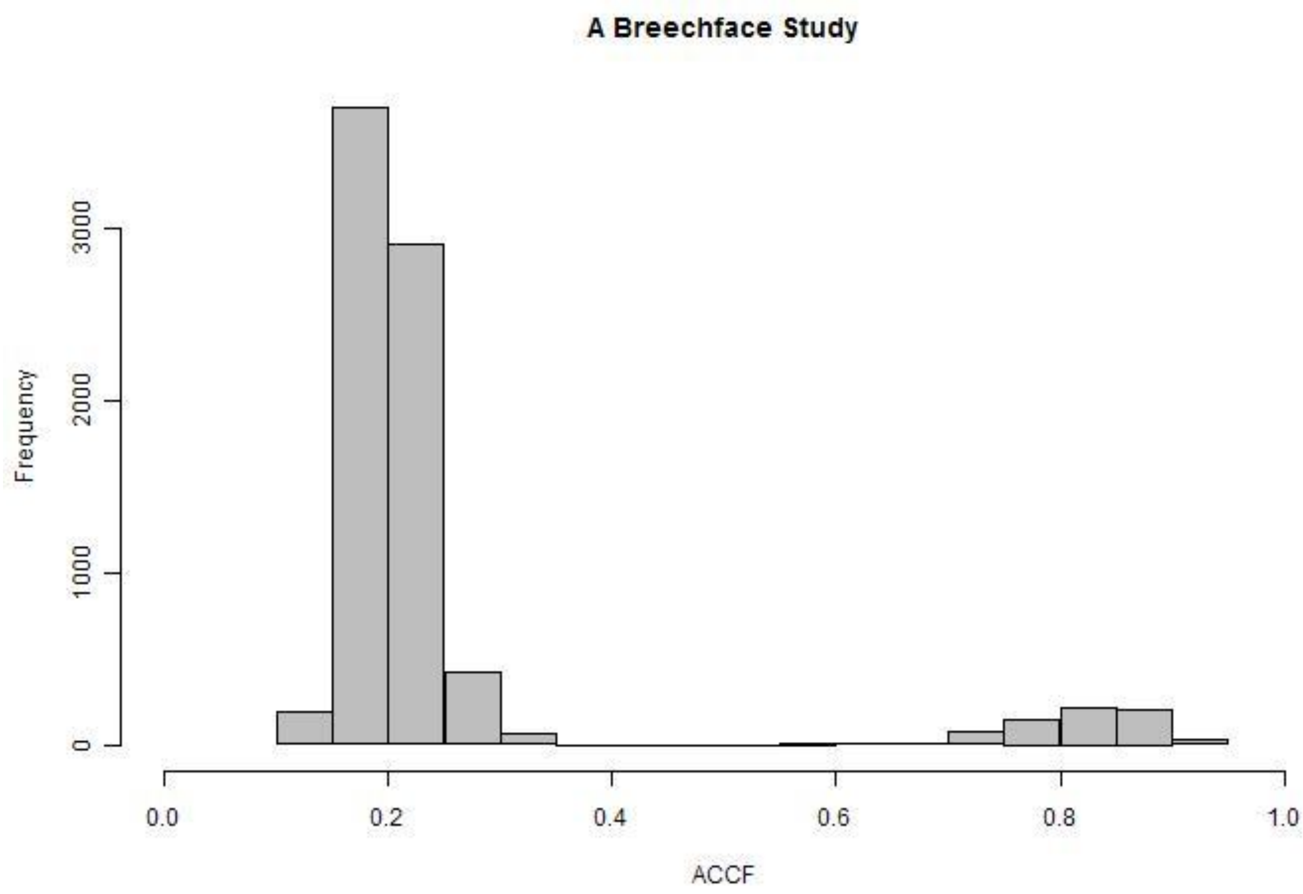

AUC Issues

- One problem with Mann-Whitney procedure: if there is no empirical overlap, estimated AUC=1
- Fitting distributions will estimate AUC<1, but depends on the distribution

- Of course, Uncertainty of AUC estimates can be estimated (bootstrap, etc.)

The Future holds…….

- Yet more advanced correlation methods (not just for database searching)
- Databases of topographies leading to databases of match/non-match distributions for various guns, ammunitions, etc.
- Theoretical models of non-match distributions
- More repeatability data for similarity measures
- In the (far) future, perhaps we can cite probability statements similar to those in DNA analysis (at least for confirmation)