Certifying Local-Realism Violation

Manny

NIST

2014

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Overview

Test Configurations and Models

Bell Functions

Anti-Local-Realism Certificates

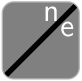
Applications to Experiments

Recommendations

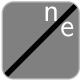
Parties, Settings and Measurements

- (2,2,2) (parties, settings choices, measurement outcomes):

A



A	00	01	10	11	B
n					n



B

Parties, Settings and Measurements

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n	0	1	0	0	n

B

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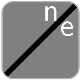
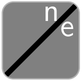


A	00	01	10	11	B
n	0	1	0	0	n
n					e



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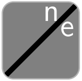
A 

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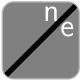
n	0	1	0	0	n
n	1	0	0	0	e
e					n



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e	0	0	0	1	n

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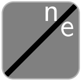

A	00	01	10	11	B
n	0	1	0	0	n
n	1	0	0	0	e
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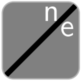

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A	00	01	10	11	B
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n	1	0	0	0	e
e	0	0	0	1	n
e	0	0	1	0	e

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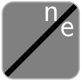
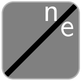


A	00	01	10	11	B
n	0	1	0	0	n
n	1	0	0	0	e
e	0	0	0	1	n
e	0	0	1	0	e



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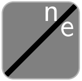

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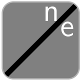
n	1	0	0	0	e
e	0	0	0	1	n
e	0	0	1	0	e



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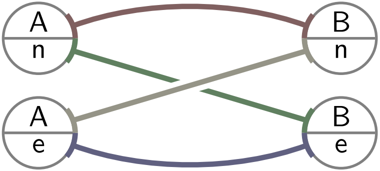
- Compatibility graph:



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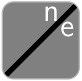
A 

A	00	01	10	11	B
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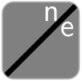
n	1	0	0	0	e
e	0	0	0	1	n
e	0	0	1	0	e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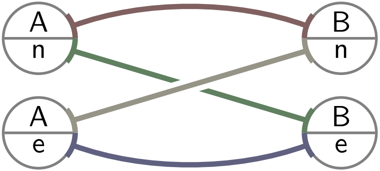

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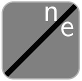
A 

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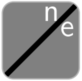
n	1	0	0	0	e
e	0	0	0	1	n
e	0	0	1	0	e



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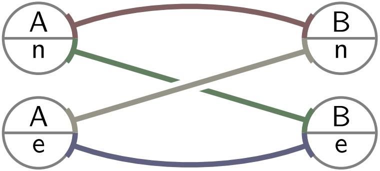

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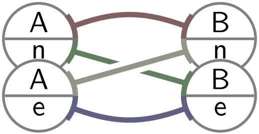
- Trial model: Prob$(O_A = o_A, O_B = o_B, S_A = s_B, S_B = s_B|\text{past})$.

Example I



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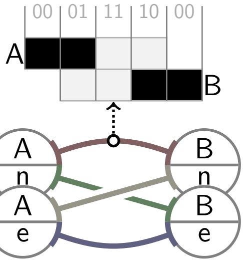

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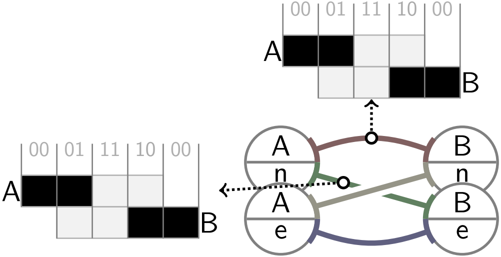

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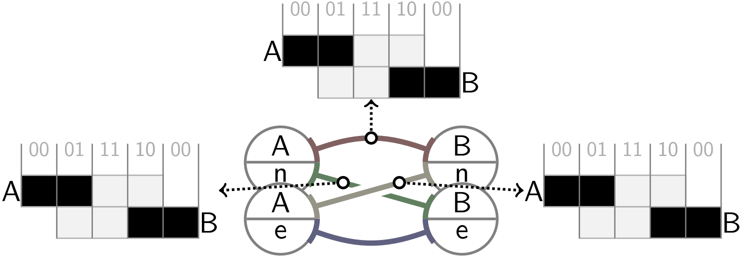

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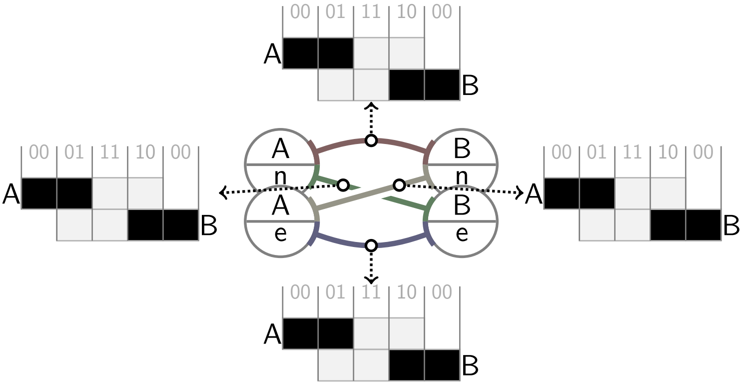
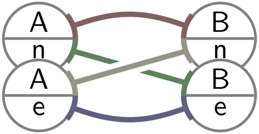

00 01 11 10 00
A
B
A
n
A
e
B
n
B
e
00 01 11 10 00
A
B
00 01 11 10 00
A
B
00 01 11 10 00
A
B

Examples II,III



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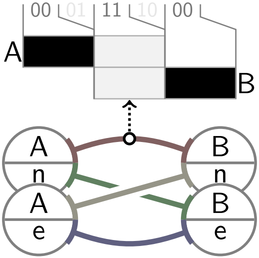

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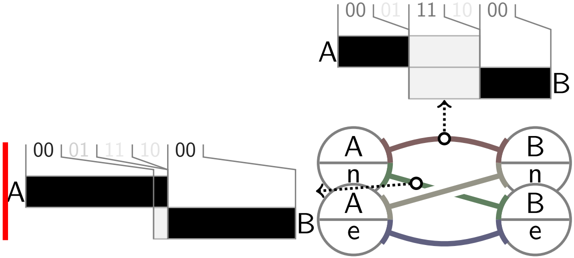

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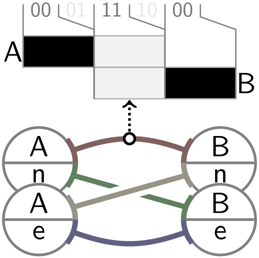

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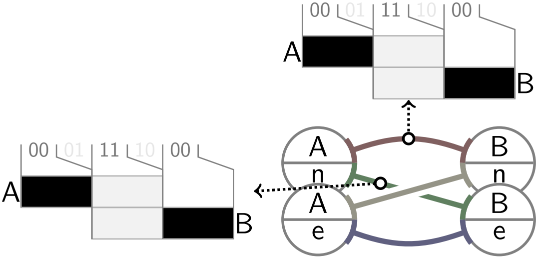

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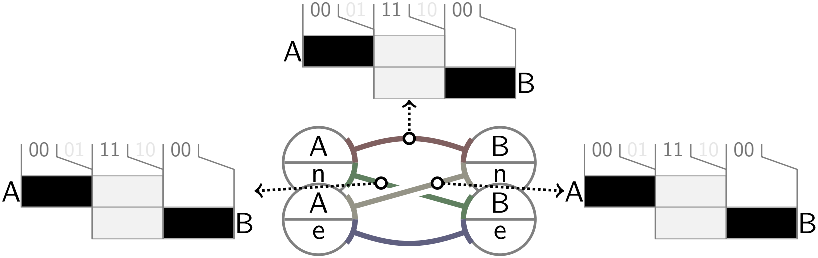

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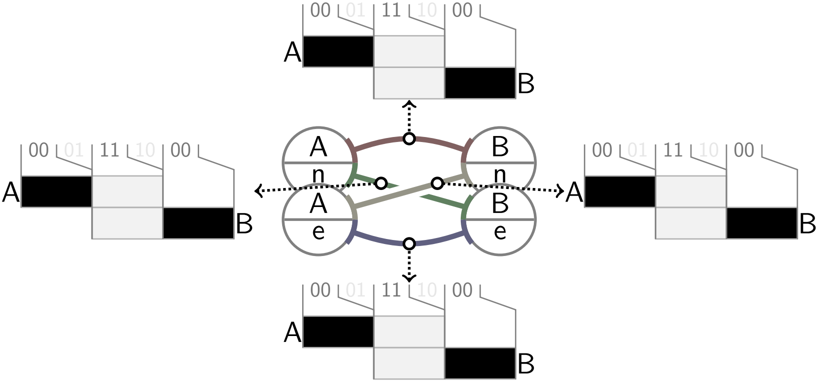

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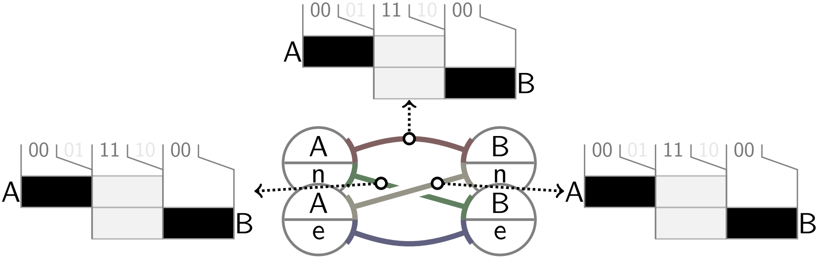

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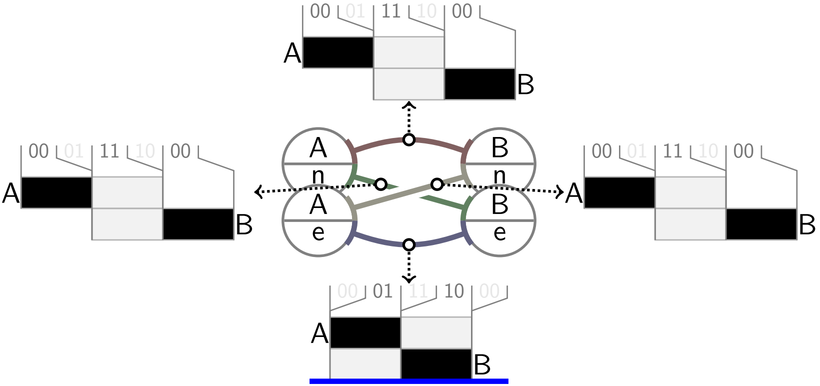
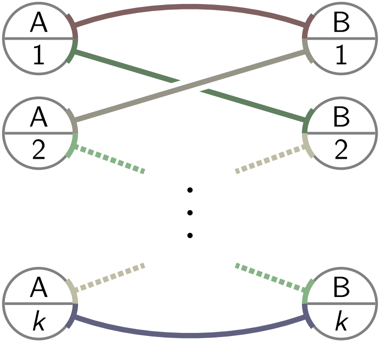


The PR Box, Popescu&Rohrlich(1997) [8].

The 2k-cycle



"chained Bell inequalities", Braunstein(1990) [1], application in Colbeck&Pironio(2011) [3]

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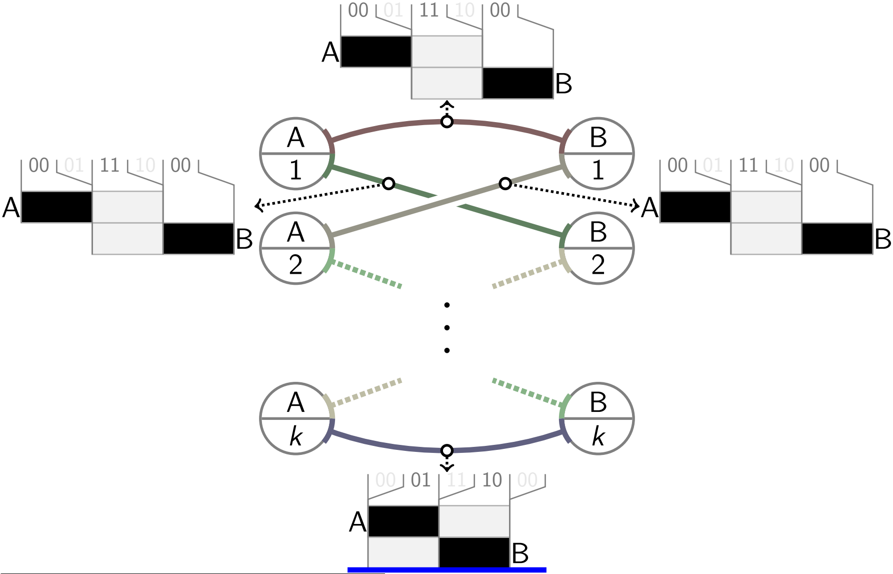

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- One Trial:
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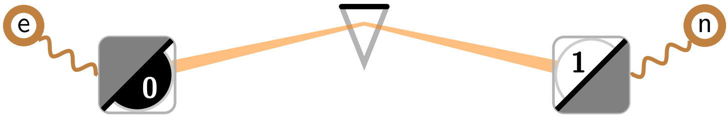
- Possible constraints:
  - Remote context independence/no-signaling/consistent marginals.
  - Remote outcome independence.
  - Definiteness given the "complete state".

Model Constraints to Consider



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CI. Remote context independence.

$$\mu(o_X | s_X, s_{\neg X}) = \mu(o_X | s_X).$$

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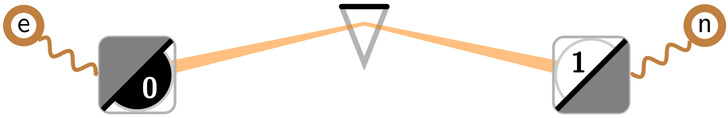

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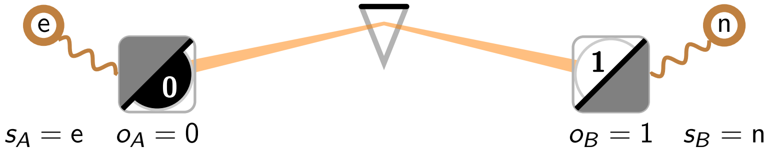

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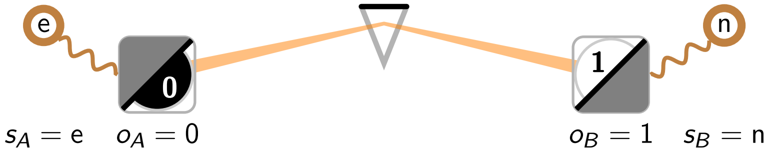

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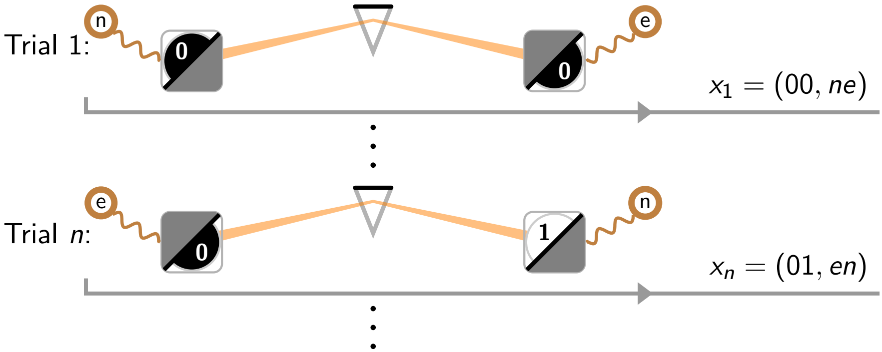
LR. Local realism, $\mathrm{CI} \wedge (\mathrm{OI} \vee \mathrm{D}) \wedge \mu(\lambda)$.

$$\mu(o, s) = \sum_{f:\text{for all } X \; f_X(s_x) = o_X} \mu(f)\mu(s).$$

Ideal Test



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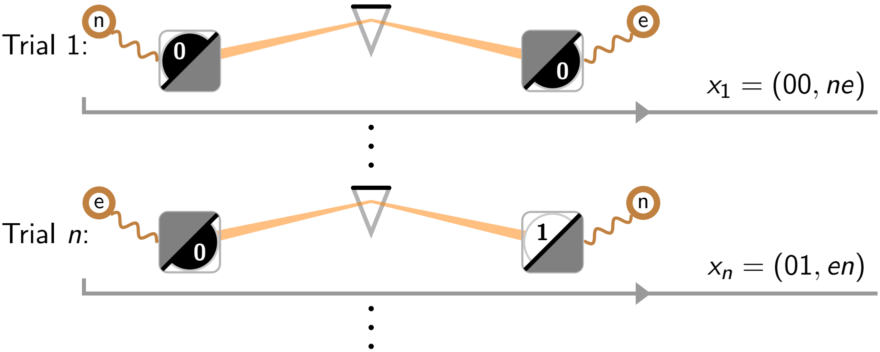

From $\mathbf{x} = (x_1, \ldots, x_n, \ldots)$ compute $C_{\neg\mathcal{P}}(x)$, a certificate for $\neg\mathcal{P}$.

...where $\mathcal{P}$ is an "unwanted" property.

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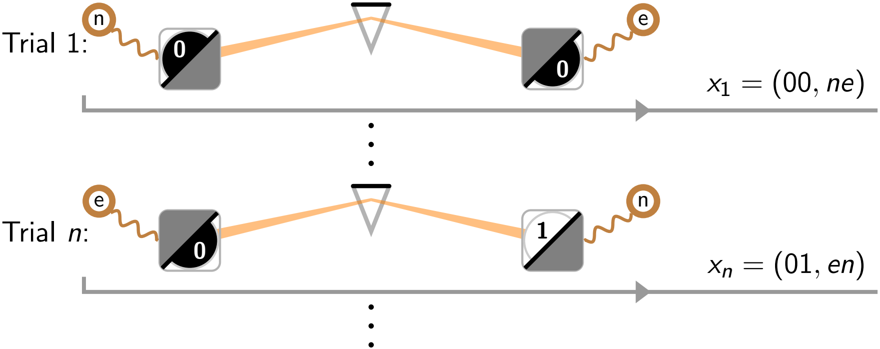

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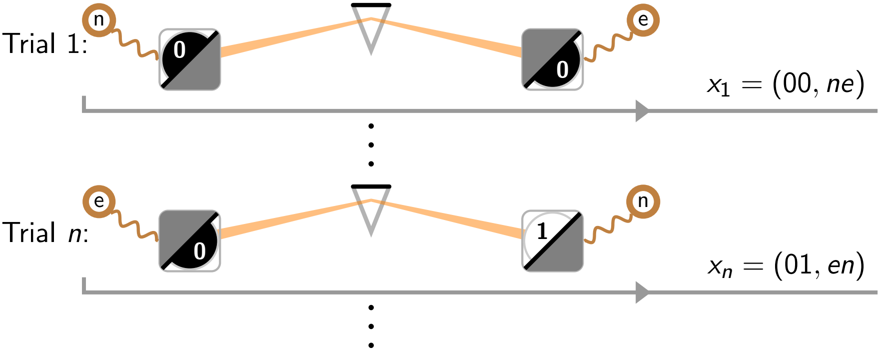

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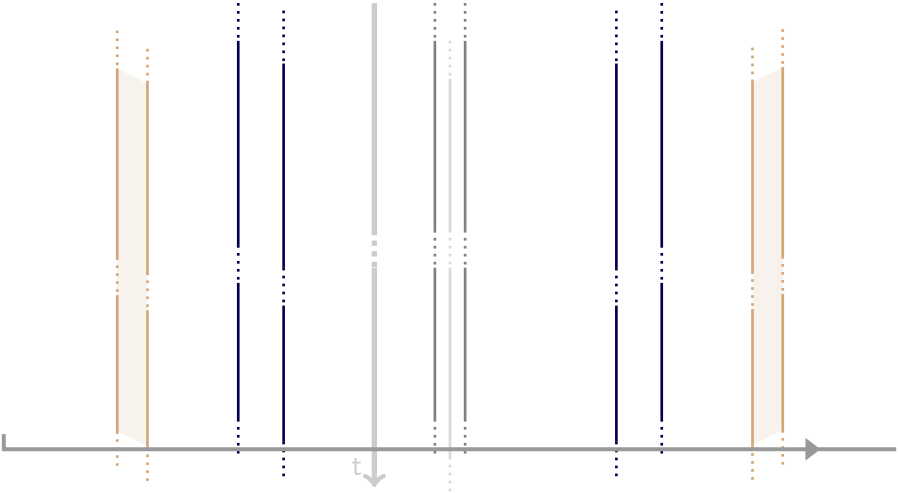
- Foundations: Constrain explanatory models.
- Protocols: Constrain hacker's access.

Common Photon-Based Trials



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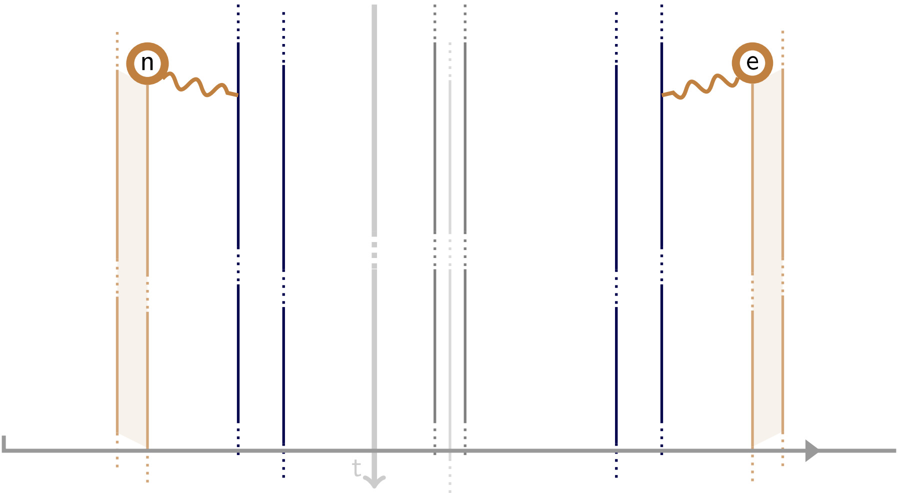

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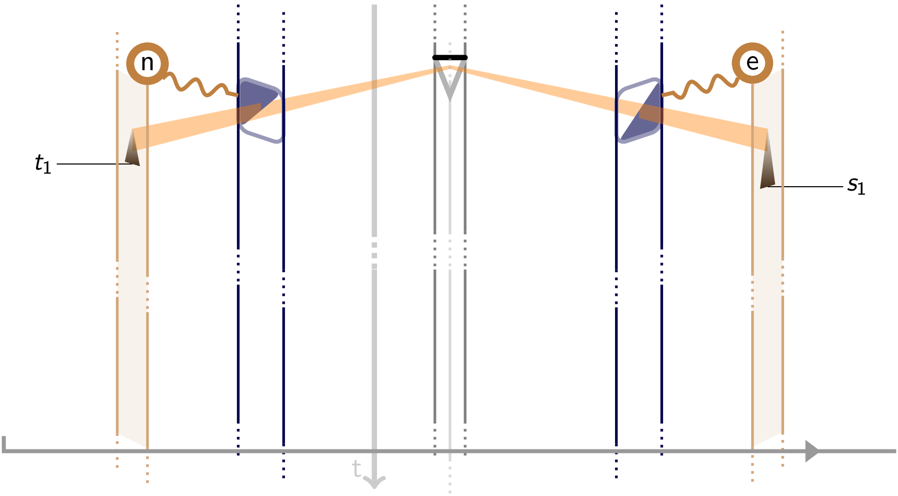

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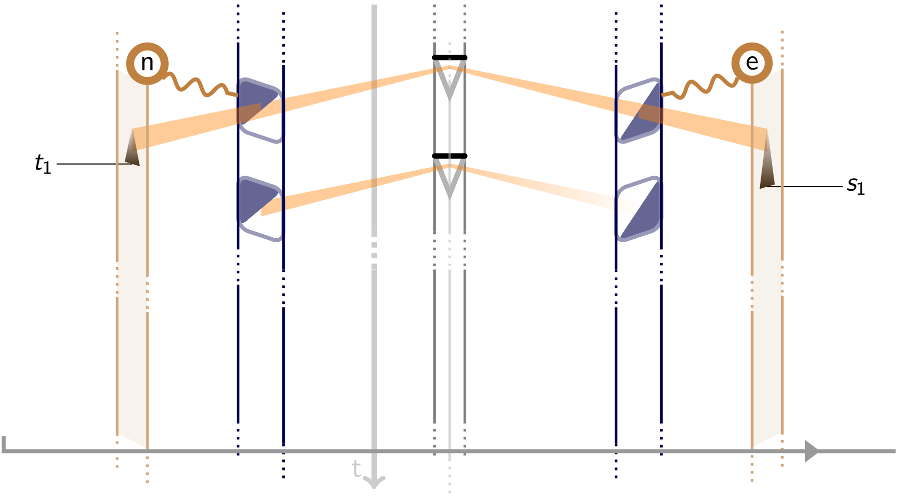

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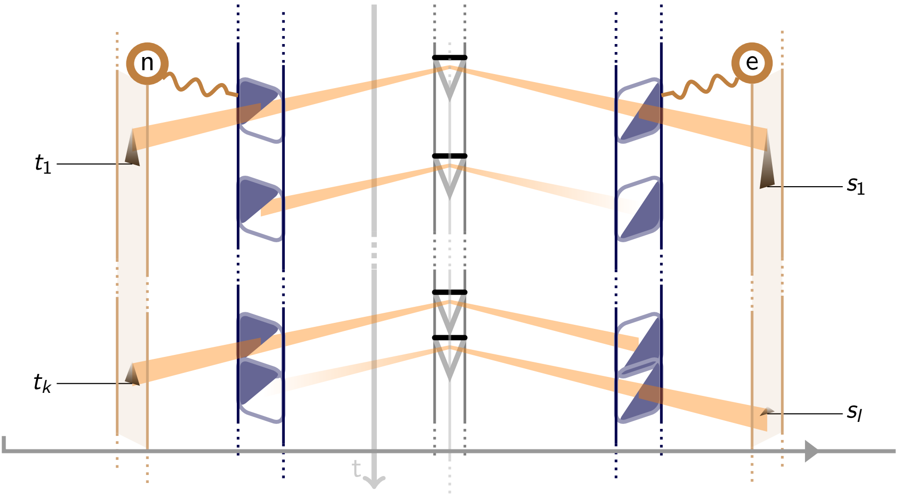

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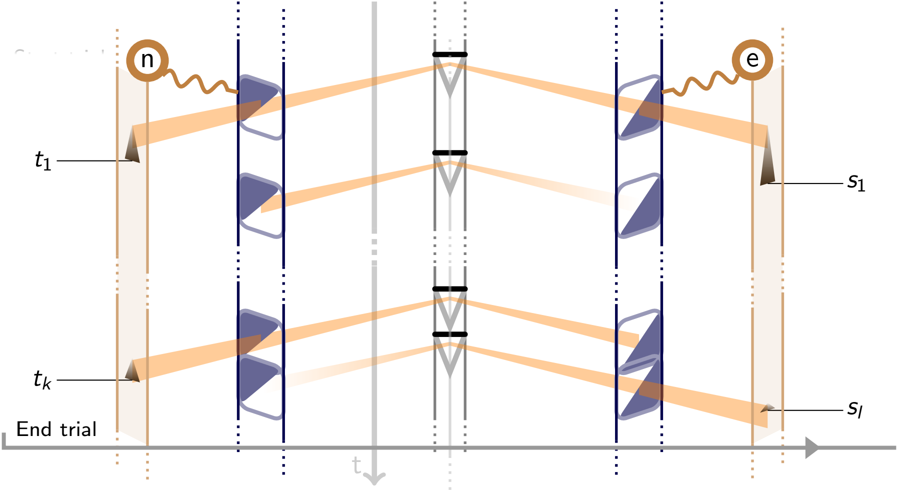

...this can be considered as *one* trial.

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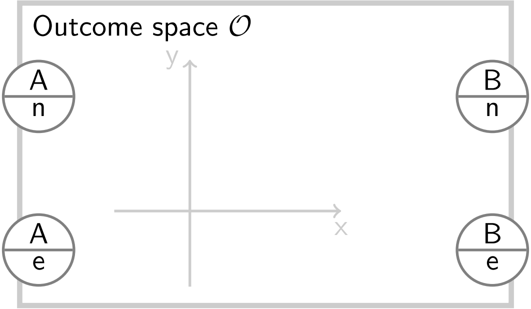
- Change settings for each trial. (Make trials last longer if needed.)
- Any randomization helps avoid auxilliary assumptions.
- Blind the trials: Automated settings choices, no tweaking when settings are "visible".
- Plan for generation of training data and confirmatory experiments.
- Compute certificates and gain rate per setting bit.
- Report: Certificate values, gain rates *and* model assumptions.

$(2, 2, \mathcal{O})$-Bell inequalities

LR:



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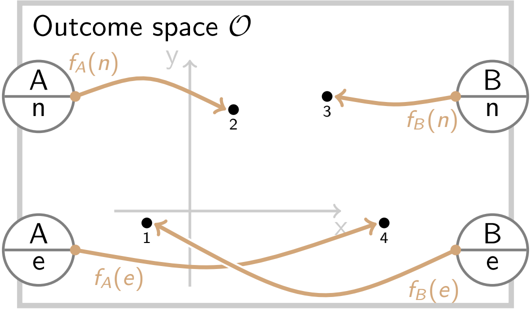

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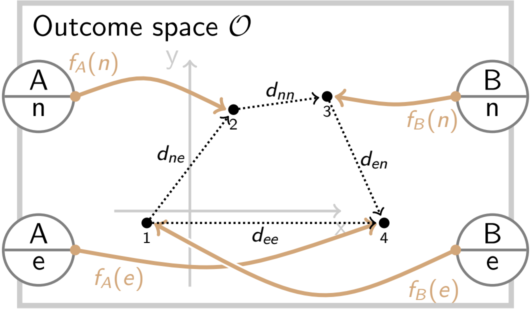

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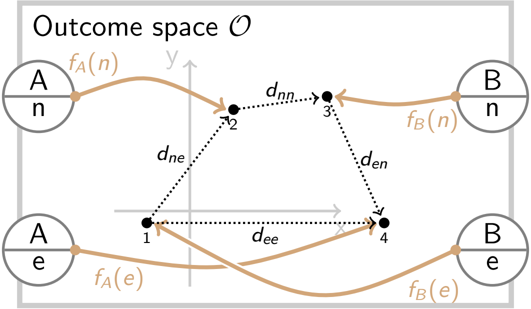

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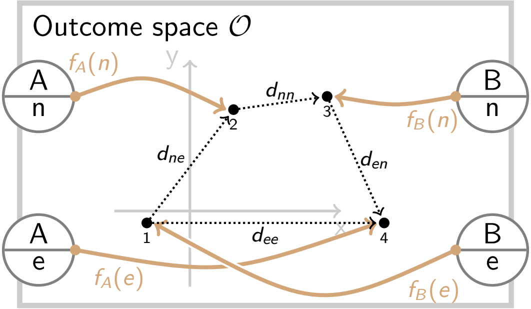

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Example. $\mathcal{O} = \{0, 1\}$: $\begin{cases} d(a,b) = |b - a| & \to \text{ CHSH variant,} \\ d(a,b) = \max(0, b - a) & \to \text{CH variant.} \end{cases}$

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Bell Functions

Assumptions and context:

- RCI must hold for each trial. RCI:
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Bell function: A function $B : (o,s) \mapsto B(o,s) \in \mathbb{R}$ satisfying

$$b_{B,p} \doteq \sup_{\mu \in \mathrm{LRI}(p)} \langle B(O,S) \rangle_\mu \quad < \quad \sup_{\mu \in \mathrm{RCI}(p)} \langle B(O,S) \rangle_\mu$$

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Interpretation:
Average Bell-values of trial states with confidence intervals.

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Not addressed:

- Fair comparison of experiments w. different configurations, Bell functions, assumptions.
- Fair comparison of implemented trials.
- Quantify ability of LRI to yield observed effects.

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- Certificate: Comparable overall strength.
- Gain rate: Comparable device/configuration strength.
- Independent of experimental details or Bell function, given model assumptions.

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Recent Experiments

- Pironio et al. (2010) [7]:
  - Entangled atoms in two iontraps at 1 m.
  - Aim: Certified random number expansion.
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- Christensen et al. (2013) [2]:
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- Blind the trials: Automated settings choices, no tweaking when settings are "visible".
- Plan for generation of training data and confirmatory experiments.
- Compute certificates and gain rate per setting bit.
- Report: Certificate values, gain rates *and* model assumptions.

PBRs: Optimizing Certificate Algorithms

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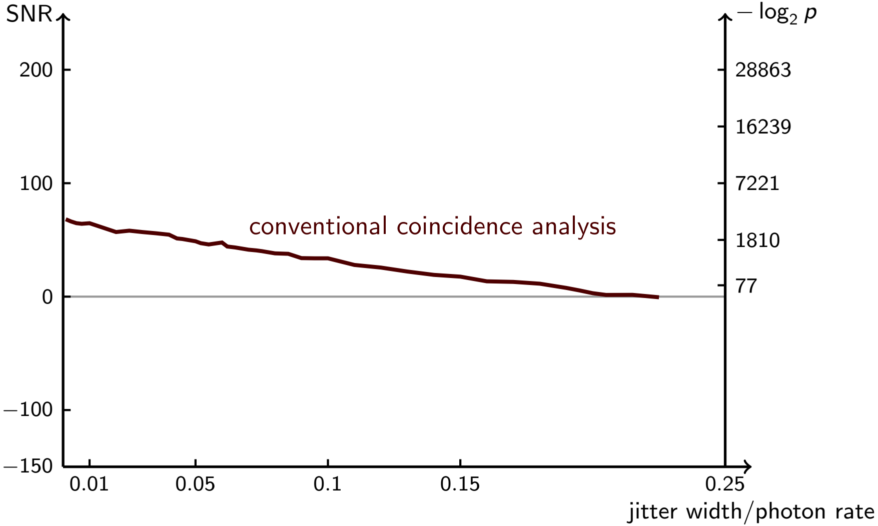
Features:

- Adapts to changing states, experimental drifts; stop anytime.
- Matches or improves other approaches (e.g. Hoeffding bounds).
- Asymptotically optimal when trials are i.i.d.
- Can automatically optimize equivalent Gaussian SNR.
- Adaptable to unbounded triangle-inequality Bell functions.

Simulation: Quantum Timetag Trials

Specs: Poisson pairs, efficiency 80 %, square jitter.
1 detector/party, CHSH optimized.



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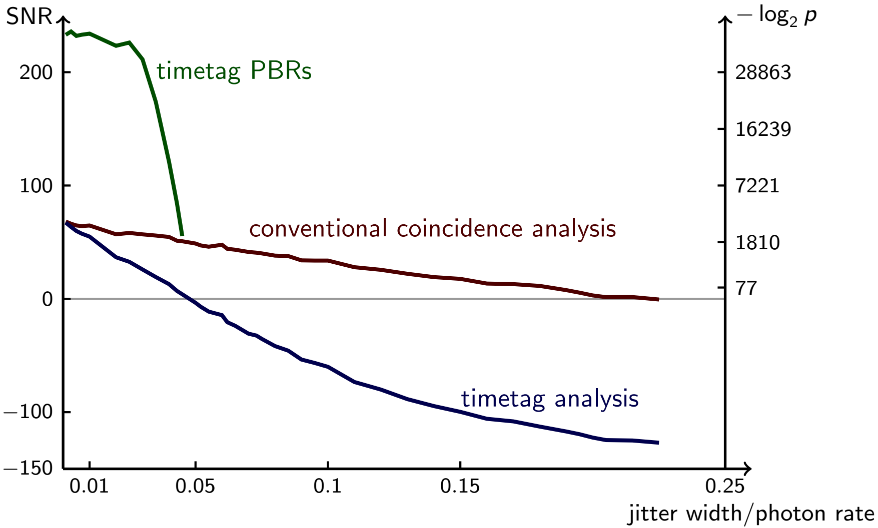
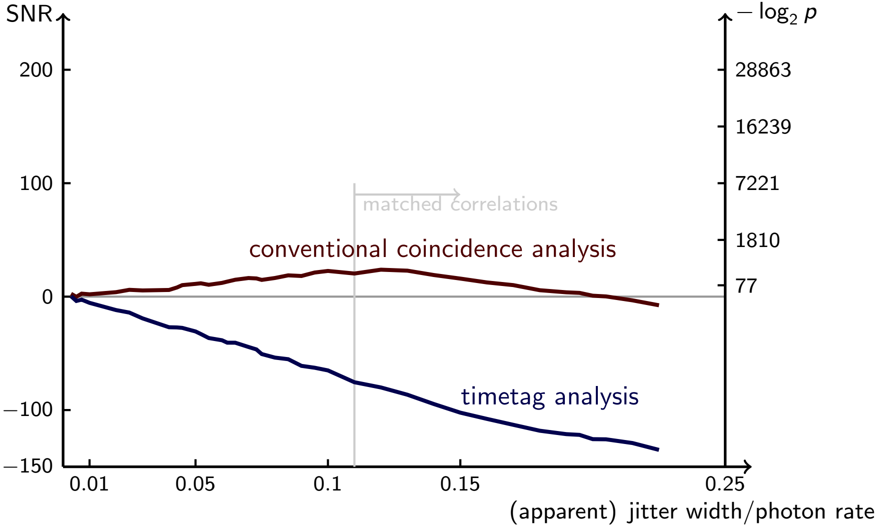


Simulation: LRI Timetag Trials

Specs: Match 1st and 2nd-order q. counting statistics at high apparent jitter.



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