

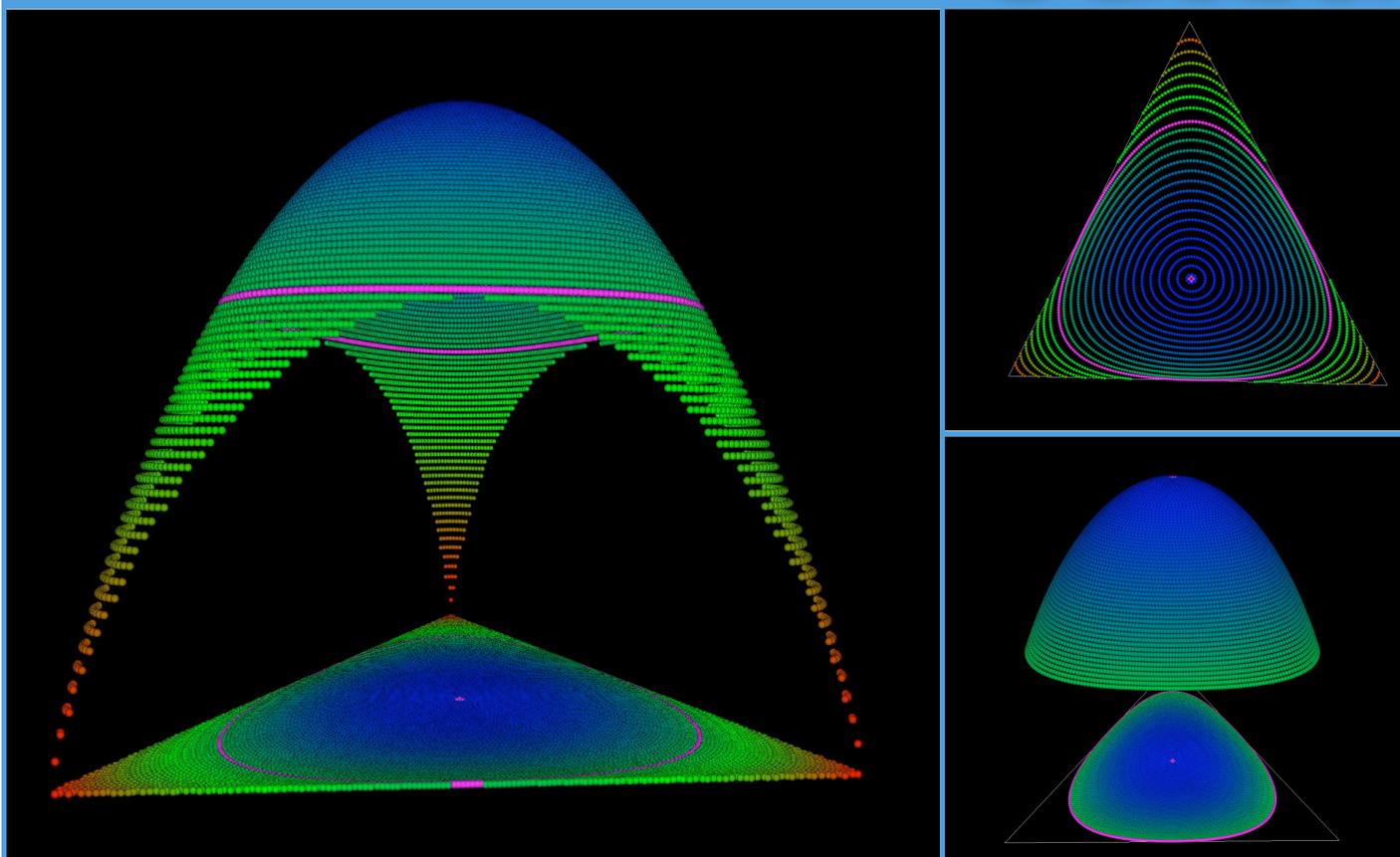
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# complex systems

IMAGE OF  
THE MONTH

# October

Possible Route Probability Distributions (now in 3D!)



In our study of TCP congestion control with allocation of paths joining a source and destination, the entropy of the route distribution is a key parameter. For a simple network with three possible paths joining a source and destination, let  $a^k$  be the fraction (non-negative) of traffic assigned to route  $k$ ,  $k=1,2,3$ . We must have  $a^1+a^2+a^3=1$ . Every such point  $(a^1,a^2,a^3)$  can be represented as a point in the equilateral triangle with unit altitude. The figure shows the surface over the triangle defined by the equation:  $H(a^1,a^2,a^3) = -a^1 \log(a^1) - a^2 \log(a^2) - a^3 \log(a^3)$ , where the height of a point on the surface above the triangle is the value of  $H$  for the point in the triangle representing  $(a^1,a^2,a^3)$ .  $H$  is the entropy function. The top of the surface is the maximum value of the entropy, i.e.  $\log 3$  which corresponds to the point  $(1/3,1/3,1/3)$  whose image in the triangle is the pink dot in the center.

The surface illustrates the changes in the level curves of  $H$  (i.e. points in the triangle with a constant entropy value) as  $H$  decreases from  $\log 3$  to 0. The level curves are closed ovals for  $H < \log 3$  and grow larger until  $H = \log 2$ . The pink circle on the surface is the intersection of the surface with the plane of

height  $\log 2$  above the triangle. The corresponding level curve is an oval that is tangent to boundaries of the triangle. If  $H < \log 2$  the level curves change. For such values the level curve is piecewise continuous and includes part of the boundary of the triangle. Points on the boundary correspond to distributions where one of the  $a_i=0$ , therefore they can only be realized by 2 routes rather than 3. These figures highlight the distributions realized by 3 routes and this accounts for the gaps in the surface. The vertices of the triangle (in red) correspond to the distribution that assigns a value 1 to one the routes and 0 to the other two. The entropy of such a distribution is  $H=0$ .



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The Complex Systems Program is part of the National Institute of Standards and Technology's Information Technology Laboratory. Complex Systems are composed of large interrelated, interacting entities which taken together, exhibit macroscopic behavior which is not predictable by examination of the individual entities. The Complex Systems program seeks to understand the fundamental science of these systems and develop rigorous descriptions (analytic, statistical, or semantic) that enable prediction and control of their behavior.

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