# Analysis of Use of 10 or 20 dB Amplitude Shifting for WWVB at 60 kHz

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## Introduction

It is shown that the WWVB signal at 60 kHz is equivalent to a binary, amplitude shift keyed (ASK) modulated signal. Then the relative performance of a binary, ASK, matched-filter receiver operating in a noise environment is examined as a function of the size of the signal amplitude shift. It is shown that the bit error probability of WWVB receivers degrades as the amplitude shift in dB is decreased. In addition, it is shown that decreasing the amplitude shift is equivalent to decreasing the transmitter's power levels by some amount. Using on-off keying as a reference, the equivalent increase in transmitter power needed to maintain the same performance for smaller amplitude shifts in dB is determined. Using this result, it is shown that changing the WWVB amplitude shift from 10 dB to 20 dB is equivalent to increasing the received signal level by 2.4 dB for matched-filter receivers.

## Signal

First, we show that this analysis is applicable to the WWVB, 60-kHz signal. The signal consists of three possible symbols transmitted one per second. The symbol for the binary 0 consists of the carrier amplitude being reduce by 10 dB at the beginning of the second and being restored to full power after 200 ms until the end of the second (800 ms). The symbol for the binary 1 consists of the carrier amplitude being reduce by 10 dB at the beginning of the second and being restored to full power after 500 ms until the end of the second (500 ms). A timing symbol is transmitted at second 0, 10, 20, 30, 40, 50, and 59 of the minute. The double timing symbol at the 59 and 0 seconds marks the start of the minute. The timing signal consists of the carrier amplitude being reduce by 10 dB at the beginning of the second and being restored to full power after 800 ms until the end of the second (200 ms).

The symbol sent thus depends only on the signal level transmitted in two consecutive 300 ms time intervals in the second at 200-500 ms and 500-800 ms. This code is presented in Table 1. Thus, a timing symbol

Table 1: Signal Codes

| Symbol | 200-500 ms | 500-800 ms |
|--------|------------|------------|
| 0 | 0 dB | 0 dB |
| 1 | -10 dB | 0 dB |
| Timing | -10 dB | -10 dB |

is composed of two amplitude shift keyed (ASK) binary symbols or bits. To successfully detect a timing symbol, the transmitter state for the two time intervals (two bits) must be correctly detected. Therefore,

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the probability of a symbol being received in error is equal to the probability of making an error in the first time interval or the second time interval or both time intervals. If the probability of an error in the first time interval is $p$, then the probability of an error in the second time interval is also $p$, since the modulation is identical in both time intervals. The probability of an error in both time intervals is $p^2$. Thus, the total probability of making a symbol error is $p+p+p^2$. It is reasonable to assume that we will want the probability of a symbol error to be small. Then, for $p \ll 1$, we have $p^2 \ll p$, so that the probability of a symbol error becomes $p + p = 2p$.

We now have the probability of a time symbol error, $2p$, in terms of the probability of a bit error, $p$, for an Amplitude Shift Keyed (ASK) binary modulation.

## Detection

One of two data symbols will be received. The ASK symbol 0 is sent with signal amplitude of $x_0$. The ASK symbol 1 is sent with signal amplitude of $x_1 = cx_0$. The value of $c$ is given by

$$c = 10^{-X/20} \tag{1}$$

where $X = 20 \log_{10}(x_0 - x_1)$ is equal to the amplitude shift in dB.

In the standard digital receiver[1] the symbol energy is integrated over a symbol duration. The received noise is also integrated during the symbol period. The resulting random voltage value, $s+n$, has a mean equal the integrated signal plus a random noise term. Since there are two possible symbols that may be sent, the integrated symbol energy may have two possible values, $s_0$ and $s_1$, which must also differ by the transmitted amplitude shift of X dB, i.e.,

$$s_1 = cs_0 \tag{2}$$

A voltage of $s_1 + n$ is developed when the transmitted is at the low power level and a voltage of $s_0 + n$ is developed if the transmitter is at the high level. See Figure 1.
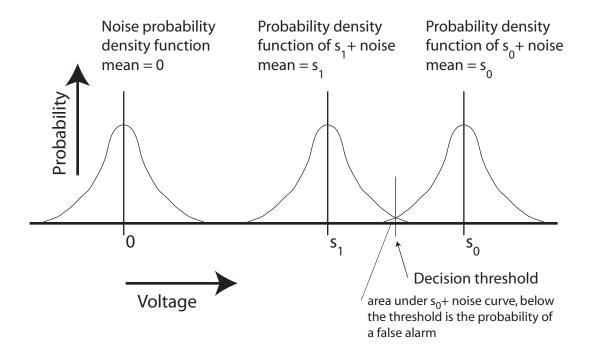


Figure 1: Diagram of voltages used to determine which ASK bit was sent.

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[1]The standard digital receiver design is optimal for reception in additive, white, Gaussian, noise (AWGN), which may not be the optimal receiver design for atmospheric noise at 60 kHz.

At the receiver, threshold is used to decide which symbol is received. When $s_0$+n is less than the threshold, a false alarm occurs and a 1 is mistakenly received. When $s_1$+n is greater than the threshold, a miss occurs and a 0 is mistakenly received.

Knowing that the minimum error rate occurs when the probability of a miss and a false alarm are equal, the threshold must be the same distance from the voltages $s_0$ and $s_1$. The separation between the voltage values $s_0$ and $s_1$ at the receiver determine the bit error rate (BER). We may assume that we require a separation distance of $y = s_0 - s_1$ for a desired BER. We may now solve for the signal level, $s_0$, necessary for $s_0$ and $s_1$ to be separated by $y$.

Using equation (2) we see that

$$s_0 = \frac{y}{(1-c)}. \tag{3}$$

Now, we can find how this required received signal level, $s_0$, depends in the amplitude shift, $X$. Say we have two amplitude shifts equal to $X_a$ and $X_b$. Then from equation (3),

$$\frac{s_0(X_a)}{s_0(X_b)} = \frac{(1-c_b)}{(1-c_a)}. \tag{4}$$

If we take $20\log_{10}$ of the ratios in equation (4) we will get the difference in dB between the received signal levels required to achieve the same bit error rate. Furthermore, we let $X_b$ be a reference shift amount, say infinity, which corresponds to keying the transmitter on and off. Then we have the signal level increase in dB required to match on-off keying given by

$$S_r(X) = -20\log_{10}(1 - 10^{-X/20}) \tag{5}$$

where equation (1) has been used to substitute for c.

Interestingly, the result given in equation (5) does not depend on the noise level or the bit error rate. This means that the improvement in the performance of the receiver, when the amplitude shift, $X$, is increased, corresponds to a fixed increase in signal power over the entire area of reception.

If we want the required increase in signal level to use a 10 dB amplitude shift instead of a 20 dB shift, we simply use (5) to get

$$S_r(10\,dB) - S_r(20\,dB) = 3.3\,dB - 0.9\,dB = 2.4\,dB.$$

Comparing an amplitude shift of 10 dB to on-off keying gives a required signal increase of 3.3 dB.
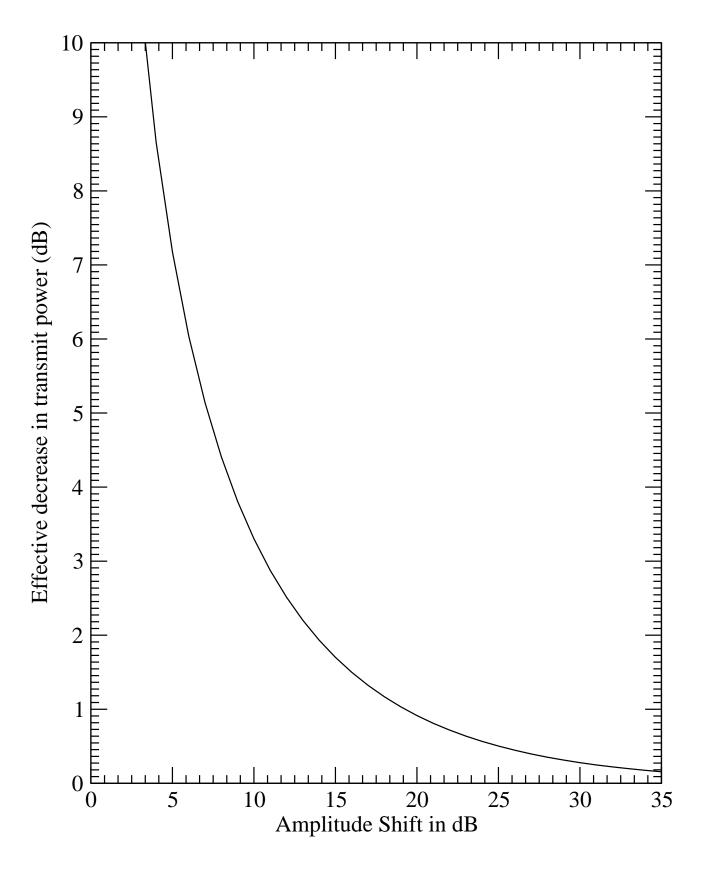
A plot of $S_r$ is given in the Figure 2.

Figure 2: The equivalent loss in signal level when an amplitude shift of $X$dB is used instead of on-off keying for binary ASK modulation.