Quantum Interference and Superradiance from entangled Atoms

R. Wiegner[1], J. von Zanthier[1], G. S. Agarwal[2]

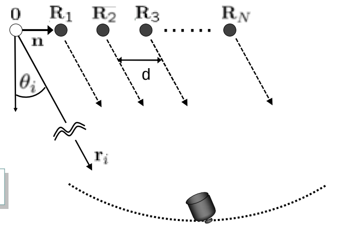
[1]Institut für Optik, Information und Photonik, Friedrich-Alexander Universität Erlangen-Nürnberg, Erlangen, Germany
[2]Department of Physics, Oklahoma State University, Stillwater, Oklahoma, USA

Recent years have seen the ability to create large varieties of entangled states among radiating particles. The question then arises how the optical properties of such entangled photon sources differ from a radiating source in a separable state. Here we investigate how entanglement of the source can lead to enhancement of the emission of radiation. We relate the enhancement to quantum interference by explicitly taking into account the various photon pathways. Our work shows how Dicke's prediction of superradiance [1] from states with zero dipole moment can be understood as an interference phenomenon.

[1] R. H. Dicke, Phys. Rev. 93, 99 (1954)

Experimental setup:

distance d typically > λ and can be arbitrarily large!



$\varphi_i \equiv \varphi(\mathbf{r}_i) = k\,d\,\sin(\theta(\mathbf{r}_i))$

I. Intensity and quantum paths of sources in a separable state vs. W-state

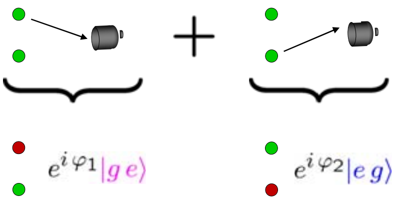
$$\mathrm{I} = |\hat{D}_N^\dagger|\Psi_\mathrm{i}\rangle|^2 \quad \text{with} \quad \hat{D}_N^\dagger = \sum_{j=1}^{N} e^{i\varphi_j}|g\rangle_j\langle e|;$$

● $\equiv |e\rangle$ (green) and ● $\equiv |g\rangle$ (red)

Example:

A. Two excited atoms in a separable state

• $|\Psi_{is}\rangle = |e\,e\rangle$



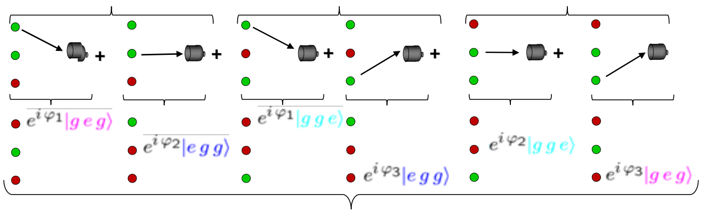
no interference terms

$$\longrightarrow \mathrm{I}_s = |\hat{D}_2^\dagger|\Psi_{is}\rangle|^2 = |e^{i\varphi_1}|g\,e\rangle|^2 + |e^{i\varphi_2}|e\,g\rangle|^2 = 2$$

B. Two excited atoms in a W-state

• $|\Psi_{IW}\rangle = \frac{1}{\sqrt{3}}(|e\,e\,g\rangle + |e\,g\,e\rangle + |g\,e\,e\rangle)$

$$|e\,e\,g\rangle \quad + \quad |e\,g\,e\rangle \quad + \quad |g\,e\,e\rangle$$



$$\mathrm{I}_W = \frac{1}{3}\left|(e^{i\varphi_1} + e^{i\varphi_2})|g\,g\,e\rangle\right|^2 +$$

$$+\frac{1}{3}\left|(e^{i\varphi_1} + e^{i\varphi_3})|g\,e\,g\rangle\right|^2 + \frac{1}{3}\left|(e^{i\varphi_2} + e^{i\varphi_3})|e\,g\,g\rangle\right|^2 =$$

$$= \frac{1}{3}\,[6 + 2(\cos(\varphi_1 - \varphi_2) +$$

$$+ \cos(\varphi_1 - \varphi_3) + \cos(\varphi_2 - \varphi_3))] =$$

MAX

$$\stackrel{\downarrow}{=} \frac{6}{3} \quad + \quad \frac{6}{3} \quad =$$

$$= [\mathrm{I}_s]^{MAX} + \textbf{extra interference terms}$$

II. Superradiance from arbitrary W-states

W-states cause additional constructive interference, namely interference of quantum paths which lead to the same final states

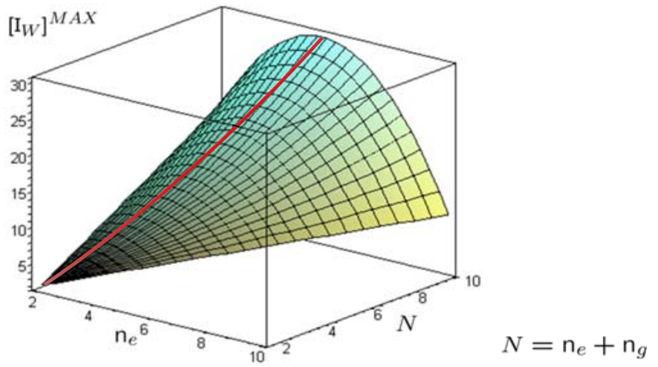
$\longrightarrow$ **enhanced emission of radiation !**

$$[\mathrm{I}_W]^{MAX} = [\mathrm{I}_s]^{MAX} + \textbf{extra interference terms}$$

(interference terms of quantum paths leading to the same final states) **x** (# final states) **x** (Normalization)

initial W-state	$[\mathrm{I}_s]^{MAX}$	+	(int. terms of QPs same final states)	x	(# final states)	x	(Norm.)	=	$[\mathrm{I}_W]^{MAX}$
$\|e\rangle$ and $(N-1)\|g\rangle$	1	+	N(N-1)	x	$\binom{N}{0}$	x	$\left[\binom{N}{1}\right]^{-1}$	=	N
$\|e\,e\rangle$ and $(N-2)\|g\rangle$	2	+	(N-1)(N-2)	x	$\binom{N}{1}$	x	$\left[\binom{N}{2}\right]^{-1}$	=	2 + 2(N-2)
$\|e\,e\,e\rangle$ and $(N-3)\|g\rangle$	3	+	(N-2)(N-3)	x	$\binom{N}{2}$	x	$\left[\binom{N}{3}\right]^{-1}$	=	3 + 3(N-3)
⋮					⋮				Dicke MAX
$\frac{N}{2}\|e\rangle$ and $\frac{N}{2}\|g\rangle$	$\frac{N}{2}$	+	$\left(\frac{N}{2}+1\right)\frac{N}{2}$	x	$\binom{N}{N/2-1}$	x	$\binom{N}{N/2}^{-1}$	=	$\frac{N}{2} + \left(\frac{N}{2}\right)^2$
⋮ $(N-1)\|e\rangle$ and $\|g\rangle$	N-1	+	2	x	⋮ $\binom{N}{N-2}$	x	$\left[\binom{N}{N-1}\right]^{-1}$	=	N-1 + N-1
$n_e\|e\rangle$ and $n_g\|g\rangle$	n_e	+	$n_g(n_g+1)$	x	$\binom{N}{n_e-1}$	x	$\left[\binom{N}{n_e}\right]^{-1}$	=	$n_e(n_g+1)$

W-state $n_e|e\rangle$ and $n_g|g\rangle \longrightarrow$ in Dicke notation $|N, m\rangle$ with $|N/2, \frac{1}{2}(n_e - n_g)\rangle$



$N = n_e + n_g$

•Similar procedure works for subradiant states, i.e., antisymmetric Dicke-states

• Quantum interference can be also used for other phenomena like nonlinear processes