Inductance Measurement Using an LCR Meter and a Current Transformer Interface

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Introduction

Traditional calibration support for commercially available four-terminal-pair (4TP) LCR meters involves the use of capacitance, resistance, and inductance standards with known frequency characteristics. This paper describes a new inductance measurement system where an unknown inductance standard is compared to known capacitance and resistance standards using a commercially available LCR meter, a commercially available audio frequency capacitance bridge, and a multi-stage, programmable-ratio, current transformer. The described measurement system is relatively simple, uses mainly commercially available instrumentation, and achieves acceptable inductance measurement uncertainty over the 50 Hz to 20 kHz frequency range.

It has been established that commercially available LCR meters have relatively good linearity characteristics over narrow measurement ranges [1], making them well suited as transfer devices when comparing impedances of nominally equal values. In recent years, highly accurate, three-terminal (3T) capacitance bridges operating over the 50 Hz to 20 kHz frequency range have become commercially available with measurement uncertainties approaching metrological levels [2]. In addition, multi-stage, amplifier-aided current transformers have been routinely developed that achieve errors below a few parts in 10⁶ over the 50 Hz to 20 kHz frequency range [3-4]. In order to improve and simplify the present inductance measurement services at NIST, an attempt has been made to exploit these three technologies by developing a system to compare an inductance standard under test to a capacitance standard having an equivalent impedance magnitude at the frequency of interest. The GR1482 inductance standards traditionally used for the dissemination of the Henry typically have large series resistance components, so equivalent impedances must be synthesized using parallel combinations of capacitance and resistance standards.

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Proposed Approach

In the proposed measurement procedure, we first make an initial, low-accuracy measurement of the inductance standard under test using an LCR meter. From this measurement, suitable resistance and capacitance standards are selected that will, when connected in parallel, result in an equivalent impedance magnitude. Since capacitors and inductors exhibit current flow with opposite phase, a current transformer interface is used to introduce a 180° phase shift in the capacitive current. The current transformer based phase shift network, a programmable capacitor, and a programmable resistor are used in order to match the measured inductance under test. The capacitance and resistance values are adjusted until the same LCR reading is achieved, within the resolution constraints of the programmable capacitor and resistor. Once the impedance is matched, the resulting resistance, C, is measured using a LCR meter (to determine its time constant) and also with a high-accuracy commercial DMM (to determine its dc resistance value). The measured parameters are then used to calculate the impedance of the inductance under test. The test procedure is outlined graphically in Figure 1.



Figure 1. Inductance Measurement Procedure

The inductor is modeled using a series resistance: $Z_L = R_S + j\omega L$. The capacitance and resistance matching network is modeled as a parallel network: $Z_{RC} = \frac{R_P}{1 + j\omega CR_P}$. Assuming that the

impedance of the inductor, Z_L , is given (by the initial LCR measurement), and that the goal is to find the equivalent capacitance that has exactly the same magnitude but with a phase shift that is exactly 180° away from the phase shift of the inductor, then the equation for the phase

relationship is:
$$\frac{\omega L}{R_s} = \omega R_p C$$
, or $C = \frac{L}{R_s R_p}$, (1)

and the equation for the magnitude relationship is:

$$\sqrt{\left(R_{s}\right)^{2} + \left(\omega L\right)^{2}} = \frac{R_{P}}{\sqrt{1 + \left(\omega R_{P}C\right)^{2}}}, \text{ or } R_{P} = R_{s} + \frac{\left(\omega L\right)^{2}}{R_{s}}.$$
(2)

Once all the measurements are completed, the impedance of the inductor is given by:

$$Z_L = \frac{Z_L}{Z_{RC}} Z_{RC},$$
(3)

where the impedances with primes are those measured with the LCR meter and $Z_{RC} = \frac{R_P}{1 + j\omega CR_P}$

is calculated from the individually measured R_p and C_p values (using the capacitance bridge, DMM, and LCR meter). Finally, the *L* and R_s values are given by:

$$L = \frac{\operatorname{Im}\{Z_L\}}{\omega}, \text{ and } R_S = \operatorname{Re}\{Z_L\}.$$
(4,5)

Present inductance calibration services at NIST support inductance values of $L = [50 \mu H, 100 \mu H, 200 \mu H, 500 \mu H, 1 mH, 2 mH, 5 mH, 10 mH, 20 mH, 50 mH, 100 mH, 200 mH, 500 mH, 1 H, 2 H, 5 H, 10 H]. Supported frequencies are 100 Hz, 400 Hz, and 1 kHz for all standards, and 10 kHz for inductors up to 100 mH. This relatively large test space would normally require adjustable resistance and capacitance standards spanning many decades of possible values. To avoid unnecessary complexity, the current transformer is wound with three primary-secondary ratios, 1:100, 10:100 and 100:100, thereby adding an additional two orders of magnitude to the matching computations. A switching network is used to apply the optimal ratio when measuring different inductors and also to control the auto calibration process. In order to achieve an optimal selection of resistors and capacitors satisfying the given design constraints, the apparent inductance seen by the LCR meter may be scaled up by a factor of 10 or 100 using appropriate windings on the current transformer.$

Software routines were developed to select the matching components for the given set of inductors. $R_{equivalent}$ and $L_{equivalent}$ are calculated using rounded values for C and R_p according to their respective adjustment resolutions. In Table I, the results are presented for the set of standard inductors at a frequency of 10 kHz, where the nominal value, L, of the inductor is used and its measured serial resistance, R_s , is given. The relative errors are given and only the values below 500µH indicate any appreciable differences between the R_s and L values and the $R_{s_{EQ}}$ and L_{EQ}

values that were calculated using the finite resolution R_P and C. This is not a problem, however, since it has been demonstrated that LCR meters of the type used in this system have linearity approaching parts in 10⁶ over very wide (+/-0.3%) operating point margins [1].

L	R_s	Z _L SF	R _p	R SF	С	C SF	$\frac{R_{S_{EQ}} - R_S}{R_S}$	$\frac{L_{EQ} - L}{L}$
[H]	[Ω]		[Ω]		[µF]		[10 ⁻⁶]	[10 ⁻⁶]
50e-6	0.17	100	75	1	3.918617	1	411.7	-341.3
100е-б	0.1	100	405	1	2.470452	1	506.3	26.4
200е-6	0.15	100	1068	1	1.248723	1	220.9	6.1
500e-6	0.61	100	1679	1	0.4882	0.1	17.7	1.4
1e-3	1.09	100	3731	1	0.2459025	0.1	31.7	2.1
2e-3	2.33	10	701	10	1.224421	1	54	-4.2
5e-3	4.62	100	21825	1	0.04958818	0.01	9.7	0.4
10e-3	7.98	10	5027	10	0.2492819	0.1	5.5	0.3
20e-3	18.7	100	86316	1	0.0123908	0.01	1.7	0.3
50e-3	45	100	223825	1	0.0049642	0.01	3.5	-0.7
100e-3	89.8	100	44861	1	0.0248232	0.01	2.1	1.7
200e-3	114	10	13966	10	0.1256177	0.01	5	-0.2
500e-3	335	10	297965	10	0.0050091	0.01	0.9	0.4
1	616	10	64704	10	0.0250891	0.01	4.2	1.7
2	1350	1	118323	1	0.01252065	0.1	0.1	-0.4
5	3600	1	277756	1	0.0050004	0.1	2	-0.4
10	8450	1	475650	1	0.00248803	0.01	0.4	0

Table I. Obtained matching parameters for the set of inductors at 10 kHz.

Results

Simulations were performed to analyze type B uncertainties associated with the various system components. A plot of the inductance measurement uncertainties over the test space of interest (50 Hz to 20 kHz, 100 μ H to 10 H) is shown in Figure 2.



Figure 2. Type B Analysis of Inductance Measurement System.

The analysis includes basic accuracy specifications for all system components including the LCR meter basic accuracy and linearity, capacitance bridge performance specifications, DMM 4-wire Ohms specifications, and current transformer performance. The errors should represent a worst case analysis, since conservative bounds were used for many of the parameters. The most significant source of error occurs at low frequencies and at low inductance settings. It is attributed to the uncertainty of the capacitance bridge estimate of the capacitor's dissipation factor. Errors in this estimate show up mathematically as the addition of a parallel resistance across R_P , thereby affecting the Z_{RC} estimate directly.

A preliminary set of measurements were taken to calibrate a 100 mH inductor at 1 kHz. The known [5] value for the inductor was $Z = j\omega(0.999879) + 86.4 [\Omega]$. The calculated matching parameters were C = 248.655 nF and Rp = 4.65 k Ω . After performing the adjustment procedure, the measured matching parameters were Cm = 248.57 nF and Rpm = 4.63 k Ω ., resulting in a measured value for the inductor of Z = $j\omega(0.999904) + 86.876 [\Omega]$. The resulting agreement between the two systems of 25.3 x10⁻⁶ is well within the measurement uncertainty of the Maxwell-Wien bridge.

Conclusions

The described inductance measurement system is relatively simple to use and relies largely on commercial instrumentation. The type B uncertainties have been shown to be comparable to existing bridge methods. Analysis of the remaining type B and type A uncertainties is under way and will be presented at the conference.

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