EXAMPLES OF UNCERTAINTY BUDGET TABLES

AIR DENSITY

SOP 2 **BUOYANCY CORRECTIONS** (NISTIR 6969)

Equation 1. OIML R 111

$$u_{bc}^2 = \left(m_s \times \frac{\rho_s - \rho_x}{\rho_s \rho_x} \times u_{\rho_a} \right)^2 + [m_s \times (\rho_a - \rho_n)]^2 \times \left(\frac{u_{\rho_s}^2}{\rho_s^4} - \frac{u_{\rho_x}^2}{\rho_x^4} \right)$$

Equation 2. SOP 2 Magnitude of Air Buoyancy Correction

$$MABC = m_0 \left(\frac{1}{\rho_x} - \frac{1}{\rho_s} \right)(\rho_a - \rho_n)$$

Component	Description	Reference
$u_{\rho a}$	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper, e.g., 0.0012 mg/cm^3
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
$u_{\rho s}$	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 % (0.00064 g/cm^3)
$u_{\rho x}$	Standard uncertainty for density of unknown (ρ_x)	Mfg. estimate, 0.05 g/cm^3

MASS

SOP 3 (DOUBLE SUBSTITUTION, EQUAL ARM) (NISTIR 6969)
SOP 4 (DOUBLE SUBSTITUTION) (NISTIR 6969)
SOP 5 (THREE IN ONE) (NISTIR 6969)
SOP 6 (TRANSPOSITION) (NISTIR 6969)
SOP 7 (SINGLE SUBSTITUTION) (NISTIR 6969)

$$C_x = C_s + d + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

$$M_x = M_s + d + \rho_a (V_x - V_s) + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control charts
u_{bc}	Standard uncertainty for buoyancy correction	
$u_{\rho a}$	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), e.g., 0.0012 mg/cm^3
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
$u_{\rho s}$	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., e.g., 0.008 % (0.00064 g/cm^3)
$u_{\rho x}$	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm^3
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111-1
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111-1
$u_{ecc\ load}$	Standard uncertainty due to eccentric loading	OIML R111-1
u_{mag}	Standard uncertainty due to magnetism	OIML R111-1
u_o	Standard uncertainty for other factors	

SOP 8 (MODIFIED SUBSTITUTION) (NISTIR 6969)

$C_x = C_s + d +$ disparity (bias/drift) + unrelated corrections

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process
u_{bc}	Standard uncertainty for buoyancy correction	Possibly: uncorrected systematic error
$u_{\rho a}$	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), e.g., 0.0012 mg/cm^3
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
$u_{\rho s}$	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 % (0.00064 g/cm^3)
$u_{\rho x}$	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm^3
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111
$u_{ecc\ load}$	Standard uncertainty due to eccentric loading	OIML R111
u_{mag}	Standard uncertainty due to magnetism	OIML R111
u_o	Standard uncertainty for other factors	

QUESTION: CAN u_{bc} BE ASSUMED TO BE NEGLIGIBLE, AT THIS LEVEL? OR IS IT SAFER TO EVALUATE IT AND THEN DECIDE THE NEGLIGIBILITY OF IT AND HAVE IT DOCUMENTED?

SOP 28 (ADVANCED WEIGHING DESIGNS) (NISTIR 5672)

$M_x = M_s + d + \rho_a (V_x - V_s)$ + disparity (bias/drift) + unrelated corrections

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control charts
u_{bc}	Standard uncertainty for buoyancy correction	
$u_{\rho a}$	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), e.g. 0.0012 mg/cm^3
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
u_{CO2}	Standard uncertainty for carbon dioxide determination	
u_{cce}	Standard uncertainty for cubical coefficient of expansion	
$u_{\rho s}$	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 % (0.00064 g/cm^3)
$u_{\rho x}$	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm^3
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111
$u_{ecc\ load}$	Standard uncertainty due to eccentric loading	OIML R111
u_{mag}	Standard uncertainty due to magnetism	OIML R111
u_o	Standard uncertainty for other factors	

LENGTH

SOP 10 (RIGID RULE) (NBS Handbook 145)

$$L_x = L_s + d + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; range charts
u_{gr}	Standard uncertainty for graduated reticle	
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the rules	
u_o	Standard uncertainty for other factors	

SOP 11 (BENCH METHOD) (NBS Handbook 145)
SOP 23 (PI TAPES) (NBS Handbook 145)

$$L_x = L_s + d + L_n\,[(t - 20)\,(\alpha - \beta)] + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; range charts
u_{gr}	Standard uncertainty for graduated reticle	
u_{tc}	Standard uncertainty for temperature correction	
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
$u_{lce\,(\alpha,\beta)}$	Standard uncertainty for linear coefficient of expansion	
u_{tw}	Standard uncertainty for tension weights	
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the tape	
u_o	Standard uncertainty for other factors	

SOP 12 (TAPE TO TAPE)

$L_x = L_s + d$ + disparity (bias/drift) + unrelated corrections

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process
u_{gp}	Standard uncertainty for graph paper	
u_{ss}	Standard uncertainty for spring scales	
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the tapes	
u_o	Standard uncertainty for other factors	

VOLUME - GRAVIMETRIC

SOP 13 (MECHANICAL BALANCE)
SOP 14 (ELECTRONIC BALANCE)
SOP 15 (EQUAL ARM BALANCE)

$$V_{t_{ref}} = (M_F - M_E)(M_S / M_R)\left(1 - \frac{\rho_a}{\rho_s}\right)\left(\frac{1}{\rho_w - \rho_a}\right)(1 - \alpha(t - t_{ref}))$$

$$V_{t_{ref}} = ((O_3 - O_2)/O_1)M_S\left(1 - \frac{\rho_a}{\rho_s}\right)\left(\frac{1}{\rho_w - \rho_a}\right)(1 - \alpha(t - t_{ref}))$$

$V_x = d_{(F-E)} + \rho_a (V_x - V_s) + [1 - \alpha (t - t_{ref})] +$ *bias/drift + other corrections*

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the measuring process to determine mass difference between filled and empty container; recorded in volume units on charts to be converted to grams by dividing a Z constant of 1.003; represents all weighing operations	Measurement assurance process; control chart or range chart and d_2^*
u_{bc}	Standard uncertainty for buoyancy correction	
$u_{\rho a}$	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), 0.0012 mg/cm^3
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
$u_{\rho s}$	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 % (0.00064 g/cm^3)
$u_{\rho x}$	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm^3
u_{tc}	Standard uncertainty for temperature correction	
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
$u_{cce\ (\alpha,\beta)}$	Standard uncertainty for cubical coefficient of expansion	NBS Report 10 081, 5 x 10^{-6} /°F (0.000009 / °C)
$u_{\rho w}$	Standard uncertainty for density of water	

u_e	Standard uncertainty for water density equation and uncorrected systematic error due to air-free vs air-saturated water densities	Metrologia, Patterson/Morris (1994), 7 ppm (0.000007 g/cm3)
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.1 °C, SOPs
$u_{read\ men}$	Standard uncertainty for reading meniscus	(ASTM E694); GMP 3
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111-1
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111-1
$u_{ecc\ load}$	Standard uncertainty due to eccentric loading	OIML R111-1
u_{mag}	Standard uncertainty due to magnetism	OIML R111-1
u_o	Standard uncertainty for other factors	

VOLUME – TRANSFER

SOP 16 (FLASKS)

$V_x = V_{SP} + V_{SB} + V\,[1 + (\alpha - \beta)\,(t - t_{ref})]$ + disparity (bias/drift) + unrelated corrections

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control chart or range chart and d_2^*
u_{tc}	Standard uncertainty for temperature correction	
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C; SOPs
$u_{cce\ (\alpha,\beta)}$	Standard uncertainty for cubical coefficient of expansion	NBS Report 10 081, 5×10^{-6} /°F (0.000009 / °C)
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
$u_{read\ men}$	Standard uncertainty for reading meniscus	(ASTM E694)
u_{drain}	Standard uncertainty due to drain time	
$u_{time\ piece}$	Standard uncertainty for time piece	
u_o	Standard uncertainty for other factors	

SOP 18 (GRADUATED NECK)

$V_x = V_s$ + disparity (bias/drift) + unrelated corrections

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control chart or range chart and d_2^*
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
$u_{read\ men}$	Standard uncertainty for reading meniscus	(ASTM E694); GMP 3
u_{drain}	Standard uncertainty due to drain time	
$u_{time\ piece}$	Standard uncertainty for time piece	
u_o	Standard uncertainty for other factors	

SOP 19 (LARGE NECK)

$$V_x = \frac{\rho_{w_s}\left\{V_s\left[1+\alpha\left(t-t_{ref}\right)\right]\right\}}{\rho_{w_x}\left[1+\beta\left(t-t_{ref}\right)\right]} + \text{disparity (bias / drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for standards	Measurement assurance process
u_{tc}	Standard uncertainty for temperature correction	
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
$u_{cce\ (\alpha,\beta)}$	Standard uncertainty for cubical coefficient of expansion	NBS Report 10 081, 5×10^{-6} /°F (0.000009 / °C)
$u_{\rho w}$	Standard uncertainty for density of water	
u_e	Standard uncertainty for water density equation	Metrologia, Patterson/Morris (1994), 7 ppm (0.000007 g/cm3)
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
$u_{read\ men}$	Standard uncertainty for reading meniscus	(ASTM E694)
u_{drain}	Standard uncertainty due to drain time	
$u_{time\ piece}$	Standard uncertainty for time piece	
u_o	Standard uncertainty for other factors	