Statistical Basis to Determine Probabilities of Occurrence of Handwriting Characteristics

NIST Meeting 2013

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Previous Research

- **1963** -- Frequency of Certain Characteristics in Handwriting, Pen Printing of 200 People," Livingston
- **1976** – A Statistical Examination of Selected Handwriting Characteristics, Muehlberger, et al
- **1990** -- "Uniqueness of Writing," Huber
- **1996** -- A Study of the Occurrence of Certain Handwriting Characteristics in a Random Population," Horton
- **1998** -- A Statistical Study of Some Differentiating Characteristics of the Handwritten Letters IT," Zlotnick
- **2013** -- "Frequency of Selected Hand Printing Characteristics Occurring within a National Population: The New International Version Bible Across America©" Bishop

Current Research

- “Statistical Examination of Handwriting Characteristics using Automated Tools,” Singer/ Srihari, SUNY

- “Development of Individual Handwriting Characteristics in ~1800 Students: Statistical Analysis and Likelihood Ratios that Emerge over an Extended Period of Time”, Lisa Hanson, Minnesota Bureau of Criminal Apprehension, Dr. Srihari, SUNY

- “Frequency Occurrence of Handwriting and Hand-Printing Characteristics” Vastrick and Whitcomb with University of Central Florida

Previous Research

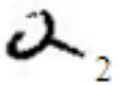
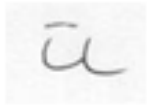
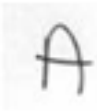
1. Height relationship of the “t” to the “h”
    a. t shorter than h (78%)
    b. t even with h (1.5%)
    c. t taller than h (5.5%)
    d. No set pattern (15%)

2. Shape of loop of “h”
    a. Retraced (27.5%)
    b. curved right side and straight left side (32%)
    c. curved left side and straight right side (2.5%)
    d. Both sides curved (17%)
    e. No set pattern (21%)

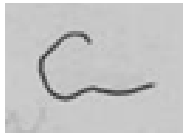
“and”

Rank	Trigram
1	the
2	and
3	tha
4	ent
5	ing

Singer/Moran

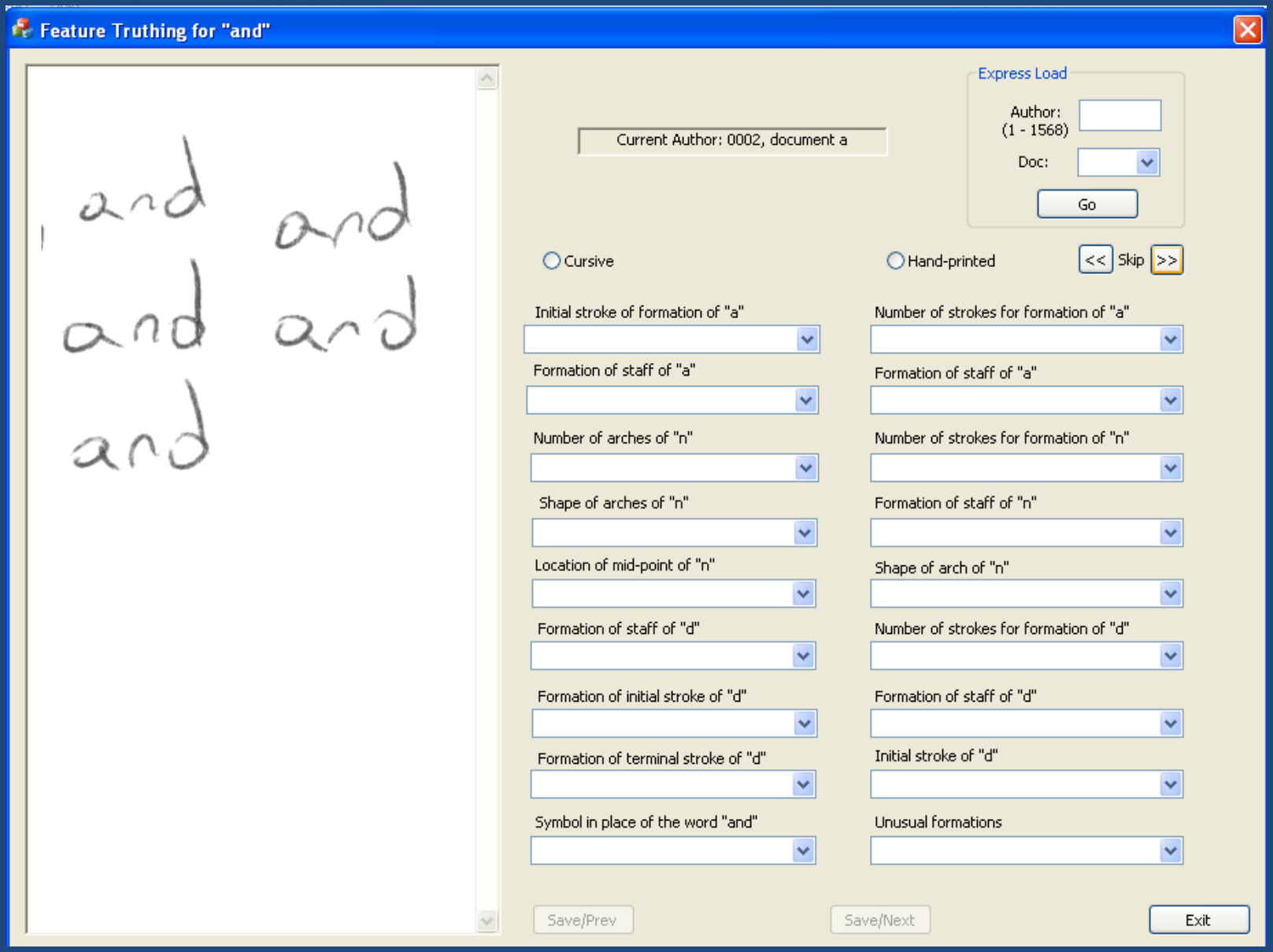
1. Number of strokes for formation of "a": (a)  (b)  2 (c)  (d) 

(a) one continuous														
(b) two strokes														
(c) three strokes														
(d) uppercase														
(e) no fixed pattern														

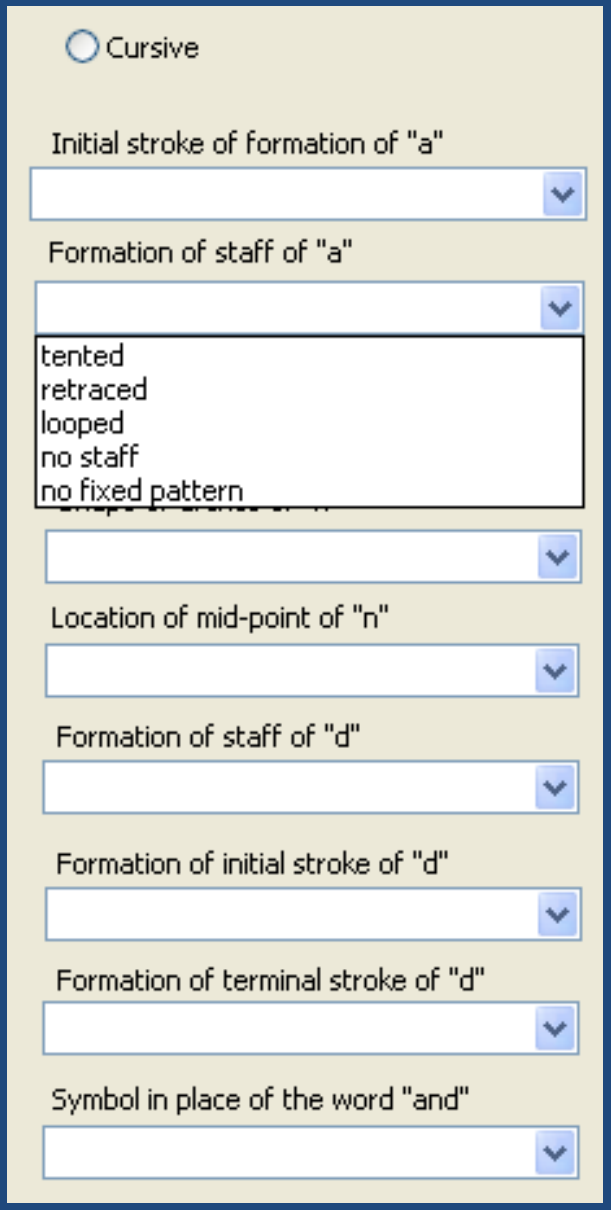
2. Formation of staff of "a": (a)  6 (b)  8 (c)  87 (d) 

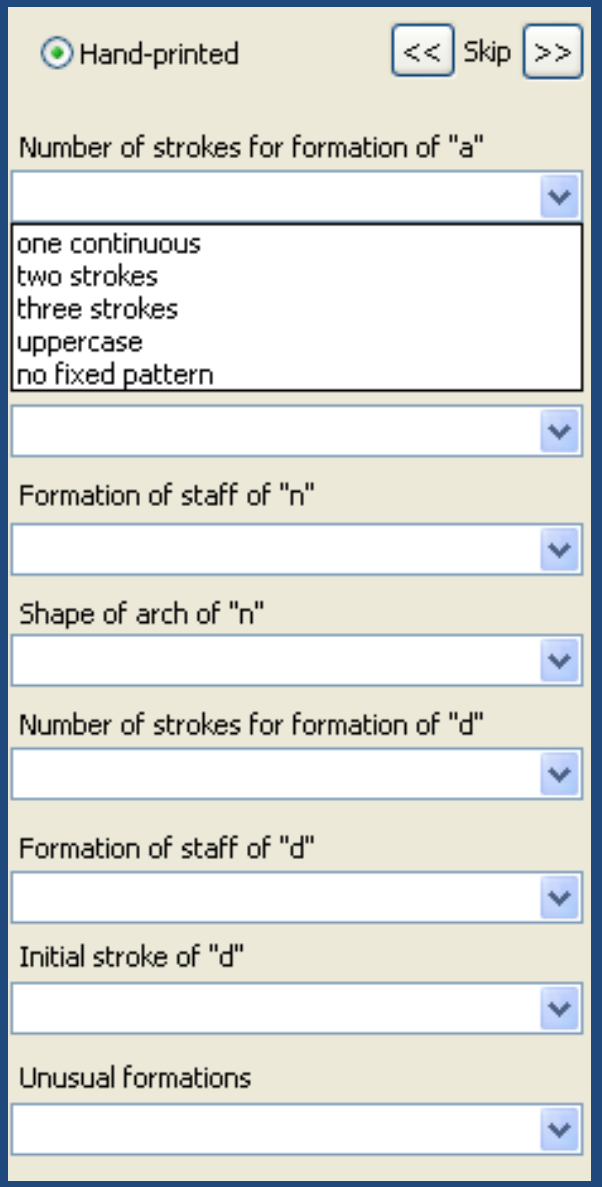
(a) tented												
(b) retraced												
(c) looped												
(d) no staff												
(e) no fixed pattern												

Truthing Tool



Truthing Tool





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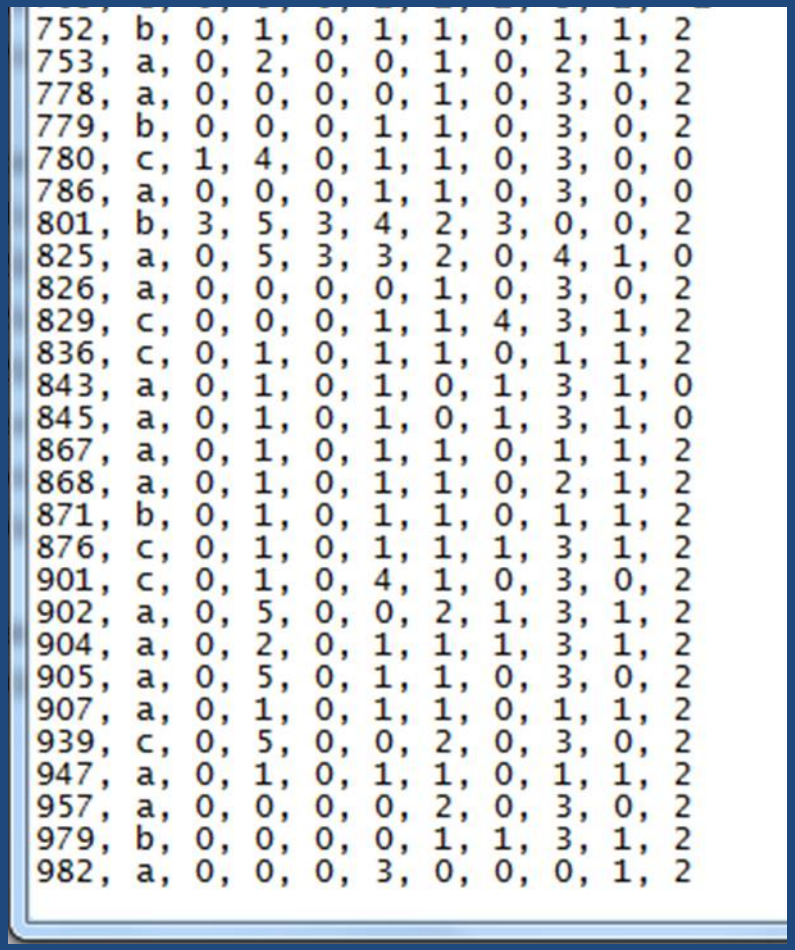

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778, a, 0, 0, 0, 0, 1, 0, 3, 0, 2
779, b, 0, 0, 0, 1, 1, 0, 3, 0, 2
780, c, 1, 4, 0, 1, 1, 0, 3, 0, 0
786, a, 0, 0, 0, 1, 1, 0, 3, 0, 0
801, b, 3, 5, 3, 4, 2, 3, 0, 0, 2
825, a, 0, 5, 3, 3, 2, 0, 4, 1, 0
826, a, 0, 0, 0, 0, 1, 0, 3, 0, 2
829, c, 0, 0, 0, 1, 1, 4, 3, 1, 2
836, c, 0, 1, 0, 1, 1, 0, 1, 1, 2
843, a, 0, 1, 0, 1, 0, 1, 3, 1, 0
845, a, 0, 1, 0, 1, 0, 1, 3, 1, 0
867, a, 0, 1, 0, 1, 1, 0, 1, 1, 2
868, a, 0, 1, 0, 1, 1, 0, 2, 1, 2
871, b, 0, 1, 0, 1, 1, 0, 1, 1, 2
876, c, 0, 1, 0, 1, 1, 1, 3, 1, 2
901, c, 0, 1, 0, 4, 1, 0, 3, 0, 2
902, a, 0, 5, 0, 0, 2, 1, 3, 1, 2
904, a, 0, 2, 0, 1, 1, 1, 3, 1, 2
905, a, 0, 5, 0, 1, 1, 0, 3, 0, 2
907, a, 0, 1, 0, 1, 1, 0, 1, 1, 2
939, c, 0, 5, 0, 0, 2, 0, 3, 0, 2
947, a, 0, 1, 0, 1, 1, 0, 1, 1, 2
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982, a, 0, 0, 0, 3, 0, 0, 0, 1, 2
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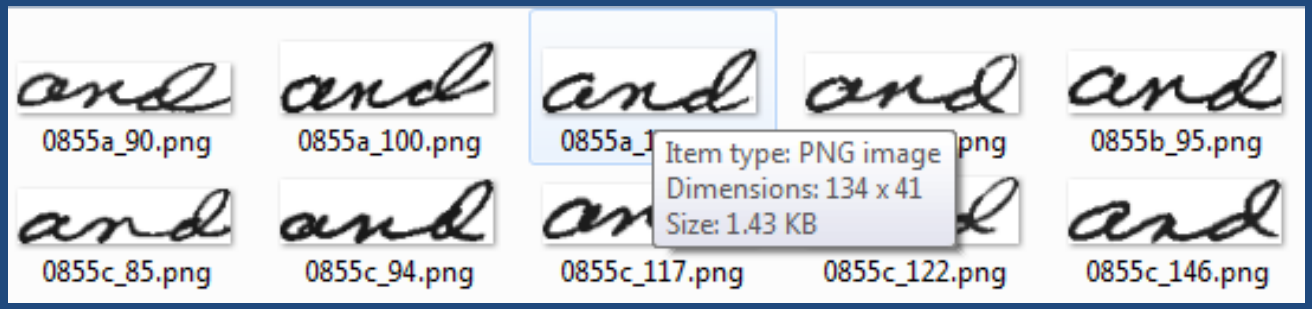
Truthing Tool



Writer ID

855 1 2 1 0 2 2 1 0 0

- 1: Initial Stroke left of staff
- 2: Looped staff
- 1: 2 Arches for “n”
- 0: Pointed arches
- 2: Arch midpoint at baseline
- 2: Looped staff of “d”
- 1: initial stroke of “d” underhand
- 0: Terminal stroke curved up
- 0: No symbol for “and”

Probabilistic Analysis




121022322

Bayesian Network Joint Probability

9.48e-005

Independent Joint Probability

1.39e-004

Joint Probability Calculation

- Calculation of probability of a given combination of characteristics is complex
- How much data is needed?
  - If we don't assume that the nine characteristics are independent, we will need to determine over a million probabilities
    - 100 million to billion samples needed
- How much time for the computation?
  - NP-hard

What if we assume independence?

True Joint Probabilities: Prob (height,weight)

P(a,b)	b^0 (heavy)	b^1 (light)	P(a) (height)
a^0 (tall)	0.6	0.05	0.65
a^1 (short)	0.05	0.3	0.35
P(b) (weight)	0.65	0.35	

Prob(tall, light) < Prob(short,light) Given that person is light, six times likely to be short

Assuming Independence

P(a,b)	b^0 (heavy)	b^1 (light)	P(a) (height)
a^0 (tall)	0.42	0.23	0.65
a^1 (short)	0.23	0.12	0.35
P(b) (weight)	0.65	0.35	

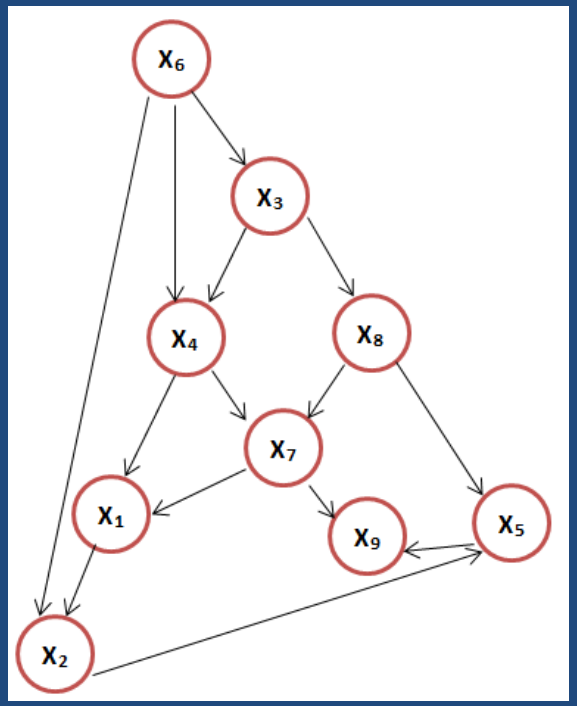
P(tall,light) > P(short,light) Given that person is light, twice likely to be tall

Compromise Solution: PGMs

- Revolution in big data analysis
- Led by statistical machine learning and probabilistic graphical models
- Exploit as to what independencies exist rather than assume everything is independent
- PGMs are directed (Bayesian Networks) or undirected (Markov networks)

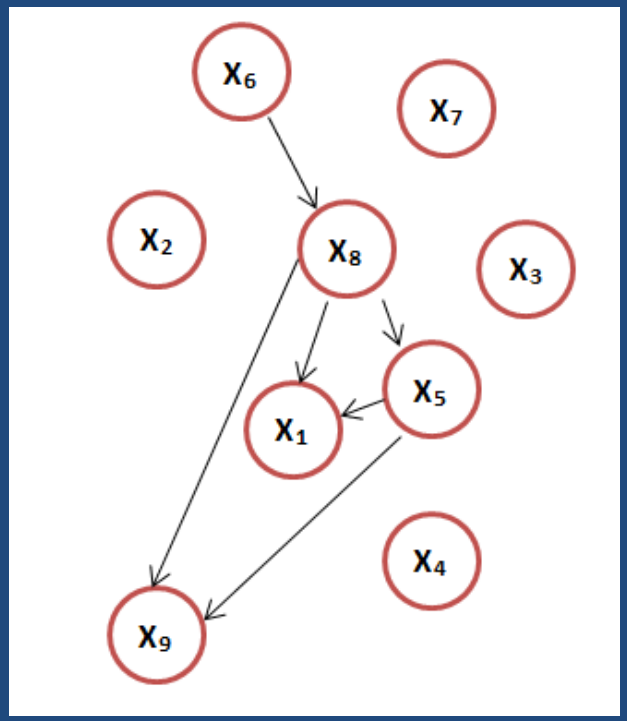
Bayesian Networks for *and*

Cursive



99 parameters

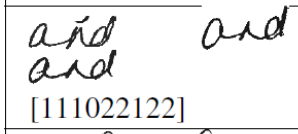
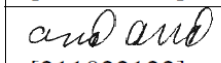
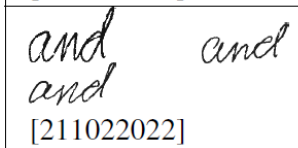
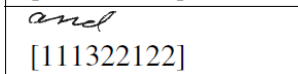
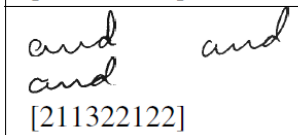
Handprint



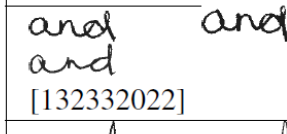
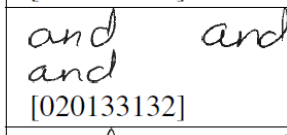
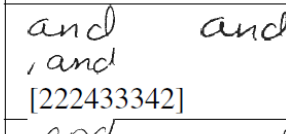
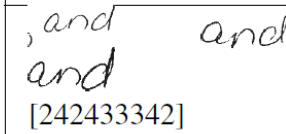
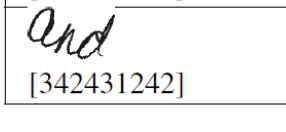
77 parameters

Common and Rare *and*

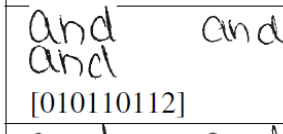
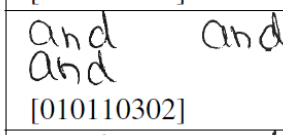
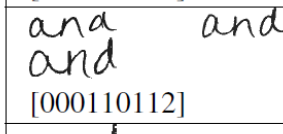
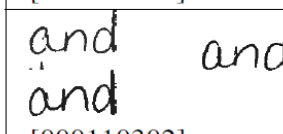
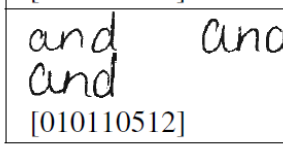
(a) Cursive-Common

Samples with Characteristics	Probability
 [111022122]	5.46×10^{-3}
 [211022122]	5.39×10^{-3}
 [211022022]	4.86×10^{-3}
 [111322122]	4.52×10^{-3}
 [211322122]	4.46×10^{-3}

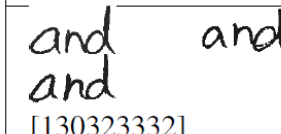
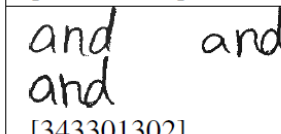
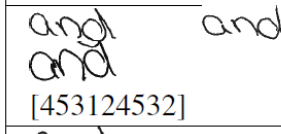
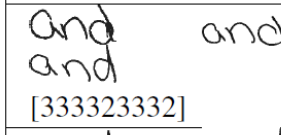
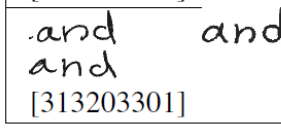
(b) Cursive-Rare

Samples with Characteristics	Probability
 [132332022]	4.15×10^{-8}
 [020133132]	4.09×10^{-8}
 [222433342]	8.64×10^{-9}
 [242433342]	7.50×10^{-9}
 [342431242]	5.75×10^{-9}

(c) Handprint-Common

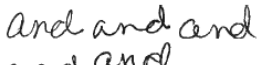
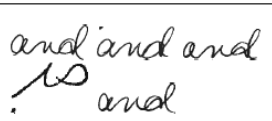
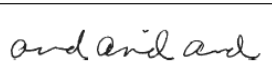
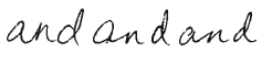
Samples with Characteristics	Probability
 [010110112]	1.51×10^{-2}
 [010110302]	1.44×10^{-2}
 [000110112]	1.21×10^{-2}
 [000110302]	1.15×10^{-2}
 [010110512]	7.42×10^{-3}

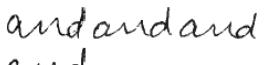
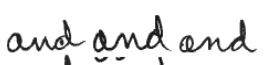
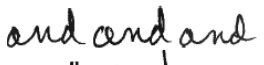
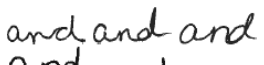
(d) Handprint-Rare

Samples with Characteristics	Probability
 [130323332]	5.90×10^{-9}
 [343301302]	4.66×10^{-9}
 [453124532]	3.75×10^{-9}
 [333323332]	7.81×10^{-10}
 [313203301]	7.23×10^{-10}

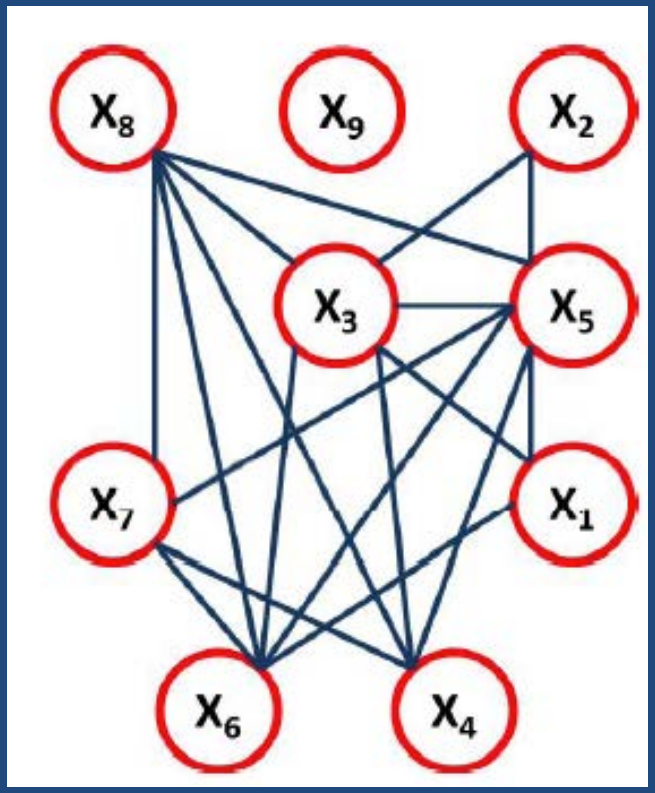
Probabilities available online

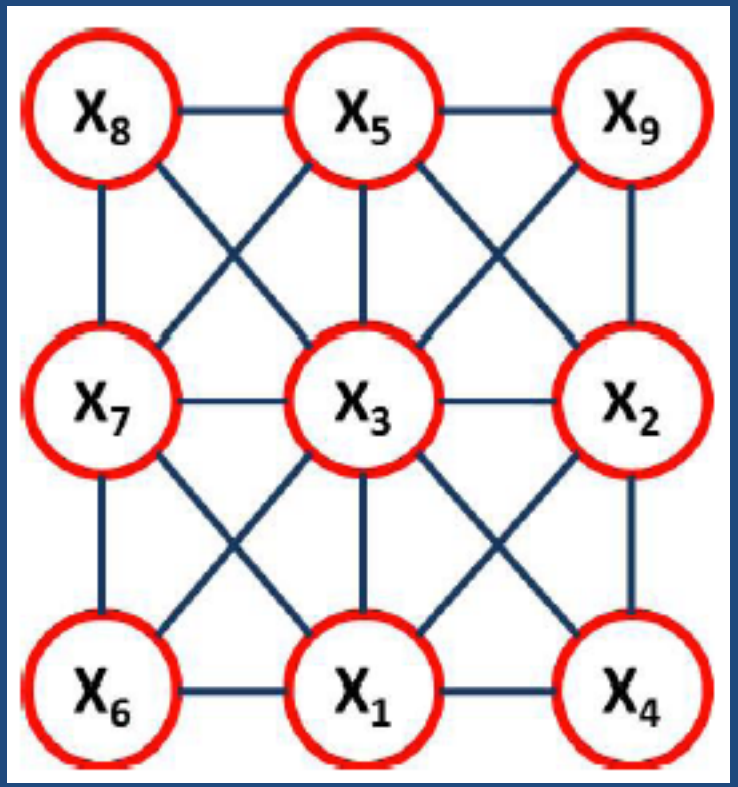
Cursive data: http://www.cedar.buffalo.edu/~srihari/HW-Stats/cursive-and
Handprint data: http:/www.cedar.buffalo.edu/~srihari/HW-Stats/handprint-and

#	Samples	ID	Characteristics	BN Joint Prob	Indep Joint Prob
986		0387b	220101021	4.70e-007	4.57e-007
987		0522c	312402032	4.68e-007	7.20e-006
988		1123a	212101121	4.16e-007	6.34e-007
989		1198a	302422042	3.76e-007	9.24e-006
990		1198b	101320221	3.65e-007	9.33e-007
991		1098a	010101111	3.38e-007	1.17e-006

#	Samples	ID	Characteristics	BN Joint Prob	Indep Joint Prob
1000		1271c	012422222	2.50e-007	3.53e-006
1001		0354c	100101322	2.49e-007	5.85e-007
1002		1091a	312422342	2.22e-007	1.12e-005
1003		0556b	020102111	2.07e-007	5.45e-007
1004		0556c	010103101	2.05e-007	3.66e-007
1005		0387a	010201222	1.87e-007	3.50e-006

Markov networks for *and*



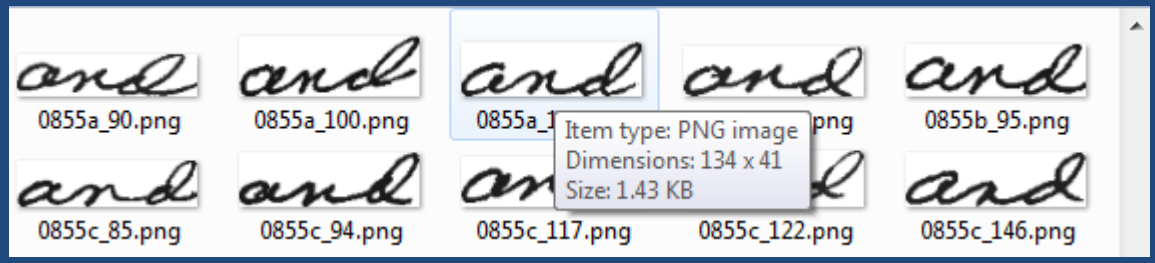


Conclusions

- FDEs defined a set of characteristics for a common word *and*
- Developed a truthing interface
- FDEs entered data using interface
- Developed learning algorithms to create statistical models
- Models used to infer probability of characteristics

Future Research

- Continue to mine existing data for information, add more individualizing characteristics
- Continue to research “th” combination
- Study the same characteristics with a more homogenous population (e.g. Durina research, twins)



855, a, 1, 2, 1, 0, 2, 2, 1, 0, -1