SOP No. 10

**Recommended Standard Operating Procedure
for
Calibration of Rigid Rules**

1. Introduction

　1.1.　Purpose

This SOP describes the procedure to be followed for the calibration of rigid rules by comparison to the 18 inch metal rule issued to each State as the State reference standard which is calibrated in the interval from 1 inch to 13 inches. Equations provided here assume that the rigid rules being compared have the same coefficient of expansions. If rigid rules having different coefficients of expansion are compared at a temperature other than 20 °C the procedure and equations of SOP 11 must be used.

The maximum length of rule that can be directly compared to the standard rule is 12 inches. However, longer lengths can be calibrated in segments of 12 inches with reference to the standard rule.

　1.2.　Prerequisites

　　1.2.1.　Valid calibration certificates with appropriate values and uncertainties must be available for all of the standards used in the calibration. All standards must have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST.

　　1.2.2.　The ocular microscope used in measuring differences in lengths must be in good operating condition and must be equipped with a graduated reticle having established traceability.

　　1.2.3.　The operator must be trained and experienced in precision measuring techniques with specific training in GMP 2, GMP 8, GMP 9, SOP 10, and SOP 29.

　　1.2.4.　Laboratory facilities must comply with following minimum conditions to meet the expected uncertainty possible with this procedure. Equilibration of supporting surface, standard rule and rule to be calibrated requires environmental stability of the laboratory within the stated limits for a minimum of 24 hours before a calibration.

Table 1. Environmental conditions.

Temperature Requirements During a Calibration	Relative Humidity (%)
Lower and upper limits: 18 °C to 22 °C Maximum changes: < ± 1 °C / 24 h and ± 0.5 °C / 1 h	40 to 60 ± 10 / 4 h

2.   Methodology

    2.1.   Scope, Precision, Accuracy

    The accuracy of calibration of standard rules is possible to within 0.000 1 inch, provided suitable calibration of standards are obtained. The precision of intercomparison and the accuracy of the standard limit the uncertainty of calibration to 0.002 inch under optimum conditions.

    2.2.   Summary

    A rigid rule (test rule) is calibrated by comparing intervals on it with intervals of the standard rule. A reticle eye piece (ocular micrometer) is used for this purpose. Test rules longer than the standard rule may be calibrated in segments, using the last calibrated graduation as the zero graduation mark for the succeeding segments. Deviations from nominal that are calculated for each successive interval are cumulative.

    2.3.   Apparatus/Equipment Required

        2.3.1.   Length bench or similar flat surface on which to lay the test rule and the standard rule.

        2.3.2.   Calibrated standard rule.

        2.3.3.   Microscope with calibrated graduated reticle having graduations spaced at intervals no greater than 0.002 inch.

2.4.   Symbols

Table 2. Symbols used in this procedure.

Symbol	Description
A, B, C, D	Calculated centers of graduations
S	Standard
X	Unknown
X_{0L}	Left edge of zero on unknown
X_{0R}	Right edge of zero on unknown
S_{0L}	Left edge of zero on standard
S_{0R}	Right edge of zero on standard
X_m	Center of graduation of unknown
S_m	Center of graduation of standard
d_i	Difference between *X* and *S*. The subscript *i*, designates the trial number, 1 or 2.
L_S	Calibrated length of standard
L_X	Calibrated length of test rule

2.5.   Procedure

2.5.1. Both the test rule and the standard rule must be in temperature equilibrium with the environment of the length laboratory.

2.5.2. Place the test rule and the standard rule on the length bench or similar flat surface, parallel to one another with the reading edges adjacent. It is not necessary to have the "zero" graduations in coincidence.

Ordinarily, this will require that one rule reads left-to-right (increasing units) while the other reads right-to-left (decreasing units). In this case, for convenience of calculation, the standard rule is placed in the right-to-left position. A worksheet to reverse the calibration on the standard rule is provided at the end of this SOP. Place shims under the rules as necessary so that the upper surfaces of both are in the same plane.

2.5.3. Place the microscope on the bench in the vicinity of the zero position and align it so that its reticle scale is parallel to the scales under test. (See GMP No. 2 for instructions on reading graduations with the microscope.)

2.5.3.1. Observe readings of left and right sides of the corresponding graduation of the standard rule and record to the nearest 0.001 inch. The average of these readings will give a value for A.

2.5.3.2. Observe readings of left and right sides of the corresponding graduation of the test rule and record to the nearest 0.001 inch. The average of these readings will give a value for B. If the

"zero" graduation is the end of the rule, only the reading for the end of the rule is taken.

2.5.4.   Move the microscope successively to each graduation to be calibrated and record readings as described in 2.5.3.1 and 2.5.3.2 identifying the readings for the standard rule as C and for the unknown as D. CAUTION! Be certain that the rules are not disturbed during movement of the microscope.

2.5.5.   Return the microscope to the zero graduation and repeat readings (2.5.3.1. and 2.5.3.2.) to verify that the rules have not been disturbed. Accept all previous data if the initial zero readings do not disagree with the final zero readings by more than 0.002 inch; otherwise, discard all previous data and repeat entire sequence of readings until a satisfactory set of zero readings are obtained.

2.5.6.   Repeat 2.5.3. thru 2.5.5. for every additional segment of the test rule requiring calibration. This will require repositioning the rules, aligning the last measured interval graduation on the test rule with the initial graduation of the standard rule. Deviations from nominal for successive segments are cumulative.

2.5.7.   Move the rules and reposition, making a second set of measurements as directed in 2.5.3, 2.5.4, and 2.5.5 for Trial 2.
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3.   Calculations

3.1.   Calculate the difference between the Initial (A, B) and Final (A$_1$, B$_1$) zero measurements to ensure that the initial and final readings agree within 0.002 inch. (2.5.5.)

$$|A - B - A_1 + B_1| \tag{1}$$

3.2.   Calculate the center of graduation for unknown, X_m, and standard, S_m, for each set of measurements and each scale interval. The center of the "starting" graduation on the standard rule is A; for the test rule it is B. The subsequent centers of graduations for the standard and the test rule are recorded as C and D, respectively, for each interval measured. For each trial, the values for A and B will remain constant for all measured intervals on that trial, and will be used to compute the measured differences between the test rule and the standard.

$$S_m = \frac{(S_L + S_R)}{2} = A, C \tag{2}$$

$$X_m = \frac{(X_L + X_R)}{2} = B, D \tag{3}$$

3.3.   Calculate the differences, d_1, and d_2 between X and S for each scale interval and each trial. (The subscript i is associated with the trial number.)

$$d_i = A - B - C_i + D_i \tag{4}$$

3.4.   Obtain the calibrated length of the standard, L_S, for the measured interval from the calibration certificate for the standard rule.

3.5.   Calculate the length of the test rule, L_X, for each interval measured using the mean of the measured differences.

$$L_X = \frac{(d_1 + d_2)}{2} + L_S \tag{5}$$

4.   Measurement Assurance

4.1.   Duplicate the process with a suitable check standard or have a suitable range of check standards for the laboratory. See NISTIR 7383 SOP 17, SOP 20 and NISTIR 6969 SOP 30. Plot the check standard length and verify it is within established limits OR a t-test may be incorporated to check the observed value against an accepted value. The mean of the check standard observations is used to evaluate bias and drift over time. Check standard observations are used to calculate the standard deviation of the measurement process which contributes to the Type A uncertainty components.

4.2   If a standard deviation chart is used for measurement assurance, the standard deviation of each combination of Trial 1 and Trial 2 is calculated and the pooled (or average) standard deviation is used as the estimate of variability in the measurement process. Note: the pooled or average standard deviation over time will reflect varying conditions of test items that are submitted to the laboratory. A standard deviation chart will be needed for each interval calibrated so that the variability resulting from transfers will be measured (the number of charts may be adjusted through analysis using F-tests).

5.   Assignment of Uncertainty

5.1.   The limits of expanded uncertainty, U, include estimates of the standard uncertainty of the length standards used, u_s, estimates of the standard deviation of the measurement process, s_p, and estimates of the effect of other components associated with this procedure, u_o. These estimates should be combined using the root-sum-squared method (RSS), and the expanded uncertainty, U, reported with a coverage factor to be determined based on degrees of freedom, which if large

enough will be 2, ($k = 2$), to give an approximate 95 percent level of confidence. See NISTIR 6969, SOP 29 (Standard Operating Procedure for the Assignment of Uncertainty) for the complete standard operating procedure for calculating the uncertainty.

5.1.1 The expanded uncertainty for the standard, U, is obtained from the calibration report. The combined standard uncertainty, u_c, is used and not the expanded uncertainty, U, therefore the reported uncertainty for the standard will usually need to be divided by the coverage factor k. When transfers are used, u_s for values after the transfer are dependent and cumulative. See NISTIR 6969, SOP 29 for handling of dependent uncertainties.

5.1.2. The standard deviation of the measurement process, s_p, is taken from a control chart for a check standard or standard deviation charts. See NISTIR 7383, SOP 17, SOP 20, and NISTIR 6969, SOP 30.

5.1.3. Uncertainty associated with bias, u_d. Any noted bias that has been determined through analysis of control charts and round robin data must be less than limits provided in SOP 29 and included if corrective action is not taken. See SOP 29 for additional details

5.1.4. Other standard uncertainties usually included at this calibration level include uncertainties associated with the ability to read the graduated reticle, only part of which is included in the process variability due to parallax and visual capabilities, and uncertainties associated with the graduations of the reticle.

Table 3. Example uncertainty budget table.

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; range charts
u_{gr}	Standard uncertainty for graduated reticle	Must be assessed experimentally or from a calibration certificate
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the rules	Must be assessed experimentally; if an interval based standard deviation chart is used u_{res} will be included in the control chart standard deviation value.
u_o	Standard uncertainty for other factors	

6. Report

Report results as described in SOP No. 1 Preparation of Calibration Certificates.

Appendix A
Rigid Rule Calibration Data Sheet

Date			Environmental parameters					
Metrologist			Before	After		Unc/ability to measure		
Test No.		Temperature			°C		°C	
s_p		in	Pressure			mmHg		mmHg	
df			Humidity			%		%	
Based on NISTIR 6969, SOP 29, Appendix A at 95.45 % probability distribution: k factor								

		ID	Range		
S					
X					

		Trial 1				Trial 2			
Rule	Graduation	Left	Right		Center of graduation	Left	Right		Center of graduation
S				A =				A =	
X				B =				B =	
S_1				C_1 =				C_1 =	
X_1				D_1 =				D_1 =	
		$d_1 = A - B - C_1 + D_1 =$				$d_2 = A - B - C_1 + D_1 =$			
		Average $d = (d_1 + d_2) / 2 =$				$L_S =$			
		Length of X, L_X = average $d + L_S =$							
S_2				C_2 =				C_2 =	
X_2				D_2 =				D_2 =	
		$d_1 = A - B - C_2 + D_2 =$				$d_2 = A - B - C_2 + D_2 =$			
		Average $d = (d_1 + d_2) / 2 =$				$L_S =$			
		Length of X, L_X = average $d + L_S =$							
S_3				C_3 =				C_3 =	
X_3				D_3 =				D_3 =	
		$d_1 = A - B - C_3 + D_3 =$				$d_2 = A - B - C_3 + D_3 =$			
		Average $d = (d_1 + d_2) / 2 =$				$L_S =$			
		Length of X, L_X = average $d + L_S =$							
S				A_1 =				A_1 =	
X				B_1 =				B_1 =	
Trial 1: Absolute difference between initial and final zero measurements = $|A - B - A_1 + B_1| =$									
Is result ≤ 0.002 inch? Yes/No								If YES, accept all data.	
Trial 2: Absolute difference between initial and final zero measurements = $|A - B - A_1 + B_1| =$									
Is result ≤ 0.002 inch? Yes/No								If YES, accept all data.	

Form A-2. Worksheet to Reverse the Calibration of the Rigid Rule Standard

Nominal Length of Interval (inches)	Calculations to be performed			Length of Interval (inches)
1	$L_1 = C_{13} - C_{12} =$		-	
2	$L_2 = C_{13} - C_{11} =$		-	
3	$L_3 = C_{13} - C_{10} =$		-	
4	$L_4 = C_{13} - C_9 =$		-	
5	$L_5 = C_{13} - C_8 =$		-	
6	$L_6 = C_{13} - C_7 =$		-	
7	$L_7 = C_{13} - C_6 =$		-	
8	$L_8 = C_{13} - C_5 =$		-	
9	$L_9 = C_{13} - C_4 =$		-	
10	$L_{10} = C_{13} - C_3 =$		-	
11	$L_{11} = C_{13} - C_2 =$		-	
12	$L_{12} = C_{13} - C_1 =$		-	

To reverse the calibration on the State standard rigid rule, subtract the lengths for each calibrated interval from the length from the 1 inch to the 13 inch graduation.

Example:

Length from 13 to 12 = Length from 1 to 13 – Length from 1 to 12
Length from 13 to 12 = 11.999 8 – 10.999 7 = 1.000 1 inch

Appendix B
Calibration of a Metric Rule Using a Standard Indicating in Inches

A. Work with the customer to determine the intervals they wish to have calibrated. Convert those interval to inches (E.g., 10 mm / 25.4 mm/in = 0.3937 in). It is helpful to make the conversions for all the interested intervals at one time. Round these conversions to the nearest 0.01 in.

B. For the first calibration interval "a", refer to the group of intervals from 1 in to 12.5 in and 1 in to 13 in on the calibration report for the lab's standard rule. Select the report's interval with the same numbers after the decimal point as the selected customer interval "a". (E.g., for the customer's 2.94 in interval, refer to the 1 in to 12.94 in interval on the report.) An interval "a" which ends in a number less than 0.5 should have a report interval selected which is 0.5 more than the listed value. (E.g., for the customer's 0.39 in interval, refer to the 1 in to 12.89 in interval on the report.) The 12.xx value which has been selected will be the point on the standard which is lined up with the 0 calibration point on the customer's rule. Record the appropriate standard length value for the referred to interval. (E.g., 11.888 4 for the 1 in to 12.89 in interval.)

C. Subtract the desired interval "a" from the 12.xx figure which was selected in step B. This difference "b" is the point on the standard rule which will be lined up with the desired calibration point on the customer's rule. Refer to the calculated difference "b" on the calibration report for the laboratory 18 in standard rule. Record the appropriate standard length value for the interval "b". (E.g., for the interval 0.39 in, subtract 0.39 in from 12.89 in to obtain a difference of 12.50 in. The 12.5 in mark on the standard rule will be lined up with an interval which is 0.39 in (10 mm) from the 0 point on the customer's rule. Refer to the 1 in to 12.50 in interval on the data sheet and record a value of 11.498 1.)

D. Subtract the value recorded in step C from the one recorded in step B to obtain the standard's length for the interval "a" which will be observed. (E.g., subtract 11.498 1 from 11.888 4 and you get a standard length of 0.390 3 in.) Use this value as your L_S in your calculations.

E. Repeat steps B through D for all of the intervals to be calibrated.

F. Set up the rules, take readings, and make calculations for each interval in accordance with this SOP. After calculating L_X in inches, convert it to metric, and report the metric values to the customer.