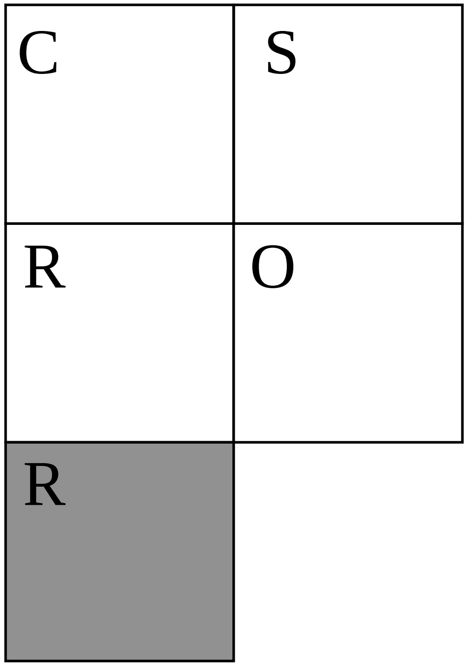
Chapter 8

Robust Designs

Robust Designs



Focus: A Few Primary Factors

Output: Best/Robust Settings

Robust Designs

C	S
R	O
R I/O Arrays Assembled Designs	

Robust Experimentation

- **Try to reduce the variation in the product.process, not just set it on target.**
- **Try to find conditions of design factors which make the product/process insensitive or robust to its environment.**
- **The idea was championed by Genichi Taguchi.**
  - **Taguchi really opened a whole area that previously had been talked about only by a few very applied people.**
  - **His methodology is heavily dependent on design of experiments, but he wanted to look at not just the mean but also the variance.**

Classification of Factors

- **Control Factors–*Design factors* that are to be set at optimal levels to improve quality and reduce sensitivity to noise**
  - **Dimensions of parts, type of material, etc**
- **Noise Factors–Factors that represent the noise that is expected in production or in use**
  - **Dimensional variation**
  - **Operating Temperature**
- **Adjustment Factor – Affects the mean but not the variance of a response**
  - **Deposition time in silicon wafer fabrication**
- **Signal Factors – Set by the user to communicate desires of the user**
  - **Position of the gas pedal**

Example: Ignition Coil

Most automotive sub-assemblies like the alternator, the ignition coil, and the electronic control module must undergo testing to determine if they are <u>resistant to salt water</u> that may be splashed on them from the read. An automotive supplier is testing an ignition coil to determine if it will withstand salt water. The following factors are tested:

Control or Noise Factor?

Factor	Low Level	High Level	Factor Type
Housing Material	Polyethylene ($9/lb)	Polypropylen ($20/lb)	C
Concentration Of Salt	10%	20%	N
Water Temperature	5ºC	15ºC	N
Water Pressure	10 PSI	20 PSI	N
Seal Material	Silicone	Vinyl	C
Seal Thickness	0.02”	0.03”	C
Exposure Time	1 hr.	5 hrs.	N
Type of Salt	Detroit Blue	Chicago Pink	N

Taguchi's Design of Experiments Ideas

- **Use Orthogonal Arrays (e.g., Resolution III Fractional Factorials) that only estimate main effects and specified interactions.**
- **Intentionally vary the noise factors so that you choose a set of conditions that will work well in the face of the noise expected in the actual application.**
- **Inner/Outer Arrays**
  - **Two statistical designs per experiment:**
    - » **A design in the *control factors* called an <u>inner array</u>**
    - » **A design in the *noise factors* called an <u>outer array</u>**
  - **Good for estimating CxN Factor Interactions.**

Taguchi's Analysis Ideas
Signal to Noise Ratio

- **A single response which makes a tradeoff between setting the mean to a desirable level while keeping the variance low.**
- **Always try to MAXIMIZE a SN Ratio**
- **There are three types:**
  - **Smaller is Better**
  - **Target is Best**
  - **Larger is Better**

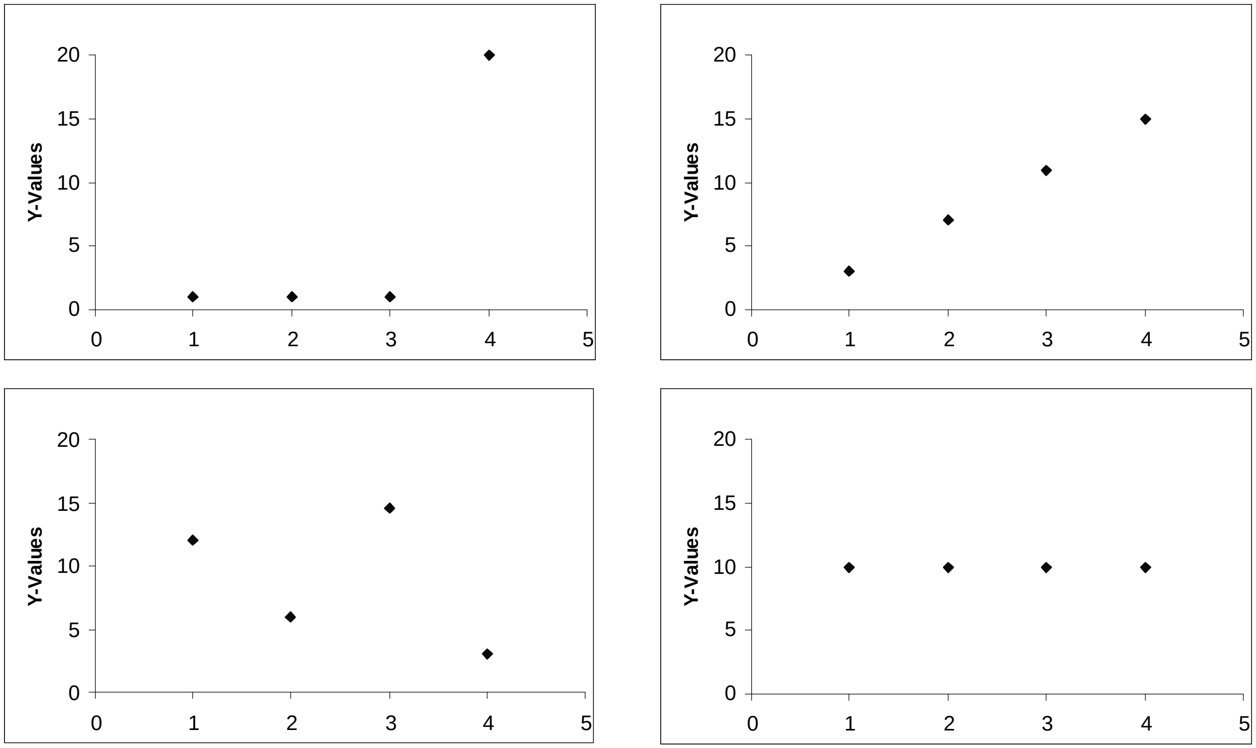
Smaller is Better

- **Maximize the signal to noise ratio**
- **Run a confirmatory experiment**

$$SN_S = -10\log\left(\sum_j \frac{y_j^2}{n}\right) = -10\log\left(\bar{y}^2 + \frac{n-1}{n}s^2\right)$$

The signal to noise ratio confounds the mean and the variance together and assumes that the variance is proportional to the mean.

Examples with the same SN_S Ratio



1	10	12	3	1
2	10	6	7	1
3	10	14.6	11	1
4	10	3	15	20
SNS	-20	-20.0234	-20.0432	-20.0325

Target is Best

- **Maximize the signal to noise ratio**
- **Adjust the mean to target using an adjustment factor that has no effect on the signal to noise ratio**

$$SN_t = -10\log\left(\frac{\bar{y}^2}{s^2}\right)$$

The signal to noise ratio does tend to prefer combinations of levels that decrease noise and are on target, but assumes that the variance is proportional to the mean. If it isn't, then the method gives an unknown compromise.

Larger is Better

- **Maximize the signal to noise ratio**
- **Run a confirmation**

$$SN_L = -10 \log\left(\sum_j \left(\frac{1}{y_j^2} \right) \Big/ n \right)$$

The signal to noise ratio again confounds the mean and the variance together and assumes that the variance is proportional to the mean.

Sources of Variance

- Measurement
- Setup
- Blocks
- Noise Factors
  - Manufacturing Variation
  - Field Conditions

Repeats

x_1	x_2	x_3	y_1	y_2	y_3
–	–	–	y_{11}	y_{21}	y_{31}
+	–	–	y_{12}	y_{22}	y_{32}
–	+	–	y_{13}	y_{23}	y_{33}
+	+	–	y_{14}	y_{24}	y_{34}
–	–	+	y_{15}	y_{25}	y_{35}
+	–	+	y_{16}	y_{26}	y_{36}
–	+	+	y_{17}	y_{27}	y_{37}
+	+	+	y_{18}	y_{28}	y_{38}

σ^2 represents measurement error

Replicates

x_1	x_2	x_3	y_1	y_2	y_3
–	–	–	y_{11}	y_{21}	y_{31}
+	–	–	y_{12}	y_{22}	y_{32}
–	+	–	y_{13}	y_{23}	y_{33}
+	+	–	y_{14}	y_{24}	y_{34}
–	–	+	y_{15}	y_{25}	y_{35}
+	–	+	y_{16}	y_{26}	y_{36}
–	+	+	y_{17}	y_{27}	y_{37}
+	+	+	y_{18}	y_{28}	y_{38}

σ^2 represents measurement and setup error

Blocks

x_1	x_2	x_3	Machine 1	Machine 2	Machine 3
–	–	–	y_{11}	y_{21}	y_{31}
+	–	–	y_{12}	y_{22}	y_{32}
–	+	–	y_{13}	y_{23}	y_{33}
+	+	–	y_{14}	y_{24}	y_{34}
–	–	+	y_{15}	y_{25}	y_{35}
+	–	+	y_{16}	y_{26}	y_{36}
–	+	+	y_{17}	y_{27}	y_{37}
+	+	+	y_{18}	y_{28}	y_{38}

σ^2 represents variance between blocks and measurement and setup error.

Noise Factors

x_1	x_2	x_3	Temp1	Temp2	Temp3
–	–	–	y_{11}	y_{21}	y_{31}
+	–	–	y_{12}	y_{22}	y_{32}
–	+	–	y_{13}	y_{23}	y_{33}
+	+	–	y_{14}	y_{24}	y_{34}
–	–	+	y_{15}	y_{25}	y_{35}
+	–	+	y_{16}	y_{26}	y_{36}
–	+	+	y_{17}	y_{27}	y_{37}
+	+	+	y_{18}	y_{28}	y_{38}

σ^2 represents variance expected in the field due to field conditions and measurement and setup error. The idea of robustness is to set the *x*'s to minimize the variance, while keeping the average response at a desired level.

I/O (crossed) Array

An Inner Array of Control Factors and an Outer Array of Noise Factors

		Length	–	+	–	+
		Temp	–	–	+	+
Gage	**Wraps**	**Geom.**	y_1	y_2	y_3	y_4
–	–	–	y_{11}	y_{21}	y_{31}	y_{41}
+	–	–	y_{12}	y_{22}	y_{32}	y_{42}
–	+	–	y_{13}	y_{23}	y_{33}	y_{43}
+	+	–	y_{14}	y_{24}	y_{34}	y_{44}
–	–	+	y_{15}	y_{25}	y_{35}	y_{45}
+	–	+	y_{16}	y_{26}	y_{36}	y_{46}
–	+	+	y_{17}	y_{27}	y_{37}	y_{47}
+	+	+	y_{18}	y_{28}	y_{38}	y_{48}

σ^2 represents variance expected in the field and measurement and setup error.

Analysis

- Taguchi suggests SN Ratios.
- **We could analyze the average and the standard deviation separately.**
- We could define some other summary response
  - Minimize the range or the curvature
- We could include effects for each y and explain each one individually like we do with blocks.

Analyzing the Mean and Standard Deviation Separately

- **For each row of y's, we can take the mean of the row as one response and the log of the standard deviation as another response.**
- **The log is used because the variance of the standard deviation is not constant as the standard deviation increases.**
- **We then try to compromise between setting the mean to a desirable level and reducing the standard deviation.**
- **Effects on the mean are called *Location Effects*.**
- **Effects on the standard deviation are called *Dispersion Effects***

Analysis of I/O (Crossed) Arrays

- **When we have I/O array there will be three types of effects**
  - **Control Main Effects and Interactions**
  - **Noise Main Effects and Interactions**
  - **Control by Noise Interactions**
- **Depending on HOW WE RUN THE EXPERIMENT, the different effects may have different variances.**
- **Suppose that there are *n* runs in the control array and *m* runs in the noise array. There will be a total of *nm* data points.**

Example: Cake Mix Example

- **Amounts of ingredients like egg, shortening and flour are the control factors (*n* levels in the control array)**
- **Time and Temperature in baking are the noise factors (*m* levels in the noise array)**

		Temp	–	+	–	+
		time	–	–	+	+
Flour	**Shrtg**	**Egg**				
–	–	–	1.3	1.6	1.2	3.1
+	–	–	2.2	5.5	3.2	6.5
–	+	–	1.3	1.2	1.5	1.7
+	+	–	3.7	3.5	3.8	4.2
–	–	+	1.6	3.5	2.3	4.4
+	–	+	4.1	6.1	4.9	6.3
–	+	+	1.9	2.4	2.6	2.2
+	+	+	5.2	5.8	5.5	6.0

How do we run the experiment?

1. Fully Randomized
2. Split Plot
3. Strip Block

1. Fully Randomized

- **Make *nm* products and test them each separately.**

2. Split Plot

- **Experiments where you can’t or don’t think it is worth it to fully randomize the experiment**
- **Used when there are certain effects that you want to estimate with high precision. (like blocking)**
- **Often needed when some factors are “hard to change” and others are “easy to change”**
- **I/O arrays are often but *not always* run as split plot experiments**

Split Plot Variance

- **Whole-plot effects have high variance.**
- **Sub-plot effects have lower variance.**
- **Only effects that have the same variance should be put on the same normal plot.**

2a. Split Plot Arrangement

Control Factors as Whole-Plot Effects

- **Mix up *n* big batches of batter**
- **Split each batch into *m* parts**
- **Bake *m* x *n* cakes in *m* x *n* oven runs**
- **All Control Effects have error due to batch error and due to random error**
- **All Noise Effects and CxN interactions have only random error**
- **2 Normal Plots**

2b. Split Plot Arrangement

Control Factors as Sub-Plot Effects

- **Mix up *m* x *n* batches of batter**
- **Bake all *n* batter types together under each set of noise conditions**
- **Bake *m* x *n* cakes in m oven runs**
- **All Noise Effects have error due to oven error and due to random error**
- **All Control Effects and CxN interactions have only random error**
- **2 Normal Plots**

3. Strip Block Arrangement

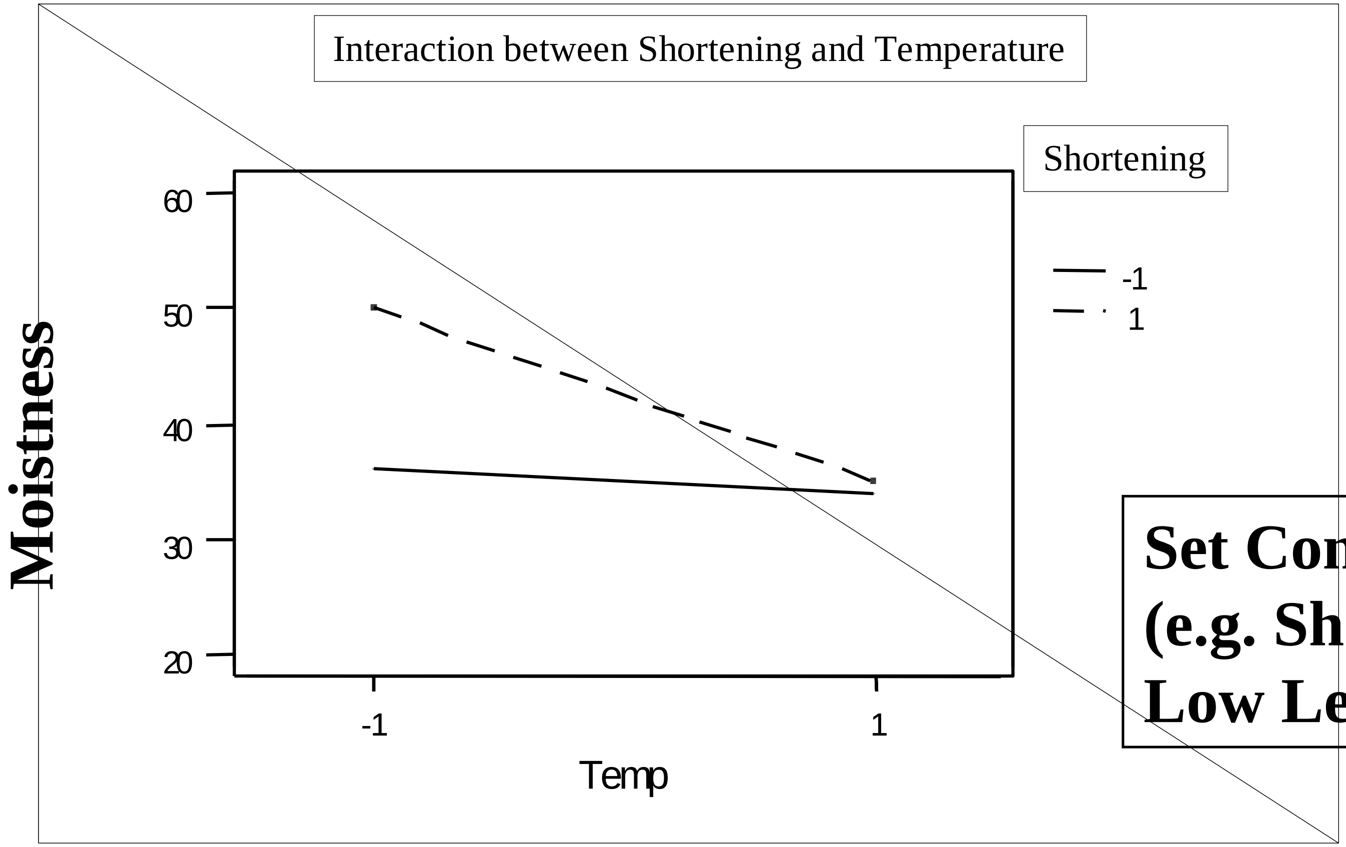
- **Mix up *n* batches of batter**
- **Split each batch into *m* parts**
- **Bake all *n* batter types together under each set of noise conditions**
- **Bake *m* x *n* cakes in *m* oven runs**
- **All Noise Effects have error due to oven error and due to random error**
- **All Control Effects have error due to batch error and due to random error**
- **CxN interactions have only random error**
- **3 Normal Plots**

The purpose of each of these configurations is to reduce the experimental cost and target certain effects to have higher precision of measurement.

HOW WE RUN THE EXPERIMENT

- **Fully Randomize**
  - **Make *nm* different products and test them each separately**
  - **One normal plot with all effects**
- **Split Plot**
  - **Make *n* different products and test each product in *nm* separate tests.**
    - » **Two normal plots (1) Control (2) Noise and CxN**
  - **Make *nm* different products and test them in groups of *n* in *m* tests.**
    - » **Two normal plots (1) Control and CxN (2) Noise**
- **Strip Block**
  - **Make *n* different products and test them each in *m* tests.**
    - » **Three normal plots (1) Control (2) CxN (3) Noise**

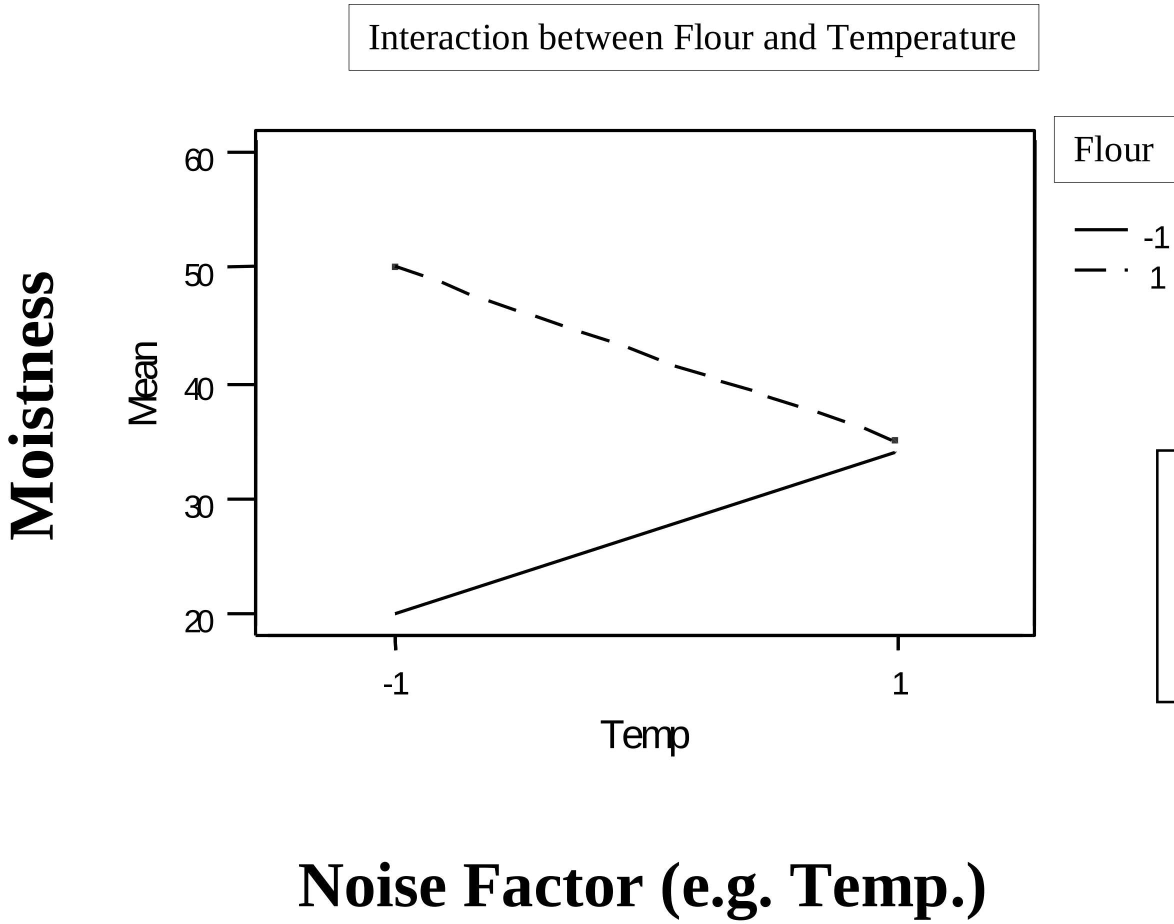
Control by Noise Factor Interactions



Set Control Factor (e.g. Shortening) to Low Level

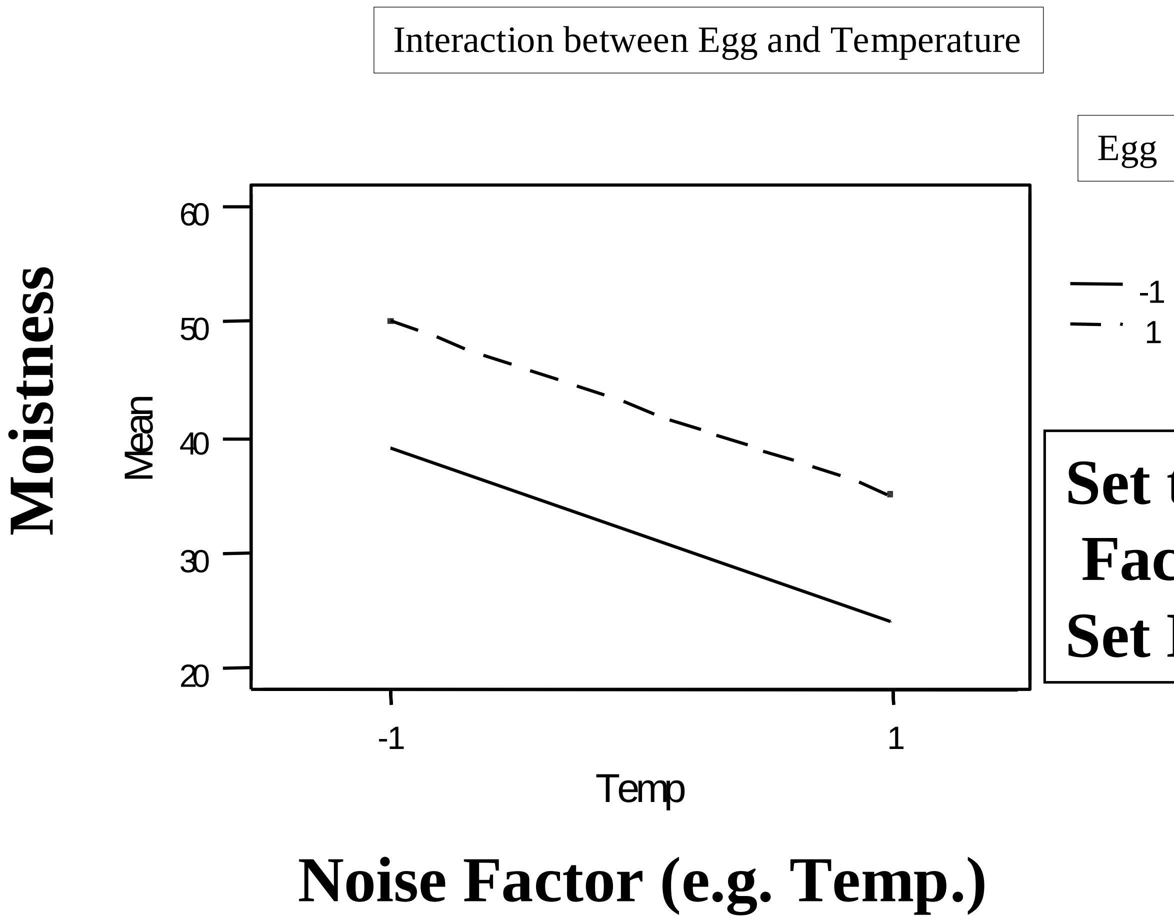
Noise Factor (e.g. Temp.)

Control by Noise Factor Interactions



Set Control Factor (e.g. Flour) to Middle Level

Control by Noise Factor Interactions



Set the Adjustment Factor (e.g. Egg) to Set Mean on Target

I/O Arrays and Control by Noise Interactions

- I/O arrays always estimate the CxN Interactions with the highest precision (lowest variance).
- Sometimes the I/O array design is larger than it needs to be to get the same resolution
- However, if we are interested in estimating CxN interactions, then an I/O array design can be a very good design.

Confounding in I/O Arrays

- Same as in standard arrays
  - Write down identity relationships from generators
  - Multiply out all possible combinations to get defining relation
  - Multiply defining relation by any effect to get alias structure
  - Equivalently we could get the individual defining relations and multiply every possible pair of terms
- **Resolution of a crossed array experiment is the minimum of the resolutions of the individual arrays**
  - **R_C = Resolution of Control Array**
  - **R_N = Resolution of Noise Array**

Defining Relation
I=ABC=PQR=ABCPQR

		P	–	+	–	+
		Q	–	–	+	+
		R=PQ	+	–	–	+
A	**B**	**C=AB**	$\mathbf{y_1}$	$\mathbf{y_2}$	$\mathbf{y_3}$	$\mathbf{y_4}$
–	–	+	y_{11}	y_{21}	y_{31}	y_{41}
+	–	–	y_{12}	y_{22}	y_{32}	y_{42}
–	+	–	y_{13}	y_{23}	y_{33}	y_{43}
+	+	+	y_{14}	y_{24}	y_{34}	y_{44}

Notice that as long as each individual array is Resolution III the shortest word containing both control and noise factors will have length 6.

Confounding Pattern

- A=BC=APQR=BCPQR
- B=AC=BPQR=ACPQR
- C=AB=CPQR=ABPQR
- P=ABCP=QR=ABCQR
- Q=ABCQ=PR=ABCPR
- R=ABCR=PQ=ABCPQ
- AP=BCP=AQR=BCQR
- AQ=BCQ=APR=BCPR
- AR=BCR=APQ=BCPQ
- BP=ACP=BQR=ACQR
- BQ=ACQ=BPR=ACPR
- BR=ACR=BPQ=ACPQ
- CP=ABP=CQR=BCQR
- CQ=ABQ=CPR=ABPR
- CR=ABR=CPQ=ABPQ

Control by Noise Factor Interactions will be confounded with interactions of order Min(R_C, R_N).

So, if both arrays have Resolution III then CxN interactions will be confounded with three factor interactions.

Summary

- **I/O arrays are always good for studying CxN interactions**
- **I/O arrays are often *but not always* run as split plot experiments**
- **Split plot is determined by how you run the experiment not how you design it.**
- **You must know how the experiment was run in order to analyze it with the correct error structure**

My Recommendations

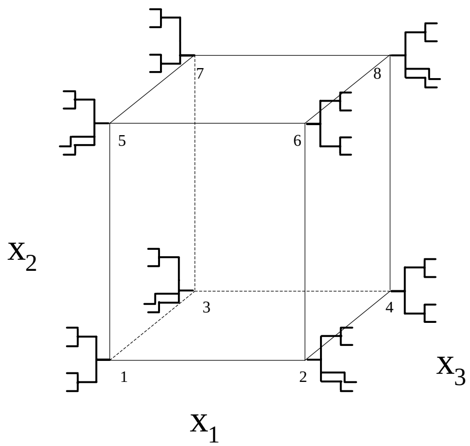
- **Think about the idea of robustness in the planning stage of your experiment and choose the response accordingly.**
- **If you choose to reduce variance, then make sure that the variance that you are reducing is the variance that you want to reduce and that it is worth the experimental effort.**
- **Analyze the mean and variance separately and make decisions concerning the tradeoffs between reducing variance and achieving the desired mean value.**
- **Use Inner and Outer Arrays to estimate Control by Noise Interactions for designing Robust Products and Processes.**

Assembled Designs

A class of experimental design for estimating location effects, dispersion effects, and variance components.

Parameters:
r = # of Design Points
n = # of observations per Treatment Combination (assume constant)
s = # of different *structures* used in the design
B_T = total number of Batches

$r = 8$
$n = 4$
$s = 2$
$B_T = 20$



"Works for anything that involves making batches and taking observations"

Notation for Assembled Designs

$$\sum_{j=1}^{s} \text{structure } j@\{\text{design points with structure } j\}$$

Must have design points ordered in some way.

Example: 2^3 design, Splitting Generator = ABC, r=8, s=2, B_1=4, B_2=3, n=6

Design Point	A	B	C	ABC	Structure
1	-	-	-	-	(2,1,1)
2	+	-	-	+	(2,2)
3	-	+	-	+	(2,2)
4	+	+	-	-	(2,1,1)
5	-	-	+	+	(2,2)
6	+	-	+	-	(2,1,1)
7	-	+	+	-	(2,1,1)
8	+	+	+	+	(2,2)

$$(2,1,1)@\{1,4,6,7\} + (2,2)@\{2,3,5,8\}$$