G with a Cryogenic Torsion Pendulum

The Good, The Bad

And thoughts for the future of G measurement instruments

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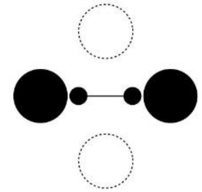
How to measure G with a torsion pendulum?

"time-of-swing" is attractive:  ...measure $\Delta\omega^2$

- Frequency is easy to measure accurately
- No precision angle measurements required
- $\Delta\omega^2$ independent of torsion constant κ (or is it?)

Alas, κ is frequency-dependent. (Quinn, Speake, Kuroda)

Kuroda: $\delta G \approx 1/(\pi Q)$ (for a particular distribution of fiber relaxation times)

UCI: $0 < \delta G < 1/(2Q)$ (for any distribution of relaxation times)

To minimize effects of torsion fiber anelasticity:

- **Feedback to avoid fiber twist** UWa, BIPM, New Zealand, PTB
- **Use a flat strip fiber (*κ* from gravity)** BIPM, New Zealand
- **Very high Q** BIPM (strip fiber), HUST (fused silica), **UCI (cryogenic)**

Why cryogenic?

- high Q   (~100,000, with electrically conducting fiber)
- low thermal noise $\tau_{noise} \propto \sqrt{\frac{k_B T}{Q}}$
- torsion constant very insensitive to T variation
- ease of T control
- superconducting magnetic shielding
- excellent vacuum

Method

Source Masses: Two copper rings, NiP plated, 59 kg each, 520 mm OD

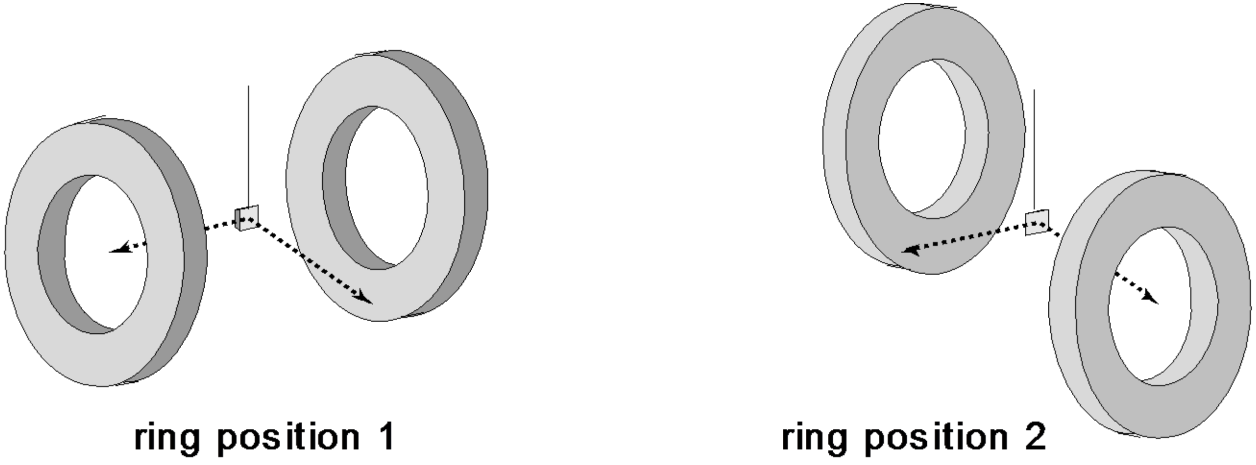
An extremely uniform central field gradient.
Ring field couples almost purely to pendulum quadrupole moment q_{22}

Pendulum: A thin fused silica plate (41 x 41 x 3 mm, 10.7 g), Al (Au) coated

q_{22}/I is very weakly dependent on size, shape, and mass distribution
3 mm position error produces $dG/G < 1$ ppm

Technique: "time-of-swing"

modulate source mass position and measure torsional frequency change

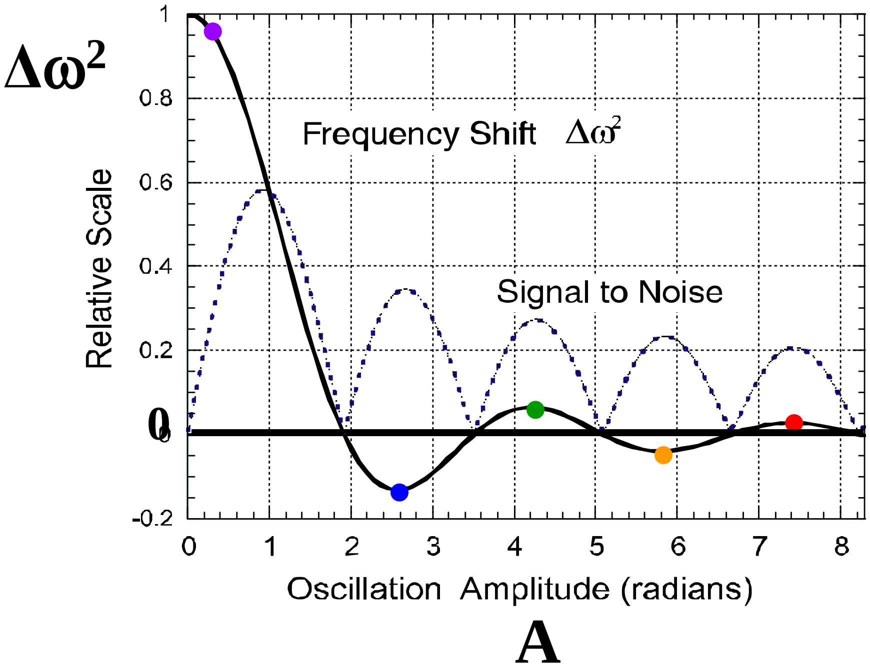


$$"signal" \, S \equiv \Delta\omega^2 \frac{A}{2J_1(2A)}$$

S = KG where K is a constant determined by mass distribution.

G = S/K

accurate to ~ 6 ppm

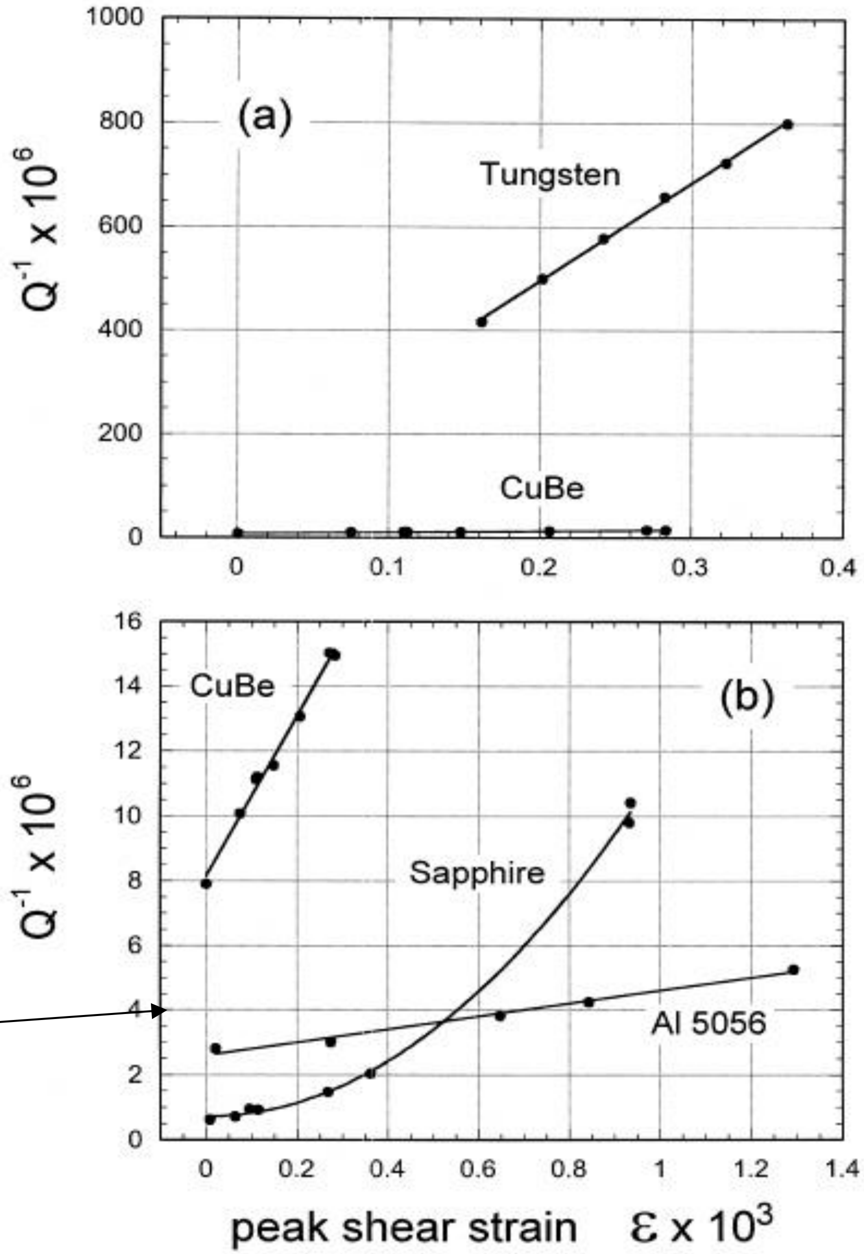


Crucial Tests:

S should be independent of:

- Oscillation amplitude
- Fiber material and treatment

Q^{-1} measurements at 4.2K



Q ~ 250,000

δG/G (Kuroda) < 2 ppm

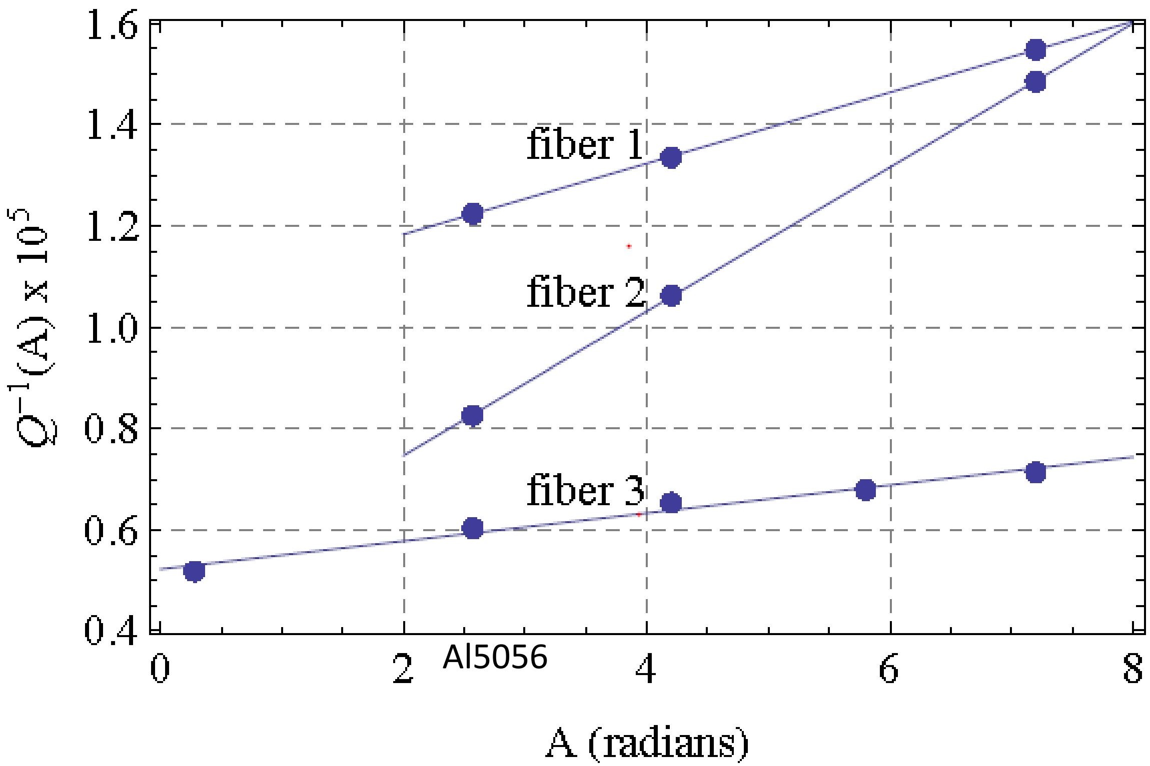
Characteristics of the stick-slip model

1. Linear variation of Q^{-1} with amplitude: $dQ^{-1}/dA = c_1$
2. Linear variation of ω^2 with amplitude: $d\,\omega^2/dA = c_2$
3. A specific ratio of these variations: $-\pi\omega_0^2\,\Delta Q^{-1}/\Delta\omega^2 = 4/3$
4. A characteristic pointy "Davidenkov" hysteresis loop
5. Frequency-independent torque: $m\ddot{\theta} = -k\theta - k_{ss}[(A^2 - \theta^2)\hat{\dot{\theta}} - 2A\theta]$

Characteristics 1-4 are clearly observed for CuBe:

M.K.Bantel, R.D. Newman, Journal of Alloys and Compounds ***310*** *(2000) 233-242*

Dependence of Q^{-1} on torsional amplitude in the UCI G measurement



		diameter	period	Q (3 K)	slope	dataset
#1	CuBe	20 μm	135 sec	80,000	0.068	2000
#2	CuBe heat treated	20 μm	130 sec	120,000	0.137	2000 and 2002
#3	Al 5056	25 μm	113 sec	170,000	0.023	2004 and 2006

“Kuroda correction” 1/πQ

ppm, based on Q(A=0):

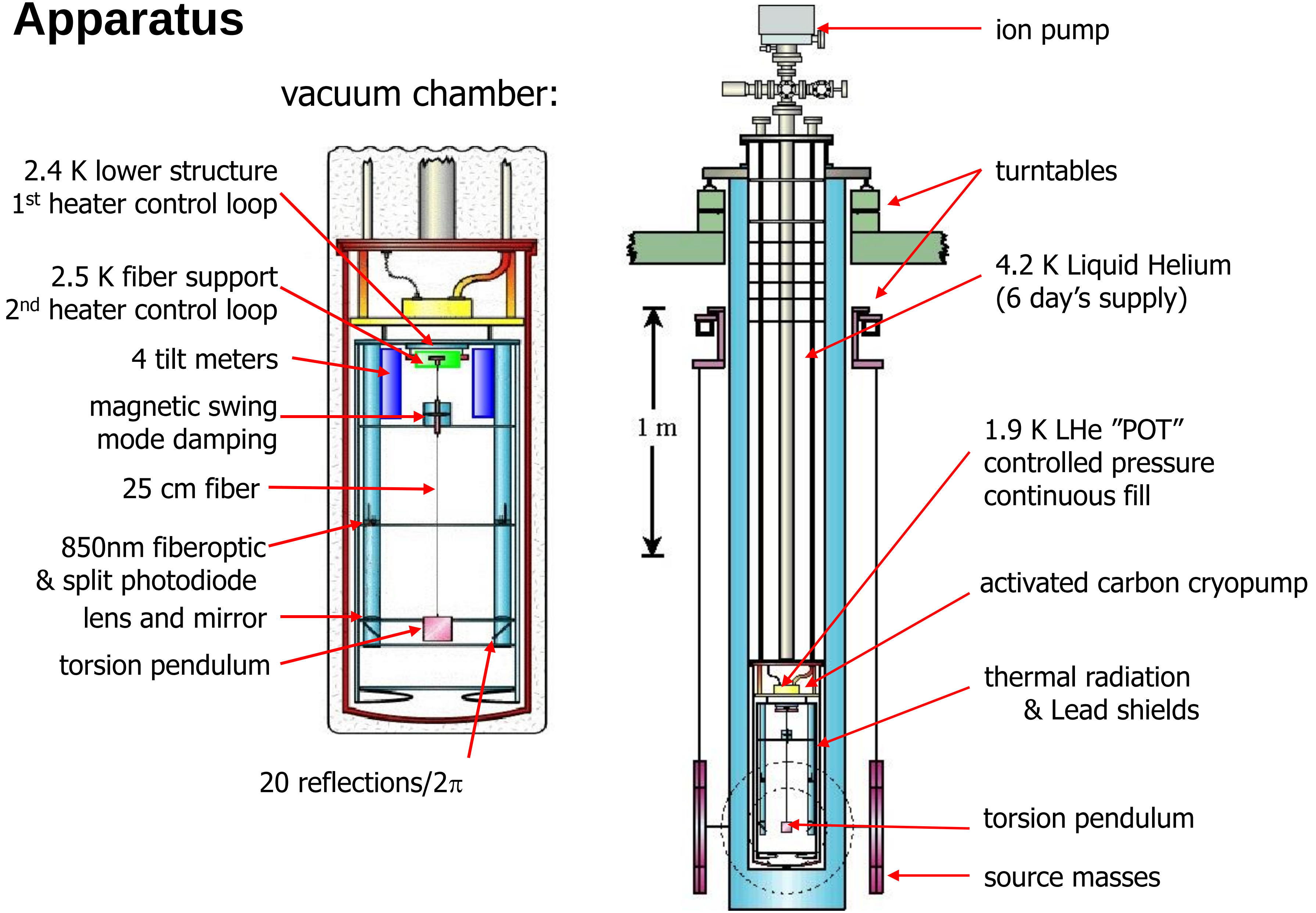
	fiber 1	fiber 2	fiber 3
ppm correction	-3.3	-1.5	-1.7
uncertainty	-1.7	-0.7	-0.9

ppm, based on Q(A):

	fiber 1	fiber 2	fiber 3
A=0.3	-3.4	-1.6	-1.8
2.6	-3.9	-2.6	-1.9
4.2	-4.2	-3.4	-2.1
5.8	-4.6	-4.1	-2.2
7.4	-4.9	-4.8	-2.3

Apparatus

vacuum chamber:



pendulum equation of motion:

$$I\ddot{\theta} = -[k_1\theta + k_2\theta^2 + k_3\theta^3 \ldots + b\dot{\theta} + \tau(\theta, \dot{\theta}) + \sum_m \alpha_m Cos(m\theta) + \sum_m \beta_m Sin(m\theta)]$$

1st order solution:

$\omega_0^2 = k_1/I$

$$\omega^2 = \omega_0^2 \left[1 + \frac{3}{4}A^2\frac{k_3}{k_1} - 2A\frac{k_{ss}}{k_1} + \frac{2}{A}\sum_m J_1(mA)\frac{\beta_m}{k_1}\right]$$

$$\frac{k_3}{k_1} \approx -5 \times 10^{-8}$$

$$\frac{k_{ss}}{k_1} \approx 3 \times 10^{-6}$$

$$\frac{\beta_2}{k_1} \approx 5 \times 10^{-5}$$

gravitational contribution:

$$\beta_m = -2m\ Re \sum_{\ell \geq m} q_{\ell m} a^*_{\ell m}$$

pendulum mass multipole moments: $q_{\ell m}$

source mass field moments:

$$\begin{aligned} a_{\ell m} &\propto \frac{\partial}{x^\ell}\Phi_{grav} \\ &\propto \vec{g} \quad \ell = 1 \\ &\propto \nabla\vec{g} \quad \ell = 2 \\ &= 0 \quad \ell = 3,\ 4,\ 5 \end{aligned}$$

magnetic coupling

1st order signal : $\mathbf{\Delta\omega^2 = \frac{4}{IA}[\beta_2 J_1(2A) + \beta_6 J_1(6A) + \beta_1 J_1(A)]}$

Second order terms in $\Delta\omega^2$

$$\boldsymbol{k_3\beta_2}$$

$$\boldsymbol{k_{ss}\beta_2}$$

$$\boldsymbol{\beta_1}(\text{fixed B})\ \boldsymbol{\beta_2}$$

$$\boldsymbol{\beta_2}(\text{fixed ambient g gradient})\boldsymbol{\beta_2}(\text{signal})$$

- Corrections depend on torsional amplitude
- Maximum correction < 5 ppm
- Corrections would be negligible in a classical (low amplitude) time-of-swing experiment

group	dates	fiber	pendulum	Period	Q	T(fiber)	runs	hours
1	9 to 11/2000	1	1a	135 sec	82,000	2.65 K	23	604
2a	12/2000	2	1b	130 sec	120,000	4.6 K	9	135
2b	3 to 5/2002	2	1b	130 sec	120,000	2.75 K	9	705
3	3 to 5/2006	3	2	113 sec	182,000	3.0 K	36	683

Initially 1669 values for $S \equiv \frac{A}{2J_1(2A)}\Delta\omega^2$

Deletions:

First three runs in group 1 (missing fiber temperature data)

First three S values in each run

Runs with fewer than three S values

149 S values (excessive ring swing)

87 S values (excessive fiber temperature variation)

Remaining for G analysis: 1085 S values

Robustness test of data processing

Three types of decisions were made in the analysis stream, each with three options:

1. Averaging S ± δS values within a run. Options:
   a. Weighted average
   b. Unweighted average
   c. Weighted average iff Allan variance analysis indicates white noise

2. How to inflate averages in other stages if χ^2/df is high
   a. By Birge ratio (forcing $\chi^2/df = 1$)
   b. Force $p = 0.05$
   c. Force $p = 0.01$

3. How to deal with outliers at various stages
   a. Delete no values
   b., c. Delete values with one of two different criteria

Total of 3x3x3 = 27 variants

G was evaluated for each fiber using each of the 27 variants.

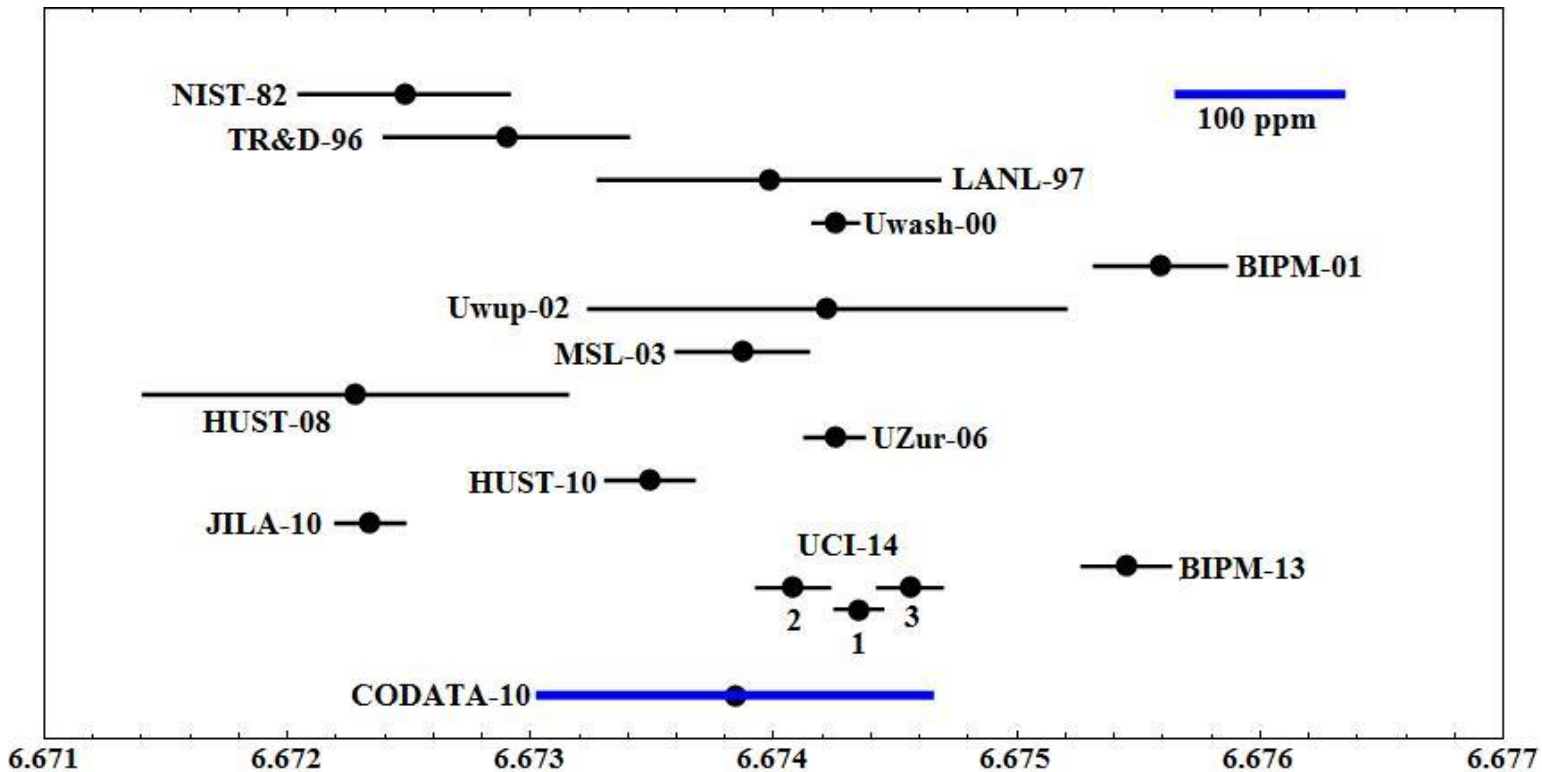
Fiber	1	2	3
Max-min G (ppm)	14	24	20
Std dev G (ppm)	5	6	6

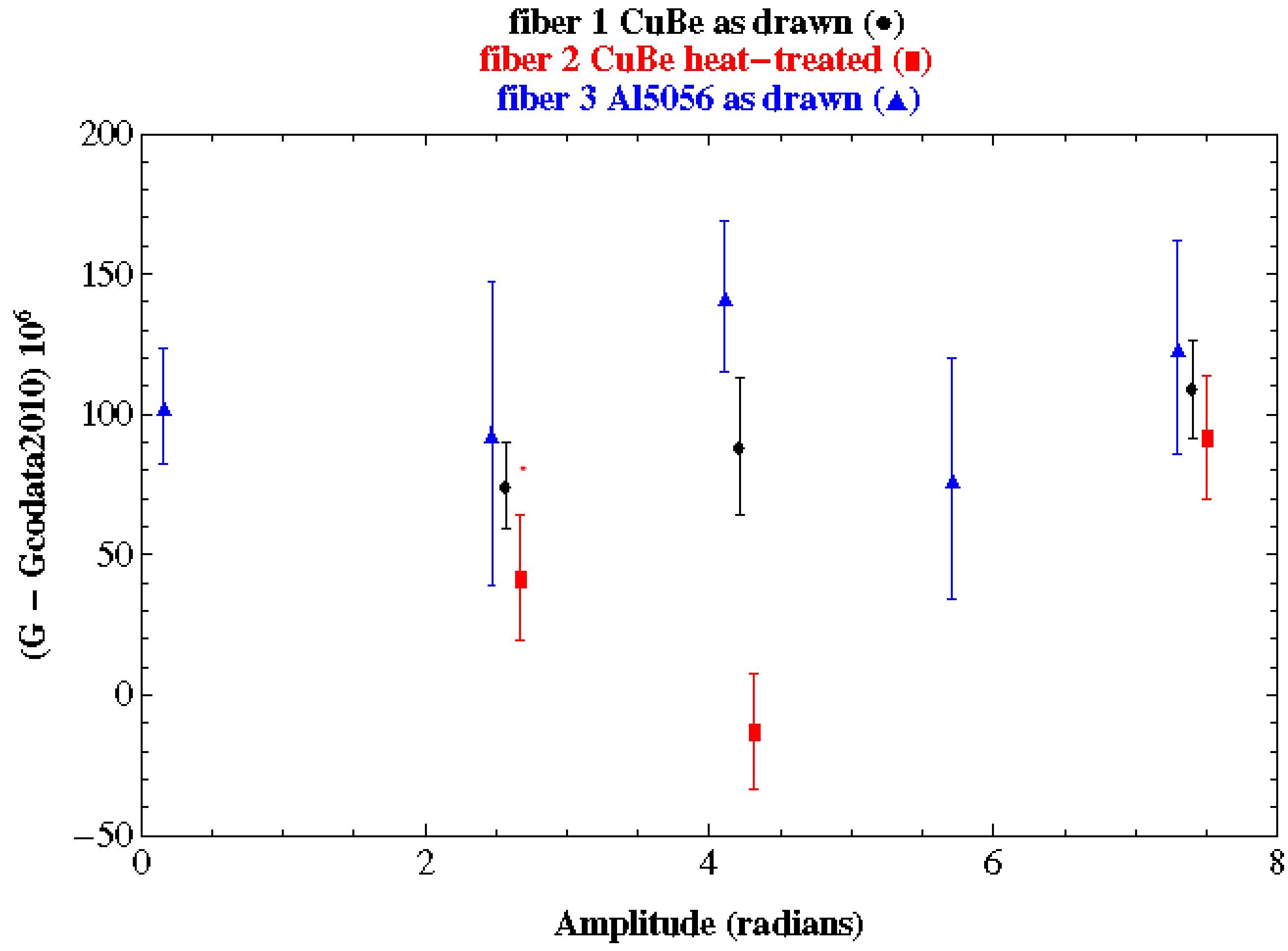
A"method" G uncertainty was assigned for each fiber equal to (max-min) /2

Final G uncertainty components for each fiber:

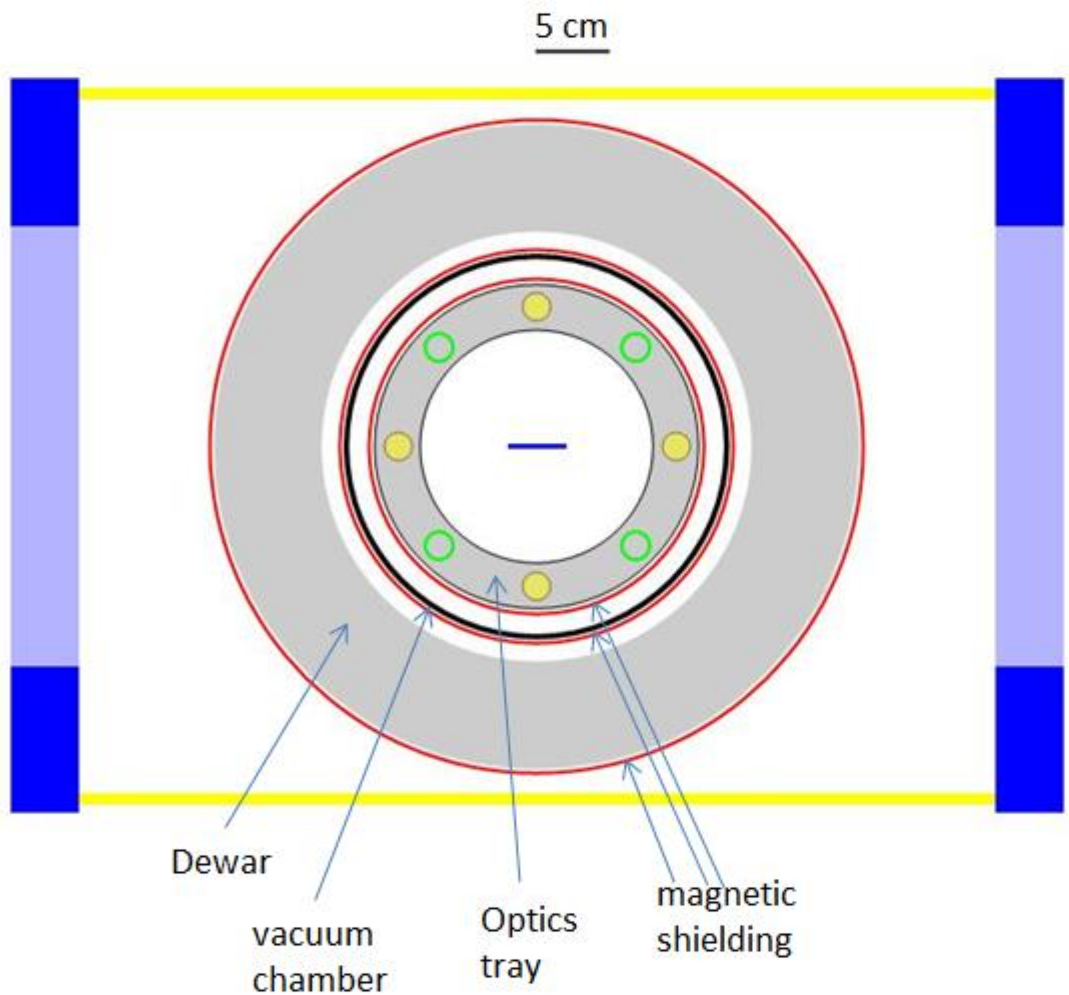
component	fiber 1	fiber 2	fiber 3
statistical	7.7	15.7	11.3
systematic	10.1	10.2	12.4
analysis method	6.9	12.0	10.2
quadrature sum	14.5	22.2	19.6

Fiber	$G \times 10^{11} m^3 kg^{-1} s^{-2}$	$\delta G (ppm)$
1	6.67435(10)	14
2	6.67408(15)	22
3	6.67455(13)	20
unwtd mean	6.67433(13)	19

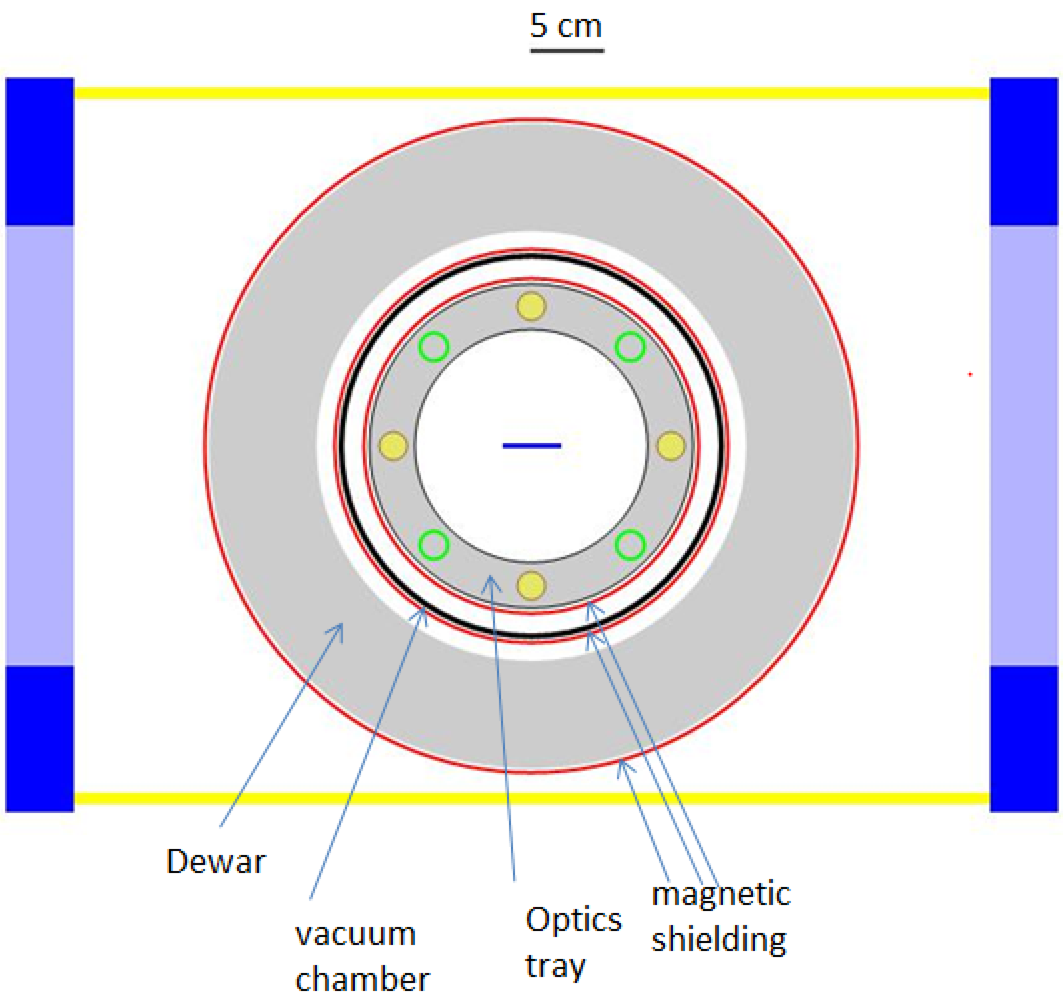

NIST-82
TR&D-96
LANL-97
Uwash-00
BIPM-01
Uwup-02
MSL-03
HUST-08
UZur-06
HUST-10
JILA-10
UCI-14
2
1
3
BIPM-13
CODATA-10
100 ppm
6.671
6.672
6.673
6.674
6.675
6.676
6.677



fiber 1 CuBe as drawn (●)
fiber 2 CuBe heat−treated (■)
fiber 3 Al5056 as drawn (▲)
(G − Gcodata2010) 10^6
Amplitude (radians)
200
150
100
50
0
−50
0
2
4
6
8


5 cm
Dewar
vacuum chamber
Optics tray
magnetic shielding

The Good!



Low sensitivity to SM inhomogeneity

reduced sensitivity to SM spacing absolute error

Extremely insensitive to pendulum placement error (1 ppm from 3 mm)

Ample room for magnetic shielding

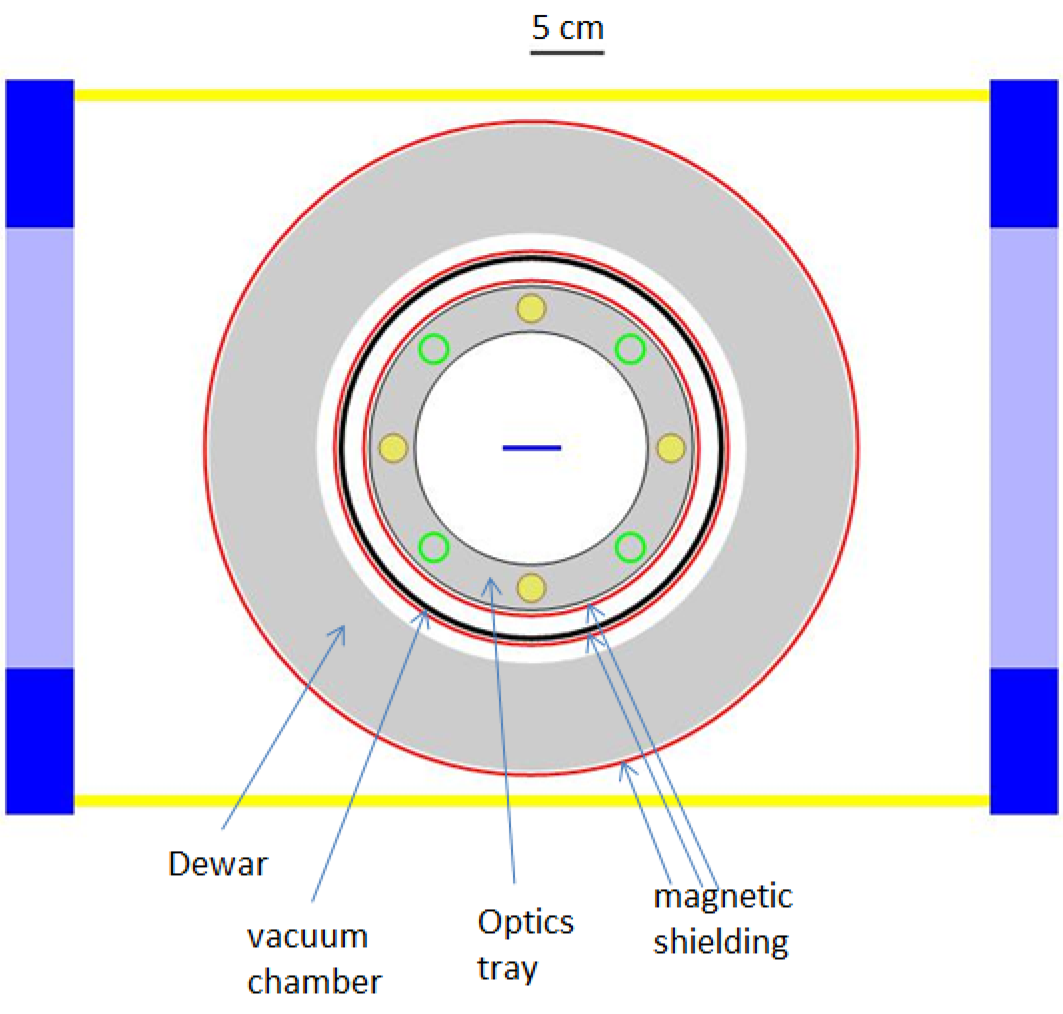
Good thermal isolation

Rapidly converging multipole analysis

Pendulum far from surrounding structure, and 2 meters from transport mechanism

G measured with 2 pendulums, 3 fibers, and at 5 amplitudes

PLUS cryogenic advantages



The Bad:

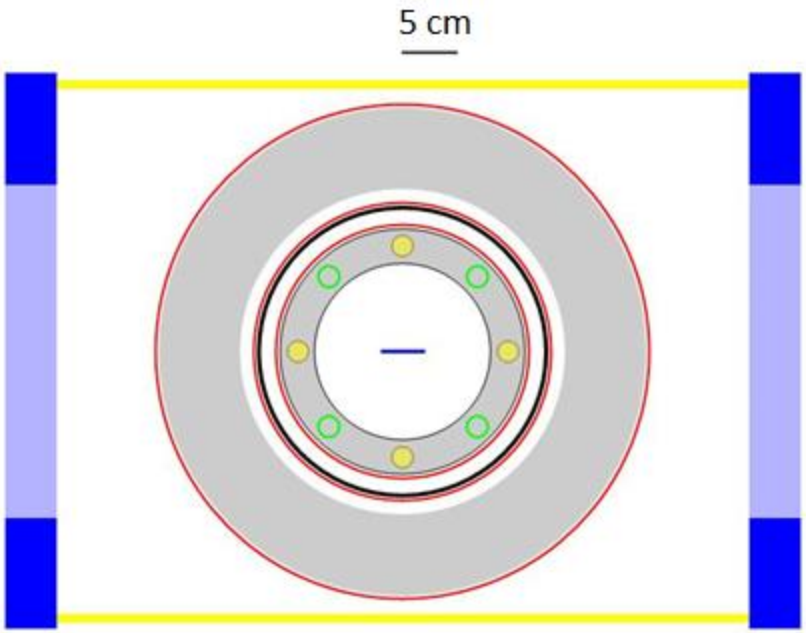
- ΔP = 1.7 to 0.2 millisec*
- Noise ~ 30 x thermal (kT)
- Technical problems:

    ring swing control

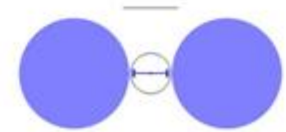
    fiber T variation

*Compare:
≈ 7 seconds (Gabe Luther)
≈ 3 seconds (HUST)

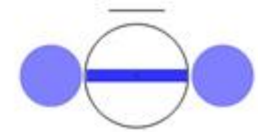
UCI



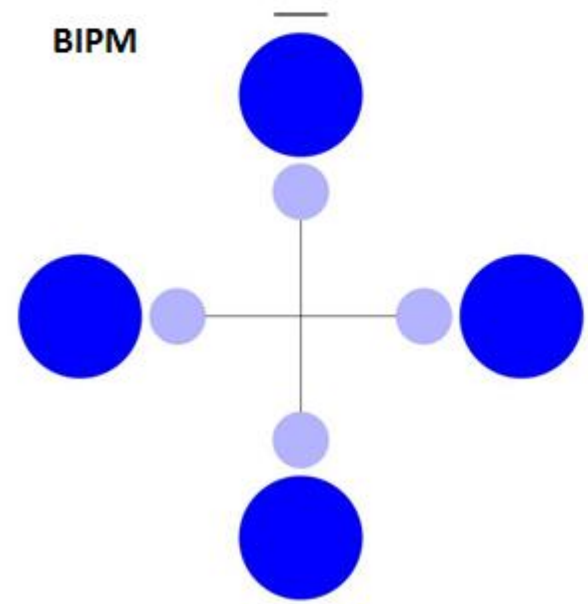
Gabe Luther, 1982



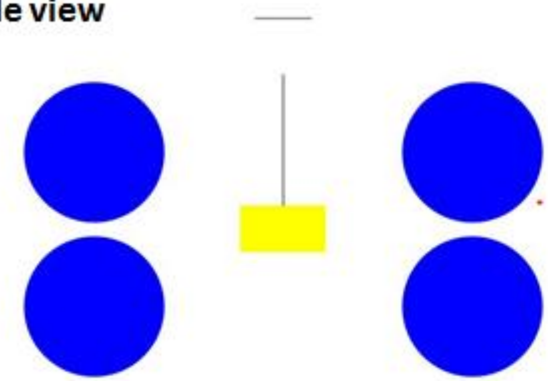
HUST



BIPM



U Wa, side view



Lab	total run time (days)	approx. minimum distance Δr SM to TM (cm)	ratio Δr/(pendulum size)	δG (statistical)	ratio, δG(stat)/ δG(systematic)	ΔP (seconds)
UCI	89	33	8.2	15	1.4	~0.001
UWa	18	7	0.9	6	0.5	
HUST	64	0.4	0.04	21	0.7	3
BIPM	14	0.6	0.1	18	0.7	
Luther	17	0.3	0.1	18	0.7	7
Faller	7	4	0.7	4	0.2	

Note:

- Short total run times
- Statistical uncertainties small compared to systematic
- Statistical uncertainties very small compared to current range in published G values

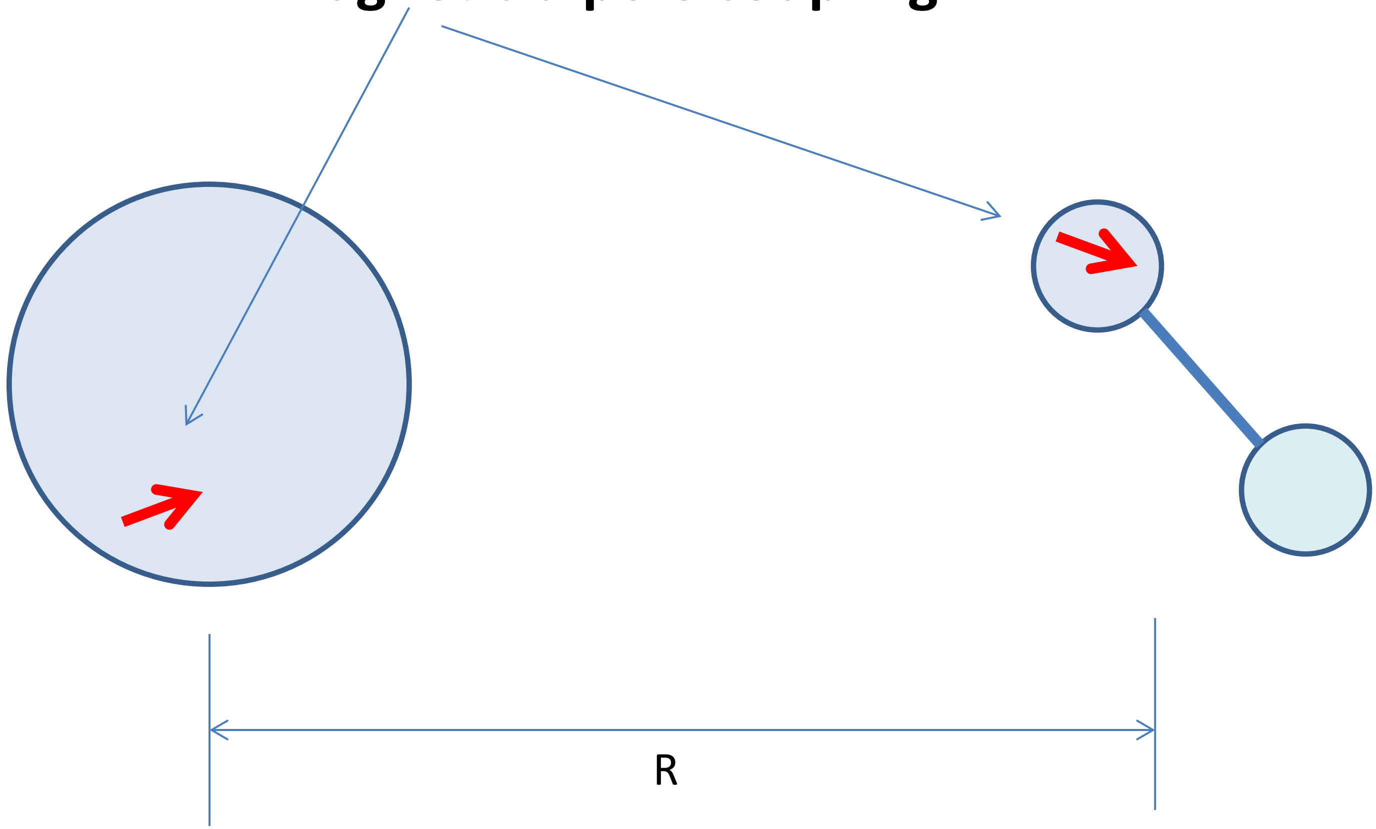
What if source mass to test mass spacings are increased?

Reduced signal strength and signal/noise ratio.
(partly compensate with longer run times)

Potentially better control of systematic error.
How would systematic errors scale?
Depends on apparatus design, and nature of the systematic;

Consider some examples:

Magnetic dipole coupling



$$\tau(magnetic) \propto \frac{1}{R^3}$$

$$\tau(gravity) \propto \frac{1}{R^3}$$

Systematic effect independent of R

Effects of torsion fiber nonlinearities, fixed gravity gradients, and fixed magnetic fields

k_3 k_{ss} β_1(B) β_2(fixed ambient g gradient)

For UCI, the effects of these torque terms on apparent G signal are proportional to products of these torque terms and the signal torque term β_2(source mass).
Thus they produce an error in G independent of signal strength.

Effects of mass inhomogeneity and error in dimensional metrology and placement

Here one gains by increasing spacing.

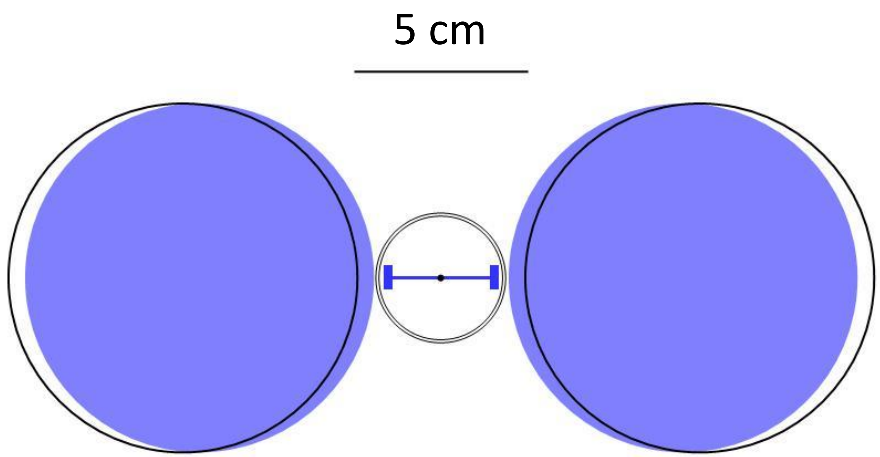
Ability to shield magnetic fields and maintain thermal control

Here one can gain dramatically.

Effect of ambient gravity gradient variation (elevators, people..)

Here reduced signal strength hurts. Compensated by longer run time.

Example (Luther 1982 instrument)

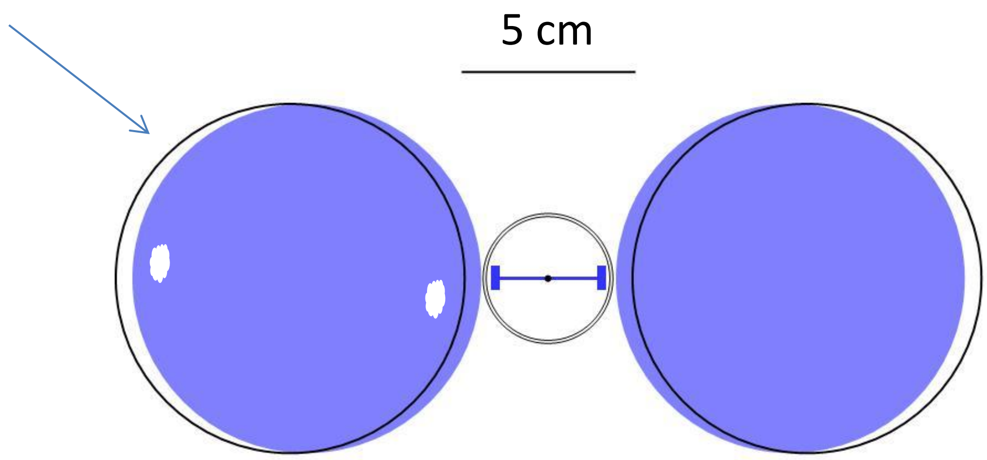


Moving spheres out 1 cm reduces signal ~25%

- allows magnetic and improved thermal shielding
- Reduces sensitivity to local mass inhomegeneity

Example (Luther 1982 instrument)

Voids invariant on 180° rotation,
CM unaffected



Moving spheres out 1 cm reduces signal ~25%

- allows magnetic shielding and improved thermal shielding
- Reduces sensitivity to local mass inhomegeneity

Advantages of a very long total run time

Partly compensates for reduced signal/noise if needed

Tests the stability of the instrument (same G in November as last May?)

Allows time for multiple variations in a single basic instrument, to systematically probe for hidden systematic error

eg, different:

Fiber materials
Source mass placements
Pendulum, source masses
Cycle intervals
Torsional oscillation amplitude
Torsional static displacement (different source masses)
Compensating electrostatic force voltages (different source masses)

etc

..as well as time for the usual tests for sensitivity to variation in
Temperature
Magnetic fields
Tilt

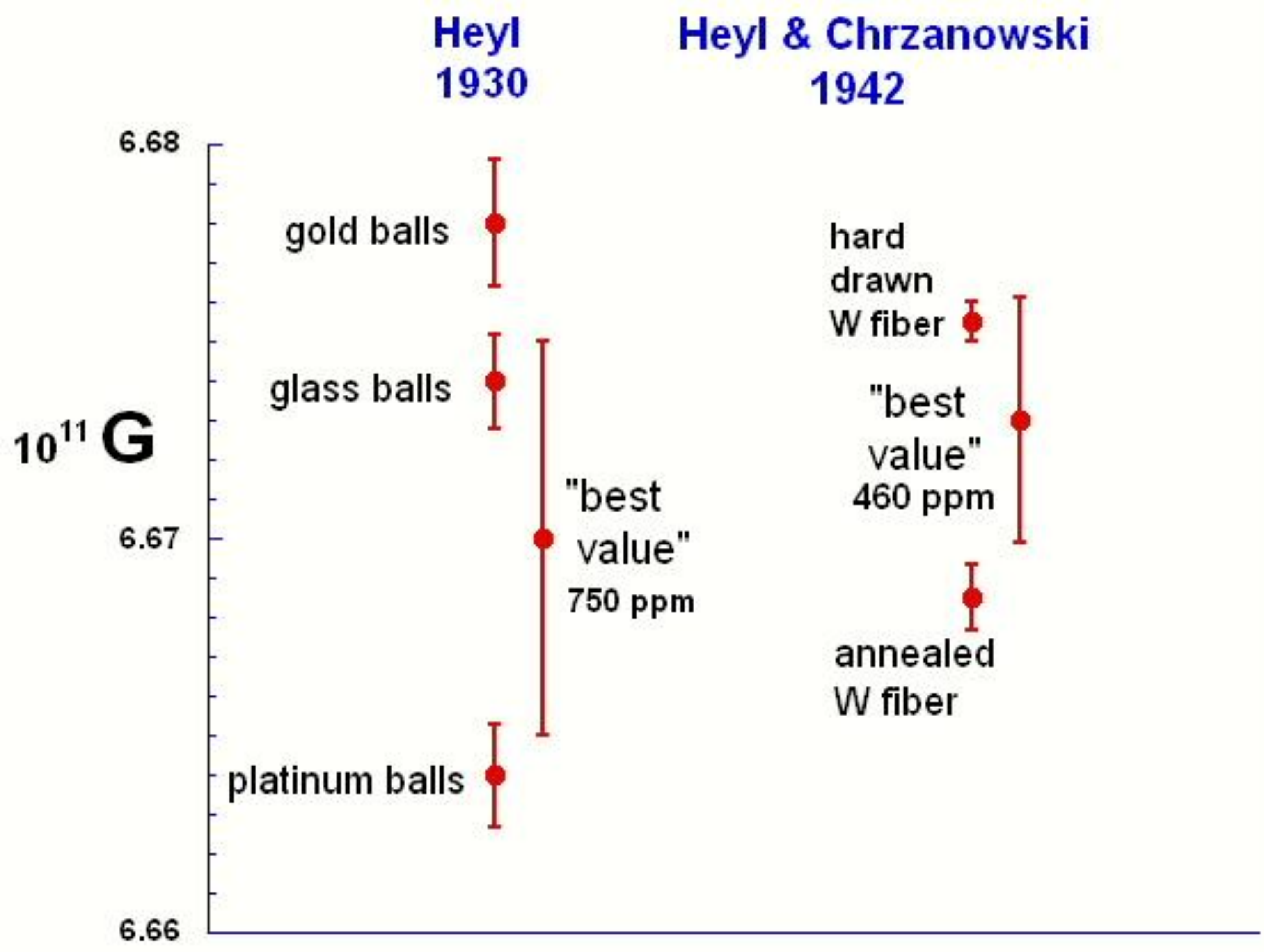
Does one sacrifice instrument quality by trying to make it versatile (eg, to accommodate different source masses and/or their positions)?

I don't think this needs to be so.

Final thoughts

- **Increase SM to TM separation (but not as much as we did!)**
- **Expect to run for years, varying many instrument characteristics one at a time.**
- **Operate at a quiet but easily accessible location where one can run 24/7 –**

  …. excavate a 20 meter deep shaft at NIST?
- **Operate blind – look at <u>change</u> in G with varying parameters, but not values.**


Heyl
1930
Heyl & Chrzanowski
1942
6.68
6.67
6.66
10^{11} G
gold balls
glass balls
platinum balls
"best value"
750 ppm
hard drawn W fiber
"best value"
460 ppm
annealed W fiber