USING MARKOV CHAIN AND GRAPH THEORY CONCEPTS TO ANALYZE BEHAVIOR IN COMPLEX DISTRIBUTED SYSTEMS

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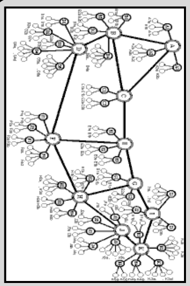
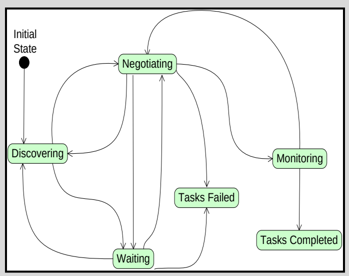
Summary

Goal: To develop scalable modeling tools for monitoring complex distributed systems and predicting catastrophic performance degradations.

- **Use *Discrete Time Markov Chain* (*DTMC*):**
  - Develop *time-inhomogeneous* model of system behavior.
  - Perturb DTMC *transition probability matrices* (TPMs) to simulate alternative system evolutions → ***Identify failure scenarios***



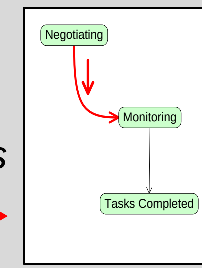
System to state model



State model to perturbable TPM

	Initial	Wait	Disc	Ngt	Mon	Comp	Failed
Initial	0.9697	0	0.0303	0	0	0	0
Wait	0	0.8363	0.0673	0.0918	0	0	0.0046
Disc	0	0.0355	0.6714	0.2931	0	0	0
Ngt	0	0.4974	0.0182	0.2882	0.1961	0	0.0001
Mon	0	0	0	0.0003	0.9917	0.0080	0
Comp	0	0	0	0	0	1.0	0
Failed	0	0	0	0	0	0	1.0

Identify failure scenarios



Problem: To design an efficient approach for analyzing DTMCs

Solution approach: Use minimal s-t cut set analysis to identify ***critical state transitions*** in a directed graph of a DTMC & relate to ***failure scenarios***

– Introduce algorithms that ***reduce search space*** to find minimal s-t cut sets

→ ***Use of combination of analysis techniques not reported before***

Outline

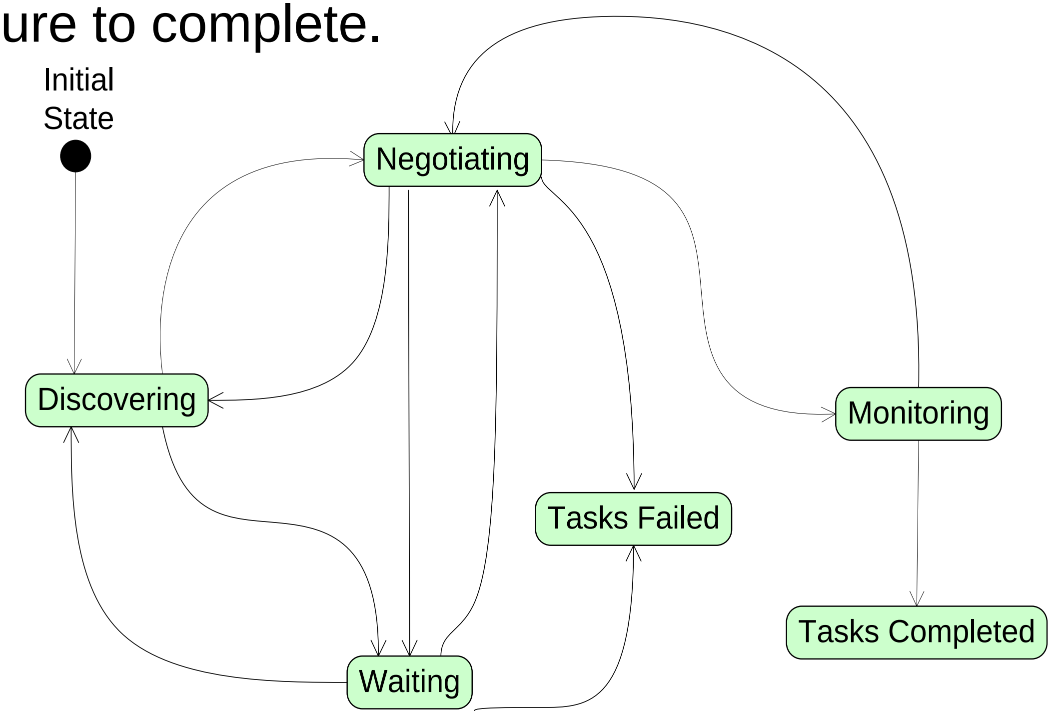
1. ***DTMC concepts and model development***
2. Perturbing a DTMC to identify a failure scenario
3. Using minimal s-t cut set analysis and node contraction to reduce search for failure scenarios
4. Conclusions

State model of a grid computing system

- Basis → Large-scale discrete-event simulation of grid computing system (Mills and Dabrowski 2008)
  - Grids "rent" compute resources – CPUs, memory, disk

- **FOCUS: *Lifecycle of a grid system task -*** stages in processing of grid task
  - Each state represents phase in processing of grid task
  - **Tasks Completed –** successful completion of a task
  - **Failed State** – failure to complete.

State Diagram of Task Lifecycle



Building a Discrete Time Markov Chain (DTMC) model

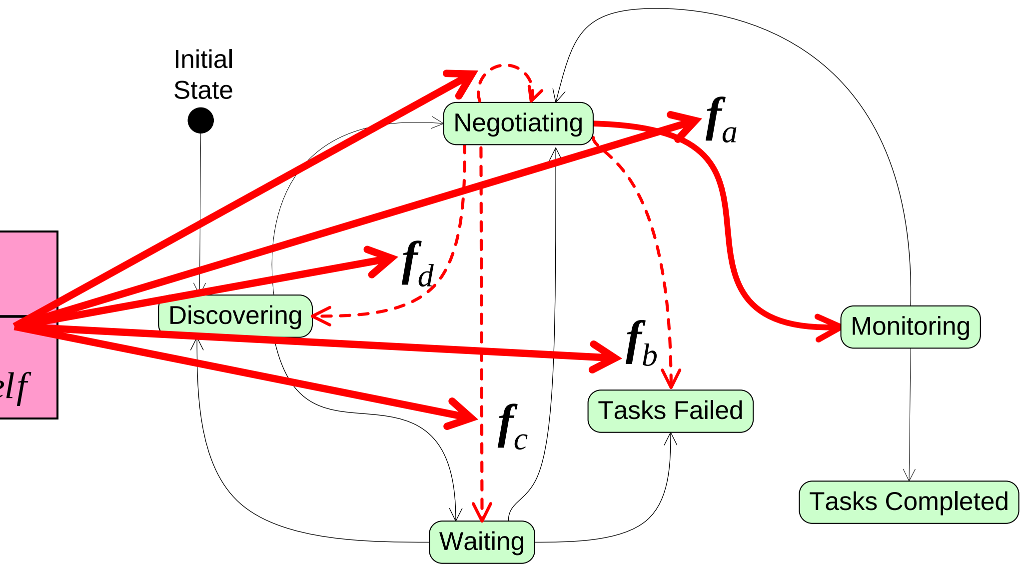
- *Markov chains* are state models where probability of transition from one state to another does not depend on past history: $\Pr(X_{n+1} = x | X_n = x_n, \ldots, X_1 = x_1) = \Pr(X_{n+1} = x | X_n = x_n)$ for sequence of states X_n, X_{n+1}, X_{n+2}…….
- In a *discrete time* Markov chain, system evolves in discrete time steps.
- Probability state i transitions to state j, $\boldsymbol{p_{ij}}$, is the proportion of total number of transitions from state i to other states, where $\boldsymbol{f_{ij}}$ are frequencies. Note: if $i = j$, a ***self transition*** occurs.

$$p_{ij} = \frac{f_{ij}}{\sum_{k=1}^{n} f_{ik}}$$

Observe system (large scale simulation) and obtain frequencies for all transitions

Example:

$$p\,(\text{Negotiating} \rightarrow \text{Monitoring}) = \boxed{\frac{f_a}{f_a + f_b + f_c + f_d + f_{self}}}$$



→ ***Produce Transition Probability Matrix (TPM)***

Result is set of TPMs for *m* time periods

- ***Key Concept:*** Observation of system over time yields series of TPMs for ***m*** successive time periods → ***a piece-wise homogenous DTMC*** (Rosenberg, Solan, and Vielle, 2004) → captures change over time.

  Transition frequencies recorded over 7200s time periods in large-scale simulation
- Summary TPMs -- weighted average of m periods

8 hours (5 time periods*)

To

From

	Initial	Wait	Disc	Ngt	Mon	Comp	Failed
Initial	0.9697	0	0.0303	0	0	0	0
Wait	0	0.8363	0.0673	0.0918	0	0	0.0046
Disc	0	0.0355	0.6714	0.2931	0	0	0
Ngt	0	0.4974	0.0182	0.2882	0.1961	0	0.0001
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Comp	0	0	0	0	0	1.0	0
Failed	0	0	0	0	0	0	1.0

Similar analysis 640hours (321 periods). See paper

- A DTMC that has ***absorbing states*** (tasks enter and never exit), e.g., Tasks Completed & Tasks Failed → ***absorbing chain***. States that can be re-entered are ***transient states*** (Waiting, Discovering, Negotiating, and Monitoring)

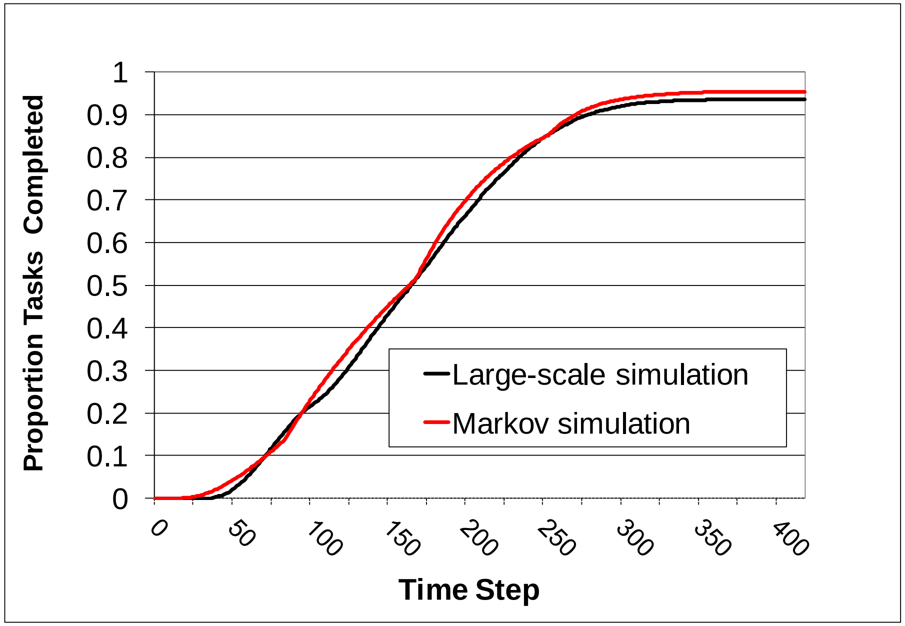
*Extra period for clean-up operations

DTMC simulates system evolution

- Set of TPMs for successive time periods (7200 s)
- System evolves in discrete time steps (85 s per step)
- Vector v_n shows system state at any step n:
  - consists of 7 elements → one for each state
- Matrix multiplication: $Q^T \cdot v_n = v_{n+1}$ with Q_i for related time period.

8-hour period

(421 time steps and 5 TPMs)



End system state vector $\mathbf{v}_{421}$ approximates result of discrete event large-scale simulation, i.e., *Tasks Completed*

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TPM perturbation

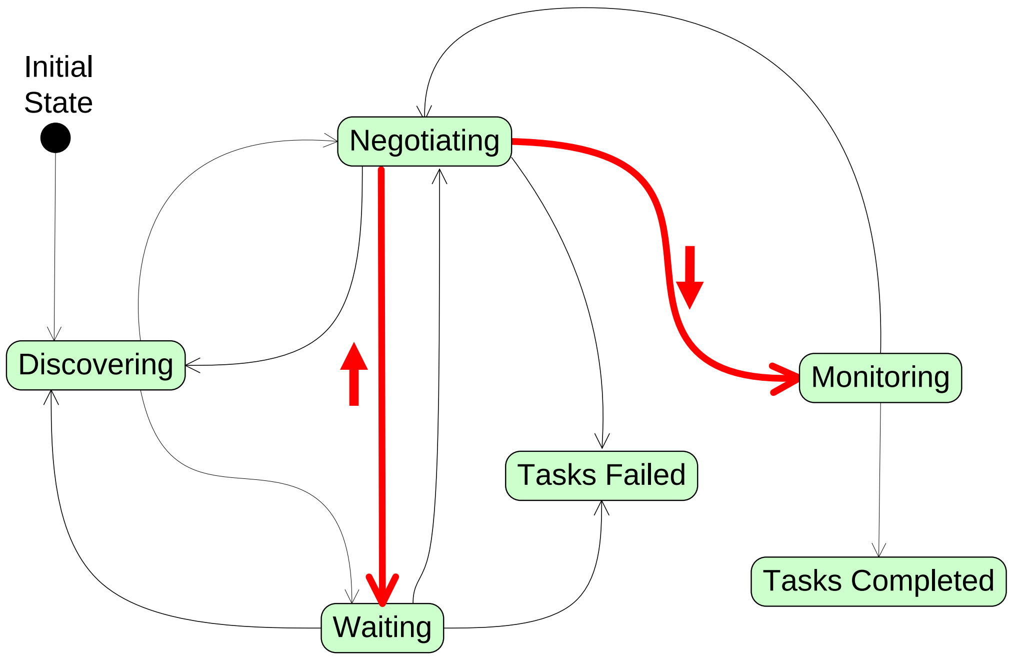
- *Modifying state transition probabilities changes behavior and outcome of Markov simulation*

EXAMPLE:

Decrease *p* (*Negotiating* → *Monitoring*)

Increase *p* (*Negotiating* → *Waiting*)

	Wait	Ngt	Mon
Wait	0.8363	0.0918	0
Ngt	0.4974	0.2882	0.1961
Mon	0	0.0003	0.9917



→ *changes proportion of requests that enter* ***Tasks_Completed*** *absorbing state*

Using critical transitions to predict failure scenarios

- Markov simulation of perturbed critical transitions over multiple time periods (time inhomogeneous evolution) drives down performance
- *Can be related to failure scenarios:*

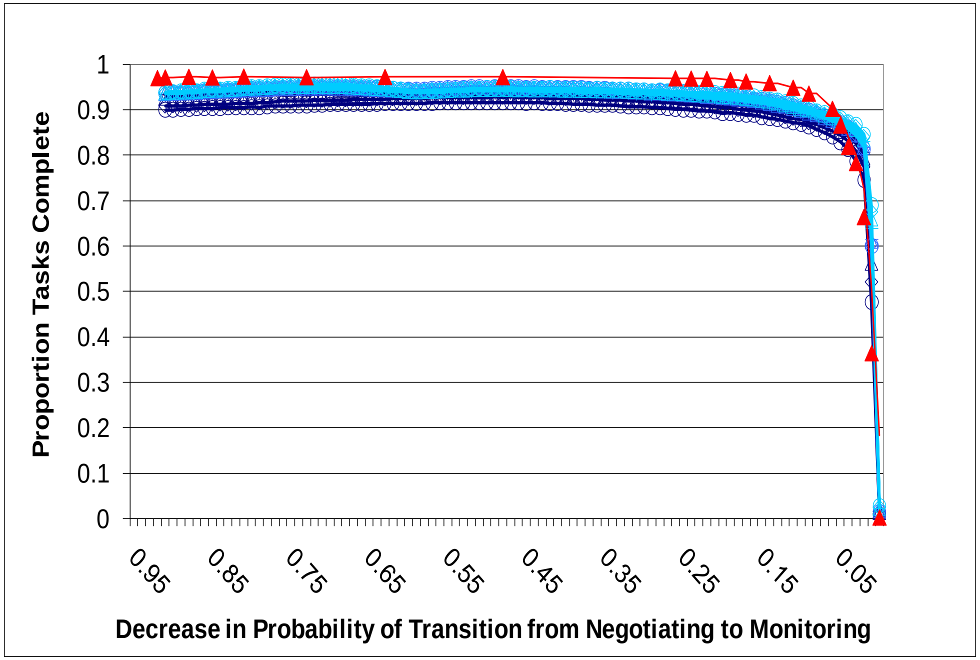
    *ex. System-wide f*ailure of Negotiation components due to spread of Trojan virus

EXAMPLE:

Decrease *p* (*Negotiating* → *Monitoring*)

Increase *p* (*Negotiating* → *Waiting*)

	Wait	Ngt	Mon
Wait	0.8363	0.0918	0
Ngt	↑0.4974	0.2882	↓0.1961
Mon	0	0.0003	0.9917



→ Predict the performance of the system being modeled.

Computability of finding critical state transitions

Unfortunately, there may be many perturbation combinations to examine in a large problem

- Developed *exhaustive perturbation algorithm* (Dabrowski and Hunt 2009) which iterates over rows of TPM representing transient states. For all columns in each row,
  - raises the transition probability of one column
  - lowers transition probabilities of one or more other non-zero columns in the same row.
- See (Dabrowski and Hunt, 2011) for analysis of larger DTMC in which multiple rows must be perturbed together to find combinations of state transitions which together are critical increases.

→ ***Exhaustive search over all perturbation combinations infeasible for larger problems***

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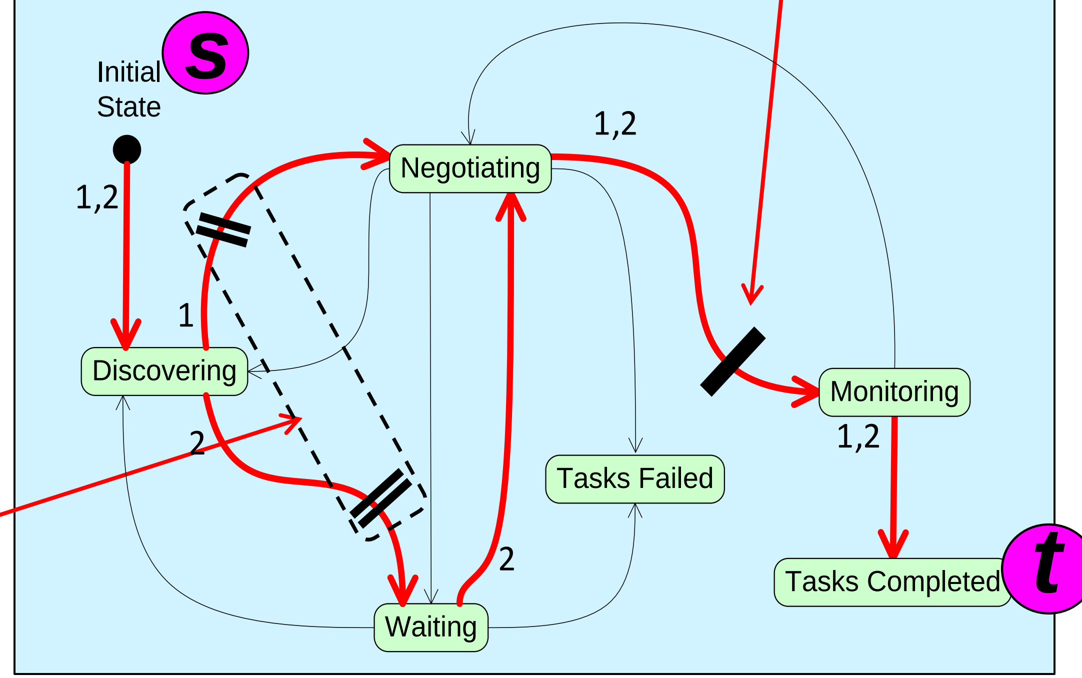
Minimal s-t cut set analysis

- A DTMC is a directed graph
- Minimal s-t cut set: edges (transitions) that disconnect all paths from vertex ***s*** (*Initial* state) to vertex ***t*** -- desired absorbing states *Tasks_Complete*

Single-transition cut

*Cut sets contain **critical Transitions** where perturbation reduces performance*



Multiple-transition cut

For two paths from s to t, there are 3 single-transition s-t cut sets and 2 two-transition s-t cut sets. For related discussion of trap states, see paper.

Result of applying minimal s-t cut set analysis

- ***Reduces perturbation combinations to examine to focus on most critical***
- ***2x magnitude less computation time*** over exhaustive perturbation algorithm
- Finds all critical transitions found by exhaustive perturbation, including those involving > 1 state, verified by large-scale simulation (bolded and shaded entries)
  - → All related to failure scenarios. See (Dabrowski and Hunt 2009)

Results of exhaustive perturbation of TPMs and minimal s-t cut set analysis for the 8- and 640-hour cases

(a) *row* = *Discovering*

	Element reduced→0	Element raised	Proportion of *Tasks Complete* 8-hour	Proportion of *Tasks Complete* 640-hour	s-t cut exists
1	*Waiting*	*Discovering*	0.957	0.935	No
2	*Waiting*	*Negotiating*	0.959	0.935	No
3	*Discovering*	*Waiting*	0.939	0.935	No
4	*Discovering*	*Negotiating*	0.963	0.935	No
5	*Negotiating*	*Waiting*	0.894	0.933	No
6	*Negotiating*	*Discovering*	0.651	0.932	No

(b) *row* = *Waiting*

	Element reduced→0	Element raised	Proportion of *Tasks Complete* 8-hour	Proportion of *Tasks Complete* 640-hour	s-t cut exists
1	*Waiting*	*Discovering*	0.974	0.937	No
2	*Waiting*	*Negotiating*	0.981	0.939	No
3	*Discovering*	*Waiting*	0.937	0.934	No
4	*Discovering*	*Negotiating*	0.963	0.936	No
5	*Negotiating*	*Waiting*	0.818	0.843	No
6	*Negotiating*	*Discovering*	0.939	0.932	No

(c) *row* = *Negotiating*

	Element reduced→0	Element raised	Proportion of *Tasks Complete* 8-hour	Proportion of *Tasks Complete* 640-hour	s-t cut exists
1	*Waiting*	*Discovering*	0.974	0.937	No
2	*Waiting*	*Negotiating*	0.985	0.938	No
3	*Waiting*	*Monitoring*	1.000	0.939	No
4	*Discovering*	*Waiting*	0.954	0.935	No
5	*Discovering*	*Negotiating*	0.957	0.935	No
6	*Discovering*	*Monitoring*	0.967	0.936	No
7	*Negotiating*	*Waiting*	0.923	0.931	No
8	*Negotiating*	*Discovering*	0.941	0.933	No
9	*Negotiating*	*Monitoring*	0.988	0.938	No
10	*Monitoring*	*Waiting*	**0.000**	**0.000**	Yes
11	*Monitoring*	*Discovering*	**0.000**	**0.000**	Yes
12	*Monitoring*	*Negotiating*	**0.000**	**0.000**	Yes

(d) *row* = *Monitoring*

	Element reduced→0	Element raised	Proportion of *Tasks Complete* 8-hour	Proportion of *Tasks Complete* 640-hour	s-t cut exists
1	*Negotiating*	*Monitoring*	0.982	0.937	No
2	*Negotiating*	*Tasks Comp*	0.982	0.938	No
3	*Monitoring*	*Negotiating*	**0.028**	**0.186**	Yes
4	*Monitoring*	*Tasks Comp*	0.980	0.949	No
5	*Tasks Comp*	*Negotiating*	**0.001**	**0.006**	Yes
6	*Tasks Comp*	*Monitoring*	**0.002**	**0.016**	Yes

(e) *row* = *Initial*

	Element reduced→0	Element raised	Proportion of *Tasks Complete* 8-hour	Proportion of *Tasks Complete* 640-hour	s-t cut exists
1	*Discovering*	*Initial*	***0***	***0***	Yes
2	*Initial*	*Discovering*	0.970	0.988	No

Monitoring → Tasks Completed

Negotiating → Monitoring

Initial → Discovering

Tractability of minimal s-t cut set analysis

However, number of potential s-t cut sets in large directed graphs poses barriers

- Implemented minimal s-t cut set enumeration algorithm described in (Provan and Shier 1996)
  - Complexity is *O* |*E*| for each s-t cut set that exists, where |*E*| is the number of edges in the graph. (Other algorithms surveyed were similar)

- Example: One Markov chain directed graph of order 40 contained > 4×10^8 minimal s-t cut sets even though related TPM was sparse (required > 193 hours to compute).

→ ***Minimal s-t cut set enumeration algorithms may not be computationally efficient for large Markov chain problems***

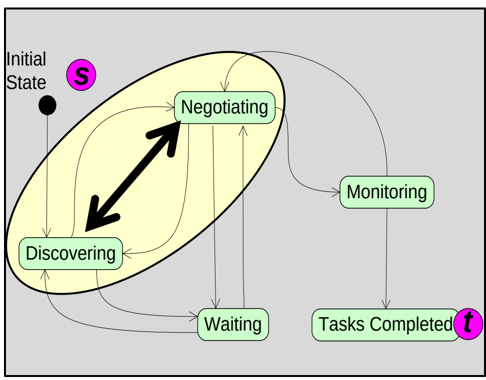
Using the node Contraction algorithm to find minimal s-t cut sets

Finds minimal s-t cut sets probabilistically → though not guaranteed to find all
(Also, <u>*finds multiple transition s-t cut sets*</u>)

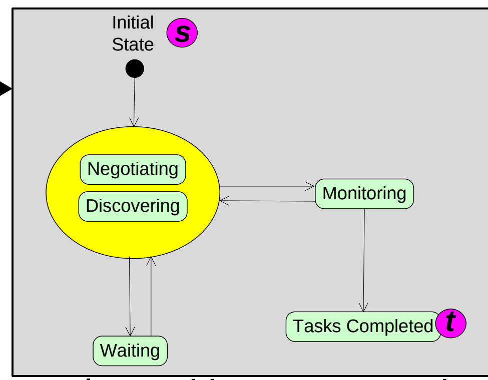
<u>Overview of steps in single repetition</u>.

Psuedo code in (Dabrowski, Hunt, and Morrison, 2010)

Minimal s-t cut set

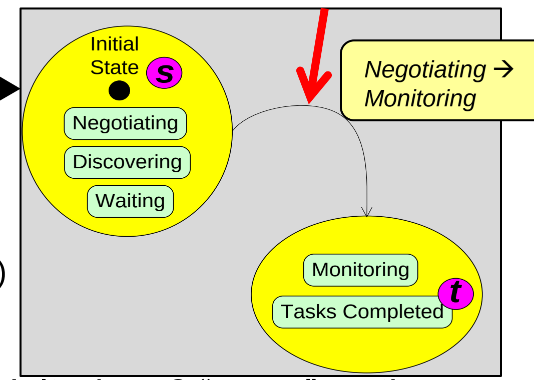


Randomly choose 2 connected vertices to contract.



Replace with new vertex that assumes edges of both contracted vertices

Repeat steps. (Ensure ***s*** and ***t*** are not in same contracted vertex)



Halt when 2 “mega” vertices remain. Assumed edges between “mega” vertices are cut set.

- Because selection of vertices to contract is random, multiple repetitions produce different results, yielding collection of cut sets
- For single repetition, best algorithms find a minimal s-t cut set in $O\ |V|^2$ *in undirected graphs* where $|V|$ is the number of vertices (Karger and Stein, 1996).
- Computational cost can be bounded by limiting number of repetitions

Experimental results of applying node contraction

- Chose four large problems, for which minimal s-t cut sets could be computed by cut set enumeration algorithm of (Provan and Shier 1996) in reasonable time.
- Compared node contraction and cut set enumeration algorithm to see if node contraction could find the ***most critical cut sets***.
- Criticality of ***minimal s-t cut sets*** determined by three ranking criteria (sorts A, B, C), based on idea that cut sets with fewest transitions were most critical → *related to most likely failure scenarios*.

		Minimal s-t cut set enumeration		Proportion (in %) of 100 top-ranked minimal s-t cut sets ranked by criteria A, B that were found by the node contraction algorithm							
Number	Order	Number of cut sets	Time (in hours)	After 10,000 repetitions				After 100,000 repetitions			
				Time	Sort A	Sort B	Sort C	Time	Sort A	Sort B	Sort C
1	50	530,432	332 s	640 s	80	100	96	---	---	---	---
2	50	28,230,288	21.6	171 s	93	98	65	1710 s	99	100	99
3	50	27,242,634	36.0	218 s	67	100	100	2288 s	88	100	100
4	40	422,060,801	193.6	106 s	30	80	62	1051 s	37	100	100

Result: node contraction could find most critical cut sets, with exceptions, with further 2x reduction in computational cost

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Conclusions

- Approach to finding critical transitions & failure scenarios in DTMCs uses combination of techniques *not previously reported*
  - Time inhomogeneous representation to capture change over time
  - Markov simulation and quantitative performance analysis (thresholds)
  - Minimal s-t cut set analysis

- Results show potential of minimal s-t cut set analysis to identify critical transitions and related failure scenarios at reduced computation cost
  - Generally 2x less than exhaustive perturbation of all combinations in TPM

→ ***Indicates potential for predictive use***

- For larger problems, node contraction algorithm shows potential to find critical transitions through reduced search, though needs further investigation

- Areas of further work
  - Investigate other approaches to finding cut sets in large problems (ex. other node contraction algorithms (Karger and Stein, 1996), min-max flow algorithms, & eigensystem analysis (Hunt, Morrison, and Dabrowski, 2011)
  - Investigate applicability to other domains.

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