How to Weigh Everything from Atoms to Apples Using the Revised SI

Jon R. Pratt

Abstract: The fact that the unit of mass might soon be derived from the Planck constant, rather than from an artifact standard, can seem daunting and downright baffling when viewed from the vantage point of our day to day perception of mass. After all, at measurement levels that register with our human senses, the connection between the quantum mechanics of Planck (atoms) and the engineering mechanics of Newton (apples) is less than obvious. However, as the physicist Richard Feynman famously observed, "there is plenty of room at the bottom", and our need to quantify the mass of objects isn't always limited to the familiar quantities we encounter in the produce section of our grocery store. Here, I explore the connection between mass and the Planck constant and suggest that a benefit of deriving the unit of mass from a fundamental constant is that it is inherently more scalable than the present artifact. For example, scientists and engineers working at the forefront of measurement science are increasingly pushing the boundary on what we consider a measurement of mass. In fact, a group now claims to have measured the mass of a cesium atom to within well below a yoctogram, which is below the mass of a single proton. This unit of mass is a submultiple of our present artifact kilogram so small that it requires 27 zeros after the decimal point before it even registers as a significant digit! How are such things possible? Why would you try? Can we even conceive of a traceable yoctogram? To begin grappling with these questions, I will attempt to guide you through the physics of Newton and Planck and, I hope, shed some light on how we can weigh everything from atoms to apples in a revised SI based on fundamental constants.

1. Introduction

The International System of Units (SI) may soon be revised so that all quantitative measurements can be based on fixed values of physical constants [1]. This proposed scheme for deriving the units of measure is a natural progression from ideas that have been with the SI since its conception (to appreciate the scope, the interested reader should consider references [2, 3] for general and entertaining accounts of the history of the SI and its coming revision).

The unit of length, for example, is already successfully tied to the speed of light, *c* (Alan Steele, the director of Measurement Science and Technology for the National Research Council, Canada walked through this as part of his presentation at the NCSLI conference in 2011 [4], and an interesting and technically thorough historical account can be found in [5], which points to a number of the seminal reviews by the giants of the field). Briefly, the process of creating our present unit of length began when general consensus emerged over the course of the last century that the speed of light in vacuum is fixed. Such consensus was attained at a time when observations of *c* with respect to the prevailing length and time standards still showed variability. The generally accepted view was that it was not actually *c* varying in the experiments, but our ability to use the standards with which *c* was being measured (talk about a measurement dominated by Type B uncertainty!). Once the notion that the speed of light is an absolute constant gained traction, its exact numerical value was established as a constant in units of meters per second, giving metrologists a very useful ratio between the units of measure for length and time with regards to the physics of light (or more generally electromagnetic waves) yielding a new definition of the meter in 1983.

This gradual evolution in thought and practice laid the foundation to measure a length in terms of meters by simply (simple being in the eye of the beholder here!) using a beam of light and a clock. Which begs the question: what do we use as a time base for our clock? Michelson, who pioneered many elements of this scheme, used the frequency of a tuning fork calibrated by the frequency of a pendulum whose oscillation period (the inverse of its frequency) was fixed in terms of the length of time of one mean solar day! Notice that a time base is simply the inverse of the frequency of some fixed periodic event. Then, the time base was the periodic behavior of planet earth. Today, the time base is the periodic behavior of energy associated with the electronic state of a Cesium atom, a so-called atomic clock. As you might imagine, measur-

Author
Jon R. Pratt
jon.pratt@nist.gov

Quantum Measurements Division
National Institute of Standards and Technology
100 Bureau Drive
Gaithersburg, MD 20899-8171

ing length by using the time of flight of light pulses measured in terms of a fluctuating atom is not always terribly convenient. In practice the fundamental relationship between the length and frequency of light waves, as expressed by $c = \lambda\nu$, is more commonly exploited, where λ is the wavelength and ν the frequency of the light, respectively. Clearly, if we know the frequency of a beam of light, then we also know its wavelength, and the light itself becomes an effective length standard by virtue of knowing the exact, agreed upon value for *c*. Importantly, the frequency of light can in turn be determined with respect to the frequency of our atomic clock, which is just the reciprocal of its period. This frequency has been defined as the primary physical invariant in our system of measurement; the ground state hyperfine splitting frequency of the cesium 133 atom, $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$ (reference [5] can also walk you through time, so to speak). This is an example where the present SI builds up a unit of measure working from physical constants, rather than physical artifacts.

A significant hurdle to duplicating this paradigm and basing all of our units of measure on physical constants is the historical problem of picking a constant for mass that is both believably invariant and technically accessible. General consensus is that the most promising approach for establishing a unit of mass in terms of a fundamental invariant of the universe is to exploit the connection between mass and the Planck constant, *h*. It is worth mentioning that most scientists now accept that *h* is in fact a constant, but that far fewer scientists are confident that we have successfully measured its value with sufficient accuracy ([6] offers a detailed review of the physics and the state of the art, parallel to what will be considered here)!

For now, let's agree that the Planck constant is likely a constant, and that its value will soon be measured to the desired accuracy within our present system of units. The questions that I wish to explore here are:

a. What is mass and how does it relate to the Planck constant, *h*?
b. How do we weigh an atom in terms of *h*?
c. How do we weigh an apple in terms of *h*?
d. Can an instrument realize and scale the unit of mass?

As it turns out, answers for many of these questions are linked to some of the most famous equations and events in science, and the impact is no less than the achievement of a system of units capable of seamlessly unifying the measurement of all mass and inertial phenomena from the subatomic to the cosmological.

2. What is Mass? And How Does it Relate to the Planck Constant, *h*?

Since Newton first described the physics and mathematics of falling apples, our concept of mass has been firmly rooted in his now familiar expression

$$F = ma, \tag{1}$$

where *F* is a force acting on an object, *m* is the inertia, or mass of the object, and *a* is its acceleration.

Mass, from the vantage point of Newtonian mechanics, is a resistance to motion. It can also be a source of gravitational attraction, according to another of Newton's famous observations about apples

$$F = \frac{Gm_1m_2}{r^2}, \tag{2}$$

where this time *F* is the gravitational force of attraction between one object of gravitational mass m_1 and another of gravitational mass m_2 separated by a distance *r*. For this discussion, we will assume the equivalence of inertial and gravitational mass, which is still the subject of experimentation, but has been proved to the level of a few parts in 10^{13}.

All we need in order to create a standard of mass from the Newtonian viewpoint of inertia is a known force, and standards of length and time with which to measure acceleration. Standards of length and time, as we saw in the introduction, have evolved to a point where they are based on fixed values of physical invariants, namely the speed of light and the atomic frequency of the cesium atom. But what can we use as a known force? This turns out to be a very difficult question to answer, and one that we will return to throughout this article. For now, simply consider that you can't use the force of gravity, since it depends on mass. Instead, the effect of gravity has been used for centuries to compare the mass of objects relative to one another by comparing the forces gravity exerts on the objects as they balance a beam about a fulcrum; the classic equal arm balance.

Armed with an equal arm balance (or its modern equivalent, the compensation balance), we can avoid the problem of finding a known force by simply declaring a certain object to be of unit mass and comparing everything else to it using a balance. Of course the object needs to be very stable (as must the balance), and there should be a recipe for making copies of it, so that this object, which we will call an artifact standard of mass, can be replaced in the event it is damaged through use. Most of you will recognize this as the approach to the measurement of mass that has been in use for over a century, with all measurement of mass defined in terms of one artifact (and its copies), the International Prototype Kilogram, or IPK.

Intuitively, mass has also always been thought of as an amount of substance, or more colloquially, an amount of "stuff". From Newton's viewpoint, the way to measure the amount of stuff in an object is to apply a force and watch the object move. As we've seen, lacking a known force, we simply balance objects with respect to a standard object using gravitational torques on lever arms. We don't even need to measure the acceleration. We simply observe that if the balance doesn't move, the gravitational force on the unknown object must be the same as the gravitational force on our standard, and therefore the unknown must have the same amount of stuff as the standard.

From a chemist's viewpoint, this is highly unsatisfactory, and a better way to measure the quantity of stuff in an object is to examine its atomic constituents, and then count how many of these "indivisible" elements there are in the whole. So many atoms of this, plus so many atoms of that, and with a little math you can assemble an entire object. From the chemist's viewpoint, the trick is to determine the mass of an atom. Everything else is just ratios. Clearly, just as we observed with regard to the Newtonian viewpoint, it is possible to declare one object, an atom in this case, as a standard (carbon-12 is the IPK of atomic mass units) and ratio ourselves to the mass of everything.

Naturally, at some point chemistry and mechanics must reconcile, and this is the path that leads to the Avogadro number, which we will take up in more detail later. For now, we simply recognize that we can't declare an atom to have a particular standard value of mass, and the IPK another. We will need an experiment that relates the two in some fashion.

Up until fairly recently, these two views, mass as inertia, or mass as amount of stuff, captured the totality of our understanding. But in the early 1900's our model of the physical universe took a relative quantum leap, so to speak, and a new way of thinking about mass in terms of energy was introduced which has gradually called into question some of our most basic ideas about matter as substance that can be discretized. In the remainder of this section we explore how this contemporary viewpoint might answer the question, "what is mass?" and introduce the central theme of how mass relates to the Planck constant.

For those of us who did not major in physics, the last encounter with the Planck constant may have been a required course somewhere in a distant academic past (I find myself rereading chapter 42, "Light and Quantum Physics," of my aging copy of Halliday and Resnick fairly regularly these days [7]). So it's useful to recall that the Planck constant (symbol h) is a physical invariant that arose early in the development of quantum mechanics, when Max Planck grappled with data from cavity radiation experiments.

Classical physics of the time could not describe the spectral radiancy of a cavity as a function of wavelength. Something new was required to circumvent the so-called ultraviolet catastrophe (models predicting infinite radiancy as wavelength decreased), and, in a bold bit of empirical insight, Planck began by assuming that the energy E observed in a cavity was governed by the atoms on the cavity walls. He postulated that the atoms behaved as tiny electromagnetic oscillators, each with a specific frequency of oscillation, emitting and absorbing energy to and from the cavity, giving rise to an equilibrium radiancy. The radical notion that he introduced was that the energy of the oscillators was not continuous, but was quantized in terms of the oscillator frequency and a constant that now bears his name. In fact, since Planck's announcement to the Berlin Physical Society in December of 1900, it has become increasingly clear that all oscillators ranging from atoms to microelectromechanical systems (see [8] for just one example of folks probing the quantum behavior of microscale objects) cannot have just any value of energy, but are constrained to have only values of energy given by

$$E = nh\nu, \tag{3}$$

where E is the energy of the oscillator, ν is its frequency, n is a number that can take on only positive integer values and here is called the quantum number, and h is, of course, the Planck constant. Planck's presentation of the equation above heralded the beginning of quantum mechanics, and provides the first piece of a puzzle connecting h to mass.

Another key piece of the puzzle connecting h to mass is found in Einstein's 1905 paper on special relativity, which yields the famous equation relating the rest mass of a particle to its total energy, or

$$E = mc^2. \tag{4}$$

As knowledge of quantum mechanics and relativity gained momentum in the early 1900's, it became evident that in certain limiting cases, the particle having mass m in Einstein's expression of relativity, might also be thought of as having a single quantum of energy, or

$$E = h\nu. \tag{5}$$

In this limiting case, we see that two famous equations for energy both describe the energy of the same object, and it is fairly easy to perform some algebra and conclude that the Planck constant and mass are linked, at least in terms of scaling, according to

$$m = \frac{h\nu}{c^2}, \tag{6}$$

where ν now corresponds to the frequency of a photon whose energy is equal to the rest mass equivalent energy of the particle in question. This is the so-called Compton frequency, and it is used in expressions from quantum mechanics that describe the conversion of mass to energy and the interaction of light (or, more specifically photons) with particles.

Summing things up, Nobel prize winner, Frank Wilczek, in his book *The Lightness of Being*: *Mass, Ether, and the Unification of Forces*, somewhat whimsically suggests that Einstein's Second Law (Chapter 3 of the book) ought to be

$$m = \frac{E}{c^2}, \tag{7}$$

and asserts that mass is really most deeply understood as energy. So according to modern physics, the answer to the question "What is mass?" is simply that mass is energy! And as we've seen, according to the quantum mechanics of Planck, energy is quantized in terms of a frequency of oscillation. To quote Jerry Lee Lewis, "There's a whole lotta shakin' goin on!"

2.1 How do we Weigh an Atom in Terms of *h*? Measure its Frequency!

The revolution in our understanding of the complex quantum nature of reality witnessed over the last century has lead to the exciting possibility that mass can be accurately expressed in terms of frequency, at least at the atomic scale. Of course it is far from trivial to measure the frequency of a matter wave, and it is only possible because of some relatively recent ground breaking work in the fields of optical trapping and frequency combs (including some Nobel prize winning work at NIST!).

To appreciate the challenge, consider that the rest mass of a very light particle, the proton, is on the order of a yoctogram, or 10^{-27} kg, so that plugging in values for c and h and solving backwards for the Compton frequency of the proton we find $\nu = 1.36 \times 10^{23}$ Hz! As a point of reference, typical red laser light has a frequency of only 4.74×10^{14} Hz. To date, the Compton frequency has been far too high to measure directly, but a team recently reported making an indirect observation of the Compton frequency of a Cesium atom by using a combination of atom interferometry and frequency comb technologies [9]. The title of this paper "A Clock Directly Linking Time to a Particle's Mass," is a harbinger of the future.

Within the context of planned revisions to the SI, where h becomes a fundamental constant of fixed numerical value, the results of Compton clock experiments [9] and atom recoil experiments (see Fig. 1 for a schematic explanation of atom recoil) that measure the ratio h / m of atomic particles [10, 11] could be used to fix the mass of atoms, with the potential for atomic clock like stability and accuracy; a traceable unit of mass at the scale of atoms. The preliminary experiments already claim the ability to measure the mass of a cesium atom in terms of a fixed value of h to four parts per billion (roughly 2.2×10^{-25} kg $\pm$ 10^{-33} kg)! The upshot is that in the proposed revised

SI, atoms can be directly and accurately weighed in terms of the SI unit of mass based on the fixed value of the Planck constant *h*, the fixed value of the speed of light *c*, and an experimentally observed frequency measured in terms of another fixed constant, the hyperfine transition frequency of the Cesium atom, $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$.

Why should we care about the mass of atoms? Because increasingly the science of molecules and atoms informs how we approach the development of new technologies. In particular, much of today's applied physics research asks the question, where do quantum and classical mechanics overlap, and how might we exploit this overlap to create new technologies? Every sensor imaginable is being re-imagined to explore how its performance can be coaxed to achieve quantum limited behavior (and beyond, in the case of quantum oddities like entanglement and squeezed states) as we shrink its size, or chill it down, or both. The manipulation of matter at the atomic scale is already an integral part of modern manufacturing (think of the semiconductor and pharmaceutical industries, e.g., your cell phone and your daily meds…), but as has been famously observed over the years, you cannot make what you cannot measure. Today, the width of a "wire" in a modern computer chip is measured by taking its image using an instrument called a Tunneling Electron Microscope and then literally counting the number of atoms across its width.

As we begin an era where technologies ranging from solar panels to targeted drug therapies will exploit the optical, electrical and mechanical properties of matter on size scales where we have literally counted and controlled the number and type of atoms involved, it becomes increasingly important to have a system of measurement that can bridge between the physics of Planck and the physics of Newton. With this in mind, it appears the proposed changes to the SI might be well and good for atoms, but what do they mean for apples?

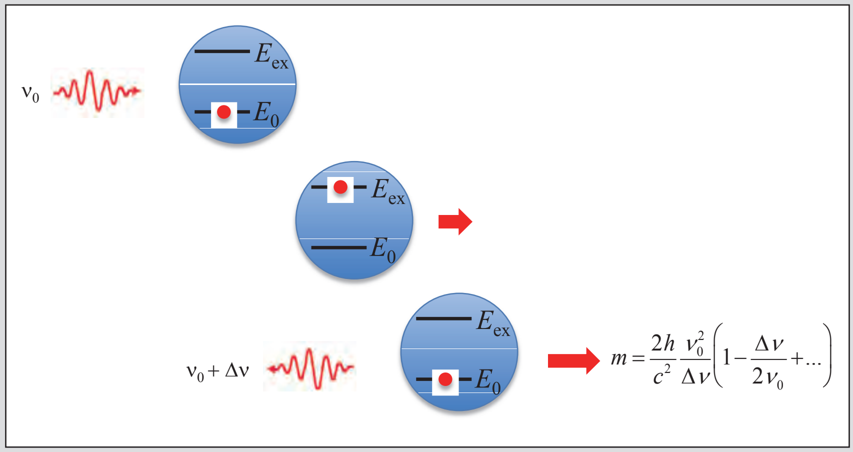

Figure 1. The above is a cartoon depiction of an atom recoil experiment to determine the mass of an atom. At the upper left of the cartoon, light at frequency ν_0 (right moving photon wave packet in the upper left hand corner) interacts with an isolated atom having energy E_0 and sitting at rest. The photon is absorbed, causing the energy of the atom to increase to E_{ex} and to begin moving to the right with a velocity indicated by the red arrow. At the lower right, the atom emits a photon and returns to initial energy E_0, but the velocity is now doubled, owing to the momentum transferred from the ejected photon (the recoil). The motion of the atom leads to a Doppler shift in the frequency of the exiting light, $\Delta\nu$, which can be used to compute the mass of the atom as shown.

3. Bridging Between Atoms and Apples in the Existing SI

We saw in Section 2 that the connection between mass and frequency at the atomic scale can be exploited to effectively weigh an atom in terms of fundamental constants of nature, namely *h*, *c*, and the hyperfine transition frequency of the Cesium atom, $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$. We also noted that modern technologies are increasingly exploiting the ability to manipulate matter at atomic scales, so that knowledge of the optical, electrical, and mechanical properties of atoms, molecules, and collections of atoms is of increasing importance to science and its ability to drive next generation technologies. But we still live in a physical world where the majority of products and measurements occur at far more classical or human scales, and the question remains as to how revising the unit of mass and basing it on the Planck constant will impact our measurement of everyday objects, like apples. Setting that question aside for the moment (we'll take it up in earnest in Section 4), it is helpful to first take a look at how we weigh an atom in terms of the existing definition of the unit of mass, in other words, by considering how we compare the mass of an atom to the International Prototype Kilogram, IPK.

3.1 Weighing with Artifacts and the Accumulation of Uncertainty

The traceability of all weighing operations in our present SI begins with an artifact, the IPK that realizes the unit of mass at a magnitude of a kilogram. This artifact and the unit of mass are one and the same, and in order to create submultiples of this unit of measure, copies of the IPK must be made, as well as a series of ever smaller artifacts that when stacked back together sum to a magnitude of a kilogram (typically beginning with a set consisting of 500 g, 200 g, 200 g, and 100 g objects). With these artifacts in place, a weighing design is executed using a mass comparator or compensation balance to record the sums and differences of these various copies and submultiples of the original artifact to compute least squares estimates for the relative mass of each object in terms of the original IPK. This process is repeated at each order of magnitude employing ever smaller series of masses. Weighing designs, such as the one illustrated in Fig. 2, which is reproduced from [12], can be performed with amazing precision and accuracy. Still, as noted in [12], uncertainty inevitably accumulates with each series, so that by the time this subdividing process reaches the level of milligram sized artifacts, the relative uncertainty in terms of the original kilogram sized artifact is on the order of a few parts in ten thousand, and speculatively might be on the order of unity by the time we would reach an artifact the size of a microgram. How then do we bridge nearly another twenty orders of magnitude to get to the mass of an atom in this system?

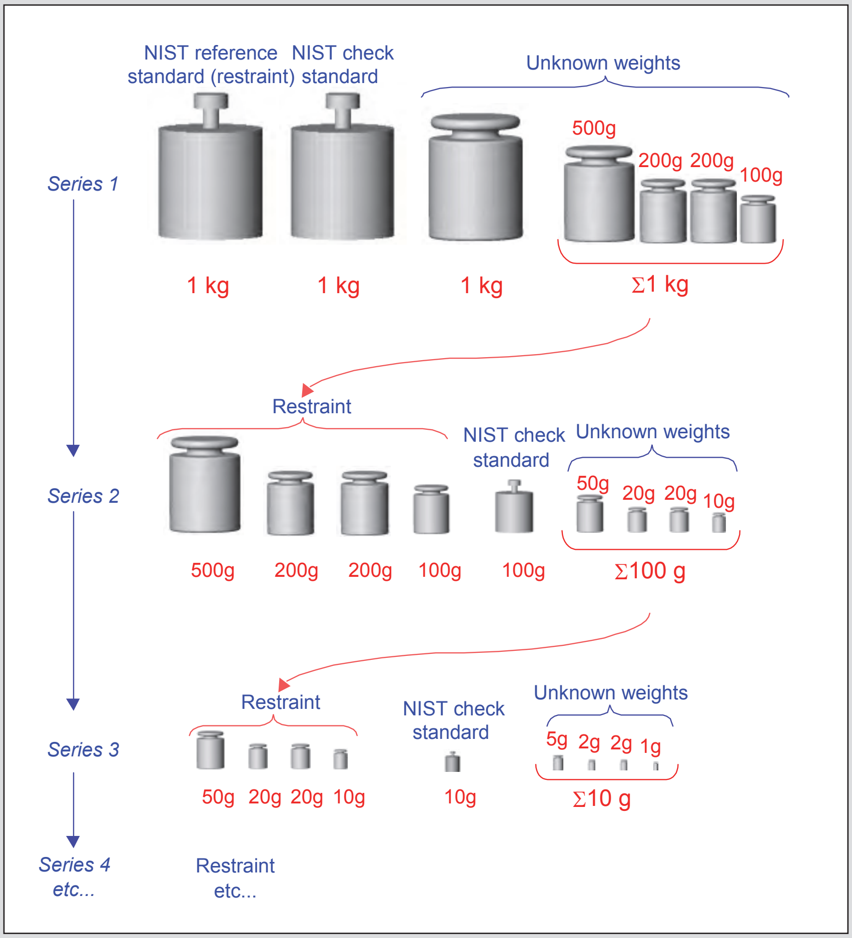

Figure 2. A schematic showing a typical weighing series reproduced from [12]. The total mass grows smaller with each series of weighings, with the unknown weights in any series being determined in terms of the known values from the previous series. At the top of the design, the reference standard can be accurate to within a few parts per billion, but as the series reaches levels of a milligram the resulting calibration is accurate to only a few parts in ten thousand.

3.2 The Avogadro Number, a Silicon Crystal, and Atoms as Artifacts

It turns out that we can still subdivide the IPK using a series of artifacts the sum of which is a kilogram. The trick is that the artifacts in this case are atoms, and the way that they stack up to a kilogram is by forming a perfect, single crystal of silicon. This is the ultimate expression of the chemist's viewpoint introduced in Section 2 and the essence of the International Avogadro project, where instead of starting at the bottom, and determining the mass of an individual atom directly (as we might do with a Compton clock), the process instead begins at the top, with a copy of the IPK that is a perfect sphere of silicon. The question then becomes "how many atoms are in the silicon sphere?", and the answer (after several decades of work [13]) is related to the Avogadro constant N_A.

The Avogadro constant relates any quantity at the atomic scale to its corresponding macroscopic scale, and it can be written as

$$N_A = \frac{nM}{\rho a^3}, \qquad (8)$$

where n in this case is again a positive integer, this time corresponding to the number of atoms per unit cell of the crystal, and r, M, and a are the density, molar mass, and lattice parameter, respectively. The determination of the Avogadro constant using carefully manufactured silicon copies of the IPK has been a long, and exceedingly challenging project involving many workers around the globe, but knowledge of this number provides a powerful link between the world of atoms and molecules and the physical world in which we live.

For example, the efforts of the Avogadro project have made it possible to state the mass of a silicon atom in SI units as

$$m_{at} = \frac{M}{N_A} \; . \qquad (9)$$

All that is required in order to compute the mass of the atom in terms of the SI is knowledge of its molar mass (more complicated than it might sound, since it involves determining the isotopic abundance of the given sample) and the Avogadro constant. The relative uncertainty of the Avogadro constant listed in the most recent CODATA recommended values of the physical constants is 45 parts per billion [14], and this is roughly the uncertainty of the mass of a silicon atom in the current SI.

3.3 Atoms, Apples, and Things in Between

As we've seen so far, all appears well and good with our knowledge of the atomic mass of silicon within the current SI, and we might rightly wonder what all the fuss is about with regards to revising the system of measurement, since it already covers everything from atoms to apples, at least as far as mass goes. This is a topic of considerable debate, but to motivate a view towards change, it helps to focus on some of the limitations of the present system.

First, it is worth noting that it took a team of national metrology institutes (NMIs) working around the globe three decades to achieve a system capable of bringing the uncertainty in the expression of the SI traceable mass of a silicon atom to the present level of 45 parts per billion. The groups involved in this work hope to push this to the level of perhaps 20 parts per billion over the next three years, but at that point the project will have pushed the work as far as is thought humanly possible (at least within the budgets the world's governments are willing to expend on such tasks; they do have countries to run). And yet, Compton clock and atom recoil experiments have already yielded an estimate of atomic mass ten times more precise, if only we accept the entirely reasonable assumption that the Planck constant is a constant having a fixed value.

A second, perhaps more practical, reason to consider rethinking the unit of mass is that subdivision of the unit of mass typically leads to the accumulation of uncertainty we discussed in Section 3.1. Through the heroic efforts of the Avogadro project it is possible to say something about the mass of atoms using a silicon sphere roughly 90 mm in diameter, but this is a scaling trick based on the ability of a silicon crystal to act as a "low-noise amplifier" [13]. Crystalline uniformity allows the Avogadro project to traverse some 27 orders of magnitude using what can creatively be interpreted as a weighing series where the summation is of an Avogadro sized number of single atom masses. Unfortunately, this atom amplification trick has yet to be exploited at other size scales, owing to the extreme technical difficulties associated with obtaining absolutely pure samples, processing the samples so as to know the fraction of the various isotopes in the sample (those neutrons add up!), controlling the purity of the surface of the crystal (everything tends to form an oxide, or as the musician Neil Young put it, "Rust Never Sleeps"), and then accurately measuring the volume of the bulk sample in terms of the meter. For now, suffice to say that the technical hurdles associated with producing Avogadro style silicon masses, other than the handful of copies of the IPK already in existence, are unlikely to be surmounted by the private sector, and that the NMIs capable of such a feat are still squarely focused on understanding things at the size scale of a kilogram.

What we have is an existing system of units that can get from apples to atoms, but that for a variety of technological reasons struggles to provide accurate, precise determinations of mass at scales of micrograms and below, at least until the submultiple of the IPK is back to being an individual silicon atom. On the other hand, the proposed new system has the potential to connect the mass of atoms to the most accurately measured quantity in recorded history, atomic frequency, via Compton clocks and atom recoil experiments. Questions remain however, as to what good this might be for measuring apples, and whether or not a revised SI offers us any hope of improving the certainty with which masses on the order of micrograms can be determined. In other words, it's time to consider how the forces of Newton can be accurately expressed in terms of the energies of Planck.

4. Balancing Newton in Terms of Planck

In Section 2 we learned that mass can be linked to frequency at the atomic scale as a consequence of the fundamental particle wave duality described by quantum mechanics. This presents the exciting possibility of fixing the Planck constant and realizing the mass of individual atoms with the resolution, stability, and accuracy of atomic clocks using the technologies developed for atom recoil and Compton clock experiments. In Section 3 we learned how the Avogadro project has scaled the existing system of mass to the level of atoms through an amazing international effort that capitalized on the natural perfection of silicon crystals to effectively count the atoms in a kilogram. At the end of Section 3, we grappled with the technological difficulties of duplicating this feat to realize the unit of mass at other nontraditional scales, such as at the level of micrograms and below. This sets the stage to consider other methods for scaling mass, or how the forces of Newton can be accurately expressed in terms of the energies of Planck.

4.1 Balances for Comparing Mass

When we think of mass in the present system of units, the concept is firmly rooted in a Newtonian viewpoint, where force equals mass times acceleration. In principle, if we can apply a known force to an object, mass can be determined simply by recording the resulting acceleration. However, known forces are hard to come by, so instead of realizing mass from a known force and a measured acceleration, the unit of mass is disseminated through a relative measurement that compares the gravitational mass of an object to the gravitational mass of the IPK, taking advantage of the weak equivalence principle which allows us to treat gravity as acceleration.

Gravimetric mass measurement is carried out using comparison weighing (see, Jabbour et al. [12]) where a balance compares the effect of gravity on an unknown artifact X to the effect of gravity on the IPK, or a suitable reference copy, R. In mathematical terms, the comparison (accounting for the buoyancy of air) can be described as

$$C_R = m_R g_R - \rho_a V_R g_R, \tag{10a}$$
$$C_X = m_X g_X - \rho_a V_X g_X, \tag{10b}$$

where C_R and C_X, m_R and m_X, V_R and V_X, and g_R and g_X are the balance reading, the mass, and the volume of the reference R and the unknown X, respectively, g_R and g_X are the local accelerations of gravity at the centers of gravity for the reference R and unknown X, respectively, and ρ_a refers to the air density during the measurement. As written here, the comparison is assumed to be aligned along the axis of gravity, and this alignment is assumed to remain the same during the comparison.

Although the acceleration of gravity can be measured accurately in terms of c and $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ [15], the goal for now is simply to compare the mass of one artifact relative to the other, not to actually quantify the forces. In most cases, g_R and g_X are sufficiently equal to be factored out of the comparison. Then the relative magnitude of the forces is obtained by considering the difference between C_R and C_X, or

$$m_X = m_R - \rho_a(V_R - V_X) - C, \tag{11}$$

where $C = C_R - C_X$, and the air density is assumed to be constant during the weighing. This is an example of the most basic weighing process in the present system of units.

Gravimetric mass metrology is simple and robust but utterly and completely wed to one artifact in Paris. As such, it involves the rather inconvenient and complicated shipping of artifacts around the globe in order to facilitate the necessary chain of comparisons back to the only actual realization of mass, the IPK. The miraculous achievements of the Avogadro Project aside, the existing artifact mass definition also scales poorly, as we saw in Section 3.1. This presents a far more genuine problem that looms for nascent nano and molecular technologies. But perhaps the most glaring flaw is that the underlying physics is showing its age. Gravimetric mass metrology enshrines notions leftover from a previous century that are now known to be patently false: that the mass of an object depends only on the number of atoms present (this ignores binding energy and relativistic effects, an inconsequential thing to Newton, a big thing to Einstein and Planck) and, more practically, that the mass of a macroscopic hunk of metal washed and handled by human hands is somehow a constant of the universe. So what is the alternative?

4.2 Balancing the Electrical and Mechanical Units: the Watt Experiment

To ease ourselves into the complex world of trying to redefine the unit of mass, it helps to dig a little deeper into some of the messy finicky details of balances used to execute a gravimetric weighing, and to appreciate that such balances are at the heart of establishing a consistent

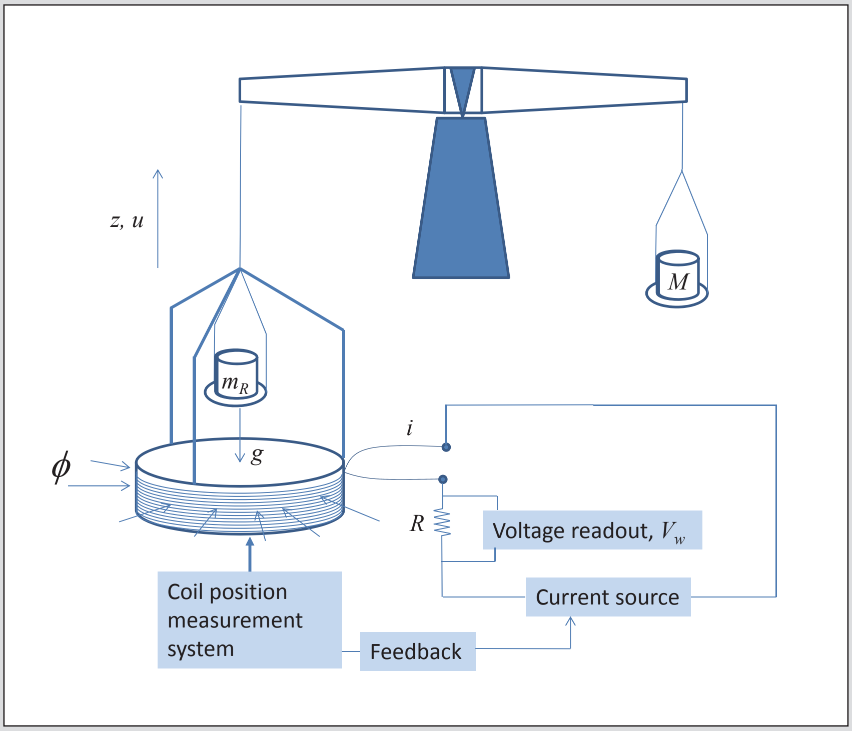

Figure 3. Above are the basic elements of a compensation balance. In this cartoon, the balance is an equal arm type where the additional weight of the moving coil system that applies the electromagnetic compensation force is offset by a tare mass *M*. When a mass, m_R is placed on the balance, the beam starts to tip about the fulcrum, causing the coil to move. A highly sensitive position sensor detects the motion immediately, signaling the feedback control to adjust the current and restore the coil position to null, effectively canceling any motion of the balance beam. The compensation current is proportional to the mass, and is read by measuring the voltage drop across the resistor, *R*.

set of units of measure. For example, consider the compensation balance shown schematically in Fig. 3.

Compensation balances are so named because the net downward force exerted on the weighing pan is compensated by an upward electromagnetic force (or electrostatic, or optical, as we'll see later in Section 5) that cancels the downward force to maintain a null position. Early on in the making of precision balances it was recognized that one of the chief limitations of mechanical balances was the fact that motion of the balance mechanics inevitably leads to hysteretic restoring forces, producing weighing errors. As shown in Fig. 3, a solution was to introduce a compensating force using a feedback control system to monitor the balance deflection. The compensation control acts to maintain null, minimizing deflections and the erroneous forces they contribute to the weighing, by controlling the current i that flows in a coil suspended in a magnetic flux f. Conveniently, this compensation current also provides a signal proportional to the mass that can be used as the readout C_x.

For the gravimetric weighing experiments described in the last section, absolute measurements of the gravity, the current in the coil, and the magnetic flux are unnecessary, since the current in the coil merely serves as a readout signal for comparing masses, the other variables being constant on the time scale of a typical comparison. But what if we did want to know the absolute value of the compensating force in terms of the electromagnetic quantities? Could we use this as a known force to directly realize the unit of mass? Can a balance be constructed so that the compensation force can be measured in terms of a set of fundamental constants? The answer is yes, of course, but it takes a good bit of explanation, and the devil is in details that I must mostly brush over here (detailed technical accounts abound, and recent examples are available by some of the pioneers of the field [16, 17]).

Imagine that the balance of Fig. 3 has been constructed so that the coil and the balance mechanics result in the magnetic and gravitational forces aligning perfectly to cancel one another along the same vertical axis in a vacuum chamber. With the buoyant forces conveniently removed by the vacuum pump, the resulting balance of forces can be written explicitly as

$$i\,\frac{\partial\varphi}{\partial z} = m_R g, \tag{12}$$

where the expression on the left hand side of the equation is recognized as a scalar component of the more general Lorentz force law, realized experimentally by painstakingly designing and fabricating a magnet system and balance mechanics that make the effects of the other force components effectively zero.

Now, if the current and magnetic flux gradient responsible for the compensating force can be measured, then the compensating force could in principle be known independent of the mass. Clearly, knowing this force and the local gravity, we could realize the mass of objects placed on the weighing pan directly from the current and gradient measurements. No need for IPK. Everyone who had such a balance could realize the unit of mass themselves, with traceability to the SI through electrical quantities, length, and time (ignoring the magnetic flux, which will conveniently disappear a little later). In principle, the scaling of mass in this scheme would be limited only by the accuracy with which we could realize and scale the ampere, perhaps offering a way to avoid the accumulation of uncertainty associated with subdividing the IPK. As we might guess, this turns out to be quite a bit trickier than it would seem at first glance.

Intuitively, a force should be a force, whether electromagnetic, or gravitational, and the "electrically" determined value of a mass should equal that determined through gravimetric comparison to the IPK. In setting up international measurement systems (well before we had electrical units it turns out, see Maxwell's paper presented to the Royal Society [18]) it has always been understood that the electrical and mechanical units of measure must be defined such that like quantities balance, regardless of their phenomenological origin. This is a big part of what it means to have a consistent set of units. However, the price for this consistency is that the base electrical unit, the ampere, is actually defined in terms of mechanical force, which derives from the IPK. So an electrically determined mass is ultimately linked to a mechanically determined mass, at least in the present system of units, and this would seem to have made for a short and circular trip. The proposed electrical mass is simply a mechanical mass in disguise, at least as far as the present system of units. Nevertheless, the idea of an electrically realized mass is still attractive. Particularly since it turns out there are ways to define an ampere that tie its value to fundamental constants, rather than the IPK. As we will see, when all is said and done, a great deal is possible if we embrace a system of units based on a set of fixed fundamental constants where the Planck constant is a constant (a familiar theme!).

The development of balances to realize the electrical units and establish the consistency of the measurement system has a long history (see [19] for a history of the development of electrical standards) beginning with Maxwell in the 1860s (e.g., [18] again) and culminating in the first absolute value of the ampere in 1934 [20]. Over the years, this so-called ampere balance experiment has evolved, and in 1975 a two-part experiment to relate the electrical and mechanical units through a virtual power measurement was conceived [21]. This experiment, known as a watt balance, began as an attempt to overcome the limitations in measuring the geometrical quantities necessary to establish the magnetic flux gradient in ampere experiments.

In addition to the weighing mode, a watt balance uses a second velocity mode, where the mass is removed and the coil is translated in the same magnetic flux at a velocity *u*. The motion of the coil induces a voltage across the now open terminals of the coil (feedback control is disconnected) such that

$$V = u\,\frac{\partial \varphi}{\partial z}, \tag{13}$$

which is simply Faraday's law of induction.

If the magnetic field present during the experiment is the same for both the weighing and velocity modes, the results can be combined to yield a virtual balance between mechanical and electrical power, so that the basic measurement equation for a watt balance becomes

$$m_R g u = iV. \tag{14}$$

Watt balance experiments make it possible to establish the consistency of the electrical units (i.e., the volt and ampere) and mechanical units (i.e., the kilogram, meter, and second) without requiring the determination of the magnetic flux gradient, which improves the realization of the ampere in terms of the IPK.

In the 1980's the question of unit consistency in watt balance experiments revolved around assuring that the representations of the electrical units (historically banks of batteries and resistors) were maintained and adjusted so that the balance of these virtual powers was as accurate as possible, with the understanding that the electrical units would always adjust to insure consistency, since length and time standards used in characterizing the mechanical power were already based on fixed fundamental constants, and the unit of mass, the IPK, is by definition a fixed quantity. It is worth reiterating that in the existing system of units, measuring the "electrical" force independent of IPK simply isn't possible, because the units used to quantify electrical measurements are themselves based on the IPK as determined in this experiment! This Gordian knot of the electrical and mechanical units loosened a bit in 1990 (or got more complicated, depending on your viewpoint), when it became practical to measure electrical quantities in terms of quantum effects.

In 1990 it became common practice to measure both voltage and resistance in terms of entities thought to be fundamental constants themselves, namely the charge on the electron, *e*, and the Planck constant *h*. Voltage and resistance standards based on the Josephson effect (inversely proportional to the Josephson constant, $K_J = 2e / h$) and the quantum Hall resistance (proportional to the von Klitzing constant, $R_K = h / e^2$) became available, and suddenly it was possible to measure voltages and resistances using very precise quantum phenomena that were codified into a system of conventional units that exist in parallel with the SI (that's right, the electrical units are effectively outside the SI!). The traditional basis of measurement, where the mechanical quantities define the electrical quantities, began to be called into question [22] and the stage was set to finally pursue a dramatic redefinition of the kilogram by fixing the value of the Planck constant.

4.3 Measuring the Planck Constant

As it stands now, in order to balance the watt balance equation, we must effectively test the equivalence, or consistency, of the SI and the quantum electrical effects, leaving the Planck constant as a free variable, since by definition, IPK is invariant (the Avogadro constant offers another approach, but that's another story). Here's how it works in terms of the physics of the electromagnetic compensation balance we have been discussing.

The weighing current *i* is observed using the voltage drop across a resistor, with the voltage measured by comparison to a quantum invariant, the Josephson effect, so

$$V_w = \frac{f_w}{K_J}, \tag{15}$$

where f_w is a microwave frequency used to excite the Josephson voltage junctions. The resistance is likewise measured in terms of a quantum invariant. In fact, we can use a so-called quantum Hall resistance standard, where the resistance of the device is expressed in terms of the quantum Hall effect, so

$$R = \frac{R_K}{n}, \tag{16}$$

where *n* again is an integer, this time having to do with the quantization of the quantum Hall resistance standard (which deep in the theory links up with quantization of energy, of course).

In electrical metrology, we routinely use Ohm's law $V = iR$ to realize, or measure, a current from the quantum standards. In this case the current during weighing is

$$i = \frac{V_w}{R} = \frac{n f_w}{K_J R_K} = \frac{f_w n e}{2}. \tag{17}$$

While the voltage induced (again measured in terms of the Josephson effect) during the velocity mode is

$$V = \frac{f_v}{K_J} = \frac{f_v h}{2e}. \tag{18}$$

Substituting for *V* and *i*, and solving the watt balance equation for the Planck constant, we find

$$h = \frac{4 m_R g u}{f_v f_w n}, \tag{19}$$

where the result, and the complex chain of measurements necessary to achieve it, provides a rationale for understanding the forces of Newton ($m_R g$) in terms of the energy of Planck. Stepping back for the moment, let us consider the relative uncertainties believed to be associated with the standards used to measure the Planck constant in this fashion, since they will determine the ultimate resolution of such an experiment.

The quantum electrical effects used as our standards for measuring the virtual electrical power are known to be exceedingly precise and stable. For instance, a review of Josephson comparison results [23] found comparisons achieving a precision of a few parts in 10^{11}, as well as no evidence contradicting that the relation $K_J = 2e / h$ is exact and universal. The situation with the quantum Hall effect is similar, with a recent comparison between a conventionally fabricated GaAs/AlGaAs heterostructure device and a new graphene device in agreement to within 8.6 parts in 10^{11} [24]. A recent theory paper [25] finds the relation $R_K = h / e^2$ is not exact, but suggests a whopping correction of perhaps 2 parts in 10^{25} may be required at the magnetic field levels employed for metrology!

On the mechanical power side of things, our ability to measure the local gravity and to likewise measure the velocity of a moving coil relies on standards of length and time already exceedingly well expressed in terms of fundamental constants, c and $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs.}}$ Optical frequencies (and hence wavelengths) derived from the cesium standard can be stable to a few parts in 10^{14}. The IPK has no uncertainty, but the IPK cannot be used directly, so one must consider the uncertainty of available artifacts, which is typically on the order of 4 to 7 parts in 10^9. Furthermore, although there is no systematic uncertainty stated for the IPK, this strains credulity, since the IPK is well known to have drifted by as much as 50 parts in 10^9 with respect to its copies.

If the Planck constant is a universal constant, then its value must correspond to some exact fixed number, and by performing enough watt balance experiments of ever increasing precision, you eventually converge on its exact value. Ultimately, such an endeavor can go only as far as the precision of the least precise quantity in the experiment, limited in the end by the stability of the standards used to make the measurements. As the procedures surrounding watt balance experiments become increasingly refined, it is more and more evident that our knowledge of the Planck constant will ultimately be limited by the mass standards used in the evaluation! If this sounds like the situation I described for the speed of light circa 1983 at the beginning of the article, it is because it is entirely analogous.

5. A New System of Mass Where Scaling can be Accomplished with "Scales"

At this point in the discussion, I will take the liberty to declare the Planck constant a constant, and begin exploring how we realize a mass from an instrument, in anticipation of a redefined system of units. In other words, from here on out, I am speaking as if we are working in the new SI.

For starters, it is a simple matter to turn the watt balance equation for the Planck constant around and write the value of any unknown mass as

$$m_X = \frac{h f_v f_W n}{4 g u}. \tag{20}$$

Here then is a method to weigh apples in terms of fundamental constants. But it is not limited to apples. We are at liberty to build other similar balances, whatever size we want, and to use them to derive a realization of the unit of mass from the Planck constant as well (think about the atom recoil experiments way back at the start of this article). And the balances need not be limited to electromagnetic force. The force could be electrostatic, or even optical. All that will change is the nature of the compensating force and the methods required to characterize its strength and direction in terms of the quantum electrical standards. The only limiting factor is our imagination. The watt balance is simply a special case of a balance between forces due to gravity, and those arising from electromagnetic fields where the characterization of the field gradient has been made accurate by a clever use of the reciprocal nature of Maxwell's equations. So far, the most successful balances have been demonstrated for measuring at the scale of apples, but there is already evidence that balances could be constructed to realize mass at the scale of say, apple seeds.

5.1 Electrostatic Force Balances and an Electrically Derived Milligram

The challenge for creating new balances to derive a realization of mass is that they must be constructed so that all the fields and forces align and cancel along the weighing axis with sufficient accuracy to maintain the precision we have come to expect from mass measurements. At the level of apples, the challenge is to do everything at the level of a few parts per billion, in order to be competitive with the existing SI. At the level of apple seeds, however, the present mass system struggles to achieve accuracy better than a few parts in 10^4. Here is a flaw in the existing system that could be addressed more directly in the new system.

Anecdotal evidence suggests there is need for improvement in the measurement of ever smaller masses, particularly to serve the pharmaceutical industries. At the 2013 NCSLI meeting in Nashville, Tennessee, a panel of mass experts from industry and governmental organizations suggested that manufacturers are increasingly interested in the accurate weighing of samples as small as a few tens of micrograms, with one paper presented on the issues of assessing balance performance at the low end of readability [26] and a second presentation that focused on proposed changes to the *ASTM E617* standard, which includes the addition of sub-milligram weights [27]. Unless something changes, we appear to be at the beginning of a new era of ever shrinking mass artifacts. This perceived pressure from the bottom of the mass scale may be causing ripples at the top, as the revisions to *ASTM E617* also include a new ultra-high precision weight class, perhaps aimed at the accumulation of uncertainty discussed in Section 3.

Extrapolating into the future, if we factor in the usual accumulation of uncertainty from all the weighing series between you and the IPK at the level of a microgram, it is quite plausible that we will need to improve the relative uncertainty of a kilogram working standard by perhaps two orders of magnitude if we want to quantify the mass of a microgram sample with even modest uncertainty. Further, we must somehow create a series of weighing designs and artifacts that can be reliably employed to calibrate instruments without risk of bias due to contamination and environmental factors, such as charged particles (think of the lint accumulated in your dryer). A microgram is roughly the size of a speck of dust, so the potential for contamination is high. For that reason, technical approaches that minimize or eliminate the need to handle such tiny mass artifacts are attractive. A balance that can directly read the traceable mass of sub-milligram samples on the factory floor might ameliorate some of these problems, particularly if it can be calibrated using only reference to robust intrinsic electrical, length, and time standards.

By the end of the 1990's, NIST was well aware that a watt balance experiment effectively enables one to compute a compensation force of 10 N with reference to intrinsic standards to within less than 50 parts per billion. Perhaps, we reasoned, a smaller balance could be constructed to realize a micronewton to within a few parts per million, since in principle all of the underlying metrology scales with little loss in relative accuracy (e.g., [28]). In other words, can a balance weigh milligrams with more accuracy by tracing to the intrinsic electrical, length and time standards than through conventional direct sub-division of an artifact kilogram? We were not seeking a mass standard at the time. What we sought was a known, calculable force that could be used as a reference for a variety of nanotechnology instruments, such as atomic force microscopes. We were trying to avoid using a mass, because we believed that at the levels of force we needed to eventually reach, nanonewtons and below, mass artifacts would never be accurate, or stable enough to serve as a deadweight reference. We were also cognizant that many new sensors were based on so-called Micro Electro Mechanical devices, or MEMs, where the readout and actuation were capacitive, and electrostatic, respectively. It seemed to us at the time that demonstrating methods for the accurate realization of electrostatic forces might prove useful to this community. The experiment which we eventually constructed is known as the NIST Electrostatic Force Balance, or EFB for short [29].

The EFB, shown schematically in Fig. 4, is used to compare mechanical and electrostatic forces that act on a cylindrical, variable capacitor in a compensation balance scheme very similar to the one described for a watt balance. Those familiar with electrical metrology in the days before the Josephson volt will recognize this as a variant on the volt balance. One electrode of this capacitor is attached to the weighing pan, and this moving electrode nests inside of another electrode fixed to the stationary balance frame. To a first approximation, the balance of forces acting on the suspended electrode can be expressed mathematically as

$$F_e = \frac{1}{2}\frac{dC}{dz}\left(V^2\right) = m_R g \;, \tag{21}$$

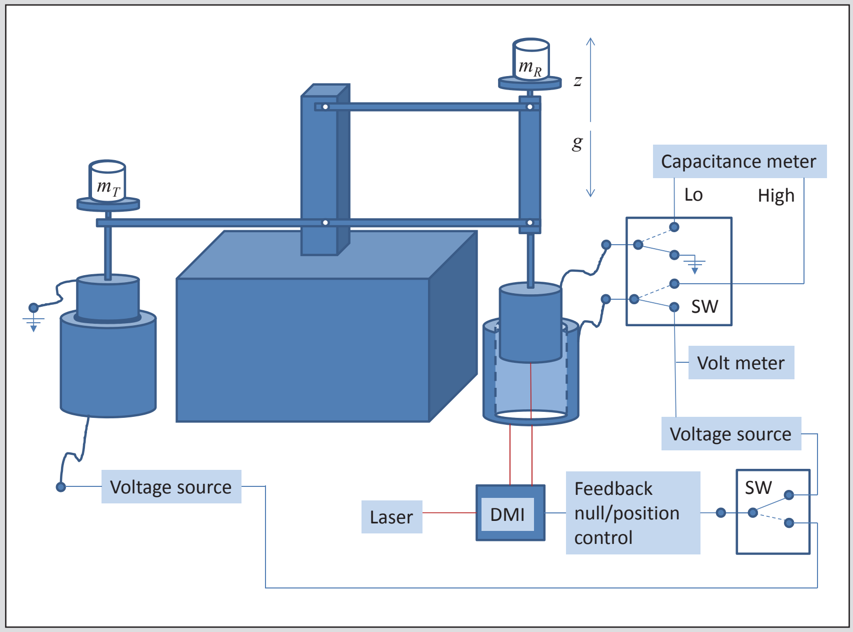

Figure 4. A schematic illustration of an Electrostatic Force Balance for the calibration of masses in terms of intrinsic electrical, length, and time standards. Operation is described in the text.

where F_e is the electrostatic force, dC / dz is the gradient of capacitance between the two cylinders, with dz being the differential displacement of the moving electrode along the balance axis and dC the corresponding differential change in capacitance, V is a voltage applied between the two electrodes, and $m_R g$ is the mechanical force exerted on the weighing pan due to an unknown mass m_R. Motion of the electrodes with respect to one another is measured along the axis z, which, as in the watt balance experiment, must be painstakingly aligned to the motion axis and to gravity, so that all the forces are acting along the same direction (one trick we use to minimize the off-axis forces is to make the capacitor as perfectly cylindrical and symmetric as possible, so that electrical forces in the x and y directions are effectively zero). The experiment is performed in vacuum to improve the accuracy of the electrical and length measurements, and has the added benefit that it eliminates the buoyant forces, which would otherwise complicate the determination of an absolute mass.

As in the discussion of compensation balances in section 4.2, we observe that the electrical quantities used to characterize the compensating force can be measured in terms of intrinsic standards. The voltage is traceable to the Josephson constant, and the capacitance is traceable to the von Klitzing constant. So the compensating force, expressed in the revised system of units, can be known without any reference to IPK. As a result, we can realize the mass of objects placed on the weighing pan. Here's how.

In the first step, or calibration mode, the switch (SW in Fig. 4) is set so that the capacitance meter records the capacitance between the suspended and fixed electrode. The capacitance gradient of the instrument is measured in order to calibrate the instrument's ability to convert, or transduce, electrical voltage into mechanical force. This is done by simply displacing the suspended electrode a measured distance with respect to the fixed electrode (SW connects the position controller to another pair of electrodes to swing the balance arm, as shown in Fig. 4) and then recording the capacitance as a function of the displacement. Of course, both the displacement and the capacitance must be measured using instruments that have been calibrated in terms of appropriate intrinsic standards. We can employ off the shelf capacitance meters and laser interferometers (such as the Displacement Measuring Interferometer, or DMI in Fig. 4) and still improve traceability compared to an artifact mass approach. This is because commercial instruments for electrical and length metrology now routinely transfer their respective dimensional quantities with accuracy and stability of a few parts per million, reliably traced to intrinsic standards across a very large dynamic range.

For the second step, the instrument is operated as a compensation balance and used to weigh objects of unknown mass. Feedback control is used to adjust the voltage on the electrodes so that the signal from the interferometer stays constant as the unknown mass is placed on and off the balance. The voltage necessary to maintain the balance at this null position serves as the readout. Since the voltage is ultimately used to compute the compensating force, it must be measured using an instrument that has been calibrated or is traceable to the appropriate intrinsic standard, a Josephson volt. Again, high-quality multimeters are often calibrated in this fashion, and are available off the shelf with an uncertainty of a few parts per million.

The third and final step converts the voltage readout to mass using the measured value of gravity (which can be estimated using global positioning coordinates to within a few parts per million using the US Geodetic Survey website [30]), the previously measured capacitance gradient, and the equation for the balance of forces. All of the references used to support the measurement are readily available and accurate to within a few parts per million (even in the present SI, it turns out), so that the combined uncertainty of the references is potentially quite low compared to the uncertainty accumulated in trying to subdivide from the IPK. Interestingly, there is no reason you could not employ this approach in the present system of units to leap frog the accumulated uncertainties. But an electrically derived mass has generated little interest among the metrology community; apparently due to the conceptual break with a century's worth of experience using artifacts.

The scaling of mass using an EFB is limited by the accuracy with which we can measure the local gravity and how well we can scale the volt, meter, and the farad. The balance mechanism and electrodes are of course also limiting factors, but their performance can, in principle, always be designed to achieve a desired sensitivity. Confounding forces, due to surface potentials, stray charge, magnetic interactions, mechanical hysteresis and the like must also be accounted for, and mitigated, as necessary. Nevertheless, in 2006, NIST succeeded in weighing a 20 mg artifact in terms of the second, volt, meter, and farad (ohm) with a combined relative uncertainty of a few parts in 10^5 [31]. So at the scale of a milligram, traceability of mass to intrinsic standards has already demonstrated precision that competes with the existing mass scale, and points the way for instruments that could weigh samples into the microgram regime with traceable accuracy, without the need to manipulate ever shrinking mass artifacts.

5.2 Back to Where We Started: Mass and Frequency

Compton clocks, watt balances and the NIST EFB experiment make a strong case that a new class of instruments can be developed to scale mass using traceability to electrical, length, and time standards that have their basis in fixed values of fundamental constants, scaled by frequency and integer multiples of the Planck constant. Broadly speaking, this situation should not surprise us. The view of modern physics is that the amount of stuff in the universe is not discretized in terms of ever smaller pieces of stuff, but in terms of energy, which is related to frequency through integer multiples of the Planck constant. Viewed in this light, it is perhaps inevitable that our system of measurement evolve to one where all dimensional quantities are linked to fixed constants through the observation of various frequencies.

Regarding mass, sometimes the link to frequency will be direct, as in the Compton clock experiments described at the beginning of the article. In other instances, the link will be less obvious, as in the balance experiments, where a variety of frequencies come into play as we quantify electrical measures in terms of the Josephson and quantum Hall effects. In the future, and between the scale extremes represented by Compton clocks and EFB's, from say yoctograms to micrograms, there will be opportunities to measure mass using shifts in the natural frequency of resonant oscillators ranging from trapped ions, to micromechanical cantilevers. Again, even in these systems, the ability to conjure up a known force will prove useful. Here's how it might work.

Recently, accurate forces on the order of piconewtons have been demonstrated in experiments using both electrostatics [32], and photon momentum [33], where the forces were ultimately traceable to intrinsic standards. The forces in these experiments were used to calibrate the absolute force sensitivity of mechanical springs using Hooke's law, or $F = kx$, where k is the unknown spring constant or sensitivity, x is the spring displacement as measured using an optical reference to the speed of light, and F is a known electrostatic force or photon pressure measured in terms of appropriate electrical and length standards. The sensors of these studies were intended to measure atomic forces, but could be used to sense the mass of very small objects. For example, such sensors can be employed to directly weigh tiny particles, using the sensor as a spring balance or scale. Of course, it is also possible to measure the sensor natural frequency by observing its thermomechanical noise spectrum. Knowing the natural frequency, we can back out the effective inertial mass of the sensor using the relation

$$m = \frac{k}{\omega^2} \;, \tag{22}$$

where ω is the natural frequency of the oscillator in radians per second. Having identified the oscillator mass, stiffness, and natural frequency, it is possible to measure the mass of objects placed on the oscillator simply by noting the resulting shift in natural frequency. This is the approach taken by quartz crystal microbalances, and by new sensors being developed to "weigh" cells, viruses, and even single molecules. At present, the traceability of such measurements is, at best, ambiguous. Accurate force, traced to intrinsic standards, offers a path towards clarity as new technologies develop to explore the measurement of mass at ever smaller scales.

6. Conclusions: Redefinition and the Outlook for the Near Future

The redefinition of mass in terms of the Planck constant is still in the future, and it remains to be seen just when plans for revision of the SI will come to pass. For the moment, all eyes are focused on results emerging from the various watt balance experiments and the corresponding efforts of the International Avogadro Collaboration (IAC). The question on most people's minds is whether or not the measured Planck values will agree to within 50 parts per billion, and whether or not one (or more?) of the experiments can succeed at measuring the constant with relative uncertainty below 20 parts per billion. These conditions have been suggested by the International Bureau of Weights and Measure's Consultative Committee on Mass as a benchmark to achieve before redefinition should proceed, and a variety of groups around the globe are working towards satisfying these requirements. We will know better the success of these ongoing endeavors within the next year, as many of the groups have plans in place to announce new numbers in 2014.

Assuming the CCM benchmarks are met before 2015, and redefinition moves forward, the SI could be revised as early as 2017 or 2018. Clearly, watt balances to realize the unit of mass at the level of a kilogram will be few and far between, and other instruments to realize the unit of mass at other scale points are unlikely to exist. How then will mass be disseminated?

It is helpful to step back and remind ourselves that in the existing system of mass we have but one realization of the unit of mass in the entire world (one mass to rule them all!). If redefinition is successful, we will immediately have at least three groups capable of realizing the unit and calibrating artifacts for use as primary standards. Plans are in place to accumulate a pool of these calibrated artifacts at the BIPM. The pool will help average variations that will inevitably occur among groups capable of deriving the unit of mass from the Planck constant, while providing the opportunity for new groups to develop and verify their own methods through comparison to the pool value. In other words, the BIPM will continue to be a source of mass calibration for those working to realize the unit, and those who are unable to derive the unit themselves. In fact, other than at the highest level of realization, the system can persist as it always has, by using artifacts, weighing designs, and compensation balances to disseminate and subdivide the unit of mass.

The day after redefinition, if all goes according to plan, nothing has to change as far as the world of mass dissemination. And yet everything can change with regards to scaling of the unit, since, as we've seen in this article, it will be possible to derive a unit of mass using whatever means is convenient and at whatever scale point desired by using intrinsic standards of length, time, and electrical quantities. More broadly, the entire system of measurement will finally be tied to a set of physical invariants, creating a basis for quantifying the physical universe across all dimensional scales. The scientific appeal is obvious, and perhaps you will agree, the ability to use instruments to realize the unit of mass and thereby weigh everything from atoms to apples has the potential to grow a new sector of metrology, where companies vie to produce economical solutions that accurately quantify ever smaller objects in terms of a vital, new SI.

7. Acknowledgements

I would like to thank NCSLI and *Measure* (in particular Carol Hockert and Michael Lombardi) for encouraging me to share my thoughts about this topic. I also thank my numerous colleagues in the watt balance and Avogadro communities for their conversations and the insights they have shared with me over the last decade. I wish to express a special thanks to Edwin Williams and Clayton Teague, mentors who, perhaps unwittingly, pointed me down this path so many years ago. Helpful reviews of this text by Carl Williams, David Newell, Stephan Schlamminger, and Peter Mohr were deeply appreciated.

8. References

[1] Resolution 1 of the 24th Meeting of the Governing Committee on Weights and Measures (CGPM), "On the possible future revision of the International System of Units," Bureau International des Poids et Mesures, Paris, France, 2011.

[2] R. Crease, *World in the Balance: The historic quest for an absolute system of measurement*, W.W. Norton and Company, Incorporated, New York, 2011.

[3] T. Quinn, *From Artefacts to Atoms the BIPM and the Search for Ultimate Measurement Standards*, Oxford University Press, New York, 2012.

[4] A. Steele, "The International System of Units: a shift to fundamental constants," presented at the *NCSL International Workshop and Symposium*, Washington, DC, 2011.

[5] L. Hollberg, C. Oates, G. Wilpers, C. Hoyt, Z. Barber, S. Diddams, W. Oskay, and J. Bergquist, "Optical frequency/wavelength references," *J. Phys. B- At. Mol. Opt.*, vol. 38, no. 9, pp. S469-S495, 2005.

[6] G. Mana and E. Massa, "The Avogadro and the Planck constants for redefinition of the kilogram," *Riv. Nuovo Cimento*, vol. 35, no. 7, pp. 353-388, 2012.

[7] D. Halliday and R. Resnick, *Fundamentals of Physics*, 2nd edition, John Wiley & Sons, New York, 1981.

[8] J. Teufel, T. Donner, D. Li, J. Harlow, M. Allman, K. Cicak, A. Sirois, J. Whittaker, K. Lehnert, and R. Simmonds, "Sideband Cooling Micromechanical Motion to the Quantum Ground State," *Nature*, vol. 475, pp. 359–363, 2011.

[9] S-Y Lan, P-C Kuan, B. Estey, D. English, J. Brown, M. Hohensee, and H. Muller, "A Clock Directly Linking Time to a Particle's Mass," *Science*, vol. 339, pp. 554-557, 2013.

[10] D. Weiss, B. Young, and S. Chu, "Precision measurement of h/mCs based on photon recoil using laser-cooled atoms and atomic interferometry," *Appl. Phys. B*, vol. 59, pp. 217-253, 1994.

[11] R. Bouchendira, P. Clade, S. Guellati-Khelifa, F. Nez, and F. Biraben, "New Determination of the Fine Structure Constant and Test of the Quantum Electrodynamics," *Phys. Rev. Lett.*, vol. 106, 080801, 2011.

[12] Z. Jabbour and S. Yaniv, "The Kilogram and Measurements of Mass and Force," *J. Res. Natl. Inst. Stan.*, vol. 106, no. 1, pp. 25-46, 2001.

[13] B. Andreas, Y. Azuma, G. Bartl, P. Becker, H. Bettin, M. Borys, I. Busch, P. Fuchs, K. Fujii, H. Fujimoto, E. Kessler, M. Krumrey, U. Kuetgens, N. Kuramoto, G. Mana, E. Massa, S. Mizushima, A. Nicolaus, A. Picard, A. Pramann, O. Rienitz, D. Schiel, S. Valkiers, A. Waseda, and S. Zakel, "Counting the atoms in a ^{28}Si crystal for a new kilogram definition," *Metrologia*, vol. 48, no. 2, pp. S1-S13, 2011.

[14] P. Mohr, B. Taylor, and D. Newell, "CODATA recommended values of fundamental physical constants: 2010," *Rev. Mod. Phys.*, vol 84, pp. 1527-1605, 2012.

[15] T. Niebauer, G. Sasagawa, J. Faller, R. Hilt, and F. Klopping, "A new generation of absolute gravimeters," *Metrologia*, vol. 32, no. 3, pp. 159-180, 1995.

[16] I. Robinson, "Towards the redefinition of the kilogram: a measurement of the Planck constant using the NPL Mark II watt balance," *Metrologia*, vol. 49, pp. 113-156, 2012.

[17] R. Steiner, "History and progress on accurate measurements of the Planck constant," *Rep. Prog. Phys.*, vol. 76, 016101, 2013.

[18] J. Maxwell, "On a method of making a direct comparison of electrostatic with electromagnetic force; with a note on the electromagnetic theory of light," *Phil. Trans.*, vol. 158, pp. 643-657, 1868.

[19] R. Elmquist, M. Cage, Y. Tang, A. Jeffery, J. Kinard, R. Dziuba, N. Oldham, and E. Williams, "The Ampere and Electrical Standards," *J. Res. Natl. Inst. Stan.*, vol. 106, pp. 65-103, 2001.

[20] H. Curtis and R. Curtis, "An Absolute Determination of the Ampere," *J. Res. Nat. Bur. Stand.*, vol. 12, pp. 665-734, 1934.

[21] B. Kibble, "A measurement of the gyromagnetic moment of the proton by using the strong field method," *Atomic Masses and Fundamental Constants*, vol. 5, J. H. Sanders and A. H. Wapstra, eds., Plenum, New York, pp. 545-551,1976.

[22] I. Mills, P. Mohr, T. Quinn, B. Taylor, and E. Williams, "Redefinition of the kilogram: a decision whose time has come," *Metrologia*, vol. 42, pp. 71-80, 2005.

[23] B. Wood and S. Solve, "A review of Josephson comparison results," *Metrologia*, vol. 46, R13, 2009.

[24] T. Janssen, N. Fletcher, R. Goebel, J. Williams, A. Tzalenchuk, R. Yakimova, S. Lara-Avila, S. Kubatkin, and V. Falko, "Graphene universality of the quantum Hall effect and redefinition of the SI system," *New J. Phys.*, vol. 13, 093026, 2011.

[25] A. Penin, "Quantum Hall effect in quantum electrodynamics," *Phys. Rev. B*, vol. 79, 113303, 2009.

[26] K. Fritsch, "GWP® - The Weighing Standard: Why We Should Challenge the Established Way We Calibrate and Test Weighing Instruments," *Proceedings of the NCSLI International Workshop and Symposium*, Nashville, Tennessee, 2013.

[27] M. Ruefenacht, "ASTM E617 Standard Specifications for laboratory weights and precision mass standards (2013 Revision)," presented at *NCSL International Workshop and Symposium*, Nashville, Tennessee, 2013.

[28] L. Howard and J. Fu, "Accurate force measurements for miniature mechanical systems: a review of progress," *P. SPIE: Microlithography and Metrology in Micromachining III*, Friedrich and Umeda, eds., vol. 3225, pp. 2-11, 1997.

[29] J. Pratt, J. Kramar, D. Newell, and D. Smith, "Review of SI traceable force metrology for instrumented indentation and atomic force microscopy," *Meas. Sci. Technol.*, vol. 16, no. 11, pp. 2129-2137, 2005.

[30] http://geodesy.noaa.gov/TOOLS/Gravity/gravcon.html

[31] J. Pratt and J. Kramar, "SI realization of small forces using an electrostatic force balance," *Proceedings of the XVIII IMEKO World Congress*, Rio de Janeiro, Brazil, 2006.

[32] K-H Chung, G. Shaw, and J. Pratt, "Accurate picoscale forces for insitu calibration of AFM," *Proceedings of the XIX IMEKO World Congress*, Lisbon, Portugal, 2009.

[33] P. Wilkinson, G. Shaw, and J. Pratt, "Determination of a cantilever's mechanical impedance using photon momentum," *Appl. Phys. Lett.*, vol. 102, 184103, 2013.