FOCUSED ISSUE FEATURE

Traveling Waves and Power Waves

Building a solid foundation for microwave circuit theory

Dylan Williams

Microwave equivalent-circuit theory is like a grand old building: elegant, carefully crafted, passed from generation to generation, and rooted in history. Traveling waves, the measureable electromagnetic field solutions of Maxwell's equations in a transmission line, form the foundation of the theory. You must descend into the basement to see them; not many people do so. It is in this "basement" that I have had the privilege of working on the foundations of the microwave circuit theory with Roger Marks, Bradley Alpert, and others. Together, we explored the underpinnings of the theory and worked toward building a better understanding of microwave equivalent-circuit theory and how it can be applied to the lossy printed transmission lines ubiquitous in modern microwave electronics.

The first floor of this grand old structure houses the retail establishments. Clean, neat, and welcoming, here you find the equivalent-circuit voltages and currents of the theory. These voltages and currents are hot items; a steady stream of clients happily snap them up. They mimic low-frequency voltages and currents in lossless transmission lines so well that most clients are not able to distinguish the real article from the microwave equivalent.

On the second floor of the building, you find the commodities exchange and business offices populated by the pseudowaves. The pseudowaves can also be referred to as "power-normalized and impedance-transformed scattering waves." They are all about commerce. Pseudowaves do the buying, selling, and converting, making sure that the equivalent-circuit voltages and currents get into the hands of customers who need them. Pseudowaves mimic traveling waves in a lossless transmission line.

Atop the building, there is a 1960s-style addition that raises the ire of the Historical Society, but is seen as a sign of progress by the young architects of the town. The power waves live here. But we get ahead of ourselves. To understand power waves, we have to understand the microwave circuit theory that lies underneath.

So let us take a short visitor's tour of this edifice. If you opt for a more in-depth tour later, [1]–[3] outline conventional microwave circuit theory. I suggest that you then look through [4] and [5] to find out more about how the theory can address lossy printed transmission

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lines. You might also look at [6] if you fancy lossy multimode transmission lines.

Traveling Waves

Let us descend into the basement and take a look at the foundation: Maxwell's equations lurk in the corners. Microwave circuit theory is often referred to as an "equivalent-circuit" theory because it constructs equivalent-circuit voltages and currents from electric and magnetic fields in a microwave circuit that mimic as closely as we can the properties of low-frequency voltages and currents. The first step in this process is to simplify the solutions for the electric and magnetic fields in the circuit itself. Otherwise, it would not be possible to describe what is going on in the circuit with a few voltages and currents.

We simplify the field solutions in a microwave circuit by forcing all of the devices in the circuit to communicate with each other via transmission lines that are uniform in their directions of propagation. We cannot construct a microwave equivalent-circuit theory without these transmission lines. Microwave equivalent-circuit theory is more than just a theory—it is also part of a very old and successful microwave design strategy based on the use of transmission lines to guide, measure and control electromagnetic energy as it propagates from device to device within a microwave circuit.

The solutions of Maxwell's equations simplify in a transmission line because they form a discrete set of separable modes there. In the classic approach, which we will follow here, the transmission lines are designed to support only one propagating mode, and are made long enough that any evanescent modes generated at the interface of a device and an interposing transmission line die out before reaching the device at the other end of the line. These are significant restrictions and correspond to the conditions assumed in [4]. Yet almost all microwave designs apply these restrictions at their input and output ports to make the circuits usable by others. Most designers apply them inside the circuit as well to simplify the design, measurement, and optimization of the circuit.

With these restrictions, the total transverse electric and magnetic fields $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$ of the single propagating mode in a transmission line can be written as

$$\boldsymbol{E}_t = c^+ e^{-\gamma z}\boldsymbol{e}_t + c^- e^{+\gamma z}\boldsymbol{e}_t; \boldsymbol{H}_t = c^+ e^{-\gamma z}\boldsymbol{h}_t - c^- e^{+\gamma z}\boldsymbol{h}_t,$$

where, following the notation of [4], $\boldsymbol{e}_t$ and $\boldsymbol{h}_t$ are the unnormalized electric and magnetic fields of the modal solution of Maxwell's equations in the transmission line, $\gamma = \alpha + i\beta$ is the complex propagation constant of the mode, z is in the direction of propagation, and c^+ and c^- are the unnormalized forward and backward amplitudes of the modes. We call these propagating modes traveling waves. The power of the simplifications we have required is that the total field due to the traveling waves in the transmission lines we use can be described by the two complex quantities c^+ and c^- at each frequency, and these are the only fields that communicate energy between the various devices in our circuit.

Notice that we have placed no restrictions on the behavior of the mode itself other than it be the sole propagating solution of Maxwell's equations in a uniform transmission line. You can connect the devices in your circuits with microstrip, coplanar waveguide, stripline, slotline, coaxial transmission line, rectangular waveguide, and a variety of other single-mode waveguides. You can also use thin metals and lossy dielectrics in their construction, and the traveling waves need not be transverse electromagnetic (TEM), or even quasi-TEM.

The traveling waves are the foundation of microwave circuit theory not only because they are real (traveling waves are the only waves that I will talk about that actually exist), but also because they can be measured directly. For example, traveling-wave reflection coefficients can be measured by observing the peaks and valleys of the electric fields of the standing wave created by the beating of incident and reflected traveling waves in a slotted-line experiment. The through-reflect-line (TRL) vector-network-analyzer calibration, a modern-day analogy of the slotted line, also measures traveling waves [4], [7], [8]. This explains why the microwave metrologist starts with a TRL calibration when he or she works to establish measurement traceability.

I also imagine that it did not escape your notice that there are only two complex quantities c^+ and c^- required to describe the forward and backward traveling waves in a transmission line. It is no accident that there are exactly two of them, just as there are two complex quantities, an equivalent-circuit voltage and an equivalent-circuit current, that we wish to construct in the line. Let us now proceed up the stairs to the first floor to study the microwave engineer's most prized commodity, equivalent-circuit voltages and currents.

Equivalent-Circuit Voltage and Current

The equivalent-circuit voltage $v(z)$ and current $i(z)$ in a transmission line can be conveniently defined with

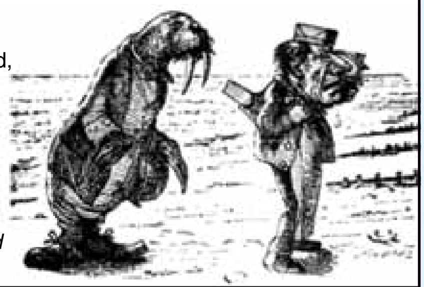
$$\boldsymbol{E}_t(z) = \frac{v(z)}{v_0}\boldsymbol{e}_t; \boldsymbol{H}_t(z) = \frac{i(z)}{i_0}\boldsymbol{h}_t,$$

where v_0 and i_0 are normalization constants that define v and i and allow them to take units of root-mean-square voltage and current [4]. For instance, to achieve a conventional voltage normalization $v = \int \boldsymbol{E}_t \cdot dl$, the normalization constant v_0 is set to the integral of the modal electric field over the same path with $v_0 = \int \boldsymbol{e}_t \cdot dl$. The trick to constructing equivalent-circuit voltages and currents that mimic low-frequency voltages and currents as closely as possible

The Walrus and the Carpenter

"The time has come," the Walrus said,
"To talk of many things:
Of shoes—and ships—and sealing-wax—
Of power—Lorentz—and vectors Poynting—
And why we use pseudo-waves—
And whether pigs have wings."

"But wait a bit," the Oysters cried,
"Before we have our chat,...



Adapted from Lewis Carroll's
Through The Looking-Glass and What Alice Found There, 1872.

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is to understand the implications of the choice of the normalization constants v_0 and i_0.

If we set the total fields in a uniform transmission line equal to the modal fields (i.e., $\boldsymbol{E}_t = \boldsymbol{e}_t$ and $\boldsymbol{H}_t = \boldsymbol{h}_t$), we see that v_0 and i_0 correspond to the voltage and current carried by a forward traveling wave. Thus, the characteristic impedance Z_0 (see Tables 1 and 2) of the traveling wave is

$$Z_0 \equiv \frac{v_0}{i_0}.$$

To ensure that the total power p transmitted in the guide is given by vi^*, we must set v_0 and i_0 so that vi^* is equal to the integral p_0 of the Poynting vector over the transmission line's cross section S. Therefore, we must impose the constraint $p_0 = v_0 i_0^* = \iint_S \boldsymbol{e}_t \times \boldsymbol{h}_t^* \cdot \boldsymbol{z}\, \mathrm{d}S$ on v_0 and i_0.

Brews first recognized in [9] that meeting the power condition requires setting the phase angle of Z_0 in the equivalent-circuit theory equal to the phase of p_0, which also places a constraint on v_0 and i_0. In lossy guides, the electric and magnetic fields are generally out of phase and typically approach $-45°$ at low frequencies when metal resistance dominates the loss. And here we begin to see the physical significance of the characteristic impedance Z_0 in microwave circuit theory. If we are to calculate power correctly from the voltages and currents, the phase of the characteristic impedance Z_0 of the traveling wave must reflect the actual difference in the phase of the electric and magnetic fields as they propagate down the waveguide.

However, the frequency-point-by-frequency-point power constraint of [4] is not enough to complete the theory; this is because [4] does not consider the temporal behavior of v and i. For example, [4] does not completely fix the phases of v_0 and i_0, which leaves ambiguity in reciprocity and other conditions, and the magnitude of the characteristic impedance is left undetermined. Most significantly, the theory of [4] does not guarantee that driving-point impedances (which include Z_0) are minimum phase, i.e., $\mathcal{H}(\ln(Z)) = \arg(Z)$, where $\mathcal{H}$ is the Hilbert transform [10], and are, therefore, causal. This is a require-

Table 1. Equivalent-Circuit Theory Terminology

Concept	Physical	Smith Chart	Continuous	Role
Traveling waves	✓	✓	✓	Solutions of Maxwell's equations; measurable
Equivalent voltage and current	✓	✓	✓	Related to E_t and H_t, mimic low-frequency v and i
Pseudowaves	✗	✓	✓	Mimic S-parameters in a lossless line
Power waves	✗	✗	✗	Tool for achieving maximum power transfer

ment for stable temporal circuit simulators, which find it very upsetting when passive devices in the circuit start responding before the stimulus is applied.

It is the causality constraint of [5] that completes the theory. The causality constraint fixes not only v_0 and i_0, but also the magnitude of the characteristic impedance to within a single real frequency-independent constant. In quasi-TEM transmission lines, this final constant is usually chosen to set the units of the characteristic impedance Z_0 to ohms at low frequencies and thus the units of the voltage v to volts and the current i to amperes. In TEM or quasi-TEM guides, intuitive power-voltage and power-current normalizations meet the power and causality constraints by construction, and suitable normalizations are not difficult to find for more complex guides such as metal-insulator-semiconductor transmission lines and rectangular waveguides [11], [12].

Here we see that the characteristic impedance Z_0 of the traveling waves plays a critical role in determining the equivalent-circuit voltages and currents in microwave circuit theory. This explains why fixing the characteristic impedance of coaxial transmission lines and rectangular waveguides with precise dimensional control and measuring the complex amplitude and phase of the characteristic impedance of lossy printed lines with vector-network-analyzer calibration methods such as [13] and [14] are so important: the theory cannot be completed and correctly applied without first determining Z_0.

Now let us visit the second floor of our historic structure, where the real commerce takes place. Here we find the pseudowaves hard at work answering phones, talking with their clients, and filling large orders. Converting traveling-wave scattering parameters to 1, 50, and 75 Ω scattering parameters is big business.

Pseudowaves

Pseudowaves and pseudoscattering parameters have long been a cornerstone of microwave circuit theory and are often referred to as impedance-normalized traveling waves and scattering parameters with a fixed, real and frequency-independent reference impedance. Scattering parameters with a 50 Ω reference impedance have long been used in coaxial metrology, for example, and scattering parameters with a reference impedance of 1 Ω are often used in rectangular waveguide metrology.

Roger Marks and I coined the term pseudowaves in [4] when we were working to extend microwave circuit theory to lossy printed transmission lines. We chose the term pseudowaves to emphasize that power-normalized and impedance-transformed waves, unlike traveling waves, are just a construction, not actual propagating solutions of Maxwell's equations.

We also used the term pseudowaves to emphasize that impedance-normalized waves and traveling waves in lossy printed transmission lines have very different properties. This was something that was not so apparent in the low-loss coaxial and rectangular waveguides used back when microwave circuit theory was first constructed.

In fact, microwave engineers have become so accustomed over the years to measuring scattering parameters in low-loss guides, that most of us have forgotten how peculiar traveling waves can be. And I, for one, cannot point to a single microwave textbook that does not sidestep this issue by assuming, either implicitly or explicitly, lossless transmission lines in its scattering-parameter formulation.

For example, we commonly assume that the power transmitted across a reference plane is equal to the difference of the power carried by the forward wave and the power carried by the backward wave. We also usually assume that the forward and backward transmission coefficients of reciprocal devices are equal. However, the transverse electric and magnetic fields of the propagating modes are usually out of phase in lossy transmission lines, and thus neither of these conditions is satisfied in general by lossy traveling waves. If we blindly insert traveling waves and traveling-wave scattering parameters into our circuit simulators, as Roger Marks and I tried to do in the early days of on-

TABLE 2. Impedances in Microwave Circuit Theory

Impedance	Symbol	Role
Characteristic impedance	Z_0	Ratio of voltage to current carried by forward wave; equal to v_0/i_0
Reference impedance	Z_{ref}	Sets the relationship between the pseudowave amplitude and v and i; usually set real (*e.g.*, to 50 Ω)
Complex port number	$\hat{Z}$	Defines relationship between power-wave amplitude and v and i; usually set to impedance of generator or load

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wafer microwave measurements, we get some very strange results indeed [15], [16].

Pseudowaves offer a solution to the complexity of traveling waves in lossy transmission lines. Pseudowaves with a real reference impedance are constructed from the voltages and currents in lossy lines with the same formula we use to construct power-normalized traveling waves from the voltages and currents of a lossless transmission line. While pseudowaves do not actually exist, they obey the same rules and formulas that apply to the lossless transmission lines most of us are accustomed to.

Pseudowaves allow us to continue the tradition in the microwave community of reporting results as impedance-transformed traveling waves, reflection coefficients, and scattering parameters. This is a practice that has been going on for decades; it is ingrained in the Smith chart [17], in fundamental concepts like conjugate matching, and in our circuit simulators and most other tools we use for microwave design. This small artifice allows microwave engineers the world over to continue to use wave representations and scattering parameters as we did in low-loss coaxial transmission lines and rectangular waveguide, even when the characteristic impedance of the lossy printed transmission lines we are using is complex and far from 50 Ω.

We define the forward and backward pseudowave amplitudes a and b as

$$a(Z_{\mathrm{ref}}) = \left[\frac{|v_0|}{v_0}\frac{\sqrt{\mathrm{Re}(Z_{\mathrm{ref}})}}{2|Z_{\mathrm{ref}}|}\right](v + iZ_{\mathrm{ref}}),$$
$$b(Z_{\mathrm{ref}}) = \left[\frac{|v_0|}{v_0}\frac{\sqrt{\mathrm{Re}(Z_{\mathrm{ref}})}}{2|Z_{\mathrm{ref}}|}\right](v - iZ_{\mathrm{ref}}),$$

where Z_{ref} is the reference impedance of the pseudowave. This formulation was developed so that when Z_{ref} is set real, a and b mimic traveling-wave amplitudes in a lossless transmission line. On the other hand, when Z_{ref} is set equal to the characteristic impedance of the guide, a and b become power-normalized versions of the traveling-wave amplitudes c^+ and c^-. Now we see another important advantage of pseudowaves: their built-in impedance transformations allow us to easily convert from the measurable and calculable traveling waves to the pseudowaves used by most microwave engineers.

Pseudowaves are defined by their reference impedance Z_{ref}. The power p transmitted by a pseudowave through the guide is given by

$$p = |a|^2 - |b|^2 + 2Im(ab^*)\frac{\mathrm{Im}(Z_{\mathrm{ref}})}{\mathrm{Re}(Z_{\mathrm{ref}})}.$$

The condition on the forward and backward transmission coefficients through a reciprocal junction is

$$\frac{S_{ji}}{S_{ij}} = \frac{K_i}{K_j}\,\frac{1 - j\,\mathrm{Im}(Z_{i,\mathrm{ref}})/\mathrm{Re}(Z_{i,\mathrm{ref}})}{1 - j\,\mathrm{Im}(Z_{j,\mathrm{ref}})/\mathrm{Re}(Z_{j,\mathrm{ref}})},$$

where $Z_{i,\mathrm{ref}}$ is the reference impedance at the ith port. After applying the causality constraints of [5] to eliminate the phase ambiguities of [4], it is easy to see that the K_i are close to one in most guides and account for the difference between the power normalization used in microwave equivalent-circuit theory and the integrals used in deriving the Lorentz reciprocity condition (see [4, Appendix D] and [18]).

The pseudowaves are equal to the traveling waves when we set Z_{ref} equal to Z_0, the characteristic impedance of the transmission line. A look at the two previous equations for a complex reference impedance equal to Z_0 makes it clear why traveling waves in lossy lines are so difficult to use.

However, pseudowaves with a real reference impedance do behave just like traveling waves in a lossless transmission line with a real characteristic impedance. For example, if we set Z_{ref} real, we see that the resulting pseudowave amplitudes a and b are normalized so that the power is just the difference between the power in the forward wave and the power in the reverse wave. That is, the power $p = |a|^2 - |b|^2$. Furthermore, for reciprocal junctions, we have $S_{ji}/S_{ij} \not\asymp 1$ [4], [18]. (Keep in mind that equivalent-circuit theories that set $S_{ji}/S_{ij} = 1$ cannot always satisfy $p = |a|^2 - |b|^2$ perfectly, and vice versa.)

Thus, pseudowaves and pseudowave scattering parameters mimic quite well (but not always perfectly) what traveling waves and their scattering parameters would do in a lossless line with a real characteristic impedance. Also, pseudowaves defined with a real reference impedance (often called waves with a 50 Ω reference impedance) are exactly what are reported by the measurement community, (see "On-Wafer Measurement") and they are what our circuit simulators, microwave designers, and product manufacturers use on a day-to-day basis.

Power Waves

The last tour stop is the new addition on the roof, where the entrepreneurial family of power waves can be found. The name "power waves" evokes an image of substance.

The power waves $\hat{a}$ and $\hat{b}$ are defined with respect to frequency-dependent "complex port numbers" $\hat{Z}$ as [19], [20]

$$\hat{a}(\hat{Z}) = \frac{v + i\hat{Z}}{2\sqrt{\mathrm{Re}(\hat{Z})}};\ \hat{b}(\hat{Z}) = \frac{v - i\hat{Z}^*}{2\sqrt{\mathrm{Re}(\hat{Z})}}.$$

The family of power waves are designed to satisfy the power relation $p = |\hat{a}|^2 - |\hat{b}|^2$ for any $\hat{Z}$, not just when $\hat{Z}$ is set real. Pseudowaves do not have this property. In practice, we are told, power waves can be applied by setting $\hat{Z}$ equal to either the complex impedance of a generator or a load. This helps us visualize power flow, and it simplifies obtaining maximum power transfer between the source and the load. With

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On-Wafer Measurement

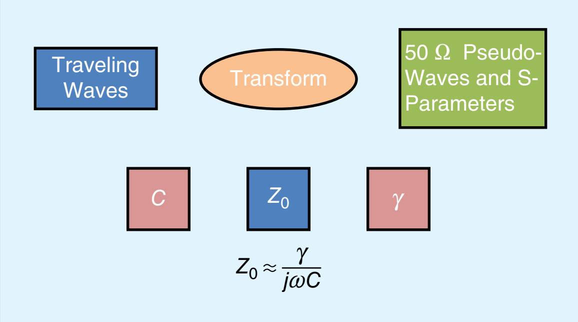

Figure S1. *Transforming traveling waves to 50 Ω pseudowaves.*

Traveling waves are the propagating solutions of Maxwell's equations in a transmission line. While the through-reflect-line (TRL) calibration algorithm [7], [8] is not the only way of measuring traveling-wave amplitudes, it is the most well-known and widely used approach and thus plays a fundamental role in microwave metrology. Figure S1 illustrates how the TRL calibration is usually applied in quasi-transverse electromagnetic (TEM) transmission lines printed on a low-loss substrate.

First, the vector network analyzer is calibrated with the TRL algorithm and measurements of a through, a pair of unknown but symmetric reflects, and a line (or multiple lines to allow averaging). This calibration sets the reflection coefficient of the lines to zero and measures traveling-wave amplitudes [4].

Then, in coaxial, microstrip, and coplanar waveguides constructed with low-loss dielectrics so that the conductance of the line is small, the characteristic impedance Z_0 of the transmission line may be estimated with the formula in Figure S1 [13] using the low-frequency capacitance C of the line [14] and the line's propagation constant determined by the TRL calibration. The final step is to transform the traveling waves and their scattering parameters with a reference impedance of Z_0 to pseudowaves and pseudowave scattering parameters with a real reference impedance (e.g., 50 Ω). The pseudowaves and their scattering parameters mimic waves in a lossless transmission line and can be used directly in computer-aided design tools.

power waves, you can dispense with conjugate matching and just iterate until the power-wave reflection coefficient is zero and you have matched your devices for maximum power transfer. These appear to be useful properties. Why would anyone settle for less?

But do the power waves make sense only as a limited tool for simplifying matching problems, or do they play a greater role in microwave circuit theory? Let's consult the Smith chart.

The Smith chart is based on a ratio of the impedance of the load and the reference impedance that defines the center of the chart. There are no conjugates in the Smith-chart formulation, and it cannot be used to find impedances from power-wave reflection coefficients. This is easily seen by considering the power-wave reflection coefficients of an open and a short. While the open resides at one, by setting the voltage equal to zero we see that the power-wave reflection coefficient of a short is found at $-\hat{Z}^*/\hat{Z}$ [21]. This point lies on the edge of the unit circle but is not on the real axis, even though a short circuit clearly cannot not store reactive energy. This unexpected behavior is the result of the appearance of $\hat{Z}^*$ in the definition of the amplitude of the reverse power wave and shows just how different power waves really are from anything we studied in school. In particular, the Smith chart does not apply. Neither do the sets of rules we learned in college about how reflection coefficients translate along circular paths inside the Smith chart when we add series or shunt reactive elements to a matching circuit as illustrated in Figure 1.

And what about simple tasks such as cascading linear circuits? The voltages and currents at a reference plane in a transmission line connecting two circuits are continuous. Thus, you can set the amplitude of a pseudowave or a traveling wave emanating from the circuit on one side of a reference plane equal to the amplitude of the wave entering the circuit on the other side of the reference plane. This is the basis of signal-flow-graph theory. Without it, the transmission matrix of a cascade of circuits could not be determined from the products of their individual transmission matrices.

However, power waves are directional. Whenever $\hat{Z} \neq \hat{Z}^*$ (i.e., whenever $\hat{Z}$ is complex), the definitions of the forward and backward power waves differ. This is illustrated in "Cascading Power Waves." As a result, the amplitude of the power wave leaving one circuit is not equal to the amplitude of the power wave entering the other [21]. Transmission matrices no lon-

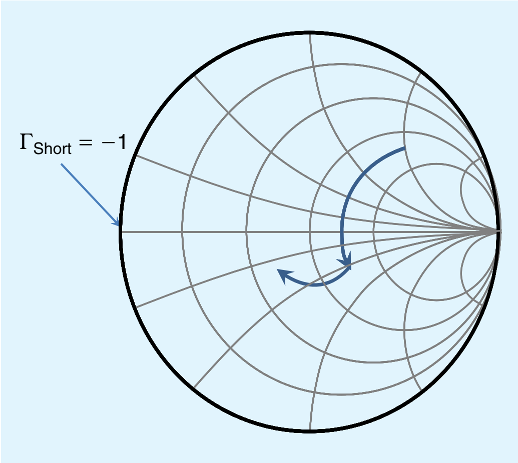

Figure 1 *The Smith Chart. Not only is the power-wave reflection coefficient of a short not found at -1, but power-wave reflection coefficients do not follow the impedance and admittance trajectories commonly used to solve conjugate-matching and other tasks with the Smith chart.* **<au: Formatting of this figure will be done in the production process.>**

ger cascade, and power waves cannot be used in many commonly used microwave-circuit design strategies. Much like a lava lamp, the power waves are comforting to look at, but do not offer the engineer the tools required for analytic design.

This ends our tour. The traveling waves in the basement turned out to be more complex than we might have imagined, but the voltages and currents on the first floor lived up to all of our expectations. The sheer volume of the commerce performed by the pseudowaves on the second floor was impressive. There we saw pseudowaves handling large bulk orders all day long, rapidly converting between impedances and serving the needs of industry. And in the end, the new addition on the roof did not add much to the grand old building.

A Product of Its Time

My father once told me that philosophers become great not because they think beyond their time, but because they address the important social and political problems of their day. To be useful, microwave equivalent-circuit theory must also adapt to the problems of our time.

During my career, this has meant exploring how microwave circuit theory can be applied to lossy printed transmission lines. Choosing equivalent-circuit voltages and currents that mimic as closely as possible the properties of low-frequency voltages and currents is of fundamental importance. Perhaps one of the most intriguing aspects of this exploration has been a shift in focus from the *means* of constructing equivalent-circuit voltages and currents to a focus on understanding the *properties* of those voltages and currents. This has led to a greater understanding of the physical meaning of the characteristic impedance of traveling waves [4], [13], power flow and reciprocity in lossy printed transmission lines [16], [18], temporal behavior [5], [11], [12], [22], and the extent to which the behavior of equivalent-circuit voltages and currents deviate from the ideal behavior we expect. These advances in our understanding have been keys to the establishment of traceable and well-understood on-wafer calibrations in the last two decades.

I expect that our understanding of microwave equivalent-circuit theory will continue to grow as we push the frontiers of microwave technology. I have certainly found my studies of microwave equivalent-circuit theory over the years to be extremely rewarding, and I look forward to watching the theory evolve. It is, after all, microwave circuit theory that distinguishes us, as microwave engineers, from the rest of the profession.

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Cascading Power Waves and the Smith Chart

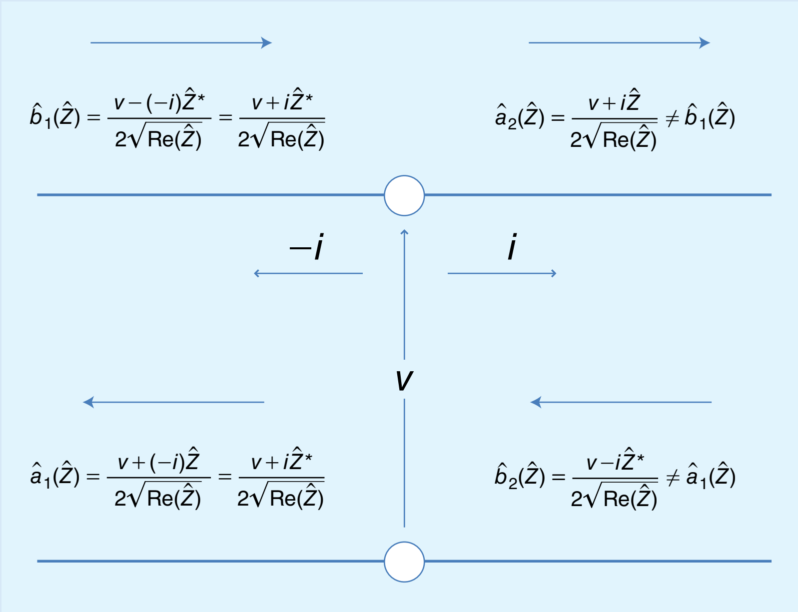

Figure S2. *Power waves are discontinuous.*

While traveling waves and pseudowaves are always continuous at the interface between circuits, power waves are not. Figure S2 illustrates a transparent interface between two identical guides, and shows the formulas for the power-wave amplitudes on each side of the junction. From the figure we see that the power wave $\hat{b}_1$ exiting the circuit on the left is not equal to the power wave $\hat{a}_2$ entering the circuit on the right, for example, unless $\hat{Z}$ is real (in which case the power waves are equal to pseudowaves with the real reference impedance $\hat{Z}$). Thus we see that, except in special case in which $\hat{Z}$ is real and the power waves reduce to the pseudowaves, the power waves are discontinuous across even a transparent junction between two identical transmission lines.

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Callouts

Pseudowaves mimic traveling waves in a lossless transmission line.

The traveling waves are the foundation of microwave circuit theory not only because they are real but also because they can be measured directly.

If we blindly insert traveling waves and traveling-wave scattering parameters into our circuit simulators, we get some very strange results.

Much like a lava lamp, the power waves are comforting to look at, but do not offer the engineer the tools required for analytic design.