Robust Face Recognition via Sparse Representation

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Face Recognition: *"Where amazing happens"*

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Figure: Steve Nash, Kevin Garnett, Jason Kidd.

Sparse Representation

Sparsity

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1. Sparsity in frequency domain

Figure: 2-D DCT transform.

2. Sparsity in spatial domain

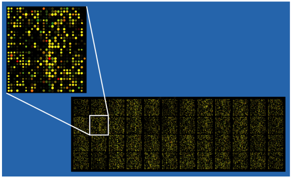
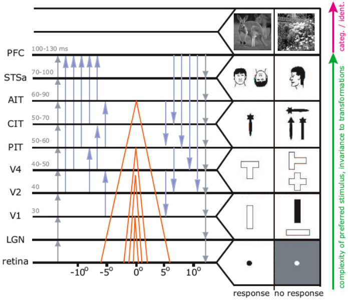


Figure: Gene microarray data.

- **Sparsity in human visual cortex** [Olshausen & Field 1997, Serre & Poggio 2006]



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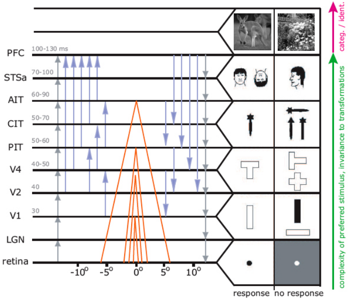
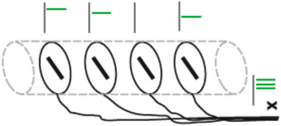


1. **Feed-forward**: No iterative feedback loop.
2. **Redundancy**: Average 80-200 neurons for each feature representation.
3. **Recognition**: Information exchange between stages is not about individual neurons, but rather how many neurons as a group fire together.



Problem Formulation

1. **Notation**
   - Training: For K classes, collect training samples $\{\mathbf{v}_{1,1}, \cdots, \mathbf{v}_{1,n_1}\}, \cdots, \{\mathbf{v}_{K,1}, \cdots, \mathbf{v}_{K,n_K}\} \in \mathbb{R}^D$.
   - Test: Present a new $\mathbf{y} \in \mathbb{R}^D$, solve for label($\mathbf{y}$) $\in [1, 2, \cdots, K]$.

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2. **Data representation** in (long) vector form via stacking

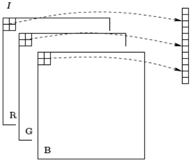

Figure: Assume 3-channel 640×480 image, $D = 3 \cdot 640 \cdot 480$.

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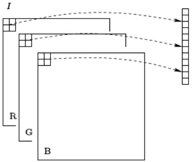

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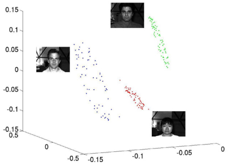
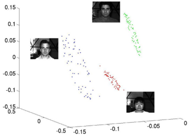
3. **Mixture subspace model** for face recognition [Belhumeur et al. 1997, Basri & Jocobs 2003]



Classification of Mixture Subspace Model

1. Assume $\mathbf{y}$ belongs to Class i



$$\begin{aligned} \mathbf{y} &= \quad {}_{i,1}\mathbf{v}_{i,1} + \quad {}_{i,2}\mathbf{v}_{i,2} + \cdots + \quad {}_{i,n_1}\mathbf{v}_{i,n_i}, \\ &= A_i \quad {}_i, \end{aligned}$$

where $A_i = [\mathbf{v}_{i,1}, \mathbf{v}_{i,2}, \cdots, \mathbf{v}_{i,n_i}]$.

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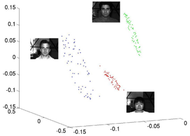

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where $A_i = [\mathbf{v}_{i,1}, \mathbf{v}_{i,2}, \cdots, \mathbf{v}_{i,n_i}]$.

2. Nevertheless, Class i is the **unknown** variable we need to solve:

Sparse representation $\quad \mathbf{y} = [A_1, A_2, \cdots, A_K] \begin{bmatrix} 1 \\ 2 \\ \vdots \\ K \end{bmatrix} = A\mathbf{x} \in \mathbb{R}^{3 \cdot 640 \cdot 480}.$

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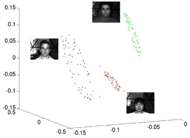

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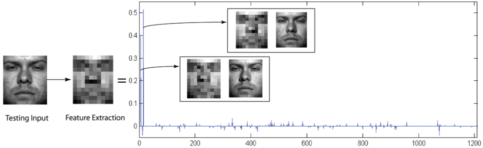
3. $\mathbf{x}_0 = [\,0 \cdots 0 \;\; {}_i^T \;\; 0 \cdots 0\,]^T \in \mathbb{R}^n$.



Sparse representation encodes membership!

Dimensionality Redunction

1. Construct linear projection $R \in \mathbb{R}^{d \times D}$, d is the **feature dimension**.

$$\tilde{\mathbf{y}} \doteq R\mathbf{y} = RA\mathbf{x}_0 = \tilde{A}\mathbf{x}_0 \in \mathbb{R}^d.$$

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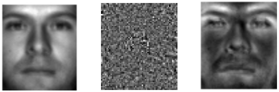
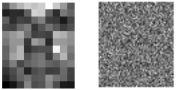
2. Holistic features
   - Eigenfaces [Turk 1991]
   - Fisherfaces [Belhumeur 1997]
   - Laplacianfaces [He 2005]



3. Partial features

4. Unconventional features
   - Downsampled faces
   - Random projections



ℓ^1-Minimization

1. Ideal solution: ℓ^0-Minimization

$$(P_0) \quad \mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0 \text{ s.t. } \tilde{\mathbf{y}} = \tilde{A}\mathbf{x}.$$

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2. **Compressed sensing**: Under mild condition, ℓ^0-minimization is equivalent to

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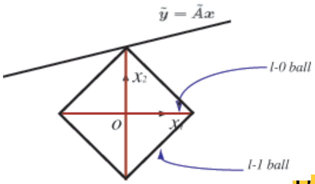
3. ℓ^1-Ball

- ℓ^1-Minimization is convex.
- Solution equal to ℓ^0-minimization.



ℓ^1-Minimization Routines

- **Matching pursuit** [Mallat 1993]
  1. Find most correlated vector $\mathbf{v}_i$ in A with $\mathbf{y}$: $i = \arg\max \langle \mathbf{y}, \mathbf{v}_j \rangle$.
  2. $A \leftarrow A^{(i)}$, $x_i \leftarrow \langle \mathbf{y}, \mathbf{v}_i \rangle$, $\mathbf{y} \leftarrow \mathbf{y} - x_i \mathbf{v}_i$.
  3. Repeat until $\|\mathbf{y}\| < \epsilon$.
- **Basis pursuit** [Chen 1998]
  1. Start with number of sparse coefficients $m = 1$.
  2. Select m linearly independent vectors B_m in A as a basis
$$\mathbf{x}_m = B_m^\dagger \mathbf{y}.$$
  3. Repeat swapping one basis vector in B_m with another vector not in B_m if improve $\|\mathbf{y} - B_m \mathbf{x}_m\|$.
  4. If $\|\mathbf{y} - B_m \mathbf{x}_m\|_2 < \epsilon$, stop; Otherwise, $m \leftarrow m+1$, repeat Step 2.
- **Quadratic solvers**: $\mathbf{y} = A\mathbf{x}_0 + \mathbf{z} \in \mathbb{R}^d$, where $\|\mathbf{z}\|_2 < \epsilon$

$$\mathbf{x}^* = \arg\min\{\|\mathbf{x}\|_1 + \lambda \|\mathbf{y} - A\mathbf{x}\|_2\}$$

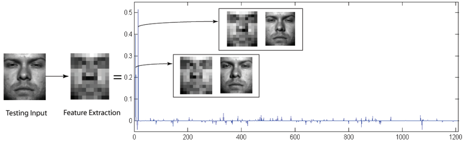
[LASSO, Second-order cone programming]: Much more expensive.

Matlab Toolboxes for ℓ^1-Minimization

- ℓ^1-**Magic** by Candes
- **SparseLab** by Donoho
- **cvx** by Boyd

Sparse Representation Classification

Solve $(P_1) \Rightarrow \mathbf{x}_1$.



1. Project $\mathbf{x}_1$ onto face subspaces:

$$\delta_1(\mathbf{x}_1) = \begin{bmatrix} {}^{1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \delta_2(\mathbf{x}_1) = \begin{bmatrix} 0 \\ {}^{2} \\ \vdots \\ 0 \end{bmatrix}, \cdots, \delta_K(\mathbf{x}_1) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ {}^{K} \end{bmatrix}. \quad (1)$$

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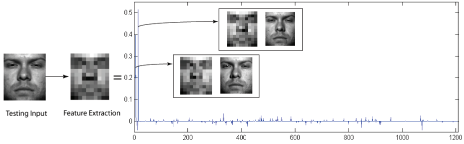

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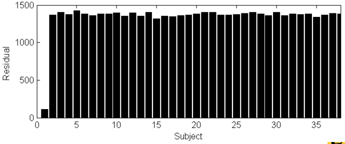
2. Define residual $r_i = \|\tilde{\mathbf{y}} - \tilde{A}\delta_i(\mathbf{x}_1)\|_2$ for Subject i:

- $\mathrm{id}(\mathbf{y}) = \arg\min_{i=1,\cdots,K}\{r_i\} \leftarrow$



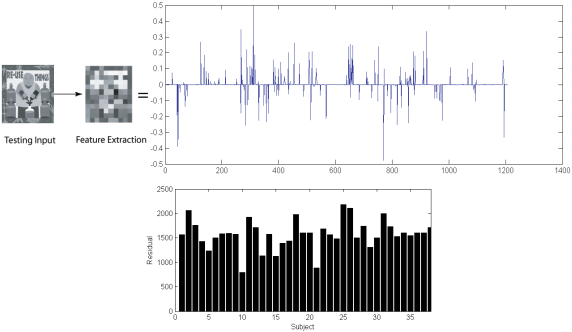
Partial Features on Extended Yale B Database

Features	Nose	Right Eye	Mouth & Chin
Dimension	4,270	5,040	12,936
SRC [%]	87.3	93.7	98.3
nearest-neighbor [%]	49.2	68.8	72.7
nearest-subspace [%]	83.7	78.6	94.4
Linear SVM [%]	70.8	85.8	95.3

SRC: sparse-representation classifier

Extension I: Outlier Rejection

- ℓ^1-Coefficients for invalid images



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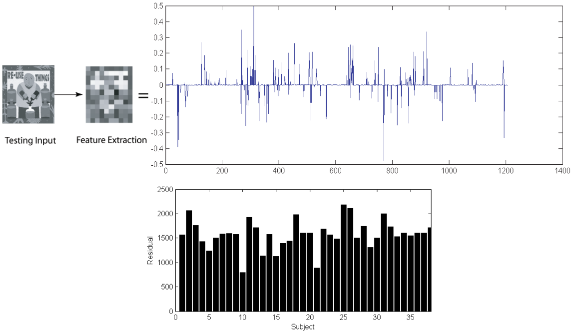


Outlier Rejection

When ℓ^1-solution is not sparse or concentrated to one subspace, the test sample is invalid.

$$\text{Sparsity Concentration Index: } \mathrm{SCI}(\mathbf{x}) \doteq \frac{K \cdot \max_i \|\delta_i(\mathbf{x})\|_1 / \|\mathbf{x}\|_1 - 1}{K - 1} \in [0, 1].$$

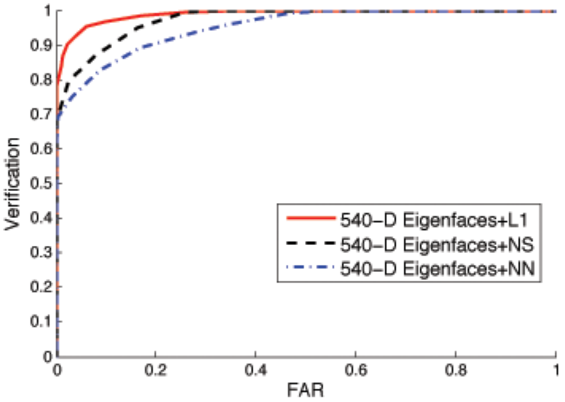
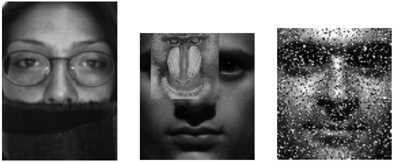


Figure: ROC curve on Eigenfaces and AR database.

Extension II: Occlusion Compensation



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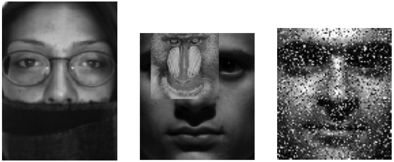
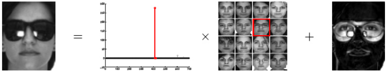


1. Sparse representation + sparse error

$$\mathbf{y} = A\mathbf{x} + \mathbf{e}$$



2. Occlusion compensation

$$\mathbf{y} = \left(A \;\middle|\; I\right)\begin{pmatrix}\mathbf{x}\\ \mathbf{e}\end{pmatrix} = B\mathbf{w}$$

AR Database

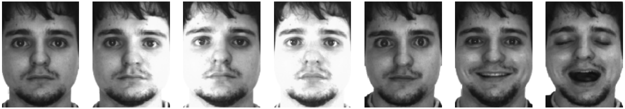


Figure: Training samples for Subject 1.

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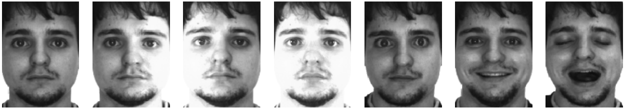

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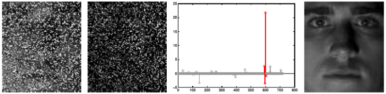
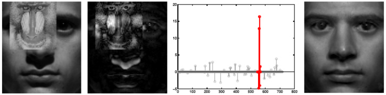


(a) random corruption



(b) occlusion

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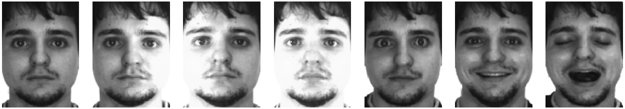

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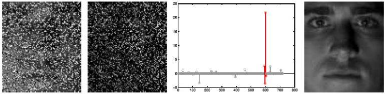

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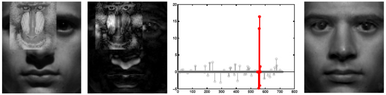

(b) occlusion

 sunglasses	 scarves
97.5%	93.5%

Future Directions

Open problems:

1. Pose variation
2. Scalability to > 1000 subjects

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Other databases:

1. Multi-PIE (about 350 subjects)
2. Chinese CASPEAL (about 1000-3000 subjects)

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Wish list: Because few algorithm succeed under all-weather conditions (illumination, expression, pose, disguise), we are looking forward to a comprehensive database

1. large number of subjects
2. carefully controlled subclasses

Acknowledgments

Collaborators

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- **UIUC**: Prof. Yi Ma, John Wright, Arvind Ganesh

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References

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- http://www.eecs.berkeley.edu/~yang