Recent Progress in Determining Gravitational Constant G at HUST

Jun Luo

Center for Gravitational Experiments,
Huazhong University of Science & Technology

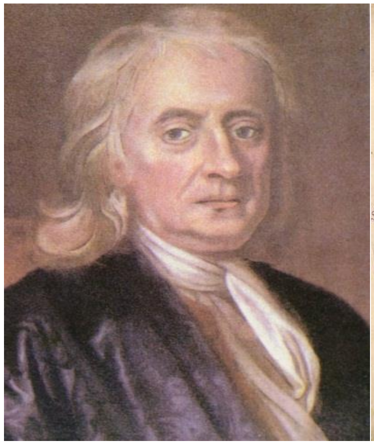
Oct. 9, 2014, NIST

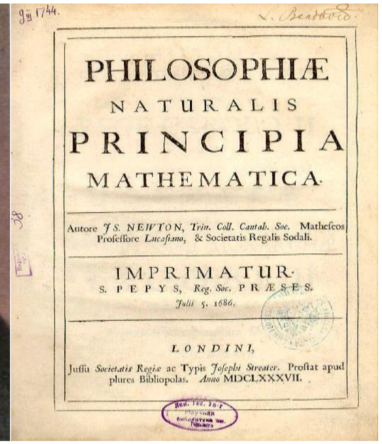
Outline

- Brief Review of G
- HUST-09 experiment
- Recent progress

Newtonian gravitational law and first G value

1687, Newton's law

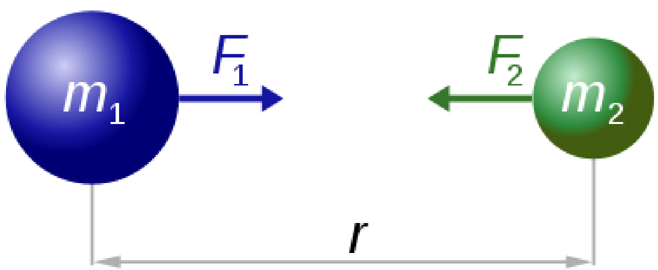



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

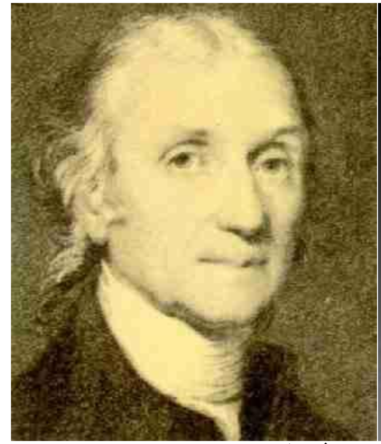
IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

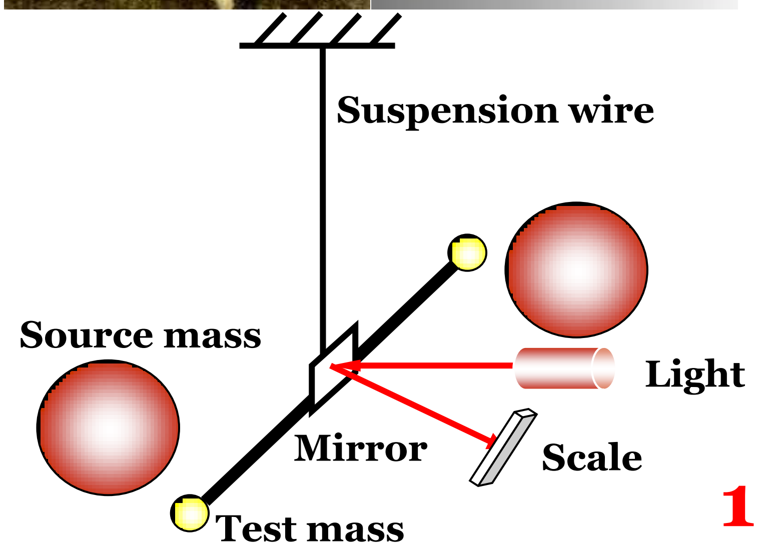
LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

First G value





1%

$$G=(6.67\pm0.07)\times10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}}$$

CODATA 2010 recommended values

G was known earliest, but its measurement precision is worst

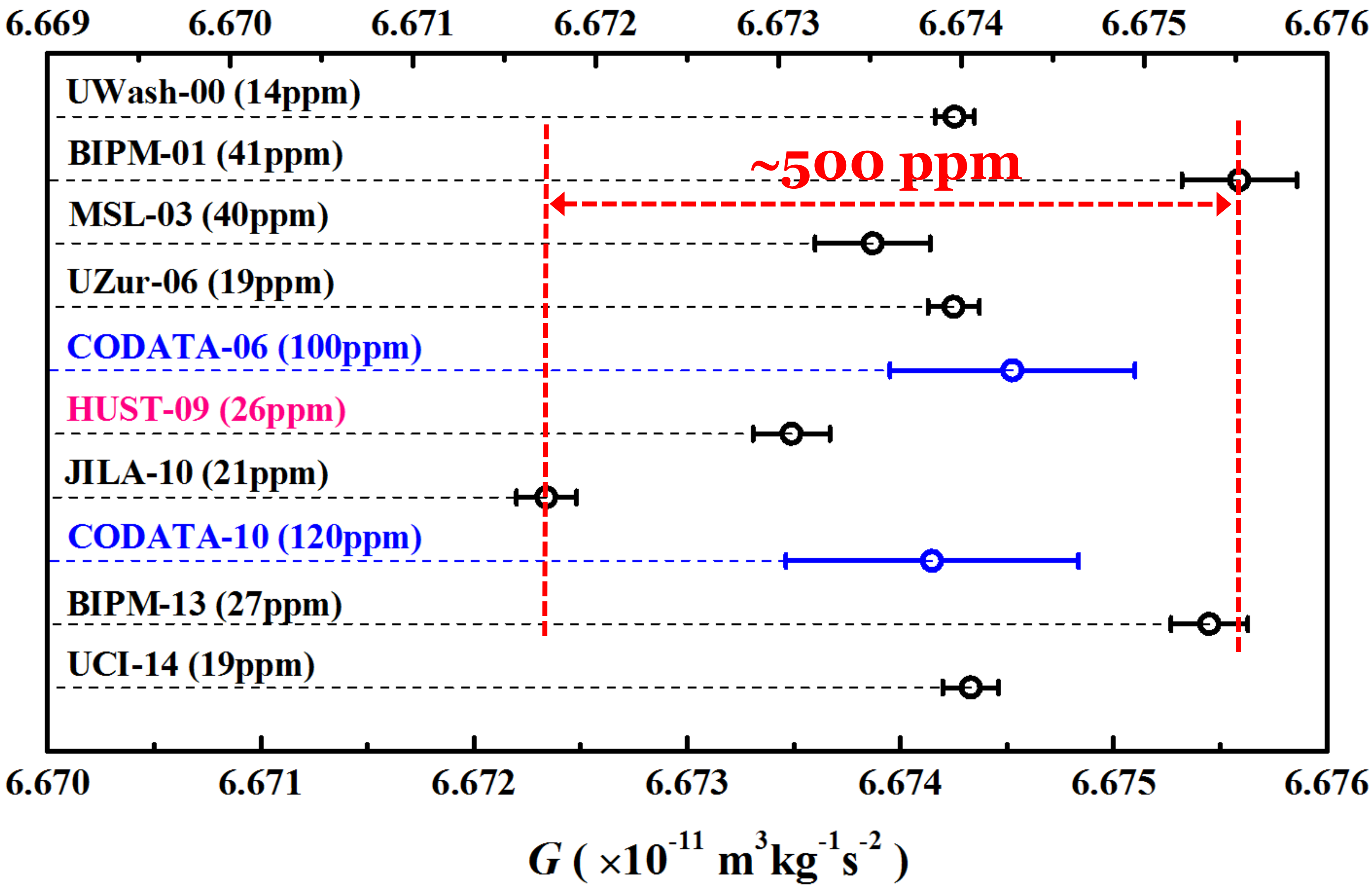
Quantity	2010 rel. std. uncert. u_r
α	3.2×10^{-10}
R_K	3.2×10^{-10}
a_0	3.2×10^{-10}
λ_C	6.5×10^{-10}
r_e	9.7×10^{-10}
σ_e	1.9×10^{-9}
h	4.4×10^{-8}
m_e	4.4×10^{-8}
m_h	4.4×10^{-8}
m_α	4.4×10^{-8}
N_A	4.4×10^{-8}
E_h	4.4×10^{-8}
c_1	4.4×10^{-8}
e	2.2×10^{-8}
K_J	2.2×10^{-8}

Quantity	2010 rel. std. uncert. u_r
F	2.2×10^{-8}
γ'_p	2.5×10^{-8}
μ_B	2.2×10^{-8}
μ_N	2.2×10^{-8}
μ_e	2.2×10^{-8}
μ_p	2.4×10^{-8}
R	9.1×10^{-7}
k	9.1×10^{-7}
V_m	9.1×10^{-7}
c_2	9.1×10^{-7}
σ	3.6×10^{-6}
G	1.2×10^{-4}

$$F_1 = F_2 = G\frac{Mm}{r^2}$$

CODATA recommended values of the fundamental physical constants: 2010
Rev. Mod. Phys. 84(2012)1527

Current situation (G values < 50 ppm)



consistent with each other at only ~500 ppm

Outline

- Brief Review of G
- HUST-09 experiment
- Recent progress

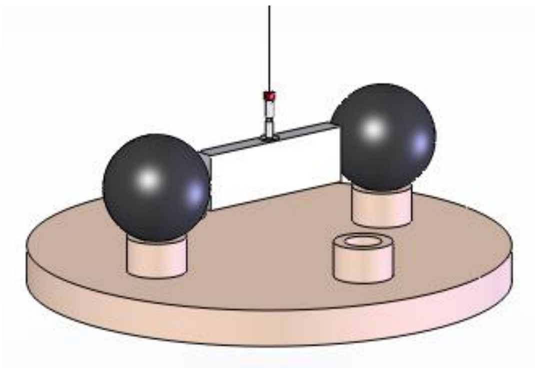
Time-of-swing method

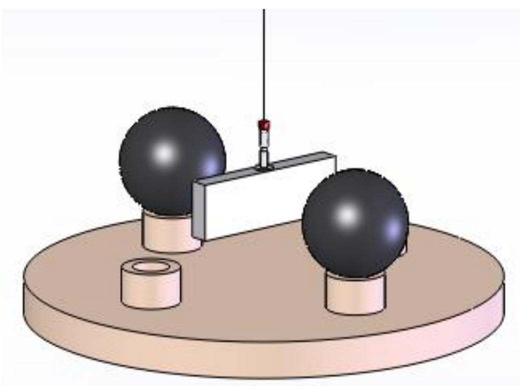
$$I\ddot{\theta}+\gamma\dot{\theta}+K\theta=\tau_G(\theta)$$

$$\tau_G(\theta)\approx\tau_0-K_G\theta$$

$$\omega_n^2=\frac{K_n+GC_{gn}}{I}\qquad \textbf{near}$$

$$\omega_f^2=\frac{K_f+GC_{gf}}{I}\qquad \textbf{far}$$





$$G=\frac{I\left(\omega_n^2-\omega_f^2\right)-\left(K_n-K_f\right)}{C_{gn}-C_{gf}}$$

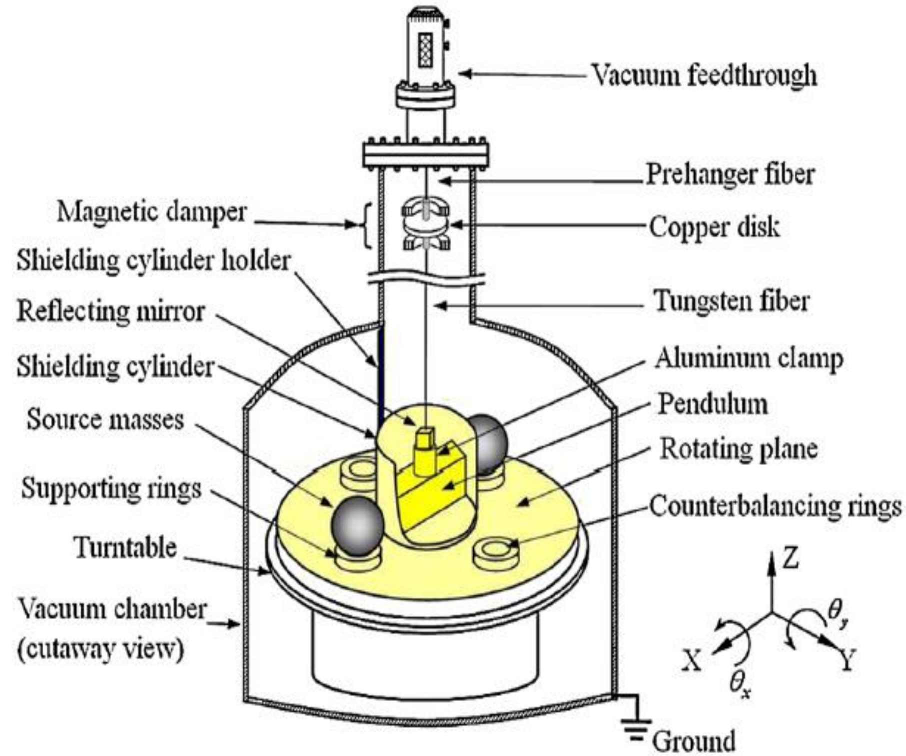
$$=\frac{I\,\Delta\omega^2-\Delta K}{\Delta C_g}$$

1. Determination of $P_g=\Delta C_g/I$
2. $\Delta\omega^2$ between near and far positions
3. Anelasticity of torsion fiber ΔK

HUST-09 experimental design

Merits：

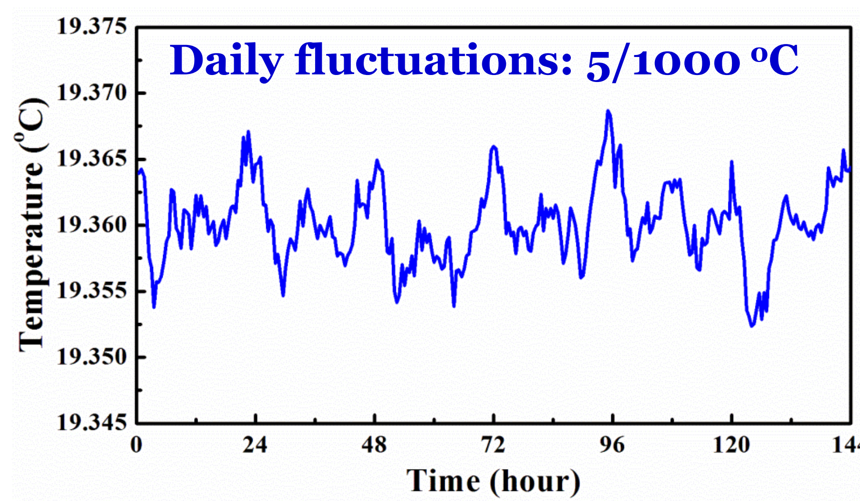
1. Spherical source mass
2. Simple pendulum
3. All in vacuum
4. Measure the anelasticity directly



Shortage：

The period (535 s) changes only ~0.6%

Experimental environment



Experimental process (almost 10 years)

*G*0: unit technologies, preliminary experiment

From 1999 to 2006

*G*1: first *G* measurement

From August, 2006 to July, 2008

*G*2: repeated *G* measurement (change the positions, orientations of the spheres and different people)

From June, 2008 to December, 2008

Final *G*: Published at 2009

PRL, 102, 240801，2009

HUST-09 *G* value

Results of G ($10^{-11}\ m^3kg^{-1}s^{-2}$)

$G_1 = (6.67352 \pm 0.00019)$

$G_2 = (6.67346 \pm 0.00021)$

consistent with 9 ppm !

Combined result:

$G = (6.67349 \pm 0.00018)$

26.3 ppm

PRL 102 (2009) 240801

PRD 82 (2010) 022001

TABLE I. One σ uncertainty budget (in units of ppm).

Error Sources	Corrections	$\Delta G/G$	
Pendulum		5.07	
Dimensions			1.95
Attitude			0.13
Nonalignment with fiber			0.45
Flatness			0.34
Clamp			1.65
Density inhomogeneity			≤0.21
Coating layer	−24.28		4.33
Edge flaw	−0.12		0.17
Source masses		10.68	
Masses			0.82
Distance of GC			9.64
Density inhomogeneity			4.50
XYZ positions			0.48
Fiber		18.76	
Nonlinearity			<0.70
Thermoelasticity	−39.83		1.52
Anelasticity	−211.80		18.69
Aging			<0.01
Gravitational nonlinearity	7.73	0.30	
Magnetic damper	17.54	0.31	
Magnetic field		0.40	
Electrostatic field		0.10	
Combined statistical $\Delta(\omega^2)$		14.18	
Total		26.33	

Problem and our tactics

Possible problems in HUST-09:

- Anelasticity of fiber : too large
- Correction to coating layers : model dependent
- Thermoelasticity of fiber : it was averaged

Our tactics:

same laboratory

↓

different methods

↓

same G value ?

- improved time-of-swing method
- angular acceleration feedback method

To evaluate the potential systematic errors

Outline

- Brief Review of G
- HUST-09 experiment
- Recent progress

1. Improved time-of-swing method

To reduce large corrections and errors, improvements are

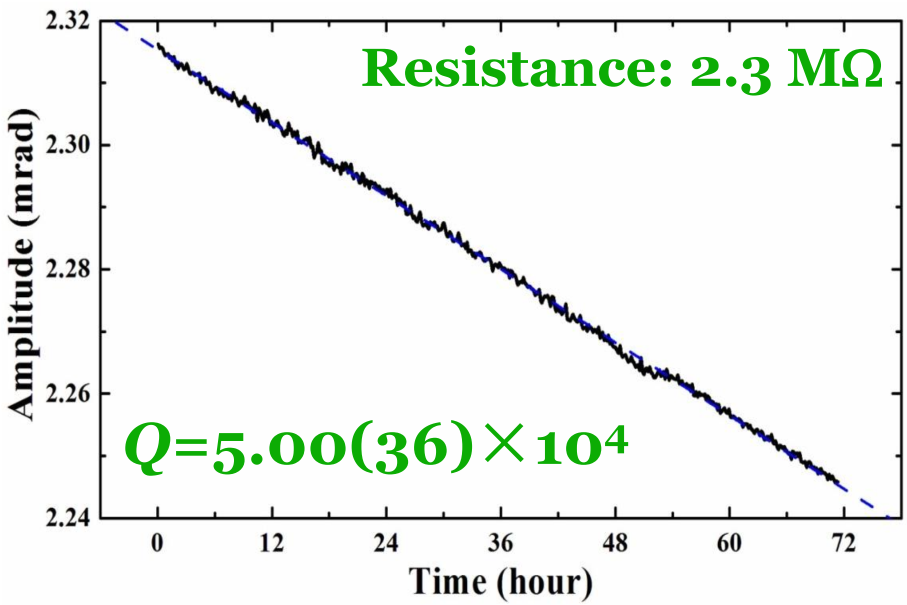
1. Aluminum layer
2. Three-point mount
3. Copper tube
4. High-Q silica fiber $Q \approx 5 \times 10^4$
5. Gravity compensation
6. Thick torsion fiber
7. Improved shielding

Error sources	Corrections (ppm)	$\delta G/G$ (ppm)
Torsion pendulum		5.05 [5.05]
Dimension		1.95
Attitude		0.13 [0.07]
Density inhomogeneity		<0.21
Chamfer property		0.34
Three chips	−0.12	0.17
Coating layer	−24.28	4.33
Clamp and ferrule		1.65
Reflecting mirror		0.03
Source masses		10.66 [10.64]
Masses		0.82
Distance of GCs		9.64 [9.61]
Density inhomogeneity		4.50
Relative positions		1.10 [1.31]
Height of pendulum		0.76 [0.40]
Height of spheres		0.48 [0.27]
Position of torsion fiber		0.63 [1.22]
Position of turntable		0.05
θ_0		0.06 [0.01]
Fiber		18.76
Nonlinearity		<0.70
Thermoelasticity	−39.83 [8.37]	1.52 [0.82]
Anelasticity	−211.80	18.69
Aging		<0.01
Gravitational nonlinearity	7.73 [4.79]	0.30 [0.20]
Magnetic damper	17.54	0.31
Magnetic field		0.40
Electrostatic field		0.10
Statistical $\Delta\omega^2$		18.43 [23.31]
Total		28.86 [32.17]
Combined		26.33

Prepare a high-Q silica fiber

Silica fiber:
Length: 900 mm
Diameter: 38-50 μm

Coated with
germanium: 8-nm-thickness
bismuth: 11-nm-thickness



3-mm rod
(SuprasilR 311)

Oxygen-natural
gas flame

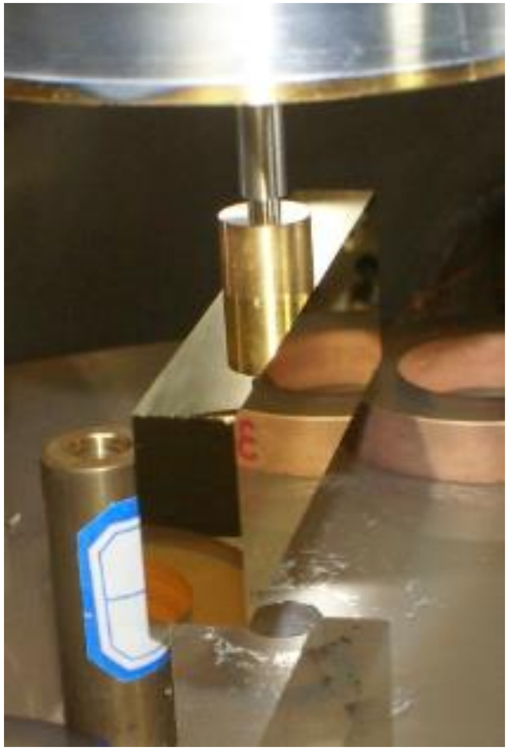
Kuroda effect:

$$\frac{\Delta G}{G} = \frac{1}{\pi Q} < 6.4\,\mathrm{ppm}$$

Use lighter coating material (Al)

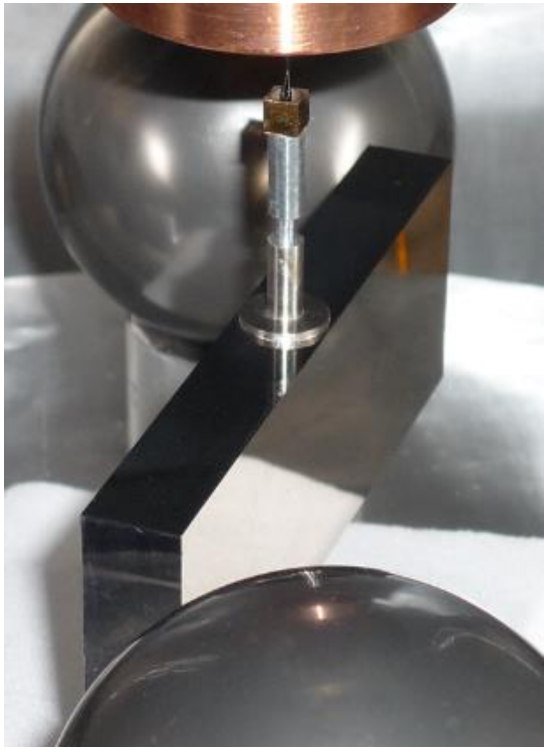
HUST-09 torsion balance

Coated with Cu+Au



Improved to

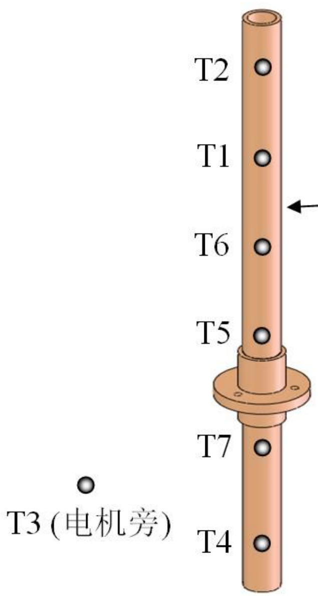
Aluminum

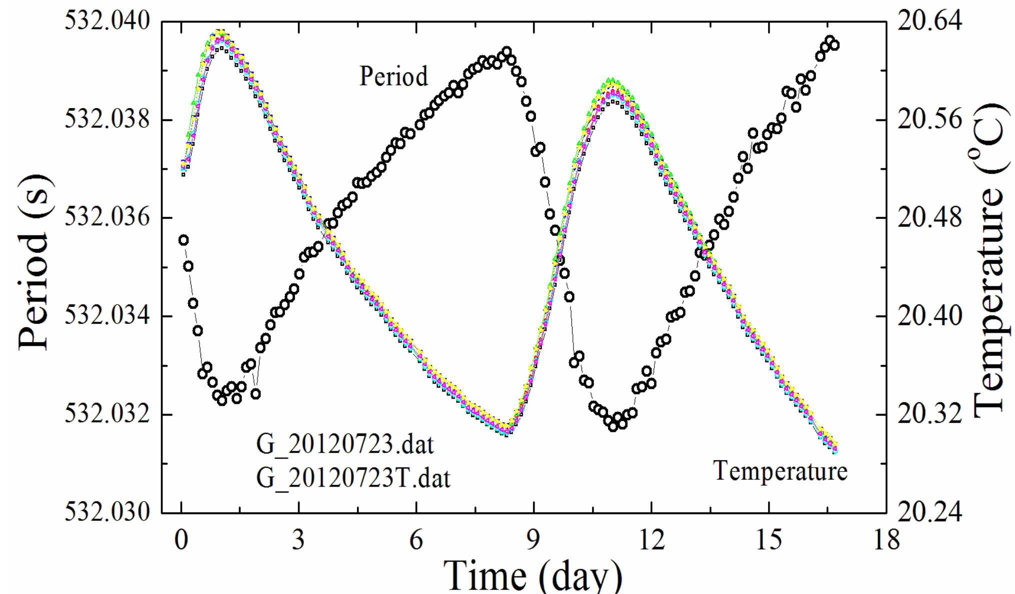


Material	Density (g/cm^3)	Mass of coating layer (mg)	Correction to G (ppm)
Cu	8.96	49.863(0.026)	-24.28(4.33)
Au	19.26		
Al	2.70	4.47(0.42)	-1.81(0.91)

Reduce the thermal gradient around fiber

A hollow Cu-tube was used to surround the fiber





Thermal modulation: Temperature vs. period

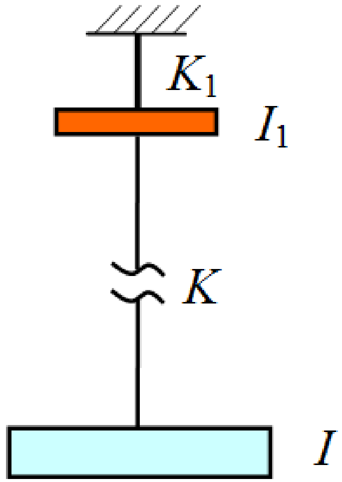
$$K = K_0(1+\alpha_K \Delta\tilde{T})$$

$$T = 2\pi\sqrt{I/K} \Rightarrow \alpha_K = \alpha_I - 2\alpha_T$$

α_K =100(3)×10^{-6} /°C

Correction to thermoelasticity of fiber：**38.50(1.50) ppm**

Redesign the magnetic damper

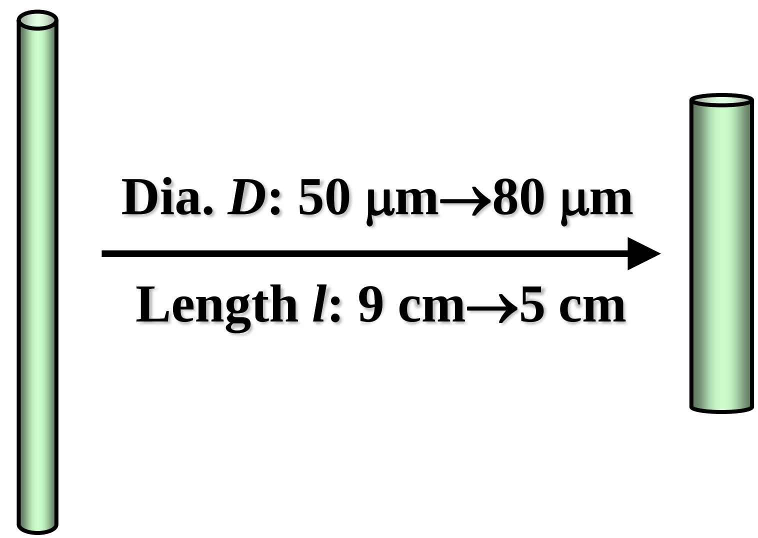


Two-stage torsion balance

$$\frac{\Delta G}{G} = \frac{I_1 K^2}{I K_1^2} \propto \frac{1}{K_1^2}$$

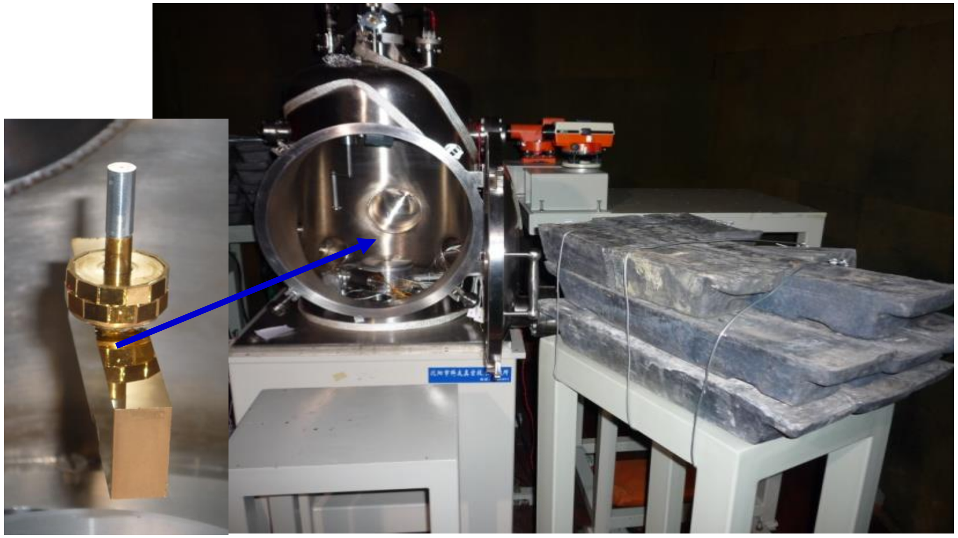
$$K_1 = \frac{\pi D^4 S}{32 l}$$

$$\frac{\Delta G}{G} \propto \frac{l^2}{D^8}$$

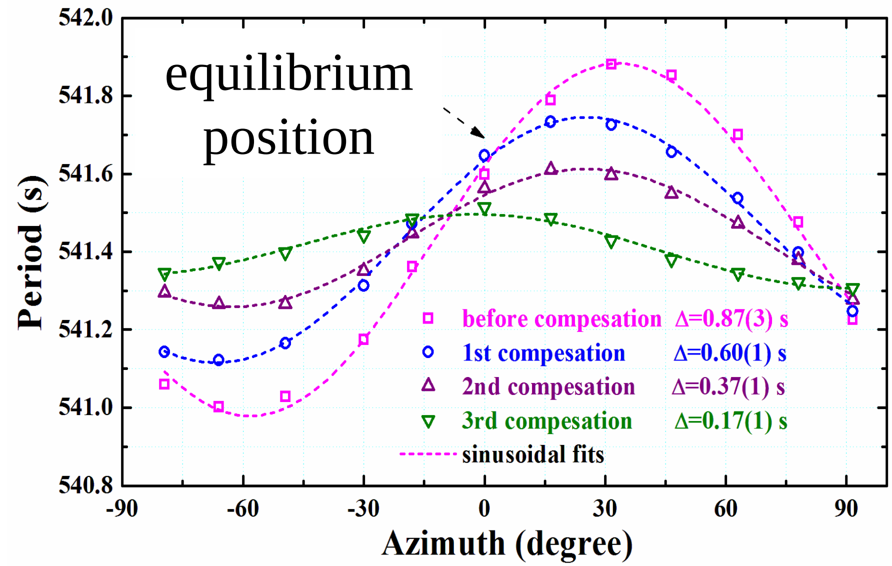


Its correction to G: 17.54(0.31) ppm to 0.13(0.01) ppm

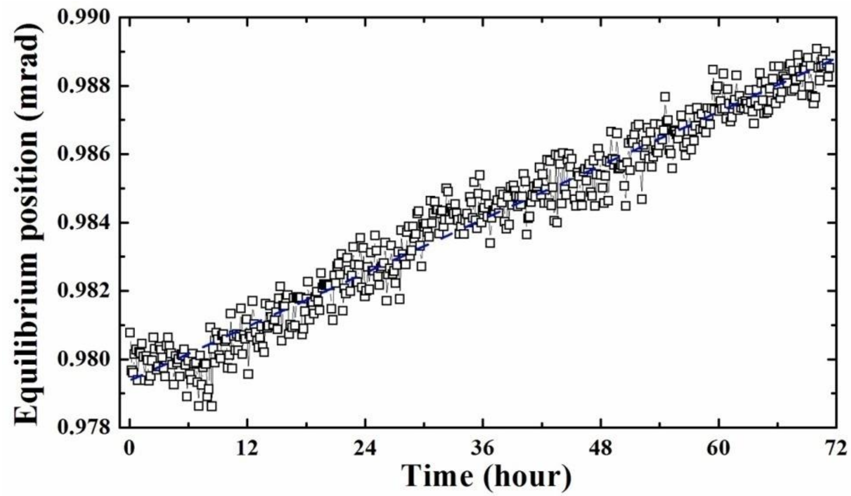
Background gravitational field



Using lead blocks to compensate



After 3 times compensation



Drift of the used silica fiber

Error to G measurements:

Single date-set(3days) <0.03 ppm

Long time (72days) <0.38 ppm

Present results of ToS method

Error sources	Corrections (ppm)	$\delta G/G$ (ppm)
Torsion pendulum		5.05 [5.05]
Dimension		1.95
Attitude		0.13 [0.07]
Density inhomogeneity		<0.21
Chamfer property		0.34
Three chips	−0.12	0.17
Coating layer	−24.28	4.33
Clamp and ferrule		1.65
Reflecting mirror		0.03
Source masses		10.66 [10.64]
Masses		0.82
Distance of GCs		9.64 [9.61]
Density inhomogeneity		4.50
Relative positions		1.10 [1.31]
Height of pendulum		0.76 [0.40]
Height of spheres		0.48 [0.27]
Position of torsion fiber		0.63 [1.22]
Position of turntable		0.05
θ_0		0.06 [0.01]
Fiber		18.76
Nonlinearity		<0.70
Thermoelasticity	−39.83 [8.37]	1.52 [0.82]
Anelasticity	−211.80	18.69
Aging		<0.01
Gravitational nonlinearity	7.73 [4.79]	0.30 [0.20]
Magnetic damper	17.54	0.31
Magnetic field		0.40
Electrostatic field		0.10
Statistical $\Delta\omega^2$		18.43 [23.31]
Total		28.86 [32.17]
Combined		26.33

-1.81(0.91) ppm

38.50(1.50) ppm

< 6.37 ppm

0.13(0.01) ppm

< 15 ppm (13 sets)

< 20 ppm (?)

2. Angular acceleration feedback method

$$I\ddot{\theta} + \gamma\dot{\theta} + k\theta = \tau_g - I\alpha_t$$

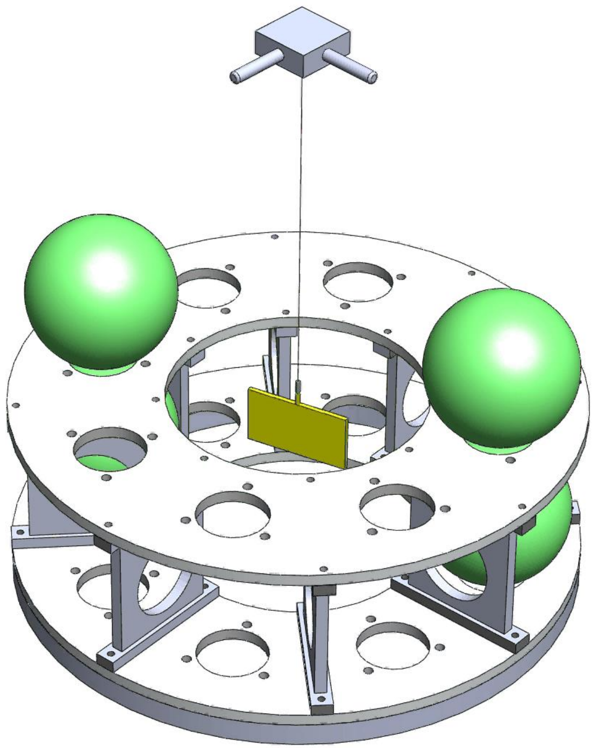
$$\underbrace{\theta = 0 \quad \dot{\theta} = 0 \quad \ddot{\theta} = 0}_{\Downarrow}$$

$$\tau_g = I\alpha_t \quad \Rightarrow \quad \alpha_g = \alpha_t$$

$$\alpha_g = \frac{\tau_g}{I} = G \cdot P_g \sin(2\omega_d t)$$

$$\Rightarrow \alpha_t = G \cdot P_g \sin(2\omega_d t)$$

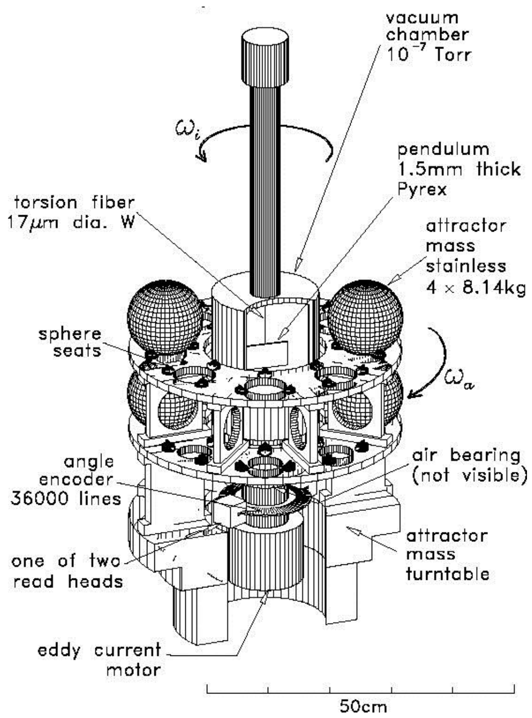
The torsion fiber does not twist



Advantages

1. Independence on torsion fiber properties
2. Pendulum mass distribution need not be determined exactly
3. Separate the environmental gravitational gradients

Uwash-2000 and our questions



Our questionable points:

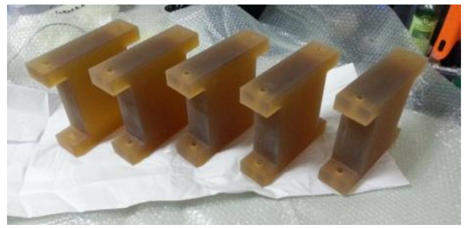
1. thermal effect of the aluminum shelf
2. gap and its stability between spheres
3. effect from slow creep deformation of fiber

$$G = (6.674255 \pm 0.000092) \times 10^{-11} m^3 kg^{-1} s^{-2} \quad 14ppm$$

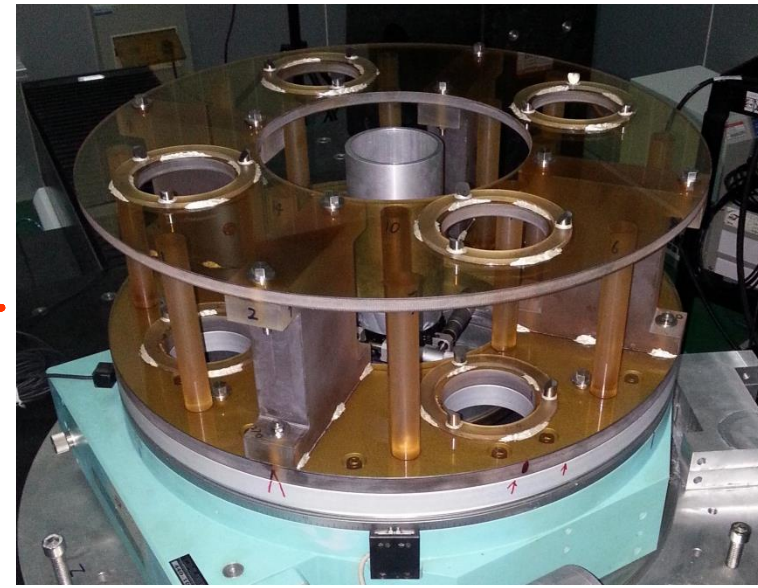
Use ULE to lower thermal effect of sphere's shelf

Al thermal coefficient ～ 20×10^{-6}/°C

ULE thermal coefficient < 0.1×10^{-6}/°C



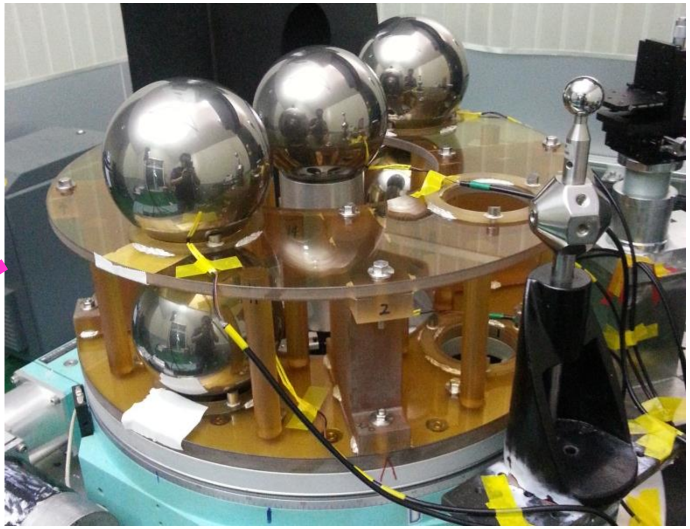
ULE Shelf

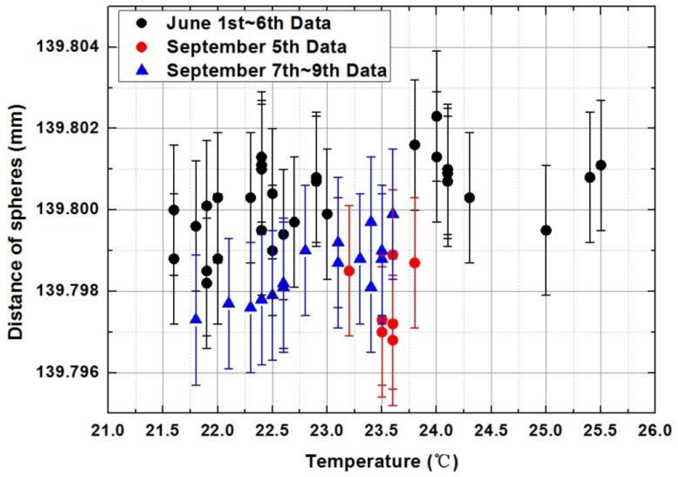


Distance (mm)	Expansion Coefficient (/°C)	Temperature Fluctuation (°C)	Distance Variation (μm)	Relative Uncertainty (ppm)
Horizontal 342.319	0.1×10^{-6}	0.7	0.02	0.10
Vertical 139.751	0.1×10^{-6}		0.01	0.04

Gap and its stability between spheres

Coordinate Measuring Machine





Distance Vs Temperature

Re-modeling the measuring equation

In torsion balance's frame，equation of motion will be

$$I\ddot{\theta}(t)+\gamma\dot{\theta}(t)+k_0\theta(t)=\tau_b(t)+\tau_g(t)-I\ddot{\Theta}(t)+\left[\gamma\dot{\theta}_{drift}(t)+k_0\theta_{drift}(t)\right]$$

Measuring equation:

$$\ddot{\Theta}(t)=\alpha_g(t)-\left[\ddot{\theta}(t)+2\beta\dot{\theta}(t)+\omega_0^2\theta(t)\right]+\alpha_b(t)+\left[2\beta\dot{\theta}_{drift}(t)+\omega_0^2\theta_{drift}(t)\right]$$

Spheres' gravitational effect

Correction to the residual deflection angle of torsion balance (～ 1ppm)

Fixed background gravitational effect (compensated)

Correction to the slow creep of fiber

Effects from slow creep of fiber

$$\theta_{dirft}(t) \approx \theta_0 + \omega_{drift} t$$

θ_0 : initial residual deflection angle.

$\omega_{drift} t$: slow linear creep deformation of fiber

Contribution to the signal frequency ω

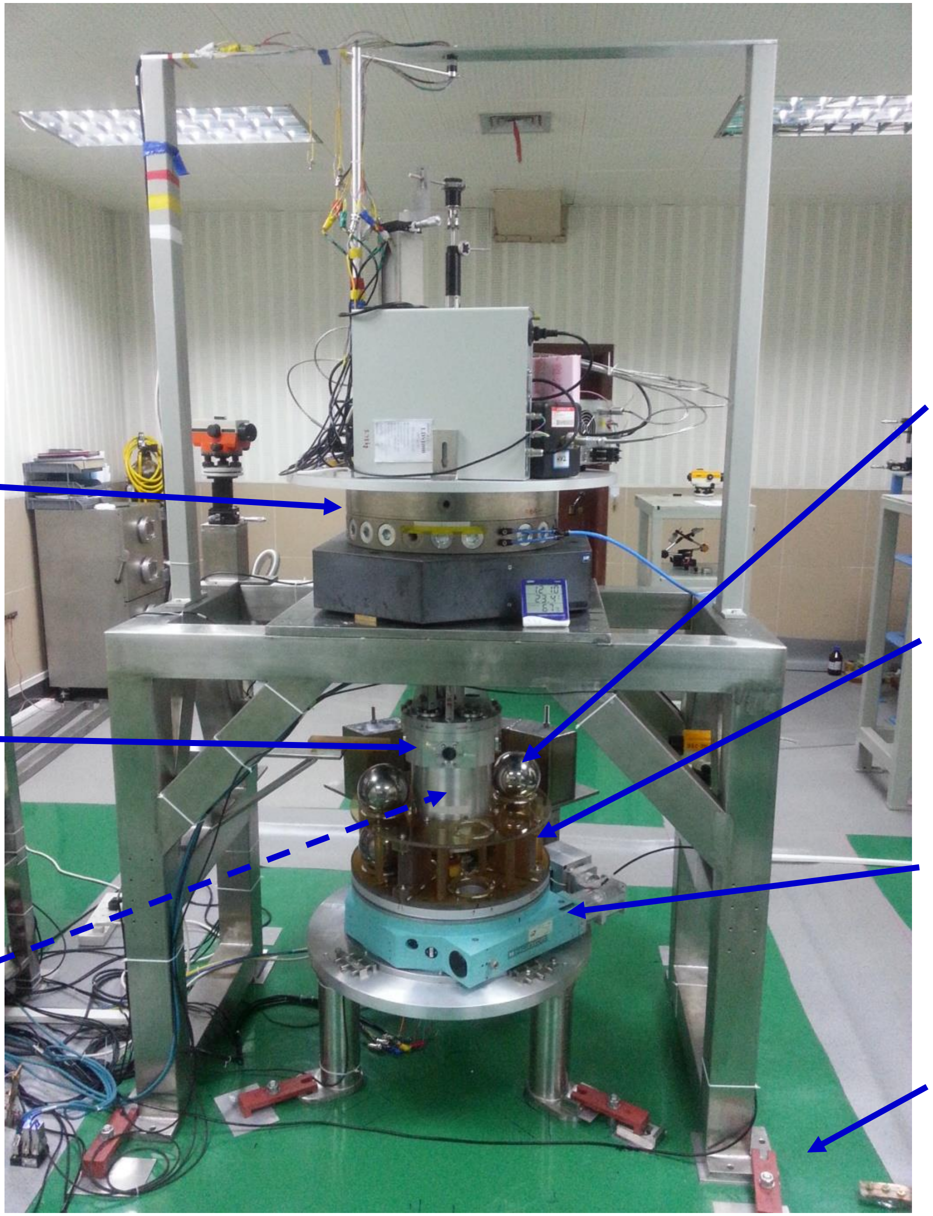
$$A_{drift} = \frac{2\omega_0^2}{mT}\int_0^{mT} \cos(\omega t + \phi)\,\theta_{dirft}(t)\,dt = \frac{2\omega_0^2 \omega_{dirft}}{\omega}\sin(\phi)$$

Compared with the signal amplitude:

$$\frac{A_{drift}}{A_{spheres}} = \frac{2\omega_0^2 \omega_{dirft}}{\omega A_{spheres}}\sin(\phi) \begin{cases} A: 462\times 10^{-9}\ \mathrm{rad/s^2} \\ \omega_0: 2\pi \times 2.4\,\mathrm{mHz} = 0.015\,\mathrm{rad/s} \\ \omega: 2\pi \times 3.0\,\mathrm{mHz} = 0.019\,\mathrm{rad/s} \\ \omega_{drift}: 2\,\mu\mathrm{rad/hour} = 5.6\times 10^{-10}\ \mathrm{rad/s} \end{cases} = 2.87\times 10^{-5}\sin(\phi)$$

Which means that the linear creep of fiber could produce 28.7ppm error maximally, and the initial phase of the signal should be measured.

Updated apparatus



Air Bearing Turntable

Vacuum Chamber

Torsion Balance (not visible)

SS316 Spheres

ULE Shelves

Gear Bearing Turntable

Shock-proof Platform

Summary

- We obtained a G value (HUST-09) by the ToS method, the relative uncertainty is about 26 ppm.

- A high-Q silica fiber is used to measure G by the ToS method, and the possible systematic errors of HUST-09 were re-considered. The present experimental results show that the uncertainty of the improved G will be less than 20 ppm.

- The angular acceleration feedback method is used, and several possible systematic errors are considered. The measured angular acceleration shows the relative uncertainty is less than 15ppm, and the expected G will also be less than 20ppm.

We hope to get the two G values with the uncertainties of less than 20 ppm very soon and ……