A Reevaluation of the NIST Low-Frequency Standards for AC-DC Difference in the Voltage Range 0.6–100 V

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***Abstract*— A reevaluation of the NIST standards of ac-dc difference was undertaken in an effort to reduce the calibration uncertainty offered by NIST for thermal voltage converters (TVC's) at frequencies below 100 Hz. This paper describes the measurements taken in support of this effort, as well as the devices used for the reevaluation process and the analysis of the uncertainty of the measurements. This reevaluation of the NIST low-frequency standards will permit a significant reduction in uncertainty for ac-dc difference calibrations at 10 Hz in the voltage range from 0.6–100 V.**

I. Introduction

RMS values of ac voltage and current are most accurately measured by comparing the heating effect of an unknown ac signal to that of a known, stable dc signal using thermal voltage converters (TVC's) consisting of one or more thermoelements (TE's), possibly in series with a range resistor, or thermal current converters (TCC's) consisting of either a TE alone or a TE in parallel with a precision shunt. The TE's are composed of one or more thermocouples arrayed along a heater structure. These thermocouples are used to monitor the temperature along the heater structure while applying a timed sequence of an ac signal and both polarities of a dc signal. By comparing the output of the TE with ac applied to the average output of the TE when both polarities of dc are applied, the unknown ac quantity may be determined in terms of the known dc quantity. This ac-dc difference is calculated using

$$\delta = \frac{V_a - V_d}{V_d} \tag{1}$$

where δ is the measured ac-dc difference, and V_a and V_d are the magnitudes of the ac and dc quantities required to produce the same thermocouple output.

It is well-known that at low frequencies the TE fails to thermally average the input ac waveform, leading to ac being present in the output EMF. The frequency at which thermal averaging ceases depends upon the thermal time constant of the TE, which in turn depends in large part on the mass of the heater and the thermal conductivity to the surroundings. Hermach [1] has approximated the ac-dc difference of a thermal converter at low frequencies to be

$$\delta = -\frac{25\left(\alpha - \beta - \frac{2p\zeta K}{ak}T_0^2 l^2 + \frac{B}{2A}\right)\theta_{dc} d^2}{(2\pi f)^2 l^4} \tag{2}$$

where: α and β are the temperature coefficient of electrical resistivity and the temperature coefficient of the thermal conductivity of the conductor, respectively.

p and a	Circumference and cross sectional area of the heater, respectively.
ζ	Emissivity.
K	Stefan-Boltzmann constant ($\mathrm{W{\cdot}cm^{-2}{\cdot}K^{-4}}$).
k	Thermal conductivity of the heater in $\mathrm{W{\cdot}cm^{-1}{\cdot}K^{-1}}$.
T_0	Ambient temperature in K.
l	One-half the length of the heater in cm.
f	Frequency of the applied rms signal.

A and B are the constants in the thermocouple EMF-characteristic equation, where the output EMF of the thermoelement is $E = A\theta + \frac{B}{2}\theta^2$ with θ the temperature of the heater

θ_{dc}	Temperature rise in K at the midpoint of the heater, for applied dc voltage.
d	Thermal diffusivity of the heater material in $\mathrm{cm^2{\cdot}s^{-1}}$.

For traditional vacuum-bulb single-junction thermal converters (SJTC's) with Evanohm heaters and constantan-copper thermocouples, the generally accepted values of A and B in (2), combined with the magnitude of the Stefan-Boltzmann constant make the last two terms in the parentheses negligible. It is evident that, for a given choice of heater material and cross section, the ac-dc difference of a thermal converter at low frequencies depends upon the fourth power of the heater length and upon the temperature rise in the heater. Traditional vacuum-bulb thermoelements have heater lengths of about 0.8 cm and cease thermal averaging at about 20–30 Hz [1]. Thin film devices [2], [3], however, may have time constants on the order of a few milliseconds and may cease thermal averaging at frequencies as high as 100 Hz or more.

This behavior of TE's produces low-frequency errors which increase the ac-dc difference of the TVC, so the uncertainties assigned by laboratories to low-frequency ac-dc measurements are generally larger than at the mid-audio frequencies due to this low-frequency error. The present NIST uncertainties

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TABLE I
PRESENT NIST CALIBRATION UNCERTAINTIES (IN μV/V) FOR TVC'S AT LOW FREQUENCIES. THE VALUE IN PARENTHESES INDICATES THE LOWEST AVAILABLE NIST UNCERTAINTY, BY SPECIAL TEST. V_a INDICATES THE APPLIED VOLTAGE. COVERAGE FACTOR IS $k = 2$

	10 Hz	20 Hz	100 Hz
Multirange TVCs (V_a >100 V)	100	30	30
Multirange TVCs ($V_a \leq$ 100 V)	100	20	20
Coaxial TVCs (V_a > 100 V)	100	20	20
Coaxial TVCs ($V_a \leq$ 100 V)	100	20	15 (5)

available to calibration customers are shown in Table I. This study was undertaken in an attempt to measure accurately the low-frequency errors of the NIST standard TVC's and to reduce calibration uncertainties at 10 and 20 Hz. While the present study covers the voltage range from 0.6–100 V, work is underway to extend this to 1000 V.

II. METHODOLOGY

Intercomparisons of TVC's were made using two identical automated measurement systems [4]. The TVC's were measured as transfer devices, with test and standard TVC's connected in parallel to ac and dc voltages in a timed sequence (dc+, ac, dc−, ac, dc+) through a tee structure with negligible ac-dc differences at these frequencies. Since the same voltage was applied to each TVC, the accuracy and long-term stability of the sources were reduced in importance. Any lead capacitance between source and tee was negligible at these frequencies. The output electromotive forces (EMF's) from the TE's, monitored using high-performance nanovoltmeters, were used to determine the ac-dc difference of the TVC under test. The calculated ac-dc difference of the standard TVC was used to match the output of the ac source to that of the dc source; consequently, the TVC's were operated with very small deviations in the input voltage, with the result that any nonlinearity of the TVC's and the imprecision in the measurement of their response characteristics were unimportant, and the differences in the output EMF of the test TVC with ac and dc applied reflected the true ac-dc difference of the device.

The reference standards for these low-frequency tests were two TVC's containing either four TE's (in the 1.25 V TVC designated $LFTE_4$) or six TE's (in the 2 V TVC designated $LFTE_6$) with their outputs in series aiding and their heaters connected in series to make the lengths of the heaters 3.2 cm in $LFTE_4$ and 4.8 cm in $LFTE_6$ [5]. By using these devices at half their rated input voltage, the temperature rise of the heater is small; increasing l and decreasing θ_{dc} in (2) significantly reduces the ac-dc differences of $LFTE_4$ and $LFTE_6$. Intercomparisons of $LFTE_4$ and $LFTE_6$ indicate that their ac-dc differences vary by less than 1 μV/V with respect to one another from 10 Hz to 1 kHz. These two thermal converters were taken as the starting points for the voltage build-up procedure used to determine the ac-dc differences of the NIST reference set of TVC's, designated the R set [6], and the NIST working standard set, designated the W set [7].

The basis of the voltage build-up is a set of intercomparisons between $LFTE_4$, $LFTE_6$, and two other TE's, R_A, and R_B at 0.6 V. R_A and R_B are the thermoelements from the NIST reference TVC set, and their ac-dc differences at 1 kHz are well-known from comparisons with the NIST primary TVC's. Intercomparisons were made with $LFTE_4$ and $LFTE_6$ to ascertain the low-frequency response of R_A and R_B from 1 kHz down to 10 Hz. By using $LFTE_4$ and $LFTE_6$ to measure only the frequency response of these TVC's, and not their ac-dc differences, the ac-dc differences of $LFTE_4$ and $LFTE_6$ themselves were not required to be known.

After finding the low-frequency response of R_A and R_B, these two TVC's, along with $LFTE_4$ and $LFTE_6$ were used to measure $R_{1\,V}$ (from the NIST reference TVC set) and $W_{1\,V}$ (from the NIST set of working standards) at 1 V. In this case, $LFTE_4$ and $LFTE_6$ were used at 1 V to check the frequency response of the 1 V TVC's, and R_A and R_B were used to perform a normal ac-dc difference measurement to assign ac-dc difference values to $R_{1\,V}$ and $W_{1\,V}$. The ac-dc differences calculated for $R_{1\,V}$ and $W_{1\,V}$ using $LFTE_4$ and $LFTE_6$, and the ac-dc differences measured using R_A and R_B were in very good agreement, and formed a solid basis for the voltage build-up.

The remainder of the voltage build-up was carried out in the manner shown in Fig. 1. Range-to-range intercomparisons were performed using as the standard a TVC which had been previously measured, and comparing the test unit to the standard at an applied voltage of from 50 to 67% of full rated voltage for the test TVC. As an example, the 10 V TVC $R_{10\,V}$ was measured at 5 V using $R_{5\,V}$, which had been measured using $R_{3\,V}$ at 3 V. In addition, cross checks were performed using TVC's formed by using either $LFTE_4$ or $LFTE_6$ in series with a range resistor. Since the frequency response of a TVC at 1 kHz or less is generally independent of the range resistor, these measurements provided a good cross-check on the values provided by the build-up procedure.

III. RESULTS AND UNCERTAINTY CALCULATIONS

Since the focus of this work is the reduction of uncertainty for NIST low-frequency TVC measurements, the newly derived corrections to the NIST standard TVC's are not presented here. Instead, the method of the calculation of uncertainties are given, as well as the new NIST uncertainties for these measurements. The uncertainties were calculated in accordance with CIPM and NIST requirements [8], which divide the uncertainty assigned to the measurements into Type A uncertainties (those evaluated by statistical means) and Type B uncertainties (those evaluated by other means) and then combine these uncertainties in a root-sum-of-squares (RSS) fashion. For ac-dc measurements in general and for this analysis in particular, the Type B uncertainties are the dominant sources of error. These uncertainties are generally based on estimates of the upper limits of the various contributions of ac-dc difference and the measurement process. The uncertainty element is the upper limit value divided by $\sqrt{3}$. The Type A uncertainties are the pooled standard deviations of intercomparisons of TVC's, and are usually small compared to the Type B uncertainties.

The primary standards of ac-dc difference at NIST are a group of multijunction thermal converters (MJTC's) [9] of

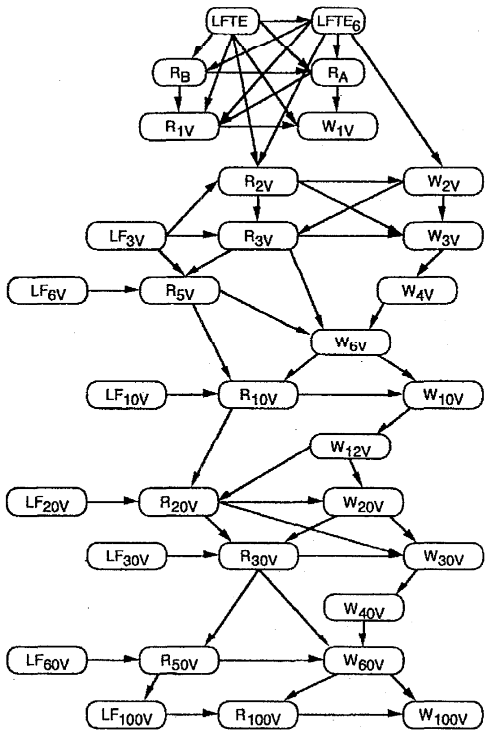

Fig. 1. Build-up paths for the low-frequency TVC intercomparisons. The tails of the arrows indicate the standard TVC for each comparison. The arrowheads point to the test TVC.

TABLE II
REFERENCE UNCERTAINTY ($k = 1$) FOR $LFTE_4$ AND $LFTE_6$ FROM 10 TO 100 Hz. S_{pa} DENOTES THE POOLED STANDARD DEVIATIONS FOR EACH UNCERTAINTY ELEMENT, b_0 REPRESENTS THE LIMITS OF THE RECTANGULAR DISTRIBUTION FOR THE UNCERTAINTY ELEMENT, AND b IS THE RESIDUAL UNCERTAINTY ($b_0/\sqrt{3}$)

Type A Uncertainties (μV/V)		Type B Uncertainties (μV/V)				
Element	S_{pa}	Element	$\pm b_0$	b		
Voltage extension (10 V to 0.6 V)	0.5	Voltage extension (10 V to 0.6 V) Each step After 5 steps	0.2	0.1 0.3		
Comparison of $LFTE_4$ and $LFTE_6$ to reference		Comparator system uncertainty			Uncertainty for $LFTE_4$ and $LFTE_6$	
Frequency	S_{pa}	Frequency	$\pm b_0$	b	Frequency	u_1
10 Hz	1.0	10 Hz	2.0	1.2	10 Hz	1.8
20 Hz	0.8	20 Hz	1.5	0.9	20 Hz	1.8
40 Hz	0.8	40 Hz	1.0	0.6	40 Hz	1.0
100 Hz	1.0	100 Hz	1.0	0.6	100 Hz	1.6

TABLE III
UNCERTAINTIES ($k = 1$) ASSIGNED TO BASE THERMOELEMENTS ($R_A, R_B, W_{1\,V}$, AND $W_{2\,V}$)

Type A Uncertainties (μV/V)		Type B Uncertainties (μV/V)				
Measurement of working standards		Comparator system uncertainty			Uncertainty for R_A, R_B, W_{1V}, W_{2V}	
Frequency	S_{pa}	Frequency	$\pm b_0$	b	Frequency	u_2
10 Hz	0.9	10 Hz	1.0	0.6	10 Hz	3.4
20 Hz	0.9	20 Hz	1.0	0.6	20 Hz	2.5
40 Hz	0.8	40 Hz	1.0	0.6	40 Hz	1.9
100 Hz	0.9	100 Hz	0.5	0.3	100 Hz	1.9
		Voltage level coefficient	$\pm b_0$	b		
		10 Hz	5.0	2.9		
		20 Hz	3.0	1.7		
		40 Hz	2.0	1.1		
		100 Hz	0.5	0.3		
		Intercomparison error	$\pm b_0$	b		
		10 Hz	1.0	0.6		
		20 Hz	1.0	0.6		
		40 Hz	0.8	0.4		
		100 Hz	0.5	0.3		

various construction and from different manufacturers. The mean of the ac-dc difference of these MJTC's is taken to be less than 0.5 μV/V from 30 Hz to 10 kHz, with an uncertainty of 0.42 μV/V ($k = 1$) [10]. This uncertainty includes estimates of both the performance of the primary standards, the MJTC comparator system, and the measurement of the reference standard used to transfer the ac-dc difference from the primary standard to a single-junction reference standard.

To obtain values for $LFTE_4$ and $LFTE_6$ at the beginning of the voltage buildup path, it was necessary to step down in voltage from the 10 V level of the MJTC to the 0.6 V level of $LFTE_4$ and $LFTE_6$ and in frequency from 1 kHz to 10 Hz. Taking the root-sum-square values for this voltage scaling and frequency extension, and the uncertainties and pooled standard deviations associated with the comparator system used to perform the extension yields the reference uncertainties for $LFTE_4$ and $LFTE_6$ shown in Table II.

The next step was to compare the thermoelements $R_A, R_B, W_{1\,V}$, and $W_{2\,V}$ to $LFTE_4$ and $LFTE_6$ from 10 to 100 Hz. These comparisons were made at both 60% and 100% of full scale input voltage to ascertain the voltage coefficients of the thermoelements, a potentially significant source of error at these frequencies. The worst voltage level coefficient, approximately 5 μV/V, was measured for $W_{1\,V}$. Table III presents the uncertainties assigned to R_A and R_B at 0.6 V, $W_{1\,V}$ at 1 V, and $W_{2\,V}$ at 2 V. These uncertainties include the pooled standard deviations for these intercomparisons, the uncertainties assigned to the comparator systems, uncertainties associated with voltage level effects, and the uncertainty assigned to a single intercomparison.

The voltage buildup from 0.6 to 100 V was performed by comparing a TVC of a particular voltage range at less than full scale against a previously-measured TVC at full scale, as illustrated in Fig. 1. So, for example, to find the value of the 10 V range of the reference set, $R_{10\,V}$ was compared to $R_{5\,V}$ at 5 V. $R_{20\,V}$ was then compared to $R_{10\,V}$ at 10 V, and so on. As can be seen from Fig. 1, the uncertainty analysis contains multiple build-up paths, which for statistical purposes were taken as independent paths. The uncertainties calculated for each step of the build-up were combined using

$$u_c = \frac{1}{N}\sqrt{\sum_{n=1}^{N} \sigma_n^2} \tag{3}$$

where u_c is the uncertainty at a particular step in the build-up, N is the number of independent build-up paths, and σ_n is the standard error of the nth step in the build-up. In this case, $N = 2$ for the reference set build-up and the working set build-up (measurements using the $LFTE_4$ and $LFTE_6$ TE's in series with working set resistors do not form a continuous build-up path). The uncertainties at each step, which include the uncertainty of the comparator system for a single measurement, depend upon the uncertainty of the

TABLE IV
UNCERTAINTIES ($k = 2$) ASSIGNED TO NIST COAXIAL REFERENCE AND WORKING TVC's

Applied Voltage	Uncertainty of NIST standards (μV/V) (k=2)			
Volts	10 Hz	20 Hz	40 Hz	100 Hz
0.6	7.2	5.0	3.8	3.8
1.0	9.3	6.2	4.5	3.8
2.0	11.0	7.2	5.1	3.9
3.0	12.5	8.1	5.7	4.0
6.0	13.8	8.9	6.2	4.1
10.0	15.0	9.6	6.7	4.2
20.0	16.1	10.3	7.1	4.3
30.0	17.2	10.9	7.5	4.3
60.0	18.1	11.5	7.9	4.4
100.0	19.0	12.0	8.3	4.5

TABLE V
UNCERTAINTIES ($k = 2$) ASSIGNED TO COAXIAL TVC's FOR ROUTINE CALIBRATIONS OF CUSTOMER UNITS

Applied Voltage	Uncertainty of coaxial TVCs (μV/V) (k=2)			
Volts	10 Hz	20 Hz	40 Hz	100 Hz
0.6	14	11	9	5
1.0	15	12	9	5
2.0	16	12	10	5
3.0	17	13	10	5
6.0	18	13	10	5
10.0	19	14	11	6
20.0	20	14	11	6
30.0	21	15	11	6
60.0	22	15	11	6
100.0	22	16	12	6

TABLE VI
UNCERTAINTIES ($k = 2$) ASSIGNED TO MULTIRANGE TVC's FOR ROUTINE CUSTOMER CALIBRATION

Applied Voltage	Uncertainty of multirange TVCs (μV/V) (k=2)			
Volts	10 Hz	20 Hz	40 Hz	100 Hz
0.6	15	13	12	10
1.0	16	14	13	10
2.0	17	15	14	10
3.0	18	16	16	10
6.0	19	17	17	10
10.0	20	18	18	11
20.0	20	19	19	11
30.0	21	20	20	11
60.0	22	21	20	11
100.0	23	22	21	11

TABLE VII
UNCERTAINTIES ($k = 2$) ASSIGNED FOR ROUTINE CALIBRATION TO THERMAL TRANSFER STANDARDS WITH SHORT THERMAL TIME CONSTANTS

Applied Voltage	Uncertainty of TVCs with short thermal time constants (μV/V) (k=2)			
Volts	10 Hz	20 Hz	40 Hz	100 Hz
0.6	21	18	16	6
1.0	22	18	16	6
2.0	23	18	16	6
3.0	23	19	16	6
6.0	24	19	17	6
10.0	25	19	17	6
20.0	26	20	17	6
30.0	26	20	17	6
60.0	27	20	17	6
100.0	28	21	17	6

previous step; these uncertainties were combined using the RSS approach for the propagation of measurement error from the 0.6 V step up through the 100 V step.

The uncertainties assigned to the NIST coaxial TVC's used as reference and working standards are shown in Table IV. The intercomparison error (Type B) has been estimated as 1.00 μV/V from 10–30 Hz, 0.75 μV/V from 40–90 Hz, and 0.5 μV/V at 100 Hz.

IV. PROPAGATION OF UNCERTAINTIES TO CUSTOMER CALIBRATIONS

The uncertainties assigned to routine calibrations were calculated using the same methods as with the NIST coaxial standards with several exceptions. The uncertainty estimates for the intercomparison (which includes the stability of the unit under test during the measurement) as well as voltage coefficient errors were expanded to cover adequately these effects on the ac-dc differences of commercially available TVC's. This analysis led to dividing the calibration uncertainties into three categories; coaxial standards similar to the NIST standards, multirange standards, and thermal transfer standards with short thermal time constants.

For coaxial TVC's, the uncertainties due to voltage level effects and frequency dependence were increased at 10 Hz and 20 Hz. In addition, the intercomparison error was increased to account for instability and drift in the unit under test. The resulting uncertainties for coaxial TVC's with a coverage factor of 2 are shown in Table V.

Since, owing to their construction, multirange transfer standards may be less stable than coaxial TVC's, the intercomparison uncertainty was increased to more accurately reflect their performance, while the effects of the voltage and frequency coefficients were estimated to be those of coaxial TVC's. The resulting calibration uncertainties are shown in Table VI.

Finally, transfer standards are available which have quite good performance at audio frequency, but which have short thermal time constants and subsequently potentially large voltage and frequency coefficients of ac-dc difference at low frequency. For these instruments, the uncertainties associated with these effects have been substantially increased from 10–40 Hz although the intercomparison uncertainty remains the same as that for coaxial TVC's. Calibration uncertainties for these devices are shown in Table VII.

It must be emphasized that the uncertainties available to NIST customers, as presented in the tables, will apply only to devices with ac-dc difference measured to be stable and free from drift. If a customer's TVC is observed to lack the necessary stability and freedom from drift to warrant these uncertainties, the uncertainties will be expanded to cover the instability of the instrument.

V. CONCLUSION

A redetermination of the ac-dc differences for NIST reference and working standard TVC's and their related uncertainties has been performed and the uncertainties calculated in accordance with the CIPM and NIST guidelines for the expression of uncertainty. A reduction in the uncertainty of the NIST standards has been realized in the frequency range from 10–100 Hz at voltages from 0.6–100 V with a significant reduction at 10 Hz.

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