Dynamic Calibration of Waveform Recorders and Oscilloscopes Using Pulse Standards

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***Abstract*—The purpose of this paper is to convince the reader of two key points. First, virtually no one calibrates oscilloscopes or waveform recorders properly and completely at present. Second, in most cases, the tools are now available to perform these complete and proper calibrations when the application requires it. After a brief introduction describing the current methods used to calibrate oscilloscopes, the problems associated with oscilloscope vertical channel bandwidth testing will be discussed and illustrated. Then, a solution will be described that involves using pulse signals and a NIST-developed deconvolution algorithm. Finally, an example of the calibration of a 20-GHz sampling oscilloscope will be presented.**

I. Introduction

OSCILLOSCOPES allow us to measure and "see" (through a CRT display and/or hardcopy plot) a functional representation of voltage versus time [1]. With the proper sensors or transducers connected to the oscilloscope, the user can measure not only voltage versus time but also almost *any* parameter of interest versus time. As we would expect with this versatility and utility, there are an enormous number of applications for these instruments in science and technology. In addition, there are an enormous number of different models available for all of these applications, with innumerable knobs, buttons and other specialized features.

With all of the available (or future) variations, however, there is really only one basic characteristic of any oscilloscope that ultimately matters to most users. That is, how well does the displayed or recorded voltage-versus-time graph represent the true shape of the signal being measured? Disregarding all of the knobs, buttons, and fancy features, how do we verify that the observed waveform accurately represents the true signal? For decades, as well as at present (with few exceptions), this verification, or "calibration" question has been addressed by performing three basic types of tests.

The first type of test is concerned with the gain and linearity of the voltage (usually *y* axis) channel. These tests are designed to verify an accurate mapping between the voltage at the oscilloscope input and the displacement of the electron beam (or other indicator). Usually they consist of applying known dc voltages to the oscilloscope input terminals and visually or computationally verifying the accuracy of this mapping over the entire range of permissible voltages.

The second type of test is concerned with the gain and linearity of the time (usually *x* axis) channel. Analogous to the voltage channel case, these tests verify the accuracy of the mapping between the passage of time and the displacement of the electron beam. Usually these time channel gain and linearity tests consist of applying a set of time-varying voltages of fixed and known period to the oscilloscope input terminals and, again, verifying the accuracy of this mapping over the entire range of permissible sweep speeds.

The third basic type of test is concerned with the speed of response (displacement) of the voltage channel. The laws of physics dictate that no realizable electronic circuit can produce an instantaneous change. As a result, every oscilloscope has some upper limit on speed of response beyond which it will no longer accurately follow the rapid voltage variations of the input signal. The most common test for verifying this speed of response is a two-step process. First, a "low" frequency sine wave of known amplitude is applied to the oscilloscope and the displayed amplitude is noted. Then, with the amplitude held constant, the frequency of this sine wave is increased until the displayed amplitude falls to some prescribed value, usually to 70.7% of its original (low-frequency) displayed amplitude. This test then determines the frequency at which the voltage channel gain falls three decibels from its "low" frequency gain. It is often called an oscilloscope "bandwidth test."

A variation of this third type of test sometimes performed is to apply a step-like baseband pulse with a known 10%–90% first transition duration (rise time) to the oscilloscope input and verify that the displayed transition duration is less than or equal to some prescribed value. This pulse test is often called an oscilloscope "rise time test." (This is technically a misnomer. According to the IEEE standards on pulses[1,2] it should be called a "transition duration test." The term "rise time" is not defined.)

These three types of tests (voltage, time, and, speed of response) constitute the heart of the calibration of any oscilloscope. Depending on the particular oscilloscope, a

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[1] "IEEE standard on pulse measurement and analysis by objective techniques," IEEE Std. 181-1977, IEEE, NY, July 1977.

[2] "IEEE standard pulse terms and definitions," IEEE Std. 194-1977, IEEE, NY, July 1977.

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number of other tests such as proper internal and external trigger circuit operation, delayed sweep circuit operation, and intensity and focus circuit operation may be required. None of these other tests is of interest here since they generally do not play a role in the fundamental question of how well the oscilloscope displays or records the waveform shape of the true signal at the oscilloscope input terminals.

The claim generally made, according to currently accepted principles, is that if you perform a suitable version of these three tests on any oscilloscope, its ability to produce a display or recording that represents the shape of the true signal within some specified accuracy is verified. This is simply not true! One of these tests is inadequate!

II. The Problem

If the gain and linearity tests of the voltage and time channels are performed properly, they are both correct and sufficient as outlined above. The problem is with the bandwidth or transition duration test; it is correct but not sufficient. If we wish to measure and view the shape of *any* voltage-versus-time signal except a sine wave, then the speed-of-response test, outlined above, is inadequate. Simply knowing the bandwidth or transition duration of the oscilloscope voltage channel is not sufficient to guarantee the accuracy of the shape of the displayed or recorded waveform.

To understand why this is true, we need only to consider the Fourier series expansion of any given voltage-versus-time signal. A nonsinusoidal, periodic signal consists of the sum of an often large number of harmonically related sinusoids, each with its own amplitude and phase. In the time domain, the shape of this signal is determined by the amplitude and phase of each of those sinusoids. Therefore, if the oscilloscope voltage channel does not accurately maintain these amplitude and phase relationships, the shape of the displayed waveform can be drastically different from that of the input signal. Simply knowing the frequency at which the voltage channel gain is reduced by some number of decibels tells us very little about the voltage channel's ability to maintain these amplitude and phase relationships. In fact, the *only* way to verify that these relationships are being maintained over some desired frequency range is to *measure* them. In other words, we have to measure the *entire complex transfer function* of the voltage channel over the frequency range of interest.

As a simple illustration of this problem, Fig. 1 is the full-scale display of a 500-mV step-like pulse (first transition duration of about 500 ps) as recorded on a very fast sampling oscilloscope. The bandwidth of this oscilloscope is 20 GHz, which far exceeds the discernable spectral content of this pulse. Therefore, we can assume that the displayed shape of this pulse in Fig. 1 is accurate.

Fig. 2 is the display of the same pulse as recorded on an oscilloscope with a specified bandwidth of 350 MHz. The waveform shapes in Figs. 1 and 2 appear to be closely related, but they visibly differ somewhat.

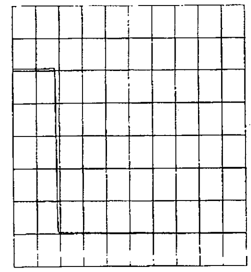

Fig. 1. 500-mV step-like pulse recorded on a 20-GHz sampling oscilloscope. Voltage scale is 100 mV/div and time scale is 5 ns/div.

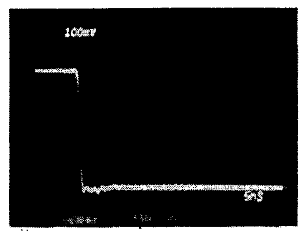

Fig. 2. The same 500-mV step-like pulse recorded on a 350-MHz oscilloscope. Voltage scale is 100 mV/div and time scale is 5 ns/div.

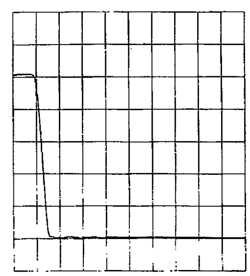

Fig. 3. The same as Fig. 1 except sweep speed has been increased to 1 ns/div.

The oscilloscopes' sweep speeds were increased by a factor of five for the recordings in Figs. 3 and 4. Now the displayed transition duration (about 500 ps) of the 20-GHz scope waveform is very different from that (about 1000 ps) displayed on the 350-MHz scope. In addition, and almost as important, the aberrations (or "wiggles") on the displayed waveform baselines are noticeably different, a sure sign that the amplitude and phase relationships are being distorted by the 350-MHz scope.

Figs. 5 and 6 are displays on the respective oscilloscopes recorded over the same time interval as that in Figs. 1 and 2. The difference is that the voltage channel input attenuators were adjusted to amplify the displays by a factor of 10. (Note that the original pulse baseline is now offscreen and no longer visible.) The displayed waveform shapes are now radically different. In fact, the display in Fig. 6 does not even look like an amplified version of that in Fig. 2. The most likely reason for this lack

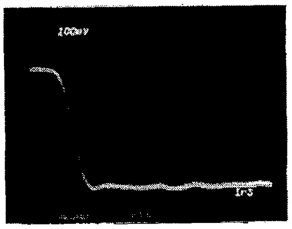

Fig. 4. The same as Fig. 2 except sweep speed has been increased to 1 ns/div.

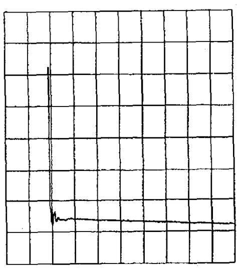

Fig. 5. The same as Fig. 1 except voltage scale has been expanded to 10 mV/div.

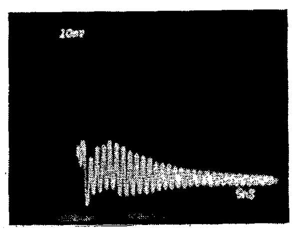

Fig. 6. The same as Fig. 2 except voltage scale has been expanded to 10 mV/div.

of agreement, even on the same oscilloscope viewing the same pulse, is that different electronic components were switched into the voltage channel when the vertical attenuator was switched, and their broadband frequency responses are quite different.

Many examples such as this could be presented, but by now the point should be clear.

III. The Solution

It should now be evident that we must know the entire complex transfer function of an oscilloscope's voltage channel for a correct and sufficient calibration. The next question is, how can this be accomplished? The answer consists of two steps. First, a known broadband signal must be measured on the oscilloscope and somehow digitized. Second, the measured, digitized signal must be deconvolved from the known signal. The complex transfer function obtained from this deconvolution may then be used to correct the shape of any subsequent waveforms with another deconvolution. What follows is a detailed discussion of each of these steps.

A. Broadband Signal Measurement

In principle, there are two ways to measure the oscilloscope's amplitude and phase response to a broad band of frequencies. The first method is to apply to the oscilloscope input terminals a large set of harmonically related sine waves of known amplitude and phase and to record the oscilloscope's amplitude and phase response for each one. The second method is to apply an appropriate pulse waveform which contains harmonic sinusoidal components over the frequency range of interest. In this case, the fast Fourier transform (FFT) could be used to obtain the desired complex frequency domain information.

In practice, the first method is usually not possible. An accurate estimate of the sine waves' amplitude responses may be obtained, but the phase responses cannot. The reason for this is that most oscilloscopes' trigger circuits are not designed to maintain a known, fixed delay between the incoming signal and the beginning of the oscilloscope's time sweep. Therefore, this method will not be considered here.

The second method, using known pulses, is feasible because both the amplitudes and phases of each Fourier harmonic are calculable using the FFT. In particular, the phases of all of the higher order harmonics are uniquely related to that of the first harmonic and are dictated by the shape of the time-domain pulse. (In general, there will be a linearly varying phase angle added to the "minimum phase" angles for each harmonic. These added phase angles are a function of the time at which the pulse is displayed within the time interval of this display. This added linear phase function does not affect the calibration and may either be subtracted out or ignored.)

The objective, then, for this step is to measure a known pulse, somehow digitize the displayed or recorded oscilloscope waveform, and then compute the FFT of both the known pulse waveform and the recorded oscilloscope response waveform for use in the deconvolution step. For a digitizing oscilloscope with a built-in computer, this step is easy and straightforward. If the oscilloscope is an analog model with only a CRT display, the problem is more difficult. Recently introduced oscilloscope-digitizing cameras make it possible, however, for even these analog oscilloscopes to be properly calibrated if so desired. In any event, a computer is required to perform the necessary FFT and deconvolution calculations.

The choice of the pulse to be used for the calibration of a particular oscilloscope is very important. For the most accurate results the pulse's spectrum should be slowly varying as a function of frequency and should contain no zeros in the frequency range of interest. Also, it should be fast enough to contain useful harmonics that well exceed the oscilloscope's bandwidth specification. Since all oscilloscopes' gain roll-off characteristics are different, there is no exact rule relating any individual oscilloscope's bandwidth to the desired transition duration of the test pulse. However, as a rough rule of thumb, the 10%–90% first transition duration of the step-like test pulse should be less than the reciprocal of six to nine times the oscilloscope's specified 3-dB bandwidth. If an impulse-like pulse is to be used then this same rough rule would apply for the impulse's 50%–50% pulse duration. (This rule was derived from the desire to ensure that the spectral content of the pulse be two to three times the 3-dB roll-off frequency of the oscilloscope's vertical channel, coupled with the rough rule that the transition duration of a step-like pulse is approximately equal to 0.35 times the reciprocal of its bandwidth. This "0.35 rule" is precise only for an *RC* circuit.)

The FFT also imposes two restrictions on the choices of test pulses available. Errors in the FFT computation will occur if the digitized waveform has different voltage values or different slopes (first differences) at the beginning and end points of the waveform array. These errors are commonly called "leakage" errors. It might appear, then, that only impulse-like pulses are allowable. However, methods developed at NIST and elsewhere [2]–[4] remove the equal-voltage-value restriction and thus allow the use of step-like pulses.

The other restriction imposed by the FFT is concerned with the pulse waveform digitization. This digitization must be performed in such a manner as to ensure that the time between waveform sample points is small enough. In particular, according to the well-known sampling theorem, the time between points must be less than or equal to the reciprocal of two times the highest frequency component present in the pulse waveform. If this condition is not satisfied, "aliasing" errors in the FFT computation will result. Once a suitable pulse (or set of pulses) has been chosen, measured, and digitized, and the two pulse spectra computed with the FFT, the next step is to deconvolve the two spectra in order to obtain the oscilloscope voltage channel's complex transfer function.

B. Deconvolution

Based on the theory developed by Tikhonov and Arsenin [5], a simple deconvolution algorithm has been developed at NIST [6], [7]. It requires only the repetitive computation of FFT's and can be performed on very small (desktop) computers. (Reference [6] contains a detailed discussion of this algorithm while [7] contains a much shorter but adequate description. The software is available from the author at NIST.) Other studies of this algorithm have also been performed [8], [9]. What follows is a simplified description of the NIST deconvolution algorithm.

If the spectrum of the standard pulse is denoted by $X(n)$, and that of the oscilloscope's response as $Y(n)$, then, in principle, the oscilloscope's complex transfer function, $H(n)$, may be obtained as

$$H(n) = \frac{Y(n)}{X(n)}, \qquad n = 0, 1, 2, \cdots, N - 1 \qquad (1)$$

where n is the Fourier harmonic number and N is the total number of points in the spectrum. Unfortunately, and especially in the presence of measurement noise, this problem is ill-conditioned. Attempting to solve this equation directly will almost always lead to solutions that diverge when we perform the inverse FFT. In order to stabilize the solution it is necessary to add a low-pass filter in the frequency domain so (1) becomes of the form,

$$H(n) = \frac{Y(n)}{X(n)} R(n), \qquad n = 0, 1, 2, \cdots, N - 1 \qquad (2)$$

where $R(n)$ is the low-pass filter function. Tikhonov and Arsenin's major contribution was to find an optimal form for this filter function, which they call a "regularization operator." The simplest form of their filter may be written,

$$R(n) = \frac{|X(n)|^2}{|X(n)|^2 + \gamma |C(n)|^2}, \qquad n = 0, 1, 2, \cdots, N - 1 \qquad (3)$$

where γ is the regularization parameter, and $|C(n)|^2$ is the squared magnitude of the discrete Fourier transform of the second difference operator and may be written as

$$|C(n)|^2 = 6 - 8 \cos \frac{2\pi n}{N} + 2 \cos \frac{4\pi n}{N}, \qquad n = 0, 1, 2, \cdots, N - 1. \qquad (4)$$

In practice, the value of γ is adjusted to yield the "best" answer for $H(n)$. This "best" answer is subjective, but it is clear that if γ is too small, then the estimate for $H(n)$ will contain too much noise, and if γ is too large, then the filter will smooth the $H(n)$ estimate too much. An iterative approach for determining the optimal value for γ has also been developed at NIST and is discussed in [5].

Once the oscilloscope's complex transfer function has been computed and stored, it may be used to correct the shape of a wide class of waveforms subsequently measured on the oscilloscope. This is accomplished by deconvolving the new measured waveform spectrum $Y_n(n)$, from the oscilloscope's complex transfer function, $H(n)$, to obtain the estimated true waveform, $X_n(n)$. Thus it is possible to calibrate an oscilloscope completely and correctly, not only for voltage and time gain and nonlinearity, but also for voltage channel speed of response.

C. Nonlinearities

Having presented a method for the correct and sufficient calibration of an oscilloscope, a few comments concerning nonlinearities, or distortion, are in order. Generally, distortions in oscilloscopes may be separated into three major classes; frequency distortion, phase distortion, and amplitude distortion. Frequency distortion is caused by the fact that the magnitude of the oscilloscope's transfer function is not constant with frequency. Thus some frequency components of the measured waveform will experience different amplification (or attenuation) than others. Similarly, phase distortion is caused by the fact that the phase of the oscilloscope's transfer function is a nonlinear function of frequency. Both of these distortions will cause the shape of the measured waveform to differ from the shape of the true pulse. However, if only these two classes of distortion are present, performing the oscilloscope calibration as described above will account for, and effectively remove the effects of these two classes of distortion.

The third class of distortion, amplitude distortion, is usually caused by nonlinearities in the characteristic curves of active devices in the oscilloscope's vertical channel. Often it is called "harmonic distortion" because harmonics of the frequency components in the true pulse are generated in the oscilloscope's vertical channel that were not present originally. Since the harmonics in the true pulse waveform may be indistinguishable from those generated in the oscilloscope circuitry, especially in the presence of frequency and phase distortion, using a pulse to measure the harmonic distortion is generally not possible. For an accurate quantitative measure of this distortion it is best to use a pure sinusoid as a stimulus and to measure the relative amplitudes of each of the generated harmonics. However, a pulse stimulus may be used to detect the input pulse amplitude beyond which the amplitude distortion becomes excessive. This may be done by measuring the response to a set of identically shaped pulses with increasing amplitudes and noting the input pulse amplitude at which excessive harmonic distortion occurs (that is, the input pulse amplitude at which excessive distortion of the pulse shape occurs).

There are many potential sources of amplitude distortion in an oscilloscope's or waveform recorder's vertical channel, such as amplifiers, cathode ray tubes, sampling circuits, and analog-to-digital converters. For the most exacting of calibrations these sources of amplitude distortion must be considered. Quite often, however, their contribution to the overall waveform distortion properties of an oscilloscope is very small and may be safely ignored for a wide range of input signal amplitudes. It should be noted that the FFT and deconvolution operations described above are valid only for linear systems and cannot be assumed to be accurate when excessive amplitude distortion is present.

IV. A Calibration Example

These techniques have been used at NIST to calibrate the working standard for fast electrical pulse measurements. This standard, the Automatic Waveform Analysis and Measurement System (AWAMS), consists of a 17.5 ps (20 GHz) sampling oscilloscope controlled by a desktop minicomputer.

Since neither a pulse standard nor a viable sampler model was available for this calibration, our strategy was to use a commercially available superconducting oscilloscope as a primary standard. The step response of this oscilloscope exhibits a first transition duration of approximately 5 ps, which is about 3.5 times faster than the AWAMS sampler. In frequency domain terms, this equates to a 3-dB bandwidth of approximately 70 GHz.

We assumed that this superconducting oscilloscope is so fast that we could consider it "perfect," that is, possessing a step response of 0 ps. This assumption was necessary because we have no information about the superconducting oscilloscope's complete, complex transfer function. However, if we make this assumption, the error

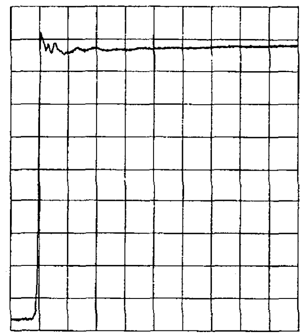

Fig. 7. Fast electrical pulse waveform as measured on a superconducting oscilloscope. Time scale is 200 ps/div and voltage scale is 30 mV/div.

in the estimate for the impulse response of our AWAMS oscilloscope should be less than ± 2 ps. Our argument for this uncertainty assertion is that, using the root-sum-of-squares approximation for cascaded transition durations, $(17.5\ \text{ps})^2 + (5.0\ \text{ps})^2 = 331.3\ \text{ps}^2$ and the square root of this is 18.2 ps. Thus the nominal error from this assumption of "perfection" is only about 0.7 ps. Furthermore, even if we assume that the superconducting oscilloscope's transition duration is 8 ps, the nominal error will be only about 1.7 ps. With no information available about the complex transfer function of the AWAMS oscilloscope, we can say that an uncertainty well within ± 2 ps is a significant improvement over an unknown uncertainty.

The method used for this AWAMS calibration was to measure a fast, commercially available electrical pulser on both the superconducting oscilloscope and the AWAMS oscilloscope. Then, by deconvolving the two scope responses to this pulse, we obtained an estimate for the impulse response (or complex transfer function) of the AWAMS oscilloscope.

Fig. 7 is a plot of the output of the fast pulser as measured on the superconducting oscilloscope. The measured (20%–80%) first transition duration was 12.3 ps. Fig. 8 is a plot of this same output as measured on the AWAMS oscilloscope. The measured (20–80%) first transition duration in this case was 14.4 ps. (Also, note that the fine structure of the two waveforms was quite different.)

Fig. 9 is a plot of the estimated impulse response for the AWAMS oscilloscope, obtained by deconvolving the waveforms shown in Figs. 7 and 8. The observed (50%–50%) pulse duration for this impulse response was 16.5 ps. (Remember that, for most circuits, the (50%–50%) impulse-response duration is *not* equal to either the (20%–80%) or the (10%–90%) step-response transition durations). The observed (20%–80%) step-response transition duration associated with this impulse-response duration, obtained by integration, was 10.9 ps.

Using this estimated impulse response for the AWAMS oscilloscope, we can deconvolve it from any subsequent measured pulse waveform to obtain an estimate for the true pulse shape. Fig. 10 is a plot of a pulse waveform from a customer-submitted fast pulser as measured on the

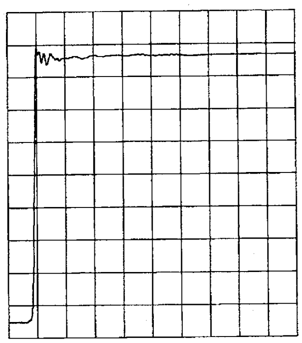

Fig. 8. Fast electrical pulse waveform as measured on the AWAMS sampling oscilloscope. Time and vertical scales are the same as those for Fig. 7.

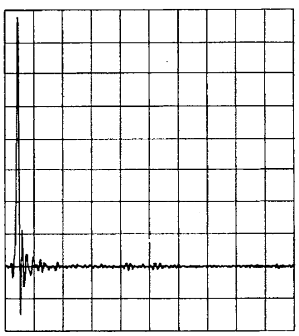

Fig. 9. Estimated impulse response for the AWAMS oscilloscope obtained from the deconvolution of the waveforms shown in Figs. 7 and 8. Same time scale as those in Figs. 7 and 8.

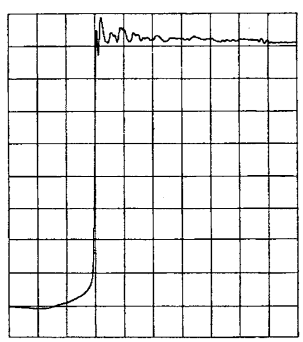

Fig. 10. Pulse waveform from a customer-supplied fast electrical pulser as measured on the AWAMS. Time scale is 200 ps/div and voltage scale is 30 mV/div. Measured (20%-80%) transition duration is 15.7 ps and measured pulse overshoot is 8.4%.

AWAMS. Its observed (20%-80%) transition duration was 15.7 ps, and its pulse overshoot was 8.4% of the pulse amplitude. After deconvolving the AWAMS estimated impulse response, the waveform shown in Fig. 11 resulted. This estimate for the true pulse shape exhibited a (20%-80%) first transition duration of 10.6 ps and a pulse overshoot of 11.1%.

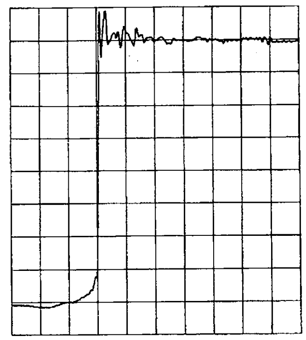

Fig. 11. Estimate of the true waveform obtained by deconvolving the measured waveform of Fig. 10 and the AWAMS impulse response of Fig. 9. Time and voltage scales are the same as in Fig. 10. Estimated true (20%-80%) transition duration is 10.6 ps and the estimated true pulse overshoot is 11.2%.

V. Conclusion

A method has been presented for the complete and correct calibration of an oscilloscope voltage channel. It is based on measuring, digitizing and deconvolving a standard (known) pulse. For the "smart" digital oscilloscopes and waveform recorders available today, the implementation of this calibration technique is quite easy and straightforward. With the availability of oscilloscope digitizing cameras, it is even possible to apply these calibration methods to analog oscilloscopes.

An example showing how NIST has used these techniques to perform a calibration of the vertical channel of the AWAMS sampling oscilloscope using the response of a superconducting oscilloscope as a primary standard has also been presented. Work is continuing at NIST to further refine and automate these calibration techniques as well as to develop better standard pulse generators.

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