plotted on a semilog graph paper to draw a curve $E(t)$ and its tangent $E_1(t)$ as shown in Fig. 7. Next, differences between the $E_1(t)$ and the curve $E(t)$ at several points of t, $E_2(t)$, were plotted again to draw a curve $E_2(t)$ and its tangent. τ_1 and τ_2 were derived by the slopes of these tangents, excepting τ_3, because measured TEMF's are too weak to analyze. The results are shown in Fig. 8.

Conclusions

Although the temperature rise was estimated up to 500°C or more, all the experiments show good agreement with the computations. The method described in the paper could be applied to analyze characteristics of VTJ in ULF range.

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High-Precision Audio-Frequency Phase Calibration Standard

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Abstract—**A high-precision standard for the calibration of audio-frequency phase meters has been designed using a microprocessor to generate the test signals. The accuracy is better than 0.01 degree over a frequency band from dc to 5 kHz and decreases to 0.1 degree at higher audio frequencies.**

I. Introduction

PHASE MEASUREMENTS are essentially a determination of the ratio of two time intervals, and as such are not dependent on any particular system of units. Therefore, in theory it should be possible to calibrate phase meters without reference to external standards; in practice, however, such calibrations are difficult especially when high precision is required. To obtain consistent measurements with various instruments, a reference standard is thus desirable.

To establish linearity and repeatability of readings of phase meters, a statistically significant number of determinations is required, and a series of measurements should ideally be taken over the frequency range of interest. For such extensive measurements, a standard capable of providing rapid calibrations is necessary. For present-day high-precision phase meters, a standard should have an accuracy approaching 0.01°.

II. Design Objectives

The phase angle calibrator was planned as the first member of a family of calibration sources for use with automatic test equipment. The unit is designed in a modular form to permit future expansion, e.g., extension of the frequency range to 50 kHz, and addition of known amounts of harmonics to the test signal. For the phase meter calibration function, it is desired to produce a source of two sinewave signals that can be set to precisely known phase differences at frequencies up to 5 kHz with a resolution of the order of a few millidegrees and with an accuracy approaching 5 millidegrees at the low end of the frequency range.

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An important consideration in the design is simplicity of operation in the manual mode, and capability of adaptation to automatic testing. This imposes requirements for rapid response to changes in the phase angle and frequency settings, while at the same time maintaining high stability once the desired operating conditions have been established. The digital method of signal generation fulfills these requirements well, because the phase difference between the outputs is determined solely by computation and is not based on the characteristics of frequency-sensitive components with their inherent aging and drift problems.

A variety of methods of digital generation of sinewaves have been described in the literature [1]–[5]. In the present system, each sine function value is calculated independently of any previous value, thereby achieving maximum flexibility and making possible a very rapid change in the phase angle.

III. Method

The output sinewave signals of the phase calibration standard are synthesized by calculating the instantaneous value of the signal at a number of equally spaced sample points along each waveform and converting these numerical values into analog voltages using two parallel digital-to-analog converters (DAC's). The outputs so obtained are stepped sinewaves which are then filtered to remove the harmonics introduced by the sampling process. The maximum sampling rate is dictated by the conversion and settling time of the DAC's, which is slightly more than 2 μs, allowing sampling rates up to 400 kHz. The number of samples per cycle is varied so that the sampling rate is always within a factor of two of the maximum, and consequently the same low-pass filter can be used for all output frequencies.

The location of sample points along the wave can be expressed in terms of angles in such a way that the initial sample point of the reference output is set at "zero," and the initial sample point of the variable output is set at the desired phase angle. A constant angular increment is then repeatedly added to each of these values to obtain subsequent sample points. Each of these angles is converted to the corresponding sine using a look-up table and interpolation technique. This conversion has an input angle resolution of 1 part in 2^{18} and an output sine value that is a 16-bit twos-complement binary number. Although the numerical values for each channel are calculated one after the other, they are temporarily stored in separate registers so that they can be applied to the two DAC's simultaneously. The timing pulse that strobes the register contents into the DAC's is derived from a pulse generator which is independent of the internal system clock. This assures precise timing and also permits the sinusoidal output to be locked to an external signal, if desired.

IV. System Architecture

Two microprocessors are used, a high-speed bit-slice version to generate the sinewave signals and a low-speed single-chip type for keyboard entry of parameters, decimal-to-binary angle conversion, and other control functions. A simplified block diagram of the system is shown in Fig. 1. Data transfer from the low-speed to the high-speed processors is carried out asynchronously via the high-speed data bus. When the high-speed processor is not using the data bus, a hardware priority circuit allows data to be transferred from an I/O port of the low-speed processor to one of three 20-bit registers in the angle computation module.

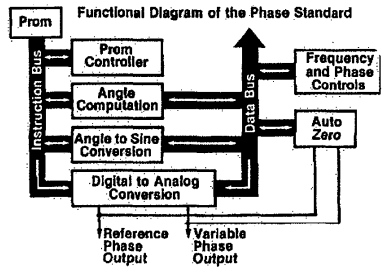


Fig. 1. Simplified diagram of phase standard. The high speed microprocessor system is comprised of the PROM controller, angle computation module, angle-to-sine conversion module, and the digital-to-analog conversion module. The low-speed microprocessor system includes the frequency and phase controls and the digital part of the auto-zero circuitry.

A 20-bit wide arithmetic-logic unit (ALU) with internal registers serves as the accumulator for the calculation of the angular sampling intervals. The inputs to the ALU are derived from four external registers three of which normally contain the initial values for the reference channel, variable phase channel, and angular increment, while the fourth one stores temporary results from the ALU output. Thus three of these registers contain the input parameters entered via the keyboard and the low-speed processor as already mentioned.

Data are transferred from the ALU to the angle-to-sine conversion module and from there to the DAC's via the high-speed bus. At each stage, the data are latched in the input register of the module. The pipeline structure permits the ALU to calculate the next angular value while the previous angle is processed in the sine conversion module. During the same time period the DAC's translate the preceding pair of output values to their analog equivalents.

The arithmetic operations and the data flow are controlled by a sequencer (PROM controller), and instructions stored in a READ-ONLY memory (PROM). The instruction bus is 16 bits wide, and the appropriate instructions are decoded in each functional module.

The sequencer, ALU, and all data instruction registers are triggered by the high-speed system clock at a rate of 5 MHz, but some pulses are delayed to allow for settling of instruction and data bits. To synchronize the instruction sequence with the timing pulse (DAC strobe), the sequencer is kept in a wait loop until the timing pulse occurs. At the beginning of each cycle of the output waveform, the parameters that have been entered through the keyboard and stored in the external ALU registers, are transferred to the working registers inside the ALU. This serves two purposes; it provides a mechanism to change the phase difference or the

TABLE I
PHASE STANDARD WAVEFORM ANALYSIS

Harmonic Number	Distortion (in ppm of rms input) 73 Hz	585 Hz	4680 Hz
2	40	61	195
3	20	<30	60
4	10	<30	74
5	20	<30	29
6*	<10	<30	<10

* The amplitudes of higher harmonics were below the sensitivity (10 ppm of input) of the wave analyzer.

frequency of the output signals, and it prevents possible errors in the data transfer from being propagated through more than one cycle.

The functions assigned to the low-speed microprocessor include the keyboard and display, as well as decimal-to-binary angle code conversion. Other tasks to be handled by this microprocessor are the algorithm of the auto-zero correction, which will be described later, and the stop, start, and single-step controls of the high-speed processor. (For convenience of software development during the construction of the phase standard, a minicomputer has been used instead of the low-speed microprocessor.)

V. Analog Output Circuitry

The digital output code is converted to analog signals by 16-bit modular DAC's which have a $\pm$ 1-mA output range and settle to their final value in approximately 2 μs. The currents are converted to voltage outputs by fast settling operational amplifiers which also perform as active filters. The feedback network is chosen to provide a low-pass, 4-pole Butterworth characteristic with a half-power frequency of 25 kHz. The effectiveness of the filter was checked with a wave analyzer at frequencies up to twice the sampling frequency. A linearity error in the response of the DAC, particularly of the more significant bits, could produce low-order harmonics in the output. Therefore, the three most significant bits were carefully adjusted. The harmonic analysis in Table I shows the effectiveness of these adjustments as well as the effectiveness of the filters.

The operational amplifiers provide an output of approximately 7-V rms, and the performance tests described were carried out at this voltage level. However, for general use of the phase calibration standard, it is planned to add high-voltage operational amplifiers to boost the output capability to 100-V rms.

VI. Auto-Zero Correction

The amplifiers, in general, cannot be adjusted so that their differential phase shift remains near zero over a range of frequencies. However, at any particular frequency, the differential phase shift remains constant (over a reasonable time), and is independent of the phase angle setting of the standard. Therefore, if the phase angle offset is determined initially, it can be applied as a correction for subsequent phase angle calibration settings.

A circuit was developed to determine this correction and apply it automatically. A quadrature phase detector similar to that described by Marzetta [6] was modified so that it was insensitive to dc offsets in either signal. The phase detector is connected to the reference and variable phase channel so that its output voltage is zero when the reference and variable phase signals are in exact quadrature. It is sensitive to a shift of less than a millidegree and provides a dc output which is a cosine function of the phase difference. The dc output is digitized using a voltage-to-frequency (V/F) conversion module and a gated counter. Superimposed on the dc output of the phase detector is a ripple of twice the frequency of the sinusoidal signal from the calibrator. At higher frequencies, this is easily filtered, but around 10 Hz the necessary filter time constants would be excessively long. Fortunately, near balance the ripple is symmetrical, and its effect is compensated by triggering the gate of the counter for two equal periods, one starting at a positive zero crossing of the ripple component, and the other at the negative crossing.

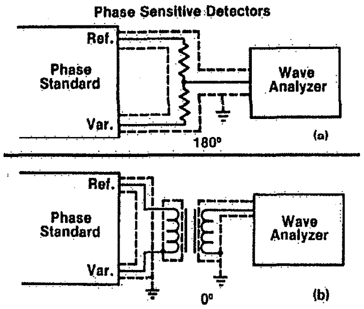
The zero correction circuit operates by setting the phase standard to 90°, digitizing the detector output, and transferring the resulting 12-bit correction to the low-speed microprocessor. The value obtained is combined with a similar measurement at 270° (−90°) by finding the difference between the two. This procedure eliminates a possible error due to a dc offset in the detector output or in the digitizing circuit. To apply this correction to the ALU calculations, it is adjusted to a 20-bit twos-complement number which is added to a (software) correction register. The updated value of that register is then used to offset the initial value of the reference channel in the ALU. The process is repeated until the amount added to the correction register is negligibly small. At that point, the correction has attained its final value and is applied during subsequent operation of the phase standard in its calibrating mode.

VII. Verification of Performance

There is no simple method to verify the performance of a phase angle standard. A combination of theoretical analysis and various experimental techniques must be used to provide confidence in the predicted results, and while it is impossible, in practice, to test every output condition results at certain key points can be indicative of the expected performance.

A. Theoretical Analysis

Near the zero crossing of a sinewave a phase shift of 0.001 corresponds to a change in voltage of 17 ppm of peak value. For a 5-kHz signal with 64 sample points per cycle, the calculated value of the largest harmonic introduced by the sampling process is 1.54 percent of the fundamental, assuming an ideal DAC and distortionless amplifiers [2]. This harmonic is near the sampling frequency. The calculated response of a 4-pole Butterworth filter (half-power frequency 25 kHz) will reduce this harmonic amplitude to 0.6 ppm, well below the amplitude that would produce a



g. 2. (a) Resistor bridge circuit to measure 180° phase angle. (b) Transformer bridge circuit to measure 0° phase angle.

pparent phase shift of 0.001°. At lower frequencies where e number of samples per cycle is larger, the harmonic ntribution is even smaller. This represents the perforance of an idealized system, in practice errors introduced various components will degrade the overall rformance.

Angle-to-Sine Conversion

Every output value of the digital conversion module was ecked against a computer-generated result and agreement as found to be within $\pm\frac{1}{2}$ of a least significant bit. Since this st was run at 100 kHz, slower than the final operating eed, a few results were verified at rated speed using a logic alyzer. No discrepancy was detected.

180° Bridge Circuit

A circuit shown in Fig. 2(a) using two matched resistors s used to establish the 180° phase angle. A wave analyzer ned to the output frequency served as a detector. A phase ift of 0.001° could easily be detected and the indications re extremely stable and repeatable. Results obtained with e resistors interchanged agreed to better than 0.001°. For mparison with the zero degree bridge circuit and the adrature phase detector described below, the phase stanrd was set to a nominal phase angle of 180.000° and the tput amplitudes of the two channels were adjusted for nimum deflection on the detector. The residual phase fference between the outputs of the filter amplifiers was en trimmed until a further minimum was obtained on the tector. (The auto-zero circuit was disabled for this test, cause somewhat better precision could be obtained with a anual adjustment.)

0° Bridge Circuit

A unity ratio double-shielded transformer was connected shown in Fig. 2(b) with the outputs from the phase ndard connected to the primary and a wave analyzer to e secondary. Test results are shown in Table II.

Quadrature Phase Detector

A phase detector similar in design [6] to the one used for e auto-zero correction circuit, but without the dc offset mpensation circuits, was used to check the phase standard performance at 90° and 270° phase settings. The results of these tests at three frequencies are shown in Table II.

TABLE II
ACCURACY VERIFICATION OF CARDINAL POINTS

Frequency	Phase standard settings required for balance of the detector			
Hz	degrees			
73	0.000	90.002	180.000*	269.998
585	0.000	90.000	180.000*	270.000
4680	0.000	90.002	180.000*	270.000

Phase offset adjusted to zero at this setting.

TABLE III
LINEARITY MEASUREMENTS OVER 1° (585 Hz)

Phase Angle Setting degrees	Deviation millidegrees
44.500	-2.3
44.600	-1.2
44.700	-0.8
44.800	0.9
44.900	2.1
45.000	-2.6
45.100	-1.3
45.200	0.1
45.300	1.
45.400	1.7
45.500	1.8
59.500	4.3
59.600	0.1
59.700	-1.1
59.800	-0.5
59.900	-0.4
60.000	3.6
60.100	1.6
60.200	-1.6
60.300	-2.9
60.400	0.7
60.500	-4.4
119.500	-6.6
119.600	-6.2
119.700	-3.6
119.800	-2.6
119.900	-2.8
120.000	-2.8
120.100	3.3
120.200	4.7
120.300	5.1
120.400	6.4
120.500	4.2

F. Linearity Tests

The output of the quadrature phase detector follows a cosine response so that it is approximately linear over small angular intervals in the region from 45° to 135°. Tests were made over 1° intervals in steps of 0.1° at several points. The results in Table III show a linearity generally better than 0.005°.

Linearity of the phase standard was checked further by measuring the incremental linearity at 10° intervals using a different method. For this purpose an adjustable, uncalibrated phase shifter was connected in series with one of the outputs. The phase standard was set to the desired phase angle and the phase shifter was adjusted until a null reading was obtained on the 180° bridge circuit (Fig. 2(b)). A second

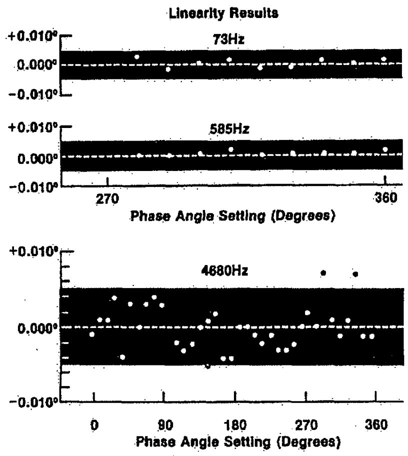


Fig. 3. Incremental linearity for three frequencies measured at 10° intervals.

fixed phase shift circuit (approximately 1°) was then inserted in series, and the bridge circuit was rebalanced by resetting the phase standard via the keyboard. The differences in phase angle settings were recorded, and the deviations from the mean difference are plotted in Fig. 3, for three frequencies. Because of the greater sensitivity of this method, less scattering of results was observed at 73 and 585 Hz. At 4.69 kHz, all but two points fall into a band of $\pm 0.005°$. Repeat tests indicated that the experimental error was not larger than 0.001°.

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Absolute Measurement of Loss Angle Using a Toroidal Cross Capacitor

JOHN Q. SHIELDS

Abstract—**An absolute measurement of loss angle has recently been completed at NBS. The measurement utilized a special toroidal cross capacitor in which the effects of dielectric films are greatly attenuated. The resulting unit of loss angle has an estimated uncertainty of 0.02 μrad at an optimum frequency of 1592 Hz.**

I. INTRODUCTION

EVALUATION of the loss angle contribution from thin dielectric films on electrode surfaces has been the major problem area in absolute determinations of loss angle in the past. In addition, the instability of dielectric films has impeded efforts to maintain the unit of loss angle obtained from absolute measurements.

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The use of a toroidal cross capacitor for the absolute measurement of loss angle had its origin in the discovery that thin uniform dielectric films on the electrode surfaces of certain types of cross capacitors result in loss angle contributions which are negligibly small. This initial work on dielectric film losses in cross capacitors was described in an earlier paper [1].

The present paper summarizes the earlier work and goes on to consider additional sources of loss angle. Emphasis is on the evaluation of uncertainties associated with the assignment of a value of loss angle to the toroidal cross capacitor and subsequently to the other reference standards.

II. TOROIDAL CROSS CAPACITOR

A cross section of the toroidal cross capacitor is shown to approximate scale in Fig. 1. It is constructed of stainless steel with each of the main electrodes I, O, T, and B supported by