New NBS Measurements of the Absolute Farad and Ohm

ROBERT D. CUTKOSKY, FELLOW, IEEE

Abstract—**A recently completed calculable cross capacitor in conjunction with a previously described collection of ac and dc bridges has made possible a highly accurate measurement of the farad and the ohm. The cross capacitor and its auxiliary equipment, as well as those components of the measurement system which have not been covered in prior publications, are described in detail. The measurements indicate that the National Bureau of Standards (NBS) unit of capacitance is given by $F_{NBS} = 1\ F + 1.787\ \mu F$, and that the NBS unit of resistance is given by $\Omega_{NBS} = 1\ \Omega - 0.819\ \mu\Omega$.**

INTRODUCTION

FOLLOWING the theoretical and experimental development of the calculable cross capacitor by Thompson and Lampard of the National Standards Laboratory of Australia [1], a research effort was initiated at the National Bureau of Standards (NBS) to utilize these principles in an absolute measurement of the farad and the ohm. The initial NBS effort in this field [2] was completed in 1960 and provided an absolute measurement of the ohm with an estimated uncertainty of about 2 parts in 10^6. These measurements provided the basis for the present NBS unit of capacitance, F_{NBS}. The NBS unit of resistance Ω_{NBS} was not adjusted at that time, because the unit in existence since 1948 was found to be substantially correct. The activities of our laboratory since 1960 have been directed toward improving the equipment and techniques which were used in the initial effort with the objective of completing an absolute ohm measurement with an uncertainty on the order of a few parts in 10^8.

Manuscript received July 3, 1974; revised August 20, 1974. This paper is a contribution of the National Bureau of Standards and is not subject to copyright.

The author is with the Electricity Division, National Bureau of Standards, Washington, D. C. 20234.

ORGANIZATION OF MEASUREMENT SYSTEM

The starting point in the NBS absolute ohm measurement is a 0.5-pF calculable cross capacitor, the capacitance of which can be determined from a measurement of its length. This instrument is used to measure the capacitance of a transportable 10-pF reference capacitor which can be carried from the basement room containing the cross capacitor to a large second floor laboratory containing the other components of the measurement system.

The second floor laboratory contains the bank of 10-pF reference capacitors that maintain the NBS unit of capacitance. An ac bridge with a nominal voltage ratio of 10:1 is used in two stages to measure two 1000-pF capacitors. These capacitors are used in turn as two arms of a special frequency-dependent bridge called a quadrature bridge for measuring the ac resistances of two 10^5-Ω resistors. These resistors are then compared with a transportable 1000-Ω resistor through the use of a 100:1 ac bridge that utilizes the 10:1 voltage ratio bridge just cited and a 10:1 current comparator [3].

The transportable 1000-Ω resistor is then carried to the laboratory in which the NBS bank of 1-Ω reference resistors is maintained. The stepup from the 1-Ω level to the 1000-Ω transportable resistor is performed in this laboratory using dc techniques with the aid of Hamon dividers and a dc comparator.

Essential special auxiliary equipment includes a low noise parametric preamplifier, instruments for measuring the voltage dependence of capacitors and other components, a transformer calibrator, and all of the equipment necessary for determing the frequency dependence of the cross capacitor and of the transportable 1000-Ω resistor. Except for the cross capacitor itself and equipment related to its use, all of the special instruments and techniques used in this work have been described in a series of publications dating back to 1962 [3]–[7]. The attention of this paper is accordingly concentrated on the construction and operation of the cross capacitor and on those aspects of the measurement system which have not already been adequately covered.

CROSS CAPACITOR

The NBS cross capacitor utilizes the now classic geometry which was first exploited by Clothier [8]. Our system contains four vertical cylindrical electrodes with diameters of 6.35 cm which were lapped to an accuracy of $\pm$ 0.1 μm and which are located in a square array with clearances of 3.60 mm between adjacent electrodes. A grounded cylindrical shield with an inside diameter of 18.16 cm surrounds the four electrodes. A movable shield rod with a diameter of 2.72 cm is partially inserted between the electrodes from the top and is located horizontally by polytetraflouroethylene rings which are placed around the shield rod and which lightly touch all four electrodes. This shield rod may by translated vertically by means of a pulley and counterweight system to vary the cross capacitance of the instrument in a controlled manner. The lower end of the shield rod is composed of an Invar fitting which is terminated with a vertical tube 22.4 mm long with an outside diameter of 10.0 mm. The geometry of this tube was chosen to eliminate the first-order dependence of capacitance upon electrode taper and angular misalignment which could otherwise seriously affect the measurements.

An optical flat is mounted inside the Invar fitting just above the misalignment compensation tube. A similar Invar fitting with a second optical flat is mounted just above the lower ends of the electrodes. The lower fitting can be adjusted either mechanically or electrically by means of a piezoelectric transducer (PZT) to make the flats precisely parallel. The PZT can also be used to displace the lower flat vertically over a range slightly larger than 1 fringe. Provision has been made to dither the lower flat over a range of a few hundredths of a fringe at a frequency of 10 Hz. The two flats form the plates of a Fabry–Perot interferometer which is illuminated by a Lamb-dip stabilized helium-neon laser. The optical system contains a polarizer and quarter-wave plate to minimize the amount of light returning to the laser, a beam expanding telescope to fill the 7-mm-diameter Fabry–Perot resonator, a 50-cm focal length lens to focus the interference fringes, and an adjustable pinhole at the focal plane of the lens to select the light from the center of the fringe pattern and to allow it to strike a phototransistor. The 10-Hz component of the phototransistor output resulting from dithering the lower flat is amplified and detected with a phase sensitive detector. The detector output is amplified and fed back to the PZT to lock the interferometer on the center of a fringe.

The shield rod contains two cylindrical sections 7 mm long which are electrically insulated from the rest of the shield rod. These sections are of the same diameter as the shield rod and are connected via sliding contacts to separate terminals on the housing of the capacitor. One of these sections is located a few millimeters above the Invar fitting containing the upper flat, and the other is about 10 cm farther up. These sections make it possible to monitor variations in the interelectrode spacing and in the angle between the shield rod and the electrode axis as the shield rod is displaced through its range of travel through measurements of the direct capacitance between each cylindrical section and each capacitor electrode. These sections are shorted to the case of the capacitor when cross-capacitance measurements are being made.

The cross capacitor becomes slightly nonlinear when the shield rod is set to produce a cross capacitance smaller than 0.2 pF, and has a maximum capacitance of 0.7 pF. Within this normal operating range of 0.5 pF, only the expected nonlinearity at the 0.2-pF end of the range has been found. The capacitance nonlinearity at the 0.2-pF end of its range was determined by comparing the rates of change of capacitance with shield rod displacement, with the shield rod set to yield various cross capacitances down to 0.06 pF. The function relating departure from linearity to normal capacitance was found to be exponential and indicated that at the 0.2-pF setting the nonlinearity amounted to 7.0×10^{-9} pF. A correction of 14 parts in in 10^9 was accordly applied to the 0.5-pF capacitance difference.

The uncertainty in cross capacitance caused by imprecise mechanical construction and alignment is estimated to be smaller than 1 part in 10^8. A correction of 3 parts in 10^9 to account for a slight error in the laser beam direction was applied. The wavelength of our present laser is unfortunately not completely stable, and at present limits our capacitance accuracy. The capacitor is normally evacuated to a sufficiently small pressure so that correction for changes either in the laser wavelenght or in the dielectric constant of the capacitor is unnecessary.

The distributed capacitances and inductances in the leads and electrodes of the calculable capacitor result in a small frequency dependence. The distributions of these parasitic impedances were determined by direct measurements coupled with some estimates of end effects. The total correction for these internal loading effects including the capacitive loading correction for the transformer used to compare the calculable capacitor with the 10-pF transportable reference capacitor was found to be $+9$ parts in 10^9 at 1592 Hz.

ALTERNATING CURRENT MEASUREMENTS

The capacitance of the calculable capacitor is compared with that of a transportable 10-pF reference capacitor in a bridge containing a special transformer which supplies 200 V at 1592 Hz to the capacitor. When the calculable capacitor is set to yield 0.2 pF, the reference capacitor is connected to a tap supplying 4 V, and when it is set to yield 0.7 pF the reference capacitor is connected to a tap supplying 14 V. The critical ratios of this transformer were measured using larger fixed capacitors which can be accurately calibrated through the use of a previously described 10:1 voltage ratio bridge [3]. The open circuit ratio correction for the special transformer was found to be -71 parts in 10^9. The remaining features of the bridge used for comparing the cross capacitor with the 10-pF reference capacitor and in determining the fringe number are similar to those described by Clothier [8].

The impedance of the transportable 10-pF reference capacitor at 1592 Hz is compared with the impedance of a 1000-Ω transportable resistor using a set of bridges described in an earlier paper [3]. The four-pair 10:1 voltage bridge was calibrated through the use of a set of eleven 10-pF capacitors by means of the permutation method [9], but modifications were made so that better advantage could be taken of the four-pair configuration. The voltage dependencies of all of the capacitors involved in the step-up and in the calibrations were measured on an absolute basis using a new version of the system described by Shields [10]. The only capacitors for which the voltage dependence corrections were larger than 1 part in 10^9 were two 1000-pF capacitors used to calibrate a 10:1 ac current comparator. This comparator was part of a 100:1 bridge used for the ac measurement of the 1000-Ω transportable resistor. Two different pairs of capacitors were used for this calibration. The voltage dependencies of the capacitors over the 20-V to 200-V range varied from 82 parts in 10^9 to 154 parts in 10^9, but after the corrections were applied, all of the measurements of the current comparator ratio agreed within 2 parts in 10^9.

TRANSPORTABLE RESISTOR

The 1000-Ω transportable resistor consists of nine 1000-Ω Evanohm resistors wound on mica cards. The individual cards are connected in series parallel and mounted in a sealed metal can nearly filled with oil. The can is in turn immersed in a temperature-regulated oil bath. This resistor is measured with both ac current and with dc current, and it is necessary to correct for any ac–dc resistance difference which may exist. Two special coaxial resistors, one of 100 Ω and one of 1000 Ω, were constructed to investigate this possibility [11]. Each of these resistors consists of a single length of Evanohm wire about 30 cm long and of suitable diameter mounted along the axis of an oil-filled tubular return conductor with a diameter of 5 cm. Shielded current and potential terminals were provided at each end. Complete calculations of the field distributions and eddy currents were made to allow an assessment of the frequency dependencies of the two resistors. A comparison of the ratios of the two resistors at 1592 Hz and at 15920 Hz showed close consistency with the calculations, both in resistance and in phase angle. On the basis of the calculations, the 1000-Ω coaxial resistor was assigned a resistance difference from dc to 1592 Hz of 8 parts in 10^{10} which is not significant, but which does not include a possible dc error attributable to the Peltier effect [12], which could arise from the use of spot welding to fasten the resistance wire to its terminals. A microscopic examination of the welded junction showed a discolored section of Evanohm wire extending out from the weld for a distance of about 0.2 mm. It was subsequently determined that annealing a piece of Evanohm wire substantially changes its thermoelectric coefficient, so that a thermoelectric junction could exist 0.2 mm from each end of the resistance wire, possibly causing the dc resistance to be abnormally high due to the Peltier effect.

A second special 1000-Ω resistor was built to determine whether or not the Peltier effect in the coaxial 1000-Ω resistor was significant. This resistor consists of a length of resistance wire about 3 m long folded twice to form a four-wire cage 75 cm long. The wires are separated by 1 cm throughout the length of the system and produce a quadrupole field, which is not strongly coupled to the metal supports and shield structure. Current and potential terminals for each end of the resistor consist of pieces of 0.07-mm diameter copper wire 1 cm long soldered to heavier copper lead wires with low thermal solder. The 0.07-mm wires were placed parallel to each other and about 3 mm apart on a beryllium oxide heat sink. The resistance wire was placed over the copper wires and perpendicular to them and clamped in place with a second beryllium oxide heat sink. The contact resistance of this purely mechanical connection was about 0.1 Ω. The resistor was easily trimmed to $\pm$ 100 ppm by releasing the clamping pressure and moving the resistance wire as required. The finished resistor was immersed in an oil bath for temperature control.

The quadrupole resistor was not expected to exhibit an appreciable Peltier effect because of the absence of thermoelectric junctions in the portions of the resistor common to the current and potential circuits. A verification of the absence of Peltier errors was made by connecting a nanovoltmeter between a normally open extension of the Evanohm resistance wire and the adjacent clamped junction. A current was then passed through the 1000-Ω section of the resistor via the innermost of the four clamped junctions. Reversing the sign of the current had no effect on the nanovoltmeter reading. Some eddy current errors could exist, but this would cause an apparent increase of resistance with frequency.

The coaxial 1000-Ω resistor is believed to be substantially free of eddy current errors, but if it were troubled with the Peltier effect, its resistance would appear to decrease with frequency. Measurements of the ratio of the two special 1000 Ω resistors at dc and at 1592 Hz agreed within 5 parts in 10^9, which is interpreted as indicating that neither resistor has a significant frequency dependence.

The ac–dc difference of the transportable resistor was determined by comparison with the coaxial 1000-Ω resistor. The difference has changed from 484 $\times 10^{-9}$ in 1970 to 535 $\times 10^{-9}$ on January 29, 1974. In February or March of 1974 the phase angle of the transportable resistor changed abruptly by about 43 μrad. Subsequently, its ac–dc difference was determined to be 460 $\times 10^{-9}$. The dc resistance of the transportable resistor was always larger than the ac resistance.

DIRECT CURRENT MEASUREMENTS

All of the dc comparisons relating the 1000-Ω transportable resistor to the NBS legal ohm as maintained with 1-Ω resistors were made directly or under the supervision of T. E. Wells of the Electrical Reference Standards Section, NBS Electricity Division. Two separate buildup processes were used. In the first process, a Hamon divider with ten 10-Ω coils was first compared in the parallel mode with the 1-Ω standard resistors using a current comparator. Then, again using the current comparator, the Hamon divider was compared in the series mode with a second Hamon divider containing ten 100-Ω coils, first to measure the series-parallel combination of nine of the coils, and second to measure the tenth coil individually. Finally, the second Hamon divider was connected in the series mode and compared with the 1000-Ω transportable resistor in a conventional Wheatstone bridge. In the second process, a Hamon divider with thirty-two 32-Ω coils was compared in the parallel mode with the 1-Ω standard resistors using a current comparator. Then it was compared in the series mode with a 1024-Ω resistor consisting of a 1000-Ω resistor in series with a 24-Ω resistor. A separate measurement of the 24-Ω resistor yielded the value of the 1000-Ω section, which was finally compared with the transportable resistor. The two processes were found to agree within the uncertainty limits of the measurements, or about 2 parts in 10^8.

MEASUREMENT RESULTS

The measurement process consisted of performing the ac calibrations relating the 10-pF transportable capacitor to the 1000-Ω transportable resistor twice on a Monday or Tuesday, and twice again on the following Thursday or Friday. On the intervening Wednesday, the 10-pF capacitor was calibrated by reference to the calculable capacitor, and the 1000-Ω resistor was compare with the bank of 1-Ω resistors maintaining the NBS legal ohm. These sequences were repeated every month or two, with the intervals between measurements being devoted to calibrating the bridges and determining the required corrections as outlined earlier

Table I summarizes the results obtained. The standard deviations associated with the average values of both F_{NBS} and Ω_{NBS} are well below one part in 10^9, which is negligible compared with the possible systematic errors. Estimates of the effects of possible systematic errors are itemized in Table II. The error in the measurement of Ω_{NBS} is almost certainly less than the direct sum of the listed systematic errors, but could easily exceed their root sum square (rss) sum. If a single number must be assigned to the uncertainly of the measurement of Ω_{NBS}, a value of 0.06 ppm (95-percent confidence level) would not be unreasonable.

A comparison between the work reported here and the absolute ohm work of the National Standards Laboratory (NSL) of Australia [13] can be made by making use of the periodic resistance intercomparisons performed at the International Bureau of Weights and Measures (BIPM) The BIPM has reported for the series of intercomparisons made between January and April 1973 that $\Omega_{NBS} = \Omega_{69-BI} + 0.2\ \mu\Omega$, and $\Omega_{NSL} = \Omega_{69-BI} + 0.33\ \mu\Omega$ [14]. Very recent work at the NSL has indicated that the resistance of the shield return lead of their ac–dc transfer resistor has increased by 0.19 ppm of the total resistance since it was last measured in 1967 [15]. This shield resistance was included in their four-pair ac measurements, but its assigned value was added to the results of their dc measurements. It is not known when the change occurred, but it is believed probable that most of the change occurred before their measurement of the transportable resistors which were involved in the BIPM resistance intercomparisons. If the entire 0.19-ppm correction is applied to the 1973 intercomparisons, the BIPM result would become $\Omega_{NSL} = \Omega_{69-BI} + 0.52\ \mu\Omega$, so that $\Omega_{NBS} = \Omega_{NSL} - 0.32\ \mu\Omega$. The NSL unit is maintained to be in accordance with the SI definition, but they had been using $c = 2\,997925 \times 10^8$ m/s for the speed of light. We have used $c = 2.99792458 \times 10^8$ m/s, as was recommended for international adoption in 1973 [16]. Since the speed of light enters the capacitance equations as $1/c^2$, the NBS unit of resistance as measured through the BIPM by NSL should be corrected by 0.280 ppm, or Ω_{NBS} (NSL) $= 1\,\Omega - 0.60\ \mu\Omega$. The discrepancy with our result, $\Omega_{NBS} = 1\,\Omega - 0.819\ \mu\Omega$, is 0.22 ppm. This is larger than expected in view of the reported NSL uncer-

TABLE I
SUMMARY OF MEASUREMENTS OF F_{NBS} AND Ω_{NBS} IN SI UNITS, TAKING $c = 2.99792458 \times 10^8$ m/s[a]

Date	10-pF Capacitor	F_{NBS}	1000-Ω Resistor	Ω_{NBS}
9/12/73	+5.522	+1.798	+9.409	−0.819
10/24/73	+5.542	+1.798	+9.553	−0.822
11/28/73	+5.523	+1.775	+9.667	−0.821
1/23/74	+5.533	+1.784	+9.832	−0.708[b]
4/3/74	+5.532	+1.782	+10.073	−0.813
	average:	+1.787	average of four:	−0.819

[a] Corrections in parts per million to indicated quantities or standards are tabulated.
[b] This measurement was discarded because the dc comparison exhibited internal discrepancies as large as 2 parts in 10^7.

TABLE II
ESTIMATES OF THE EFFECTS OF POSSIBLE SYSTEMATIC ERRORS ON THE MEASUREMENT OF Ω_{NBS} (95-PERCENT CONFIDENCE LEVEL)

Possible Systematic Errors	Error Estimates (ppm)
Drift in laser wavelength between calibrations	0.010
Imperfection of calculable capacitor electrode straightness and alignment	0.010
Misalignment of laser beam in capacitor	0.002
Diffraction error due to laser aperture diameter	0.001
Load corrections for calculable capacitor	0.007
Residual gas in calculable capacitor	0.004
Drift between calibrations and measurement uncertainties of transformer ratios	0.007
Uncertainty in capacitance voltage dependence measurement	0.003
Temperature uncertainty of transportable resistor	0.004
Frequency dependence of transportable resistor	0.005
Self-heating effects in transportable resistor	0.004
Uncertainty in dc stepup, 1 to 1000 Ω	0.050
direct sum:	0.107
rss sum:	0.054

tainty of 0.2 ppm and our own estimated uncertainty of 0.06 ppm, and in view of the fact that a direct intercomparison of capacitance standards between NBS and NSL indicates agreement within 0.02 ppm. This implies that very little of the discrepancy can be attributed to the calculable capacitors.

A second intercomparison of the NBS unit of resistance with that of the BIPM was made in May 1973 and indicated that $\Omega_{NBS} = \Omega_{69-BI} - 0.17\ \mu\Omega$ [17]. This result was not used in the formal assignment of the NBS unit of resistance in terms of the BIPM unit because the first intercomparison seemed to be in closer agreement with the trend through measurements dating back to 1957, and because some comparisons of 10^4-Ω resistors also indicated closer agreement with the first series of intercomparisons. If the second intercomparison is taken to be correct, one would obtain, using the same corrections as above, Ω_{NBS} (NSL) = 1Ω–0.97 μΩ, which differs from the NBS result by 0.15 ppm, but with the opposite sign.

It is clear that further study of the resistance transfer problem must be made. Although the precise cause of the discrepancy is not yet known, it has been noticed that the NBS transportable resistors do not return immediately to their original rates after their return from other laboratories. It has been suggested that the problem may be related to the fact that NBS normally measures its resistors at 25°C and determines their values at 20°C through a fairly quick series of temperature coefficient measurements, whereas the BIPM normally measures the resistors after a rather long stabilization period at 20°C. This difference in technique cannot explain all of the discrepant results, but points to the resistance transfer as the least reliable part of the intercomparison process. A direct interchange of resistance standards between NBS and NSL which is now underway may help resolve the discrepancies.

REFERENCES

[1] A. M. Thompson and D. G. Lampard, "A new theorem in electrostatics and its application to calculable standards of capacitance," *Nature*, vol. 177, p. 888, 1956.
[2] R. D. Cutkosky, "Evaluation of the NBS unit of resistance based on a computable capacitor," *NBS J. Res.*, vol. 65A, pp. 147–158, May–June 1961.
[3] ——, "Techniques for comparing four-terminal-pair admittance standards," *NBS J. Res.*, vol. 74C, pp. 63–78, July and Dec. 1970.
[4] ——, "Four-terminal-pair networks as precision admittance and impedance standards," *Commun. Elec.*, vol. 70, pp. 19–22, Jan. 1964.
[5] R. D. Cutkosky and L. H. Lee, "Improved ten-picofarad fused silica dielectric capacitor," *NBS J. Res.*, vol. 69C, pp. 173–179, July–Sept. 1965.
[6] R. D. Cutkosky, "A varactor null detector for audio frequency capacitance bridges," *IEEE Trans Instrum. Meas.*, vol. IM-17, pp. 232–238, Dec. 1968.
[7] ——, "An ac resistance thermometer bridge," *NBS J. Res.*, vol. 74C, pp. 15–18, Jan. and June 1970.
[8] W. K. Clothier, "A calculable standard of capacitance," *Metrologia*, vol. 1, pp. 36–55, Apr. 1965.
[9] R. D. Cutkosky and J. Q. Shields, "The precision measurement of transformer ratios," *IRE Trans. Instrum.*, vol. I-9, pp. 243–250, Sept. 1960.
[10] J. Q. Shields, "Voltage dependence of precision air capacitors," *NBS J. Res.*, vol. 69C, pp. 265–274, Oct.–Dec. 1965.
[11] R. J. Haddad, "A resistor calculable from dc to ω = 10^5 rad/s," M.S. thesis, School Eng. Appl Sci., George Washington Univ., Apr., 1969.
[12] C. G. M. Kirby and M. J. Laubitz, "The error due to the Peltier effect in direct-current measurements of resistance," *Metrologia*, vol. 9, pp. 103–106, Sept. 1973.
[13] A. M. Thompson, "An absolute determination of resistance based on a calculable standard of capacitance," *Metrologia*, vol. 4, pp. 1–7, Jan. 1968.
[14] G. Leclerc, *Rapport sur la 13e comparison des etalons nationaux de resistance electrique*, Bur. Int. Poids Mesures, Sèvres, France, Jan.–Apr. 1973.
[15] Private communication, June 7, 1974.
[16] J. Terrien, "International agreement on the value of the velocity of light," *Metrologia*, vol 10, no. 1, p. 9, 1974.
[17] Private communication, Nov. 21, 1973.