Wide-Band Two-Stage Current Transformers of High Accuracy

T. MICHAEL SOUDERS

Abstract—The design and performance of two high-accuracy transformers, identified as amplifier-aided two-stage transformers, are described. Each operates from 50 Hz to 10 kHz, supporting burdens up to 1 Ω. Self-contained ratios span from 5/5 to 100/5, with a rated secondary current of 5 A.

Results indicate the errors at 10 kHz are within 30 and 15 ppm for the respective transformers, decreasing to less than 1 ppm at 1 kHz.

Analysis of high- and low-frequency errors is presented, as well as a detailed description of a capacitance trimming technique effecting a factor-of-ten reduction of capacitive errors.

Also described is a relatively simple step-up calibration system for determining the transformer errors. Calibration results are included.

I. Introduction

RECENT advances in the field of power and energy measurements at both power and audio frequencies have placed increasingly stringent accuracy requirements on current transformers. While techniques have been developed during the past decade permitting very accurate measurements of current transformer errors, the transformers themselves seldom warrant the increased accuracy. Most transformers today exhibit relatively large errors, which are critical functions of the operating current, secondary burden, and frequency. This paper discusses the development of two multirange current transformers whose accuracies approach those of the latest measuring techniques and describes a step-up calibration system affording a relatively fast determination of the transformer errors.

Two transformers were constructed, both following the same basic design. Transformer 1, while intended primarily for use at power frequencies, also served as a model for studying the feasibility of a capacitance trimming technique. Transformer 2, intended for operation at high accuracy over the frequency range of 50 Hz to 10 kHz, has half as many turns as transformer 1, and less space between primary and secondary windings, to reduce the leakage impedances.

Both transformers can accommodate burdens up to one ohm (resistive or inductive) and secondary currents from 0.5 to 10 A. (It is worth noting that if a transformer were designed exclusively for low burdens, a significant improvement at the higher frequencies could be realized through a further reduction in its number of turns and physical size.)

Self-contained ratios of 5/5, 10/5, 20/5, 25/5, 50/5, and 100/5 are provided by each, and additional ratios up to 1200/5

Manuscript received June 17, 1972; revised July 21, 1972.
The author is with the National Bureau of Standards, Washington, D.C.

(600/5 for transformer 2) are available by using feedthrough primary turns.

Transformer 2 is shown after completion in Fig. 1. Transformer 1 is similarly cased. Construction details are given in the Appendix.

II. Design Considerations

It has long been recognized that the major sources of errors in current transformers differ for low- and high-frequency operation. At power frequencies the magnetizing current required to provide core excitation accounts for most of the error, while at high frequencies circulating capacitance currents driven by the voltage drops between and across windings are the predominant source of error. Usual methods for reducing the magnetizing current include using a core material of high permeability, increasing the core area, and increasing the number of turns in the ratio windings. The latter two methods, unfortunately, are not consistent with the requirements for low-capacitive errors, since they tend to increase both the leakage impedances of the windings (accordingly increasing the respective voltage drops) and the capacitances associated with these windings. The first method, on the other hand, has been limited by the core materials available.

A. The Amplifier-Aided Two-Stage Transformer

Various techniques have been described in which feedback amplifiers have been employed to increase, in effect, the permeability of transformer cores. Such techniques, particularly that used for the self-balancing current comparator [1], have been quite successful in reducing low-frequency errors. However, the effective permeability achieved by these methods is proportional to the amplifier gain, which, because of stability considerations, is sometimes required to roll off at a rate of 40 dB/decade. The resulting magnetizing impedance decreases then at a rate of at least 20 dB/decade, so that errors increase, rather than decrease, with frequency.

The transformer-amplifier combination discussed here avoids this limitation. The instrument, identified as an amplifier-aided two-stage transformer is shown schematically in Fig. 2.[1]

The feature making this device more attractive than other transformer–amplifier combinations is the direct-coupled feedback loop, which assures much better stability than do magnetically coupled feedback loops. Standard 20 dB/decade gain roll-off can be used, allowing the gain-error term to remain essentially constant over a wide frequency band.

The advantages of a two-stage transformer, with the tertiary winding looking into a very low voltage, have been known for many years [2]. Unfortunately, this requirement has severely limited the usefulness of the two-stage principle. With the addition of a feedback amplifier, however, the burden voltage, as seen by the tertiary winding, is attenuated by the amplifier gain, so that the transformer operates under nearly ideal conditions. The error ϵ of such a transformer is given approximately by

$$\epsilon = \frac{(Z_B + Z_2)Z_3}{Z_{m1}\, Z_{m2}} + \frac{Z_B}{G\, Z_{m2}}$$

where

Z_B	transformer burden,
Z_2, Z_3	leakage impedances of the secondary and tertiary windings,
Z_{m1}, Z_{m2}	magnetizing impedances of the first and second stages,
G	amplifier gain.

The first term is the product of the errors of each stage, assuming infinite gain, and the second term is the finite gain error.

[1] This amplifier-aided two-stage transformer circuit was conceived and developed by the author in 1970. It was learned within the last year that a similar circuit had been disclosed by P. Miljanic in U.S. Patent 3 534 247, issued in 1970. No analysis of Miljanic's work has been published, so that no references are available in the technical literature.

Fig. 1. Completed transformer number 2.

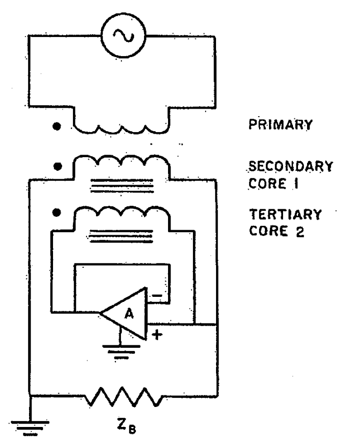

Fig. 2. Amplifier-aided two-stage transformer.

In addition to the inherently wide bandwidth afforded by this device, it shares with other transformer–amplifier combinations the following additional advantages.

1) The number of turns in the ratio windings can be reduced, without sacrificing low-frequency accuracy. This in turn reduces high-frequency capacitive errors.

2) Larger burdens (in this case up to 1 Ω) can be accommodated without appreciable loss in accuracy.

3) A capacitance trimming technique to further reduce capacitive errors is made possible by the presence of a second stage.

4) As in the current comparator, the second stage can be shielded from the leakage flux of the ratio windings by carefully designed mumetal and copper (eddy current) shields enveloping the tertiary winding. These shields reduce the magnetic errors of the second stage to less than 1×10^{-6} over the frequency range of 50 Hz to 10 kHz.

B. Shield Arrangement and Trimming Technique

The similarity of a two-stage transformer to a current comparator suggested the possibility of employing a capacitance trimming technique similar to one proposed by Miljanic [3]. The technique requires that each ratio winding be enclosed by an electrostatic shield, connected as shown in Fig. 3. Under these conditions, the effects of winding-to-shield capacitance can be balanced by an appropriate value of shunt capacitance (C_s or C_p in the figure). Furthermore, capacitances C_s and C_p are not dependent to first order on the operating parameters (burden, current level, and frequency) and hence remain fixed components of the transformer.

It can be shown that the capacitive error for a transformer having this shield configuration, a fixed number of secondary turns, and a primary winding arranged for series-parallel connection, is given by

$$\epsilon = j\omega Z_B\left(\frac{C_{ss}}{6} - C_s'\right) + j\omega Z_{sl}\frac{C_{ss}}{2} + j\omega (Z_B + Z_p + Z_s)\left[\left(\frac{C_{ps}}{6} - C_p\right)\frac{1}{n^2} - \frac{C_2}{n} - \frac{C_3}{N}\right] + j\omega Z_{pl}\frac{C_{ps}}{2}$$

where

- Z_B burden impedance,
- Z_p, Z_s primary (all sections in series) and secondary leakage impedances,
- Z_{pl}, Z_{sl} equivalent impedances of the primary and secondary leads involved in the ratio definition,
- C_{ps}, C_{ss} capacitances to shields of the primary and secondary windings,
- C_s' internal secondary shunt capacitance plus the secondary trimming capacitance C_s,
- C_p primary trimming capacitance,
- C_2 capacitance between any two adjacent sections of the multisection primary, assumed to be the same for all sections,
- C_3 capacitance between any two adjacent turns within a primary section, assumed to be the same for all turns,
- n nominal or turns ratio,
- N number of turns of the primary with all sections in series.

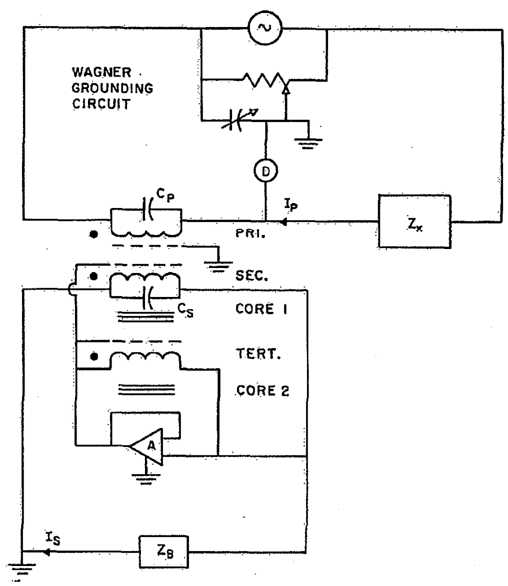

Fig. 3. Transformer showing shield arrangement and trimming capacitors.

This equation assumes that the capacitances and leakage impedances are uniformly distributed and that the ratio is defined as follows. The secondary current is the burden current (I_s in Fig. 3) entering the lead connected to the marked secondary terminal when this lead is connected to ground (amplifier common). The primary current is the current entering the lead connected to the unmarked primary terminal when this lead is brought to a virtual ground with a Wagner grounding circuit or its equivalent.

The terms $(C_{ss}/6)$ and $(C_{ps}/6)$ in the equation make by far the greatest contribution to the error. Reducing to zero only these terms ($C_s' = (C_{ss}/6)$, $C_p = (C_{ps}/6)$) can effect at least a factor of 10 reduction in capacitive error.

On the basis of this equation, calculations of C_s and C_p can be made using measurements of the natural winding-to-shield and shunt capacitances. However, their values can be determined somewhat more accurately by using, during construction, the circuits of Fig. 4 that simulate the conditions of actual operation.

This experimental determination is conducted as follows. After the second stage is completed with its tertiary winding and shields, the secondary winding (equal in number of turns to the tertiary) and its shield are put in place and connected as illustrated in Fig. 4(a). In this circuit source E excites core 1, and capacitance currents circulate as they would in normal operation. Were it not for the magnetization due to these capacitance currents, core 2 (experiencing no net ampere turns) would not be excited, and the detector would indicate null. (The voltage drop across the tertiary winding leakage impedance is negligible at an operating frequency of 10 kHz.) By

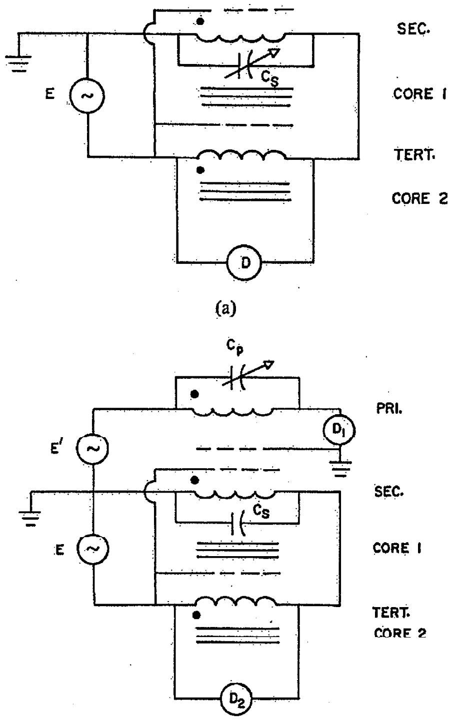

Fig. 4. (a) Circuit for trimming secondary winding; (b) circuit for trimming primary winding.

adjusting trimmer C_s the net capacitive error is reduced to zero, as indicated by a detector null.

Following this adjustment, the primary winding and its shield are added, and the transformer is connected as in Fig. 4(b). After applying voltage E, source E' is adjusted to balance detector D_1. This again establishes the conditions under which the transformer ratio is defined. Trimmer C_p is then adjusted to null D_2, bringing to zero the capacitive error associated with the primary winding.

This trimming technique was applied to both transformers. It is interesting to note that calculated values for the trimmers were within 5 percent of the experimentally determined values, indicating that significant reduction of high-frequency errors could be achieved without the necessity of an experimental determination.

III. Method of Calibration

Two circuits were developed for calibrating the transformers. One is a self-calibration circuit in which values for the 5/5 ratio are obtained for each transformer. The other is a step-up comparison circuit in which each succeeding ratio of one transformer is measured in terms of a lower ratio of the other. The transformer ratio, thus measured, is given by

$$\frac{I_s}{I_p} = \frac{1}{n}(1+\epsilon)$$

where n is the turns ratio and ϵ the measured error. I_s and I_p are as previously defined.

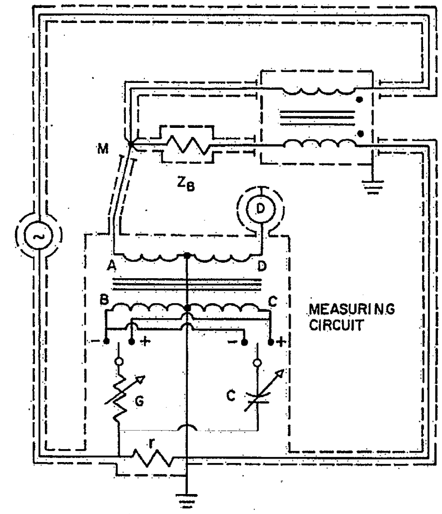

Fig. 5. Self-calibration circuit.

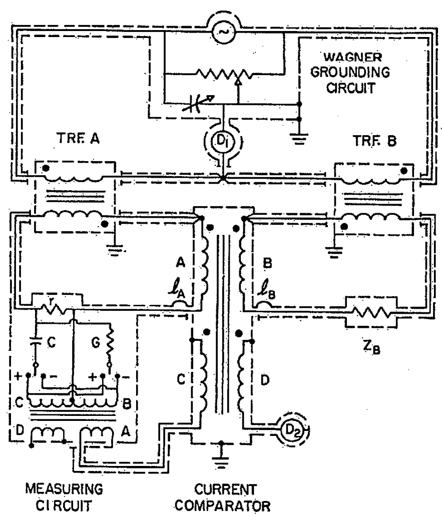

Fig. 6. Step-up comparison circuit.

A. Self-Calibration Circuit

Self-calibration of the 5/5 ratio is accomplished by comparing the primary and secondary currents as shown in Fig. 5. The difference current $I_p - I_s$ is directed to ground through a low-impedance winding (A) of the current comparator type measuring circuit. There, its magnitude and phase are determined in terms of the known currents through the admittances G and C, established by the 0.1-Ω reference resistor r. Point M, as the ratio definition requires, is at a virtual ground.

B. Step-Up Comparison Circuit

The two transformers are compared using a current comparator connected as shown in the circuit of Fig. 6. By limiting the current comparator ratios to 1/1, 2/1, and 2.5/1, it was possible to use a maximum of only ten turns for any ratio winding, causing both winding impedances and distributed capacitances to be very small. Elaborate and careful magnetic shielding assures that the magnetic errors of the comparator are less than 1×10^{-7}. A one-to-one self-calibration of this comparator indicated the errors to be less than 3×10^{-7} at 10 kHz, decreasing to less than 1×10^{-7} between 50 Hz and 1 kHz. While space limitations prohibit inclusion of a detailed analysis of this comparator, a thorough treatment of audio-frequency current comparators can be found in [4].

The comparison measurement involves three steps: a Wagner earth and the main balance, and the measurement of two shield currents. Because shield currents (comprising a portion of the defined secondary currents) shunt the windings of the current comparator, they must be accounted for separately. While their values can be calculated, based on measurements of the shunted impedances and the conductor-to-shield admittances, they can be determined most accurately by direct measurement. The three steps proceed as follows.

1) The Wagner arms are adjusted to balance detector D_1 to less than a few millivolts.

2) Admittances G and C of the measuring circuit are adjusted for a null on D_2 for the main balance.

3) Shields are broken at links l_A and l_B in turn, and winding "A" of the measuring circuit is substituted for the link. With a detector connected to winding D of the measuring circuit, the magnitude and phase of both shield currents are established by readjusting G and C for a detector null.

IV. Results of Calibration

The errors, as established for both transformers (including the shield current corrections), are given in Table I. The secondary current during these tests was between 1 and 5 A, depending on the ratio and the position of the transformer (i.e., position A or B) in the test circuit. Additional tests at these frequencies showed that the error variation with secondary current does not exceed 1×10^{-7} for transformer 1 and 2×10^{-7} for transformer 2 for currents between 0.5 and 5 A. Tests at the 100/5 ratio have not been completed because of supply limitations.

A. Cross Checks

While cross checks of the base ratio (5/5) measurements would require an independent method, it is possible to make cross checks of the step-up process by making one-to-one comparisons of the two transformers at each given ratio. The differences measured in such a test should agree with the differences between the errors given in Table I. Any disagreement is an indication of the uncertainties in the step-up process.

Such cross checks were made at each ratio for each frequency given in Table I. The maximum differences at each frequency are given in Table II, where $\epsilon_2 - \epsilon_1$ is computed from Table I and ϵ_{2-1} is the value obtained from the one-to-one comparison tests. Symbols α and β designate in-phase and quadrature components of the error. It is felt that the following two sources contribute most to these differences.

1) While the transformer in position A sees as a burden the 0.1-Ω reference resistor and the comparator winding A $[(.02 + j\omega \times 10^{-6})\ \Omega]$, its zero-burden error was used in the step-up calculations. This source of error can be reduced in future tests.

2) The primary leakage impedance of transformer 1 is excessively high (approximately 25 Ω for the series connection) at 10 kHz, so that small spurious capacitances, shunting the primary, for instance, can cause appreciable changes in the defined ratio. For the lead arrangement used in these tests, shielding is incomplete at the lead ends. Changes in position of these leads from measurement to measurement show corresponding changes in ratio of as much as 4×10^{-6} at 10 kHz. Transformer 2, with a leakage impedance of less than 4 Ω at 10 kHz, is much more stable, exhibiting a maximum change of only 0.7×10^{-6}.

B. Low-Frequency Errors

At frequencies below 400 Hz, capacitive errors tend to be overshadowed by errors of magnetic origin. These magnetic errors are inversely proportional to frequency and are essentially independent of transformer ratio. Errors of the one-to-one ratio, measured at 60 Hz, are given in Table III.

Appendix
Construction Details

A. Core 2

Transformer 1: 23-cm outside diameter × 15-cm inside diameter × 2.5-cm height–0.01 cm (4 mil) superperm 80-in aluminum case–permeability > 45 000 at 20 G, 60 Hz.

Transformer 2: Same dimensions and material–permeability > 32 000 at 20 G, 60 Hz.

B. Tertiary Winding

Transformer 1: 240 turns number 15 AWG magnet wire, uniformly distributed in one layer.

Transformer 2: 2 interleaved sections connected in parallel, each with 120 turns number 15 wire, uniformly distributed in one layer.

C. Magnetic Shield

Transformer 1: Annealed 0.127-cm (50 mil) Mumetal box, toroidally concentric with tertiary winding (also used as tertiary electrostatic shield).

Transformer 2: Same as for transformer 1.

D. Core 1

Transformer 1: Same as core 2, transformer 1, placed directly on shielded second stage.

Transformer 2: Same as core 2, transformer 2, positioned as in transformer 1.

E. Eddy Current Shield

Transformer 1: Toroidal box of 0.318-cm ($\frac{1}{8}$-in) thick copper enclosing core 1 and shielded second stage (also used as bottom half of secondary electrostatic shield).

Transformer 2: Same as in transformer 1.

TABLE I
CALIBRATION RESULTS (PARTS PER MILLION)

Transformer 1

f (Hz)	Z_B(Ω)	1/1	2/1	4/1	5/1	10/1
10^4	1	+16.5 -j15.6	+29.8 -j9.8	+21.6 -j6.6	+15.4 -j5.8	+12.9 -j1.2
	0	+ 0.6 -j9.6	+14.5 -j3.2	+ 5.9 -j0.5	- 0.9 -j0.1	- 4.1 +j4.8
5×10^3	1	+ 5.5 -j5.0	+ 8.5 -j3.4	+ 5.6 -j2.5	+ 4.2 -j1.9	+ 3.0 -j0.9
	0	+ 1.3 -j3.1	+ 4.3 -j1.2	+ 1.3 -j0.5	- 0.3 +j0.0	- 1.4 +j0.9
10^3	1	+ .22 -j .50	+ .28 -j .43	+ .08 -j .25	+ .03 -j .15	+ .06 -j .15
	0	+ .11 -j .22	+ .14 -j .12	- .05 +j .02	- .14 +j .10	- .10 +j .07
4×10^2	1	+ .04 -j .13	- .02 -j .15	- .07 -j .06	- .04 -j .05	- .08 -j .07
	0	.00 -j .04	- .01 -j .04	- .05 +j .06	- .06 +j .02	- .07 +j .02

Transformer 2

f (Hz)	Z_B(Ω)	1/1	2/1	4/1	5/1	10/1
10^4	1	+ 0.1 +j1.8	+ 4.2 +j0.6	+6.1 +j3.5	+4.9 +j3.9	+5.7 +j1.7
	0	-11.6 +j3.5	- 6.8 +j2.3	-5.4 +j4.3	-7.1 +j5.0	-6.8 +j3.0
5×10^3	1	- 0.3 +j0.6	+ 0.2 +j0.6	+1.5 +j0.8	+1.1 +j1.0	+1.2 +j0.5
	0	- 3.2 +j0.9	- 2.3 +j0.5	-1.6 +j0.8	-2.1 +j1.2	-2.0 +j0.9
10^3	1	- .11 +j .09	- .11 +j .15	.00 +j .17	- .03 +j .18	- .12 +j .21
	0	- .15 +j .08	- .16 +j .14	- .12 +j .09	- .10 +j .13	- .24 +j .16
4×10^2	1	- .02 +j .14	- .04 +j .20	- .05 +j .17	- .03 +j .15	- .09 +j .20
	0	- .02 +j .03	- .02 +j .09	- .05 +j .08	- .03 +j .05	- .11 +j .10

TABLE II
RESULTS OF CROSS CHECKS
$\delta = (\epsilon_2 - \epsilon_1) - \epsilon_{2-1}$

f (Hz)	10^4	5×10^3	10^3	4×10^2
α (ppm)	+2.8	-1.2	+0.18	+0.15
β (ppm)	+3.5	-1.8	-0.14	+0.10

TABLE III
ERRORS AT 60 Hz

I_s	Z_B(Ω)	Transformer 1 (ppm)	Transformer 2 (ppm)
5	1	0.0 + j 0.1	+0.6 + j 0.9
	0	0.0 + j 0.0	+0.4 + j 0.2
0.5	1	+0.1 + j 0.1	+3.3 + j 1.7
	0	+0.2 + j 0.1	+0.7 + j 0.3

F. Secondary Winding

Transformer 1: 240 turns number 12 wire wound one layer on outside, two layers on inside, uniformly distributed around circumference with successive turns alternating between layers.

Transformer 2: 120 turns number 12 wire uniformly distributed in one layer.

G. Secondary Shield (Top Half)

Transformer 1: 0.159-cm ($\frac{1}{16}$-in) copper toroidal box.

Transformer 2: Conducting silver paint over 0.127-cm (50-mil) insulation.

H. Primary Shield (Bottom Half)

Transformer 1: Conducting silver paint over 0.127-cm (50-mil) insulation.

Transformer 2: Same as in transformer 1.

I. Primary Winding

Transformer 1: 240 turns number 12 wire uniformly distributed in 20 equal sections, wound as in secondary.

Transformer 2: 120 turns number 12 wire uniformly distributed in one layer in 20 equal sections.

J. Primary Shield (Top Half)

Transformer 1: Brass sheet over 0.25-cm insulation.

Transformer 2: Aluminum transformer case (1 in or greater from primary winding) used as shield.

K. Trimmer Capacitors

Transformer 1: $C_s = 337$ pF, $C_p = 248$ pF.

Transformer 2: $C_s = 817$ pF, $C_p = 220$ pF.

L. Operational Amplifier

Transformer 1: Commercial 20-V 50-mA output; dc gain $> 5 \times 10^5$ with a corner frequency at approximately 10 Hz.

Transformer 2: Same as for transformer 1.

REFERENCES

[1] O. Petersons, "A self-balancing current comparator," *IEEE Trans. Instrum. Meas.*, vol. IM-15, pp. 62–71, Mar./June 1966.

[2] H. B. Brooks and F. C. Holtz, "The two-stage current transformer," *AIEE Trans.*, vol. 41, pp. 382–391, June 1922.

[3] P. N. Miljanic, "Capacitive error in current comparators," *IEEE Trans. Instrum. Meas.*, vol. IM-13, pp. 210–216, Dec. 1964.

[4] N. L. Kusters and W. J. M. Moore, "The development and performance of current comparators for audio frequencies," *IEEE Trans. Instrum. Meas.*, vol. IM-14, pp. 178–190, Dec. 1965.