Guidelines on the Calibration of Non-Automatic Weighing Instruments

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Mass and Related Quantities

Authorship and Imprint

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Guidance Publications

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Version 4.0 (11/2015)

Guidelines on the Calibration of Non-Automatic Weighing Instruments

Purpose

This document has been produced to enhance the equivalence and mutual recognition of calibration results obtained by laboratories performing calibrations of non-automatic weighing instruments.

Content

1 INTRODUCTION

Non-automatic weighing instruments (NAWI) are widely used to determine the value of a load in terms of mass. For some applications specified by national legislation, NAWI are subject to legal metrological control – i.e. type approval, verification etc. – but there is an increasing need to have their metrological quality confirmed by calibration, e.g. where required by ISO 9001 or ISO/IEC 17025 standards.

2 SCOPE

This document contains guidance for the static calibration of self-indicating, non-automatic weighing instruments (hereafter called "instrument"), in particular for

1. measurements to be performed,
2. calculation of the measuring results,
3. determination of the uncertainty of measurement,
4. contents of calibration certificates.

The object of the calibration is the indication provided by the instrument in response to an applied load. The results are expressed in units of mass. The value of the load indicated by the instrument will be affected by local gravity, the load temperature and density, and the temperature and density of the surrounding air.

The uncertainty of measurement depends significantly on properties of the calibrated instrument itself, not only on the equipment of the calibrating laboratory; it can to some extent be reduced by increasing the number of measurements performed for a calibration. This guideline does not specify lower or upper boundaries for the uncertainty of measurement.

It is up to the calibrating laboratory and the client to agree on the anticipated value of the uncertainty of measurement that is appropriate in view of the use of the instrument and in view of the cost of the calibration.

While it is not intended to present one or few uniform procedures the use of which would be obligatory, this document gives general guidance for establishing of calibration procedures the results of which may be considered as equivalent within the EURAMET Member Organisations.

Any such procedure must include, for a limited number of test loads, the determination of the error of indication and of the uncertainty of measurement assigned to these errors. The test procedure should as closely as possible resemble the weighing operations that are routinely being performed by the user – e.g. weighing discrete loads, weighing continuously upwards and/or downwards, use of tare balancing function.

The procedure may further include rules how to derive from the results advice to the user of the instrument with regard to the errors, and assigned uncertainty of measurement, of indications which may occur under normal conditions of use of the instrument, and/or rules on how to convert an indication obtained for a weighed object into the value of mass or conventional value of mass of that object.

The information presented in this guideline is intended to serve, and should be observed by

1. bodies accrediting laboratories for the calibration of weighing instruments,
2. laboratories accredited for the calibration of non-automatic weighing instruments,

3. test houses, laboratories, or manufacturers using calibrated non-automatic weighing instruments for measurements relevant for the quality of production subject to QM requirements (e.g. ISO 9000 series, ISO 10012, ISO/IEC 17025).

3 TERMINOLOGY AND SYMBOLS

The terminology used in this document is mainly based on existing documents

- JCGM 100 [1] for terms related to the determination of results and the uncertainty of measurement,
- OIML R76 [2] (or EN 45501 [3]) for terms related to the functioning, to the construction, and to the metrological characterisation of non-automatic weighing instruments,
- OIML R111 [4] for terms related to the standard weights,
- JCGM 200 [5] for terms related to the calibration.

Such terms are not explained in this document, but where they first appear, references will be indicated.

Symbols whose meanings are not self-evident, will be explained where they are first used. Those that are used in more than one section are collected in Appendix D.

4 GENERAL ASPECTS OF THE CALIBRATION

4.1 Elements of the calibration

Calibration consists of

1. applying test loads to the instrument under specified conditions,
2. determining the error or variation of the indication, and
3. evaluating the uncertainty of measurement to be attributed to the results.

4.1.1 Range of calibration

Unless requested otherwise by the client, a calibration extends over the full weighing range [2] (or [3]) from zero to the maximum capacity Max. The client may specify a certain part of a weighing range, limited by a minimum load Min' and the largest load to be weighed Max', or individual nominal loads, for which he requests calibration.

On a multiple range instrument [2] (or [3]), the client should identify which range(s) shall be calibrated. The paragraph above may be applied to each range separately.

4.1.2 Place of calibration

Calibration is normally performed in the location where the instrument is being used.

If an instrument is moved to another location after the calibration, possible effects from

1. difference in local gravity acceleration,
2. variation in environmental conditions,
3. mechanical and thermal conditions during transportation

are likely to alter the performance of the instrument and may invalidate the calibration. Moving the instrument after calibration should therefore be avoided, unless immunity to these effects of a particular instrument, or type of instrument has been clearly demonstrated. Where this has not been demonstrated, the calibration certificate should not be accepted as evidence of traceability.

4.1.3 Preconditions, preparations

Calibration should not be performed unless

1. the instrument can be readily identified,
2. all functions of the instrument are free from effects of contamination or damage, and functions essential for the calibration operate as intended,
3. presentation of weight values is unambiguous and indications, where given, are easily readable,
4. the normal conditions of use (air currents, vibrations, stability of the weighing site etc.) are suitable for the instrument to be calibrated,
5. the instrument is energized prior to calibration for an appropriate period, e.g. as long as the warm-up time specified for the instrument, or as set by the user,
6. the instrument is levelled, if applicable,
7. the instrument has been exercised by loading approximately up to the largest test load at least once, repeated loading is advised.

Instruments that are intended to be regularly adjusted before use should be adjusted before the calibration, unless otherwise agreed with the client. Adjustment should be performed with the means that are normally applied by the client, and following the manufacturer's instructions where available. Adjustment could be done by means of external or built-in test loads.

The most suitable operating procedure for high resolution balances (with relative resolution better 1×10^{-5} of full scale) is to perform the adjustment of the balance immediately before the calibration and also immediately before use.

Instruments fitted with an automatic zero-setting device or a zero-tracking device [2] (or [3]) should be calibrated with the device operative or not, as set by the client.

For on site calibration the user of the instrument should be asked to ensure that the normal conditions of use prevail during the calibration. In this way disturbing effects such as air currents, vibrations, or inclination of the measuring platform will, so far as is possible, be inherent in the measured values and will therefore be included in the determined uncertainty of measurement.

4.2 Test load and indication

4.2.1 Basic relation between load and indication

In general terms, the indication of an instrument is proportional to the force exerted by an object of mass *m* on the load receptor

$$I = k_s mg(1 - \rho_a / \rho) \qquad (4.2.1\text{-}1)$$

with

- g local gravity acceleration
- ρ_a density of the surrounding air
- ρ density of the object
- k_s adjustment factor

The terms in the brackets account for the reduction of the force due to the air buoyancy of the object.

4.2.2 Effect of air buoyancy

It is state of the art to use standard weights that have been calibrated to the conventional value of mass m_c [1], for the adjustment and/or the calibration of weighing instruments. In principle, at the reference air density ρ_0 = 1,2 kg/m^3, the balance should indicate the conventional mass m_c of the test object.

The adjustment is performed at an air density ρ_{as} and is such that the effects of g and of the actual buoyancy of the adjustment weight having conventional mass m_{cs} are included in the adjustment factor k_s. Therefore, at the moment of the adjustment, the indication I_s is

$$I_s = m_{cs} \quad (4.2.2\text{-}1)$$

This adjustment is performed under the conditions characterized by the actual values of g_s, $\rho_s \neq \rho_c$, and $\rho_{as} \neq \rho_0$, identified by the suffix "s", and is valid only under these conditions. For another body of conventional mass m_c with $\rho \neq \rho_s$, weighed on the same instrument but under different conditions: $g \neq g_s$ and $\rho_a \neq \rho_{as}$ the indication is in general (neglecting terms of 2nd or higher order) [6]

$$I = m_c(g/g_s)\{1-(\rho_a-\rho_0)(1/\rho-1/\rho_s)-(\rho_a-\rho_{as})/\rho_s\} \quad (4.2.2\text{-}3)$$

If the instrument is not displaced, there will be no variation of g, so $g/g_s = 1$. This is assumed hereafter.

The indication of the balance will be exactly the conventional mass of the body, only in some particular cases, the most evident are

- $\rho_a = \rho_{as} = \rho_0$.
- the weighing is performed at $\rho_a = \rho_{as}$ and the body has a density $\rho = \rho_s$.

The formula simplifies further in situations where some of the density values are equal

a) weighing a body in the reference air density: $\rho_a = \rho_0$, then

$$I = m_c\left[1-(\rho_a-\rho_{as})/\rho_s\right] \quad (4.2.2\text{-}4)$$

b) weighing a body of the same density as the adjustment weight: $\rho = \rho_s$, then again (as in case a))

$$I = m_c\left[1-(\rho_a-\rho_{as})/\rho_s\right] \quad (4.2.2\text{-}5)$$

c) weighing in the same air density as at the time of adjustment: $\rho_a = \rho_{as}$, then

$$I = m_c\left[1-(\rho_a-\rho_0)(1/\rho-1/\rho_s)\right] \quad (4.2.2\text{-}6)$$

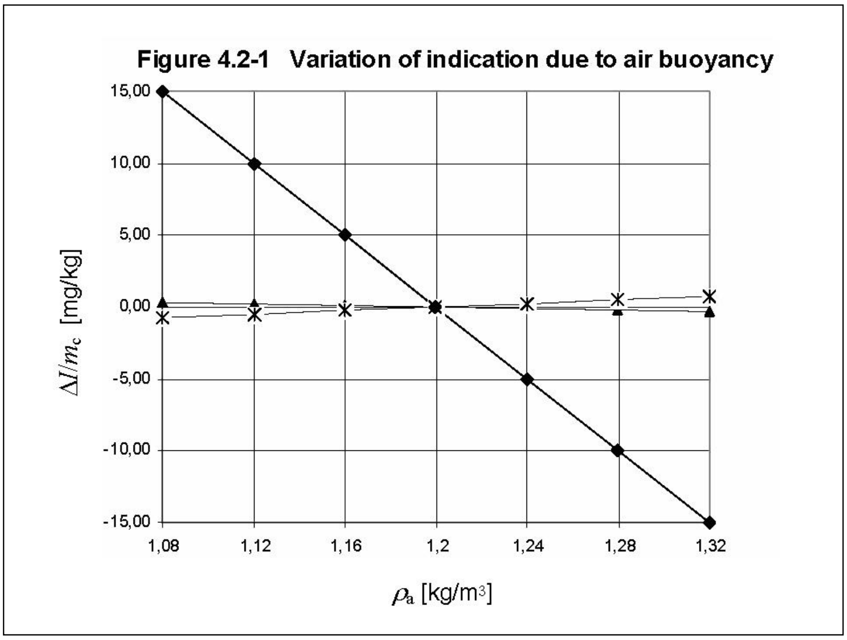
Figure 4.2-1 shows examples for the magnitude of the relative changes

[1]The conventional value of mass m_c of a body has been defined in [4] as the numerical value of mass m of a weight of reference density ρ_c = 8000 kg/m³ which balances that body at 20 °C in air of density ρ_0:

$$m_c = m\{(1-\rho_0/\rho)/(1-\rho_0/\rho_c)\} \quad (4.2.2\text{-}2)$$

with ρ_0 = 1,2 kg/m³ = reference value of the air density

$\Delta I / m_c = (I - m_c) / m_c$ for an instrument adjusted with standard weights of $\rho_s = \rho_c$, when calibrated with standard weights of different but typical density.



Line ▲ is valid for a body of $\rho =$ 7 810 kg/m³, weighed in $\rho_a = \rho_{as}$ (as for case c above)

Line ✖ is valid for a body of $\rho =$ 8 400 kg/m³, weighed in $\rho_a = \rho_{as}$ (as for case c above)

Line ◆ is valid for a body of $\rho = \rho_s = \rho_c$ after adjustment in $\rho_{as} = \rho_0$ (as for case b above)

It is obvious that under these conditions, a variation in air density has a far greater effect than a variation in the body density.

Further information on air density is given in Appendix A, and on air buoyancy related to standard weights in Appendix E.

4.2.3 Effects of convection

Where weights have been transported to the calibration site they may not be at the same temperature as the instrument and its environment. The temperature difference ΔT is defined as the difference between the temperature of a standard weight and the temperature of the surrounding air. Two phenomena should be noted in this case:

- An initial temperature difference ΔT_0 may be reduced to a smaller value ΔT by acclimatisation over a time Δt; this occurs faster for smaller weights than for larger ones.
- When a weight is put on the load receptor, the actual difference ΔT will produce an air flow about the weight leading to parasitic forces which result in an apparent change Δm_{conv} on its mass. The sign of Δm_{conv} is normally opposite to the sign of ΔT, its value being greater for large weights than for small ones.

The relations between any of the quantities mentioned: ΔT_0, Δt, ΔT, m and Δm_{conv} are nonlinear, and they depend on the conditions of heat exchange between the weights and their environment – see [7].

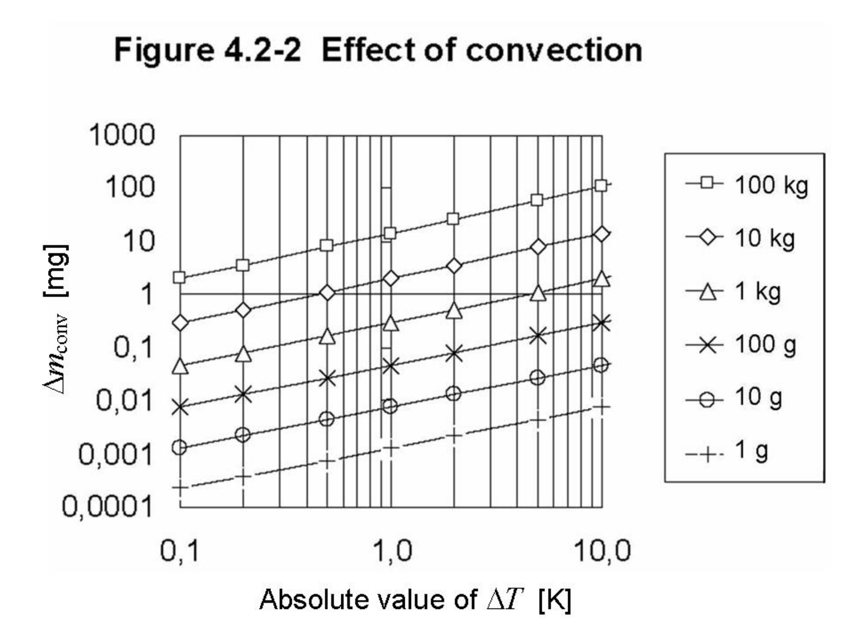


Figure 4.2-2 gives an impression of the magnitude of the apparent change in mass in relation to a temperature difference, for some selected weight values.

This effect should be taken into account by either letting the weights acclimatise to the extent that the remaining change Δm_{conv} is negligible in view of the uncertainty of the calibration required by the client, or by considering the possible change of indication in the uncertainty budget. The effect may be significant for weights of high accuracy, e.g. for weights of class E_2 or F_1 in R 111 [4].

More detailed information is given in Appendix F.

4.2.4 Buoyancy correction for the reference value of mass

To determine the errors of indication of an instrument, standard weights of known conventional value of mass m_{cCal} are applied. Their density ρ_{Cal} is normally different from the reference value ρ_{c} and the air density ρ_{aCal} at the time of calibration is normally different from ρ_0.

The error E of indication is

$$E = I - I_{\mathrm{ref}} \tag{4.2.4-1}$$

where I_{ref} is the reference value of the indication of the instrument, further called reference value of mass, m_{ref}. Due to effects of air buoyancy, convection, drift and others which may lead to minor correction terms δm_x, m_{ref} is not exactly equal to m_{cCal}, the conventional value of the mass

$$m_{\mathrm{ref}} = m_{\mathrm{cCal}} + \delta m_{\mathrm{B}} + \delta m.. \tag{4.2.4-2}$$

The correction for air buoyancy δm_{B} is affected by values of ρ_{s} and ρ_{as}, that were valid for the adjustment but are not normally known. It is assumed that weights of the reference density $\rho_{\mathrm{s}} = \rho_{\mathrm{c}}$ have been used. From (4.2.2-3) the general expression for the correction is

$$\delta m_B = -m_{cCal}[(\rho_{aCal} - \rho_0)(1/\rho_{Cal} - 1/\rho_c) + (\rho_{aCal} - \rho_{as})/\rho_c] \qquad (4.2.4\text{-}3)$$

For the air density ρ_{as} two situations are considered. If the instrument has been adjusted immediately before the calibration, then $\rho_{as} = \rho_{aCal}$. This simplifies (4.2.4-3) to

$$\delta m_B = -m_{cCal}(\rho_{aCal} - \rho_0)(1/\rho_{Cal} - 1/\rho_c) \qquad (4.2.4\text{-}4)$$

If the instrument has been adjusted independent of the calibration, in unknown air density ρ_{as}, it is not possible to perform the correction for the last term of equation (4.2.4-3), which intrinsically forms part of the error of indication. The correction to be applied should also be (4.2.4-4) [10].

The suffix “Cal” will from now on be omitted unless where necessary to avoid confusion.

4.3 Test loads

Test loads should preferably consist of standard weights that are traceable to the SI unit of mass. However, other test loads may be used for tests of a comparative nature – e.g. test with eccentric loading, repeatability test – or for the mere loading of an instrument – e.g. preloading, tare load that is to be balanced, substitution load.

4.3.1 Standard weights

The traceability of weights to be used as standards shall be demonstrated by calibration [8] consisting of

1. determination of the conventional value of mass m_c and/or the correction δm_c to its nominal value m_N: $\delta m_c = m_c - m_N$, together with the expanded uncertainty of the calibration U_{95}, or
2. confirmation that m_c is within specified maximum permissible errors mpe: $m_N - (mpe - U_{95}) \leq m_c \leq m_N + (mpe - U_{95})$

The standards should further satisfy the following requirements to an extent appropriate to their accuracy:

3. density ρ_s sufficiently close to ρ_c = 8 000 kg/m³,
4. surface finish suitable to prevent a change in mass through contamination by dirt or adhesion layers,
5. magnetic properties such that interaction with the instrument to be calibrated is minimized.

Weights that comply with the relevant specifications of the International Recommendation OIML R 111 [4] should satisfy all these requirements.

The maximum permissible errors, or the uncertainties of calibration of the standard weights, shall be compatible with the scale interval *d* [2] (or [3]) of the instrument and/or the needs of the client with regard to the uncertainty of the calibration of the instrument.

4.3.2 Other test loads

For certain applications mentioned in 4.3, 2nd sentence, it is not essential that the conventional value of mass of a test load is known. In these cases, loads other than standard weights may be used, with due consideration of the following

1. shape, material, composition should allow easy handling,

2. shape, material, composition should allow the position of the centre of gravity to be readily estimated,
3. their mass must remain constant over the full period they are in use for the calibration,
4. their density should be easy to estimate,
5. loads of low density (e.g. containers filled with sand or gravel), may require special attention in view of air buoyancy. Temperature and barometric pressure may need to be monitored over the full period the loads are in use for the calibration.

4.3.3 Use of substitution loads

A test load, of which the conventional value of mass must be known, should be made up entirely of standard weights. But where this is not possible, or where the standard weights are not sufficient to calibrate the normal range of the instrument or the range agreed with the customer, any other load which satisfies 4.3.2 may be used for substitution. The instrument under calibration is used as a comparator to adjust the substitution load L_{sub} so that it brings about approximately the same indication I as the corresponding load L_{St} made up of standard weights.

A first test load L_{T1} made up of standard weights m_{ref} is indicated as

$$I(L_{\mathrm{St}}) = I(m_{\mathrm{ref}}) \tag{4.3.3-1}$$

After removing L_{St} a substitution load L_{sub1} is put on and adjusted to give approximately the same indication

$$I(L_{\mathrm{sub1}}) \approx I(m_{\mathrm{ref}}) \tag{4.3.3-2}$$

so that

$$L_{\mathrm{sub1}} = m_{\mathrm{ref}} + I(L_{\mathrm{sub1}}) - I(m_{\mathrm{ref}}) = m_{\mathrm{ref}} + \Delta I_1 \tag{4.3.3-3}$$

The next test load L_{T2} is made up by adding m_{ref}

$$L_{\mathrm{T2}} = L_{\mathrm{sub1}} + m_{\mathrm{ref}} = 2m_{\mathrm{ref}} + \Delta I_1 \tag{4.3.3-4}$$

m_{ref} is again replaced by a substitution load of ≈ L_{sub1} with adjustment to ≈ $I(L_{\mathrm{T2}})$.

The procedure may be repeated, to generate test loads L_{T3}, ...,$L_{\mathrm{T}n}$

$$L_{\mathrm{T}n} = nm_{\mathrm{ref}} + \Delta I_1 + \Delta I_2 + \ldots + \Delta I_{n-1} \tag{4.3.3-5a}$$

With each substitution step however, the uncertainty of the total test load increases substantially more than if it were made up of standard weights only, due to the effects of repeatability and resolution of the instrument. – cf. also 7.1.2.6[2].

If the test load L_{T1} is made up of more than one standard weight, it is possible to first use

[2] Example: for an instrument with *Max* = 5000 kg, *d*= 1 kg, the standard uncertainty of 5 t standard weights of accuracy class M1 – based on their nominal value, and using (7.1.2-3) – is around150 g, while the standard uncertainty of a test load made up of 1 t standard weights and 4 t substitution load, using (7.1.2-16a), will be about 1,2 kg. In this example, uncertainty contributions due to buoyancy and drift were neglected. Equally, it was assumed that the uncertainty of the indication only comprises the rounding error at no-load and at load.

the standard weights to create *N* individual test loads $m_{\mathrm{ref},k}$ (*k* = 1,…, *N*) with the condition

$$m_{\mathrm{ref},1} < m_{\mathrm{ref},2} < \ldots < m_{\mathrm{ref},N} = m_{\mathrm{ref}} = L_{\mathrm{T1}} . \qquad (4.3.3\text{-}6)$$

Afterwards, L_{T1} is substituted by a substitution load L_{sub1}, and then the test loads $m_{\mathrm{ref},k}$ can again be added consecutively. The individual test loads shall be referred to as $L_{\mathrm{T}n,k}$ with

$$L_{\mathrm{T}n,k} = (n-1)m_{\mathrm{ref}} + m_{\mathrm{ref},k} + \Delta I_1 + \Delta I_2 + \ldots + \Delta I_{n-1} . \qquad (4.3.3\text{-}5b)$$

4.4 Indications

4.4.1 General

Any indication *I* related to a test load is basically the difference of the indications I_L under load and I_0 at no-load, before the load is applied

$$I = I_L - I_0 \qquad (4.4.1\text{-}1a)$$

It is preferable to record the no-load indications together with the load indications for any test measurement. In the case that the user of the instrument takes into account the zero return of any loading during normal use of the instrument, e.g. in the case of a substantial drift, the indication can be corrected according to equation (4.4.1-1b)[3]. However, recording the no-load indications may be redundant where a test procedure calls for a balance to be zeroed before a test load is applied.

For any test load, including no-load, the indication *I* of the instrument is read and recorded only when it can be considered as being stable. Where high resolution of the instrument, or environmental conditions at the calibration site prevent stable indications, an average value should be estimated and recorded together with information about the observed variability (e.g. spread of values, unidirectional drift).

During calibration tests, the original indications should be recorded, not errors or variations of the indication.

4.4.2 Resolution

Indications are normally obtained as integer multiples of the scale interval *d*.

At the discretion of the calibration laboratory and with the consent of the client, means to obtain indications in higher resolution than in *d* may be applied, e.g. where compliance to a specification is checked and smallest uncertainty is desired. Such means may be

1. switching the indicating device to a smaller scale interval $d_{\mathrm{T}} < d$ (“service mode”). In this case, the indications are obtained as integer multiple of d_{T}.
2. applying small extra test weights in steps of $d_{\mathrm{T}} = d/5$ or $d/10$ to determine more precisely the load at which an indication changes unambiguously from I' to $I' + d$ (“changeover point method”). In this case, the indication *I’* is recorded

[3]In case of linear drift the corrected reading is given by

$$I = I_L - (I_0 + I_{0i})/2 \qquad (4.4.1\text{-}1b)$$

where I_0 and I_{0i} are the no-load indications before and after the load is applied.

together with the amount ΔL of the *n* additional small test weights necessary to increase I' by one d .

The indication I_L is

$$I_L = I' + d/2 - \Delta L = I' + d/2 - nd_{\mathrm{T}} \quad (4.4.2\text{-}1)$$

Where the changeover point method is applied, it is advised to apply it for the indications at zero as well as for the indications at load.

5 MEASUREMENT METHODS

Tests are normally performed to determine

- the repeatability of indications,
- the errors of indications,
- the effect of eccentric application of a load on the indication.

A Calibration Laboratory deciding on the number of measurements for its routine calibration procedure should consider that, in general, a larger number of measurements tends to reduce the uncertainty of measurement but increase the cost.

Details of the tests performed for an individual calibration may be fixed by agreement of the client and the Calibration Laboratory, in view of the normal use of the instrument. The parties may also agree on further tests or checks which may assist in evaluating the performance of the instrument under special conditions of use. Any such agreement should be consistent with the minimum numbers of tests as specified in the following sections.

5.1 Repeatability test

The test consists of the repeated deposition of the same load on the load receptor, under identical conditions of handling the load and the instrument, and under constant test conditions.

The test load(s) need not be calibrated nor verified, unless the results serve for the determination of errors of indication as per 5.2. The test load should, as far as possible, consist of one single body.

The test is performed with at least one test load L_{T} which should be selected in a reasonable relation to Max and the resolution of the instrument, to allow an appraisal of the instrument performance. For instruments with a constant scale interval *d* a load of about $0{,}5Max \le L_{\mathrm{T}} \le Max$ is quite common; this is often reduced for instruments where L_{T} would amount to several 1000 kg. For multi-interval instruments [2] (or [3]) a load below and close to Max_1 may be preferred. For multiple range instruments, a load below and close to the capacity of the range with the smallest scale interval may be sufficient. A special value of L_{T} may be agreed between the parties where this is justified in view of a specific application of the instrument.

The test may be performed at more than one test point, with test loads $L_{\mathrm{T}j}$, $1 \le j \le k_L$ with k_L = number of test points.

Prior to the test, the indication is set to zero. The load is to be applied at least 5 times, or at least 3 times where L_T ≥ 100 kg.

Indications I_{Li} are recorded for each deposition of the load. After each removal of the load, the indication should be checked, and may be reset to zero if it does not show zero; recording of the no-load indications I_{0i} may be advisable as per 4.4.1. In addition, the status of the zero-setting or zero-tracking device if fitted should be recorded.

5.2 Test for errors of indication

This test is performed with k_L ≥ 5 different test loads L_{Tj}, 1 ≤ *j* ≤ k_L, distributed fairly evenly over the normal weighing range or at individual test points agreed upon as per 4.1.1. Examples for target values

- k_L = 5: zero or *Min*; 0,25 *Max*; 0,5 *Max*; 0,75 *Max;Max*. Actual test loads may deviate from the target value up to 0,1 *Max*, provided the difference between consecutive test loads is at least 0,2 *Max*,
- k_L = 11: zero or *Min*, 10 steps of 0,1 *Max* up to *Max*. Actual test loads may deviate from the target value up to 0,05 *Max,* provided the difference between consecutive test loads is at least 0,08 *Max*.

The purpose of this test is an appraisal of the accuracy of the instrument over the whole weighing range.

Where a significantly smaller range of calibration has been agreed, the number of test loads may be reduced accordingly, provided there are at least 3 test points including Min' and Max', and the difference between two consecutive test loads is not greater than $0{,}15 Max$.

It is necessary that test loads consist of appropriate standard weights or of substitution loads as per 4.3.3.

Prior to the test, the indication is set to zero. The test loads L_{Tj} are normally applied once in one of these manners

1. increasing by steps with unloading between the separate steps – corresponding to the majority of uses of the instruments for weighing single loads,
2. continuously increasing by stepswithout unloading between the separate steps; this may include creep effects in the results but reduces the amount of loads to be moved on and off the load receptor as compared to 1,
3. continuously increasing and decreasing by steps – procedure prescribed for verification tests in [2] (or [3]), same comments as for 2,
4. continuously decreasing by steps starting from Max - simulates the use of an instrument as hopper weigher for subtractive weighing, same comments as for 2.

On multi-interval instruments – see [2] (or [3]), the methods above may be modified for load steps smaller than Max, by applying increasing and/or decreasing tare loads, taring the instrument, and applying a test load close to but not larger than Max_1 to obtain indications with d_1.

On a multiple range instrument [2] (or [3]), the client should identify which range(s) shall be calibrated (see 4.1.1, 2nd paragraph).

Further tests may be performed to evaluate the performance of the instrument under special conditions of use, e.g. the indication after a tare balancing operation, the variation of the indication under a constant load over a certain time, etc.

The test, or individual loadings, may be repeated to combine the test with the repeatability test under 5.1.

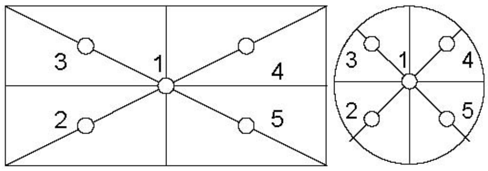
Indications I_{Lj} are recorded for each load. In the case that the loads are removed, the zero indication should be checked, and may be reset to zero if it does not show zero; recording of the no-load indications I_{0j} may be advisable as per 4.4.1.

5.3 Eccentricity test

The test comprises placing a test load L_{ecc} in different positions on the load receptor in such a manner that the centre of gravity of the applied load takes the positions as indicated in Figure 5.3-1 or equivalent positions, as closely as possible.

Fig. 5.3-1 Positions of load for test of eccentricity

1. Centre
2. Front left
3. Back left
4. Back right
5. Front right



There may be applications where the test load cannot be placed in or close to the centre of the load receptor. In this case, it is sufficient to place the test load at the remaining positions as indicated in Figure 5.3-1. Depending on the platter shape, the number of the off-centre positions might deviate from figure 5.3-1.

The test load L_{ecc} should be about $Max/3$ or higher, or $Min' + (Max' - Min')/3$ or higher for a reduced weighing range.

Advice of the manufacturer, if available, and limitations that are obvious from the design of the instrument should be considered – e.g. see OIML R76 [2] (or EN 45501 [3]) for special load receptors.

For a multiple range instrument [2] (or [3]) the test should only be performed in the range with the largest capacity identified by the client (see 4.1.1, 2nd paragraph).

The test load need not be calibrated or verified, unless the results serve to determine the errors of indication as per 5.2.

The test can be carried out in different manners:

1. Prior to the test, the indication is set to zero. The test load is first put on position 1, is then moved to the other 4 positions in arbitrary order. Indications I_{Li} are recorded for each position of the load.
2. The test load is first put on position 1, then the instrument is tared. The test load is then moved to the other 4 positions in arbitrary order. Indications I_{Li} are recorded for each position of the load.
3. Prior to the test, the indication is set to zero. The test load is first put on position 1, removed, and then put to the next position, removed, etc. until it is removed

from the last position. Indications I_{Li} are recorded for each position of the load. After each removal of the load, the indication should be checked, and may be reset to zero if it does not show zero; recording of the no-load indications I_{0i} may be advisable as per 4.4.1.

4. The test load is first put on position 1, then the instrument is tared. The test load is then moved to the next position and moved back to position 1, etc. until it is removed from the last position. The center indication I_{L1} is recorded individually for all off-centre indications I_{Li} .

Method 3 and 4 are suggested for instruments that show a substantial drift during the time of the eccentricity test.

For methods 2 and 4 zero-setting or zero-tracking devices must be switched off during the complete eccentricity test.

5.4 Auxiliary measurements

The following additional measurements or recordings are recommended, in particular where a calibration is intended to be performed with the lowest possible uncertainty.

In view of buoyancy effects – cf. 4.2.2:

The air temperature in reasonable vicinity to the instrument should be measured, at least once during the calibration. Where an instrument is used in a controlled environment, the span of the temperature variation should be noted, e.g. from a thermograph, from the settings of the control device etc.

Barometric pressure or, by default, the altitude above sea-level of the site may also be useful.

In view of convection effects – cf. 4.2.3:

Special care should be taken to prevent excessive convection effects, by observing a limiting value for the temperature difference between standard weights and instrument, and/or recording an acclimatisation time that has been executed. A thermometer kept inside the box with standard weights may be helpful, to check the temperature difference.

In view of effects of magnetic interaction:

On high resolution instruments a check is recommended to see if there is an observable effect of magnetic interaction. A standard weight is weighed together with a spacer made of non-metallic material (e.g. wood, plastic), the spacer being placed on top or underneath the weight to obtain two different indications.

If the difference between these two indications is significantly different from zero, this should be mentioned as a warning in the calibration certificate.

6 MEASUREMENT RESULTS

The procedures and formulae in chapters 6 and 7 provide the basis for the evaluation of the results of the calibration tests and therefore require no further description on a test report. If the procedures and formulae used deviate from those given in the guide, additional information may need to be provided in the test report.

It is not intended that all of the formulae, symbols and/or indices are used for presentation of the results in a Calibration Certificate.

The definition of an indication I as given in 4.4 is used in this section.

6.1 Repeatability

From the *n* indications I_{ji} for a given test load L_{Tj}, the standard deviation s_j is calculated

$$s_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(I_{ji} - \bar{I}_j\right)^2} \tag{6.1-1}$$

with

$$\bar{I}_j = \frac{1}{n}\sum_{i=1}^{n} I_{ji} \tag{6.1-2}$$

Where only one test load has been applied, the index j may be omitted.

6.2 Errors of indication

6.2.1 Discrete values

For each test load L_{Tj}, the error of indication is calculated as follows

$$E_j = I_j - m_{\mathrm{ref}j} \tag{6.2-1}$$

Where an indication I_j is the mean of more than one reading, I_j is understood as being the mean value as per (6.1-2).

The reference value of mass m_{ref} could be approximated to its nominal value $m_{\mathrm{N}j}$

$$m_{\mathrm{ref}j} = m_{\mathrm{N}j} \tag{6.2-2}$$

or, more accurately, to its actual conventional value m_c

$$m_{\mathrm{ref}j} = m_{\mathrm{c}j} = \left(m_{\mathrm{N}j} + \delta m_{\mathrm{c}j}\right) \tag{6.2-3}$$

If a test load is made up of more than one weight, $m_{\mathrm{N}j}$ is replaced by $\left(\sum m_{\mathrm{N}}\right)_j$ and $\delta m_{\mathrm{c}j}$ is replaced by $\left(\sum \delta m_{\mathrm{c}}\right)_j$ in the formulae above.

Furthercorrections as per (7.1.2-1) might apply.

6.2.2 Characteristic of the weighing range

In addition, or as an alternative to the discrete values I_j, E_j, a characteristic, or calibration curve may be determined for the weighing range, which allows estimation of the error of indication for any indication I within the weighing range.

A function

$$E = f(I) \tag{6.2-4}$$

may be generated by an appropriate approximation which should, in general, be based on the “least squares” approach

$$\sum v_j^2 = \sum \left(f\left(I_j\right) - E_j\right)^2 = \text{minimum} \tag{6.2-5}$$

with

v_j = residual
f = approximation function

The approximation should further

- take account of the uncertainties $u\left(E_j\right)$ of the errors,
- use a model function that reflects the physical properties of the instrument, e.g. the form of the relation between load and its indication $I = g\left(L\right)$,
- include a check as to whether the parameters found for the model function are mathematically consistent with the actual data.

It is assumed that for any $m_{\mathrm{N}j}$ the error E_j remains the same if the actual indication I_j is replaced by its nominal value $I_{\mathrm{N}j}$. The calculations to evaluate (6.2-5) can therefore be performed with the data sets $m_{\mathrm{N}j}$, E_j, or $I_{\mathrm{N}j}$, E_j.

Appendix C offers advice for the selection of a suitable approximation formula and for the necessary calculations.

6.3 Effect of eccentric loading

From the indications I_i obtained in the different positions of the load as per 5.3, the differences ΔI_{ecc} are calculated.

For method 1 and 2 as per 5.3

$$\Delta I_{\mathrm{ecc}i} = I_{Li} - I_{L1} \tag{6.3-1}$$

For method 3 as per 5.3

$$\Delta I_{\mathrm{ecc}i} = \left(I_{Li} - I_{0i}\right) - I_{L1} \tag{6.3-2}$$

For method 4 as per 5.3

$$\Delta I_{\mathrm{ecc}i} = I_{Li} - I_{L1i} \tag{6.3-3}$$

where for each off-centre indication I_{Li} the respective centre indication I_{L1i} is taken for the calculation.

7 UNCERTAINTY OF MEASUREMENT

In this and the following sections, there are uncertainty terms assigned to small corrections, which are proportional to a specified mass value or to a specified indication. For the quotient of such an uncertainty divided by the related value of mass or indication, the abbreviated notation u_{rel} will be used.

Example: let

$$u\left(\delta m_{\mathrm{corr}}\right) = m \cdot u\left(corr\right) \tag{7-1}$$

with the dimensionless term $u(corr)$, then

$$u_{\mathrm{rel}}(\delta m_{\mathrm{corr}}) = u(corr) \tag{7-2}$$

Accordingly, the related variance will be denoted by $u_{\mathrm{rel}}^2(\delta m_{\mathrm{corr}})$ and the related expanded uncertainty by $U_{\mathrm{rel}}(\delta m_{\mathrm{corr}})$.

For the determination of uncertainty, second order terms have been considered negligible, but when first order contributions cancel out, second order contributions should be taken into account (see JCGM 101 [9], 9.3.2.6).

7.1 Standard uncertainty for discrete values

The basic formula for the calibration is

$$E = I - m_{\mathrm{ref}} \tag{7.1-1}$$

with variance

$$u^2(E) = u^2(I) + u^2(m_{\mathrm{ref}}) \tag{7.1-2}$$

Where substitution loads are employed, see 4.3.3, m_{ref} is replaced by $L_{\mathrm{T}n}$ or $L_{\mathrm{T}n,k}$ in both expressions.

The terms are further expanded hereafter.

7.1.1 Standard uncertainty of the indication

To account for sources of variability of the indication, (4.4.1-1) is amended by correction terms δI_{xx} as follows

$$I = I_L + \delta I_{\mathrm{dig}L} + \delta I_{\mathrm{rep}} + \delta I_{\mathrm{ecc}} - I_0 - \delta I_{\mathrm{dig}0} + \dots \tag{7.1.1-1}$$

Further correction terms may be applied in special conditions (temperature effects, drift, hysteresis,..), which are not considered hereafter.

All these corrections have the expectation value zero. Their standard uncertainties are

7.1.1.1 $\delta R_{\mathrm{dig}0}$ accounts for the rounding error of no-load indication. Limits are $\pm d_0/2$ or $\pm d_{\mathrm{T}}/2$ as applicable; rectangular distribution is assumed, therefore

$$u(\delta I_{\mathrm{dig}0}) = d_0 / \left(2\sqrt{3}\right) \tag{7.1.1-2a}$$

or

$$u(\delta I_{\mathrm{dig}0}) = d_{\mathrm{T}} / \left(2\sqrt{3}\right) \tag{7.1.1-2b}$$

respectively.

Note 1: cf. 4.4.2 for significance of d_{T}.

Note 2: on an instrument which has been type approved to OIML R76 [2] (or EN 45501 [3]), the rounding error of a zero indication after a zero-setting or tare balancing operation is limited to $\pm d_0/4$, therefore

$$u(\delta I_{\text{dig0}}) = d_0 / (4\sqrt{3}) \tag{7.1.1-2c}$$

7.1.1.2 $\delta I_{\text{dig}L}$ accounts for the rounding error of indication at load. Limits are $\pm d_{\text{I}}/2$ or $\pm d_{\text{T}}/2$ as applicable; rectangular distribution to be assumed, therefore

$$u(\delta I_{\text{dig}L}) = d_{\text{I}} / 2\sqrt{3} \tag{7.1.1-3a}$$

or

$$u(\delta I_{\text{dig}L}) = d_{\text{T}} / 2\sqrt{3} \tag{7.1.1-3b}$$

Note: on a multi-interval instrument, d_I varies with *I*.

7.1.1.3 δI_{rep} accounts for the repeatability of the instrument; normal distribution is assumed, estimated as

$$u(\delta I_{\text{rep}}) = s(I_j) \tag{7.1.1-5}$$

where $s(I_j)$ is determined in 6.1.

If the indication I is a single reading and only one repeatability test has been performed, this uncertainty of repeatability can be considered as representative for the whole range of the instrument.

Where an indication I_j is the mean of *N* indications performed with the same test load during the error of indication test, the corresponding standard uncertainty is

$$u(\delta I_{\text{rep}}) = s(I_j) / \sqrt{N} \tag{7.1.1-6}$$

Where several s_j ($s_j = s(I_j)$ in abbreviated notation) values have been determined with different test loads, the greater value of s_j for the two test points enclosing the indication whose error has been determined, should be used.

For multi-interval and multiple range instruments where a repeatability test was carried out in more than one interval/range, the standard deviation of each interval/range may be considered as being representative for all indications of the instrument in the respective interval/range.

Note: For a standard deviation reported in a calibration certificate, it should be clear whether it is related to a single indication or to the mean of *N* indications.

7.1.1.4 δI_{ecc} accounts for the error due to off-centre position of the centre of gravity of a test load. This effect may occur where a test load is made up of more than one body. Where this effect cannot be neglected, an estimate of its magnitude may be based on these assumptions:

- the differences ΔI_{ecc} determined by (6.3-1) are proportional to the distance of the load from the centre of the load receptor,
- the differences ΔI_{ecc} determined by (6.3-1) are proportional to the value of the load,

- the effective centre of gravity of the test loads is not further from the centre of the load receptor than half the distance between the load receptor centre and the eccentricity load positions, as per figure 5.3-1.

Based on the largest of the differences determined as per 6.3, δI_{ecc} is estimated to be

$$\delta I_{ecc} \leq \left\{\left|\Delta I_{ecci}\right|_{max} / \left(2L_{ecc}\right)\right\} I \qquad (7.1.1\text{-}9)$$

Rectangular distribution is assumed, so the standard uncertainty is

$$u\left(\delta I_{ecc}\right) = I\left|\Delta I_{ecci}\right|_{max} / \left(2L_{ecc}\sqrt{3}\right) \qquad (7.1.1\text{-}10)$$

or, in relative notation

$$u_{rel}\left(\delta I_{ecc}\right) = \left|\Delta I_{ecci}\right|_{max} / \left(2L_{ecc}\sqrt{3}\right) \qquad (7.1.1\text{-}11)$$

7.1.1.5 The standard uncertainty of the indication is normally obtained by

$$u^2(I) = d_0^2/12 + d_I^2/12 + u^2\left(\delta I_{rep}\right) + u_{rel}^2\left(\delta I_{ecc}\right)I^2 \qquad (7.1.1\text{-}12)$$

Note 1: the uncertainty $u(I)$ is constant only where s is constant and no eccentricity error has to be considered.

Note 2: the first two terms on the right hand side may have to be modified in special cases as mentioned in 7.1.1.1 and 7.1.1.2.

7.1.2 Standard uncertainty of the reference mass

From 4.2.4 and 4.3.1 the reference value of mass is

$$m_{ref} = m_N + \delta m_c + \delta m_B + \delta m_D + \delta m_{conv} + \delta m_{...} \qquad (7.1.2\text{-}1)$$

The rightmost term stands for further corrections which may be necessary to apply under special conditions.These are not considered hereafter.

The corrections and their standard uncertainties are

7.1.2.1 δm_c is the correction to m_N to obtain the conventional value of mass m_c ; given in the calibration certificate for the standard weights, together with the uncertainty of calibration U and the coverage factor k . The standard uncertainty is

$$u(\delta m_c) = U/k \qquad (7.1.2\text{-}2)$$

Where the standard weight has been calibrated to specified tolerances Tol , e.g. to the mpe given in OIML R111 [4], and where it is used its nominal value m_N , then δm_c = 0, and rectangular distribution is assumed, therefore

$$u(\delta m_c) = Tol/\sqrt{3} \qquad (7.1.2\text{-}3)$$

Where a test load consists of more than one standard weight, the standard uncertainties are summed arithmetically not by a sum of squares, to account for assumed correlation. For test loads partially made up of substitution loads see 7.1.2.6.

7.1.2.2 δm_B is the correction for air buoyancy as introduced in 4.2.4. The value depends on the density ρ of the calibration weight and on the assumed range of air density ρ_a at the laboratory.

$$\delta m_B = -m_N(\rho_a - \rho_0)(1/\rho - 1/\rho_c) \tag{7.1.2-4}$$

with relative standard uncertainty

$$u_{rel}^2(\delta m_B) = u^2(\rho_a)(1/\rho - 1/\rho_c)^2 + (\rho_a - \rho_0)^2 u^2(\rho)/\rho^4 \qquad \text{(7.1.2-5a)}^4$$

As far as values for ρ, $u(\rho)$, ρ_a and $u(\rho_a)$, are known, these values should be used to determine $u_{rel}(\delta m_B)$.

The density ρ and its standard uncertainty may, in the absence of such information, be estimated according to the state of the art or based on information provided by the manufacturer. Appendix E1 offers internationally recognized values for common materials used for standard weights.

The air density ρ_a and its standard uncertainty can be calculated from temperature and barometric pressure if available (the relative humidity being of minor influence), or may be estimated from the altitude above sea-level.

Where conformity of the standard weights to OIML R111 [4] is established, and no information on ρ and ρ_a is at hand, recourse may be taken to section 10 of OIML R111[5]. No correction is applied, and the relative uncertainties are

If the instrument is adjusted immediately before calibration

$$u_{rel}(\delta m_B) \approx mpe/\left(4m_N\sqrt{3}\right) \tag{7.1.2-5c}$$

If the instrument is not adjusted before calibration

$$u_{rel}(\delta m_B) \approx \left(0{,}1\rho_0/\rho_c + mpe/(4m_N)\right)/\sqrt{3} \tag{7.1.2-5d}$$

If some information can be assumed for the temperature variation at the location of the instrument, equation (7.1.2-5d) can be substituted by:

$$u_{rel}(\delta m_B) \approx \sqrt{1{,}07\times10^{-4} + 1{,}33\times10^{-6}\,\mathrm{K}^{-2}\Delta T^2}\cdot\rho_0/\rho_c + mpe/\left(4m_N\sqrt{3}\right) \tag{7.1.2-5e}$$

where ΔT is the maximum variation of environmental temperature that can be assumed for the location (see appendixes A2.2 and A3 for details).

From the requirement in footnote 5, the limits of the value of ρ can be derived: e.g. for class E_2: $|\rho - \rho_c| \leq$ 200 kg/m³, and for class F_1: $|\rho - \rho_c| \leq$ 600 kg/m³.

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4 A more accurate formula for (7.1.2-5a) would be [10]

$$u_{rel}^2(\delta m_B) = u^2(\rho_a)(1/\rho - 1/\rho_c)^2 + (\rho_a - \rho_0)\left[(\rho_a - \rho_0) - 2(\rho_{a1} - \rho_0)\right]u^2(\rho)/\rho^4 \tag{7.1.2-5b}$$

where ρ_{a1} is the air density at the time of the calibration of the standard weights. This formula is useful when the instrument is located at high altitude above sea level, otherwise the uncertainty could be overestimated.

5The density of the material used for weights shall be such that a deviation of10 % from the specified air density (1.2 kg/m³) does not produce an error exceeding one quarter of the maximum permissible error.

Note: Due to the fact that the density of materials used for standard weights is normally closer to ρ_c than the OIML R111 limits would allow, the last 3 formulae may be considered as upper limits for $u_{rel}(\delta m_B)$. Where a simple comparison of these values with the resolution of the instrument (d / Max) shows they are small enough, a more elaborate calculation of this uncertainty component based on actual data may be superfluous.

7.1.2.3 δm_D corresponds to the possible drift of m_c since the last calibration. A limiting value D is best assumed, based on the difference in m_c evident from consecutive calibration certificates of the standard weights.

D may be estimated in view of the quality of the weights, and frequency and care of their use, to at least a multiple of their expanded uncertainty $U(\delta m_c)$

$$D = k_D U(\delta m_c) \quad (7.1.2\text{-}10)$$

where k_D is a chosen value between 1 and 3.

In the absence of information on drift the value of D will be chosen as the *mpe* according to OIML R 111 [4].

It is not advised to apply a correction but to assume even distribution within $\pm D$ (rectangular distribution). The standard uncertainty is then

$$u(\delta m_D) = D/\sqrt{3} \quad (7.1.2\text{-}11)$$

Where a set of weights has been calibrated with a standardised expanded relative uncertainty $U_{rel}(\delta m_c)$, it may be convenient to introduce a relative limit value for drift $D_{rel} = D/m_N$ and a relative uncertainty for drift

$$u_{rel}(\delta m_D) = D_{rel}/\sqrt{3} \quad (7.1.2\text{-}12)$$

7.1.2.4 δm_{conv} corresponds to the convection effects as per 4.2.3. A limiting value Δm_{conv} may be taken from Appendix F, depending on a known difference in temperature ΔT and on the mass of the standard weight.

It is not advised to apply a correction but to assume even distribution within $\pm \Delta m_{conv}$. The standard uncertainty is then

$$u(\delta m_{conv}) = \Delta m_{conv}/\sqrt{3} \quad (7.1.2\text{-}13)$$

It appears that this effect is only relevant for weights of classes F_1 or better.

7.1.2.5 The standard uncertainty of the reference mass is obtained from – cf. 7.1.2

$$u^2(m_{ref}) = u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{conv}) \quad (7.1.2\text{-}14)$$

with the contributions from 7.1.2.1 to 7.1.2.4.

7.1.2.6 Where a test load is partially made up of substitution loads as per 4.3.3, and the test loads are defined per (4.3.3-5a), the standard uncertainty for the sum $L_{Tn} = nm_{ref} + \Delta I_1 + \Delta I_2 + \ldots + \Delta I_{n-1}$ is given by the following expression

$$u^2(L_{Tn}) = n^2u^2(m_{ref}) + 2\left[u^2(I_1) + u^2(I_2) + \ldots + u^2(I_{n-1})\right] \qquad (7.1.2\text{-}15a)$$

with $u(m_{ref})$ from 7.1.2.5, and $u(I_j)$ from 7.1.1.5 for $I = I(L_{Tj})$

Where a test load is partially made up of substitution loads as per 4.3.3, and the test loads are defined per (4.3.3-5b), the standard uncertainty for the sum $L_{Tn,k} = (n-1)m_{ref} + m_{ref,k} + \Delta I_1 + \Delta I_2 + \ldots + \Delta I_{n-1}$ is given by the following expression

$$u^2(L_{Tn,k}) = \left[(n-1)u(m_{ref}) + u(m_{ref,k})\right]^2 + 2\left[u^2(I_1) + u^2(I_2) + \ldots + u^2(I_{n-1})\right] \qquad (7.1.2\text{-}15b)$$

with $u(m_{ref})$ from 7.1.2.5, and $u(I_j)$ from 7.1.1.5 for $I = I(L_{Tj})$

Note: The uncertainties $u(I_j)$ also have to be included for indications where the substitution load has been adjusted in such a way that the corresponding ΔI becomes zero.

Depending on the kind of the substitution load, it may be necessary to add further uncertainty contributions

- for eccentric loading as per 7.1.1.4 to some or all of the actual indications $I(L_{Tj})$
- for air buoyancy of the substitution loads, where these are made up of low density materials (e.g. sand, gravel) and the air density varies significantly over the time the substitution loads are in use.

Where $u(I_j)$ = const, the expression (7.1.2-15a) simplifies to

$$u^2(L_{Tn}) = n^2u^2(m_{ref}) + 2\left[(n-1)u^2(I)\right] \qquad (7.1.2\text{-}16a)$$

and the expression (7.1.2-15b) simplifies to

$$u^2(L_{Tn,k}) = \left[(n-1)u(m_{ref}) + u(m_{ref,k})\right]^2 + 2\left[(n-1)u^2(I)\right] \qquad (7.1.2\text{-}16b)$$

7.1.3 Standard uncertainty of the error

The standard uncertainty of the error is, with the terms from 7.1.1 and 7.1.2, as appropriate, calculated from

$$\begin{aligned} u^2(E) &= u^2(\delta I_{dig0}) + u^2(\delta I_{digI}) + u^2(\delta I_{rep}) + u^2(\delta I_{ecc}) \\ &+ u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{conv}) \end{aligned} \qquad (7.1.3\text{-}1a)$$

or, where relative uncertainties apply, from

$$\begin{aligned} u^2(E) &= u^2(\delta I_{dig0}) + u^2(\delta I_{digI}) + u^2(\delta I_{rep}) + u^2_{rel}(\delta I_{ecc})I^2 \\ &+ \left\{u^2_{rel}(\delta m_c) + u^2_{rel}(\delta m_B) + u^2_{rel}(\delta m_D)\right\}m^2_{ref} + u^2(\delta m_{conv}) \end{aligned} \qquad (7.1.3\text{-}1b)$$

In the case of using substitution loads

$$u^2(E_{n,k}) = u^2(\delta I_{dig0}) + u^2(\delta I_{digI}) + u^2(\delta I_{rep}) + u^2(\delta I_{ecc}) + u^2(L_{Tn,k}) \qquad (7.1.3\text{-}1c)$$

where *n* is related to the number of substitution steps and *k* is the number of standard weights.

All input quantities are considered to be uncorrelated, therefore covariances are not considered.

The index " j "has been omitted.

In view of the general experience that errors are normally very small compared to the indication, or may even be zero, in (7.1.3-1b) the values for m_{ref} and I may be replaced by I_{N} .

The terms in (7.1.3-1b) may then be grouped into a simple formula which better reflects the fact that some of the terms are absolute in nature while others are proportional to the indication

$$u^2(E) = \alpha^2 + \beta^2 I^2 \qquad (7.1.3\text{-}2)$$

7.2 Standard uncertainty for a characteristic

Where an approximation is performed to obtain a formula $E = f(I)$ for the whole weighing range as per 6.2.2, the standard uncertainty of the error per 7.1.3 has to be modified to be consistent with the method of approximation. Depending on the model function, this may be

- a single variance which is added to (7.1.3-1), or
- a set of variances and covariances which include the variances in (7.1.3-1).

The calculations should also include a check whether the model function is mathematically consistent with the data sets E_j , I_j , $u(E_j)$.

The minimum χ^2 approach, which is similar to the least squares approach, is proposed for approximations. Details are given in Appendix C.

7.3 Expanded uncertainty at calibration

The expanded uncertainty of the error is

$$U(E) = k\, u(E) \qquad (7.3\text{-}1)$$

The coverage factor k should be chosen such that the expanded uncertainty corresponds to a coverage probability of 95,45 %.

Further information on how to derive the coverage factor is given in Appendix B.

7.4 Standard uncertainty of a weighing result

Chapter 7.4 and 7.5 provide advice on how the measurement uncertainty of an instrument could be estimated in normal usage, thereby taking into account the measurement uncertainty at calibration. Where a calibration laboratory offers to its clients such estimates which are based upon information that has not been measured by the laboratory, the estimates must not be presented as part of the calibration certificate. However, it is acceptable to provide such estimates as long as they are clearly separated from the calibration results.

The user of an instrument should be aware of the fact that in normal usage, the situation is different from that at calibration in some if not all of these aspects

1. the indications obtained for weighed bodies are not the ones at calibration,
2. the weighing process may be different from the procedure at calibration
    a. generally only one reading is taken for each load, not several readings to obtain a mean value,
    b. reading is to the scale interval d, of the instrument, not to a higher resolution,
    c. loading is up and down, not only upwards – or vice versa,
    d. load may be kept on load receptor for a longer time, not unloading after each loading step – or vice versa,
    e. eccentric application of the load,
    f. use of tare balancing device, etc.
3. the environment (temperature, barometric pressure etc.) may be different,
4. for instruments which are not readjusted regularly e.g. by use of a built-in device, the adjustment may have changed, due to drift or to wear and tear. Unlike items 1 to 3, this effect should therefore be considered in relation to a certain period of time, e.g. for one year or the normal interval between calibrations,
5. the repeatability of the adjustment.

In order to clearly distinguish from the indications I obtained during calibration, the weighing results obtained when weighing a load L on the calibrated instrument, these terms and symbols are introduced

- R_L = reading when weighing a load L on the calibrated instrumentobtained after the calibration.
- R_0 = reading without load on the calibrated instrumentobtained after the calibration.

Readings are taken to be single readings in normal resolution (multiple of d), with corrections to be applied as applicable.

For a reading taken under the same conditions as those prevailing at calibration, the result may be denominated as the weighing result under the conditions of the calibration $W*$

$$W* = R_L + \delta R_{\mathrm{dig}L} + \delta R_{\mathrm{rep}} + \delta R_{\mathrm{ecc}} - \left(R_0 + \delta R_{\mathrm{dig}0}\right) - E \qquad (7.4\text{-}1a)$$

with the associated uncertainty

$$u(W*) = \sqrt{\left\{u^2(E) + u^2\left(\delta R_{\mathrm{dig}0}\right) + u^2\left(\delta R_{\mathrm{dig}L}\right) + u^2\left(\delta R_{\mathrm{rep}}\right) + u^2\left(\delta R_{\mathrm{ecc}}\right)\right\}} \qquad (7.4\text{-}2a)$$

To take account of the remaining possible influences on the weighing result, further corrections are formally added to the reading in a general manner resulting in the general weighing result

$$W = W* + \delta R_{\mathrm{instr}} + \delta R_{\mathrm{proc}} \qquad (7.4\text{-}1b)$$

where $\delta R_{\mathrm{instr}}$ represents a correction term due to environmental influences and δR_{proc} represents a correction term due to the operation of the instrument.

The associated uncertainty is

$$u(W) = \sqrt{u^2(W^*) + u^2(\delta R_{\text{instr}}) + u^2(\delta R_{\text{proc}})} \qquad (7.4\text{-}2b)$$

The added terms and the corresponding standard uncertainties are discussed in 7.4.3 and 7.4.4. The standard uncertainties $u(W^*)$ and $u(W)$ are finally presented in 7.4.5.

Sections 7.4.3 and 7.4.4, 7.4.5 and 7.5, are meant as advice to the user of the instrument on how to estimate the uncertainty of weighing results obtained under their normal conditions of use. They are not meant to be exhaustive or mandatory.

7.4.1 Standard uncertainty of a reading in use

To account for sources of variability of the reading, (7.1.1-1) applies, with I replaced by R

$$R = R_L + \delta R_{\text{dig}L} + \delta R_{\text{rep}} + \delta R_{\text{ecc}} - (R_0 + \delta R_{\text{dig}0}) \cdots \qquad (7.4.1\text{-}1)$$

The corrections and their standard uncertainties are

7.4.1.1 $\delta R_{\text{dig}0}$ accounts for the rounding error at zero reading. 7.1.1.1 applies with the exception that the variant $d_T < d$, is excluded, so

$$u(\delta R_{\text{dig}0}) = d_0 \big/ \sqrt{12} \qquad (7.4.1\text{-}2)$$

7.4.1.2 $\delta R_{\text{dig}L}$ accounts for the rounding error at load reading. 7.1.1.2 applies with the exception that the variant $d_{\text{T}} < d_L$ is excluded, so

$$u(\delta R_{\text{dig}L}) = d_L \big/ \sqrt{12} \qquad (7.4.1\text{-}3)$$

7.4.1.3 δR_{rep} accounts for the repeatability of the instrument. 7.1.1.3 applies, the relevant standard deviation s for a single reading is to be taken from the calibration certificate, so

$$u(\delta R_{\text{rep}}) = s \quad \text{or} \quad u(\delta R_{\text{rep}}) = s(R) \qquad (7.4.1\text{-}4)$$

Note: The standard deviation not the standard deviation of the mean should be used for the uncertainty calculation.

7.4.1.4 δR_{ecc} accounts for the error due to off-centre position of the centre of gravity of a load.

$$u_{\text{rel}}(\delta R_{\text{ecc}}) = |\Delta I_{\text{ecc}i}|_{\max} \big/ \left(2L_{\text{ecc}}\sqrt{3}\right) \qquad (7.4.1\text{-}5)$$

7.4.1.5 The standard uncertainty of the reading is then obtained by

$$u^2(R) = d_0^2/12 + d_L^2/12 + s^2(R) + \left(|\Delta I_{\text{ecc}i}|_{\max} \big/ \left(2L_{\text{ecc}}\sqrt{3}\right)\right)^2 R^2 \cdots \qquad (7.4.1\text{-}6)$$

7.4.2 Uncertainty of the error of a reading

Where a reading R corresponds to an indication $I_{\text{cal}j}$ reported in the calibration certificate, $u(E_{\text{cal}j})$ may be taken from there. For other readings, $u(E)$ may be calculated by (7.1.3-2) if α and β are known, or it results from interpolation, or from an approximation formula as per 7.2.

The uncertainty $u(E)$ is normally not smaller than $u(E_{\mathrm{cal}j})$ for an indication I_j that is close to the actual reading R, unless it has been determined by an approximation formula.

Note: the calibration certificate normally presents $U_{95}(E_{\mathrm{cal}})$ from which $u(E_{\mathrm{cal}})$ is calculated by dividing $U_{95}(E_{\mathrm{cal}})$ by the coverage factor k stated in the certificate.

7.4.3 Uncertainty from environmental influences

The term $\delta R_{\mathrm{instr}}$ accounts for up to 3 effects δR_{temp}, δR_{buoy} and δR_{adj}, which are discussed hereafter. Except for the contribution due to buoyancy, they do normally not apply to instruments which are adjusted directly before they are actually used. Other instruments should be considered as appropriate. No corrections are actually applied, the corresponding uncertainties are estimated, based on the user's knowledge of the properties of the instrument.

7.4.3.1 The term δR_{temp} accounts for a change in the characteristic of the instrument caused by a change in ambient temperature. A limiting value can be estimated to be $\delta R_{\mathrm{temp}} = K_T \Delta T R$ where ΔT is the maximum temperature variation at the instrument location and K_T is the sensitivity of the instrument to temperature variation. When the balance is controlled by a temperature triggered adjustment by means of the built-in weights then ΔT can be reduced to the trigger threshold.

Normally there is a manufacturer's specification such as $K_T = \left[\partial I(Max)/\partial T\right]/Max$, in many cases quoted in 10^{-6}/K. By default, for instruments with type approval under OIML R76 [2] (or EN 45501 [3]), it may be assumed $|K_T| \leq mpe(Max)/(Max\Delta T_{\mathrm{Approval}})$ where $\Delta T_{\mathrm{Approval}}$ is the temperature range of approval marked on the instrument; for other instruments, either a conservative assumption has to be made, leading to a multiple (3 to 10 times) of the comparable value for instruments with type approval, or no information can be given at all for a use of the instrument at other temperatures than that at calibration.

The range of variation of temperature ΔT (full width) should be estimated in view of the site where the instrument is being used, as discussed in Appendix A2.2.

Rectangular distribution is assumed, therefore the relative uncertainty is

$$u_{\mathrm{rel}}(\delta R_{\mathrm{temp}}) = K_T \Delta T / \sqrt{12} \qquad (7.4.3\text{-}1)$$

7.4.3.2 The term δR_{buoy} accounts for a change in the adjustment of the instrument due to the variation of the air density; no correction to be applied.

When the balance is adjusted immediately before use and some assumption for the variation in air density with respect to the air density value at the calibration time $\Delta\rho_a$ can be made, the uncertainty contribution could be [10]

$$u_{\mathrm{rel}}(\delta R_{\mathrm{bouy}}) = \frac{\Delta\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}^{2}} u(\rho_{\mathrm{s}}) \qquad (7.4.3\text{-}2)$$

where $u(\rho_s)$ is the uncertainty of the density of the reference weight used for adjustment (built-in or external).

When the balance is not adjusted before use and some assumption for the variation in density $\Delta\rho_a$ can be made, the uncertainty contribution could be

$$u_{\mathrm{rel}}\left(\delta R_{\mathrm{bouy}}\right) = \frac{\Delta\rho_{\mathrm{a}}}{\rho_{\mathrm{c}}\sqrt{3}} \qquad (7.4.3\text{-}3)$$

If some assumptions can be made for the temperature variation in the location of the balance, equation (7.4.3-3) can be approximated by

$$u_{\mathrm{rel}}\left(\delta R_{\mathrm{bouy}}\right) = \frac{\sqrt{1{,}07\times10^{-4} + 1{,}33\times10^{-6}\,\mathrm{K}^{-2}\Delta T^2}\cdot\rho_0}{\rho_{\mathrm{c}}} \qquad (7.4.3\text{-}4)$$

where ΔT is the maximum assumed variation for the temperature in the location of the balance (see appendixes A2.2 and A3 for details).

If no assumption about the density variation can be made the most conservative approach would be

$$u_{\mathrm{rel}}\left(\delta R_{\mathrm{bouy}}\right) = \frac{0{,}1\rho_0}{\rho_{\mathrm{c}}\sqrt{3}} \qquad (7.4.3\text{-}5)$$

7.4.3.3 The term δR_{adj} accounts for a change in the characteristics of the instrument since the time of calibration due to drift, or wear and tear.

A limiting value may be taken from previous calibrations where they exist, as the largest difference $\left|\Delta E(Max)\right|$ in the errors at or near Max between any two consecutive calibrations. By default, $\Delta E(Max)$ should be taken from the manufacturer's specification for the instrument, or may be estimated as $\Delta E(Max) = mpe(Max)$ for instruments conforming to a type approval under OIML R76 [2] (or EN 45501 [3]). Any such value can be considered in view of the expected time interval between calibrations, assuming fairly linear progress of the change with time.

Rectangular distribution is assumed, therefore the relative uncertainty is

$$u_{\mathrm{rel}}\left(\delta R_{\mathrm{adj}}\right) = \left|\Delta E(Max)\right| / \left(Max\sqrt{3}\right) \qquad (7.4.3\text{-}6)$$

7.4.3.4 The relative standard uncertainty related to errors resulting from environmental effects is calculated by

$$u^2_{\mathrm{rel}}\left(\delta R_{\mathrm{instr}}\right) = u^2_{\mathrm{rel}}\left(\delta R_{\mathrm{temp}}\right) + u^2_{\mathrm{rel}}\left(\delta R_{\mathrm{buoy}}\right) + u^2_{\mathrm{rel}}\left(\delta R_{\mathrm{adj}}\right) \qquad (7.4.3\text{-}7)$$

7.4.4 Uncertainty from the operation of the instrument

The correction term δR_{proc} accounts for additional errors (δR_{Tare}, δR_{time} and δR_{ecc}) which may occur where the weighing procedure(s) is different from the one(s) at calibration. No corrections are actually applied but the corresponding uncertainties are estimated, based on the user's knowledge of the properties of the instrument.

7.4.4.1 The term δR_{Tare} accounts for a net weighing result after a tare balancing operation [2] (or [3]). The possible error and the uncertainty assigned to it should be estimated considering the basic relation between the readings involved

$$R_{\mathrm{Net}} = R'_{\mathrm{Gross}} - R'_{\mathrm{Tare}} \quad (7.4.4\text{-}1)$$

where the R' are fictitious readings which are processed inside the instrument, while the visible indication R_{Net} is obtained directly, after setting the instrument indication to zero with the tare load on the load receptor. The weighing result, in this case, is, in theory

$$W_{\mathrm{Net}} = R_{\mathrm{Net}} - [E(Gross) - E(Tare)] + \delta R_{\mathrm{instr}} + \delta R_{\mathrm{proc}} \quad (7.4.4\text{-}2)$$

consistent with (7.4-1). The errors at gross and tare would have to be taken as errors for equivalent R values as above. However, the tare values – and consequently the gross values – are not normally recorded.

The error may then be estimated as

$$E_{\mathrm{Net}} = E(Net) + \delta R_{\mathrm{Tare}} \quad (7.4.4\text{-}3)$$

where $E(Net)$ is the error for a reading R_{Net} and δR_{Tare} is an additional correction for the effect of non-linearity of the error curve $E_{\mathrm{cal}}(I)$. To quantify the non-linearity, recourse may be taken to the first derivative of the function $E = f(R)$, if known, or the slope q_E between consecutive calibration points may be calculated by

$$q_E = \frac{\Delta E}{\Delta I} = \frac{E_{j+1} - E_j}{I_{j+1} - I_j} \quad (7.4.4\text{-}4)$$

The largest and the smallest values of the derivatives or of the quotients are taken as limiting values for the correction δR_{Tare}, for which rectangular distribution may be assumed. This results in the relative standard uncertainty

$$u_{\mathrm{rel}}(\delta R_{\mathrm{Tare}}) = (q_{E\,\max} - q_{E\,\min}) / \sqrt{12} \quad (7.4.4\text{-}5)$$

To estimate the uncertainty $u(W)$, $R = R_{\mathrm{Net}}$ is considered. For $u(E)$ it is valid to assume $u(E(Net)) = u(E(R = Net))$ because there is full correlation between the quantities contributing to the uncertainties of the errors of the fictitious grossand tarereadings.

7.4.4.2 The term δR_{time} accounts for possible effects of creep and hysteresis, in situations such as

a) loading at calibration continuously upwards, or continuously upwards and downwards (method 2 or 3 in 5.2), so the load remains on the load receptor for a certain period of time; this is quite significant where the substitution method has been applied, usually with high capacity instruments. When in normal use, a discrete load to be weighed is put on the load receptor and is kept there just as long as is necessary to obtain a reading or a printout, the error of indication may differ from the value obtained for the same load at calibration.

Where tests were performed continuously up and down, the largest difference of errors ΔE_j for any test load m_j may be taken as the limiting value for this effect, leading to a relative standard uncertainty

$$u_{\mathrm{rel}}(\delta R_{\mathrm{time}}) = \Delta E_{j\max} / \left(m_j \sqrt{12}\right) \quad (7.4.4\text{-}6)$$

Where tests were performed only upwards, the error on return to zero E_0, if determined, may be used to estimate a relative standard uncertainty

$$u_{\mathrm{rel}}(\delta R_{\mathrm{time}}) = E_0 / \left(Max\sqrt{3}\right) \quad (7.4.4\text{-}7)$$

In the absence of such information, the limiting value may be estimated for instruments with type approval under OIML R76 [2] (or EN 45501 [3]) as

$$\Delta E(R) = R\, mpe(Max)/Max \quad (7.4.4\text{-}8)$$

For instruments without such type approval, a conservative estimate would be a multiple (*m* = 3 to 10 times) of this value.

The relative standard uncertainty is

$$u_{\mathrm{rel}}(\delta R_{\mathrm{time}}) = mpe(Max)/\left(Max\sqrt{3}\right), \quad (7.4.4\text{-}9a)$$

for instruments with type approval and

$$u_{\mathrm{rel}}(\delta R_{\mathrm{time}}) = m\, mpe(Max)/\left(Max\sqrt{3}\right) \quad (7.4.4\text{-}9b)$$

for instruments without type approval.

b) loading at calibration with unloading between load steps, loads to be weighed in normal use are kept on the load receptor for a longer period. In the absence of any other information – e.g. observation of the change in indication over a typical period of time – recourse may be taken to (7.4.4-9) as applicable.

c) loading at calibration only upwards, discharge weighing is performed in use. This situation may be treated as the inverse of the tare balancing operation – see 7.4.4.1 - combined with point b) above. (7.4.4-5) and (7.4.4-9) apply.

Note: In case of discharge weighing, the reading R shall be taken as a positive value although it may be indicated as negative by the weighing instrument.

7.4.4.3 δR_{ecc} accounts for the error due to off-centre position of the centre of gravity of a load.

(7.4.1-5) applies with the modification that the effect found during calibration should be considered in full, so

$$u_{\mathrm{rel}}(\delta R_{\mathrm{ecc}}) = \left|\Delta I_{\mathrm{ecc}i}\right|_{\max} / \left(L_{\mathrm{ecc}}\sqrt{3}\right) \quad (7.4.4\text{-}10)$$

7.4.5 Standard uncertainty of a weighing result

The standard uncertainty of a weighing result is calculated from the terms specified in 7.4.1 to 7.4.4, as applicable.

For the weighing result under the conditions of the calibration

$$u^2(W^*) = d_0^2/12 + d_L^2/12 + s^2(R) + u_{\mathrm{rel}}^2(\delta R_{\mathrm{ecc}})R^2 + u^2(E) \quad (7.4.5\text{-}1a)$$

For the weighing result in general

$$u^2(W) = u^2(W*) + \left[u_{\mathrm{rel}}^2(\delta R_{\mathrm{temp}}) + u_{\mathrm{rel}}^2(\delta R_{\mathrm{bouy}}) + u_{\mathrm{rel}}^2(\delta R_{\mathrm{adj}}) + u_{\mathrm{rel}}^2(\delta R_{\mathrm{Tare}}) + u_{\mathrm{rel}}^2(\delta R_{\mathrm{time}})\right]R^2 \quad (7.4.5\text{-}1b)$$

The many contributions to $u(W)$ may be grouped in two terms α_W^2 and β_W^2

$$u^2(W) = \alpha_W^2 + \beta_W^2 R^2 \quad (7.4.5\text{-}2)$$

where α_W^2 is the sum of squares of all absolute standard uncertainties, and β_W^2 is the sum of squares of all relative standard uncertainties.

7.5 Expanded uncertainty of a weighing result

7.5.1 Errors accounted for by correction

The complete formula for a weighing result which is equal to the reading corrected for the error determined by calibration, is

$$W* = R - E(R) \pm U(W*) \quad (7.5.1\text{-}1a)$$

or

$$W = R - E(R) \pm U(W) \quad (7.5.1\text{-}1b)$$

as applicable.

The expanded uncertainty $U(W)$ is to be determined as

$$U(W*) = k\,u(W*) \quad (7.5.1\text{-}2a)$$

or

$$U(W) = k\,u(W) \quad (7.5.1\text{-}2b)$$

with $u(W*)$ or $u(W)$ as applicable from 7.4.5.

For $U(W*)$ the coverage factor k should be determined as per 7.3.

For $U(W)$ the coverage factor k will, in most cases be equal to 2 even where the standard deviation s is obtained from only few measurements, and/or where $k_{cal} > 2$ was stated in the calibration certificate. This is due to the large number of terms contributing to $u(W)$.

7.5.2 Errors included in uncertainty

It may have been agreed by the calibration laboratory and the client to derive a "global uncertainty" $U_{\mathrm{gl}}(W)$ which includes the errors of indication such that no corrections have to be applied to the readings in use

$$W = R \pm U_{\mathrm{gl}}(W) \quad (7.5.2\text{-}1)$$

Unless the errors are more or less centred around zero, they form a one-sided contribution to the uncertainty which can only be treated in an approximate manner. For the sake of simplicity and convenience, the "global uncertainty" is best stated in the format of an expression for the whole weighing range, instead of individual values stated for fixed values of the weighing result.

Let $E(R)$ be a function, or E^0 be one value representative for all errors stated over the weighing range in the calibration certificate. The combination with the uncertainties in use may then, in principle, take on one of these forms

$$U_{gl}(W) = k\sqrt{u^2(W) + (E(R))^2} \quad (7.5.2\text{-}2a)$$

$$U_{gl}(W) = k\sqrt{u^2(W) + (E^0)^2} \quad (7.5.2\text{-}2b)$$

$$U_{gl}(W) = k\sqrt{u^2(W) + (E^0)^2 \left(R/Max\right)^2} \quad (7.5.2\text{-}2c)$$

$$U_{gl}(W) = ku(W) + |E(R)| \quad (7.5.2\text{-}3a)$$

$$U_{gl}(W) = ku(W) + |E^0| \quad (7.5.2\text{-}3b)$$

$$U_{gl}(W) = ku(W) + |E^0| R/Max \quad (7.5.2\text{-}3c)$$

Quite frequently, (7.5.2-3a) is taken as basis for the statement of the global uncertainty. Thereby, $U(W) = k\,u(W)$ is often approximated by the following formula

$$U(W) \approx U(W = 0) + \left\{ \left[U(W = Max) - U(W = 0) \right] / Max \right\} R \quad (7.5.2\text{-}3d)$$

and $E(R)$ is often approximated by $E(R) = a_1 R$ as per (C2.2-16) and (C2.2-16a) so that

$$U_{gl}(W) \approx U(W = 0) + \left\{ \left[U(W = Max) - U(W = 0) \right] / Max \right\} R + |a_1| R \quad (7.5.2\text{-}3e)$$

For further information on alternative generation of the formulae $E(R)$ or the representative value E^0 see Appendix C.

In analogy to (7.5.2-3d), for multi-interval instruments $U(W)$ is indicated per interval as

$$U(W) \approx U(Max_{i-1}) + \left\{ \frac{\left[U(Max_i) - U(Max_{i-1})\right]}{Max_i - Max_{i-1}} \right\} \cdot (R - Max_{i-1}) \quad (7.5.2\text{-}3f)$$

and for multiple range instruments $U(W)$ is indicated per range.

It is important to ensure that $U_{gl}(W)$ retains a coverage probability of not less than 95 % over the whole weighing range. For $U_{gl}(W)$ the coverage factor k will, in most cases be equal to 2 even where the standard deviation s is obtained from only few measurements, and/or where $k_{cal} > 2$ was stated in the calibration certificate. This is due to the large number of terms contributing to $u(W)$.

7.5.3 Other ways of qualification of the instrument

A client may expect from, or have asked the Calibration Laboratory for a statement of conformity to a given specification, as $|W - R| \leq Tol$ with Tol being the applicable tolerance. The tolerance may be specified as “$Tol = x\%$ of R”, as “$Tol = n\ d$”, or the like.

Conformity may be declared, in consistency with ISO/IEC 17025 under condition that

$$|E(R) + U(W(R))| \leq Tol(R) \qquad (7.5.3\text{-}1)$$

either for individual values of R or for any values within the whole or part of the weighing range.

Within the same weighing range, conformity may be declared for different parts of the weighing range, to different values of *Tol*.

If the user defines a relative weighing accuracy requirement, then Appendix G “Minimum weight” provides further advice.

8 CALIBRATION CERTIFICATE

This section contains advice regarding what information may usefully be provided in a calibration certificate. It is intended to be consistent with the requirements of ISO/IEC 17025, which take precedence.

8.1 General information

Identification of the calibration laboratory,
reference to the accreditation (accrediting body, number of the accreditation),
identification of the certificate (calibration number, date of issue, number of pages),
signature(s) of authorised person(s).

Identification of the client.

Identification of the calibrated instrument,
information about the instrument (manufacturer, kind of instrument, Max, d, place of installation).

Warning that the certificate may be reproduced only in full unless the calibration laboratory permits otherwise in writing.

8.2 Information about the calibration procedure

Date of measurements,
site of calibration,
conditions of environment and/or use that may affect the results of the calibration.

Information about the instrument (adjustment performed: internal or external adjustment and in the case of external adjustment what weight has been used, any anomalies of functions, setting of software as far as relevant for the calibration, etc.).

Reference to, or description of the applied procedure, as far as this is not obvious from the certificate, e.g. constant time interval observed between loadings and/or readings.

Agreements with the client e.g. over limited range of calibration, metrological specifications to which conformity is declared.

Information about the traceability of the measuring results.

8.3 Results of measurement

Indications and/or errors for applied test loads, or errors related to indications – as discrete values and/or by an equation resulting from approximation,
details of the loading procedure if relevant for the understanding of the above,
standard deviation(s) determined as related to a single indication,
information about the eccentricity test if performed,
expanded uncertainty of measurement for the error of indication results.

Indication of the coverage factor k, with comment on coverage probability, and reason for $k \neq 2$ where applicable.

Where the indications/errors have not been determined by normal readings - single readings with the normal resolution of the instrument - a warning should be given that the reported uncertainty is smaller than would be found with normal readings.

8.4 Additional information

Additional information about the uncertainty of measurement expected in use, inclusive of conditions under which it is applicable, may be attached to the certificate without becoming a part of it.

Where errors are to be accounted for by correction, this formula could be used

$$W = R - E(R) \pm U(W) \tag{8.4-1}$$

accompanied by the equation for $E(R)$.

Where errors are included in the "global uncertainty", this formula could be used

$$W = R \pm U_{gl}(W) \tag{8.4-2}$$

A statement should be added that the expanded uncertainty of values from the formula corresponds to a coverage probability of at least 95 %.

Optional:
Statement of conformity to a given specification, and range of validity where applicable.

This statement may take the form

$$W = R \pm Tol \tag{8.4-3}$$

and may be given

in addition to the results of measurement, or
as stand-alone statement, with reference to the results of measurement declared to be retained at the calibration laboratory.

The statement may be accompanied by a comment indicating that all measurement results enlarged by the expanded uncertainty of measurement, are within the specification limits.

Information about the minimum weight values for various weighing tolerances as per appendix G may be provided.

For clients that are less knowledgeable, advice might be provided where applicable, on

the definition of the error of indication,
how to correct readings in use by subtracting the corresponding errors,
how to interpret indications and/or errors presented with fewer digits than the scale interval d .

It may be useful to quote the values of $U(W^*)$ for either all individual errors or for the function $E(R)$ resulting from approximation.

9 VALUE OF MASS OR CONVENTIONAL VALUE OF MASS

The quantity W is an estimate of the conventional value of mass m_c of the object weighed[6]. For certain applications it may be necessary to derive from W the value of mass m , or a more accurate value for m_c .

The density ρ or the volume V of the object, together with an estimate of their standard uncertainty, must be known from other sources.

9.1 Value of mass

The mass of the object is

$$m = W[1 + \rho_a (1/\rho - 1/\rho_c)] \qquad (9.1\text{-}1)$$

Neglecting terms of second and higher order, the relative standard uncertainty $u_{rel}(m)$ is given by

$$u_{rel}^2(m) = \frac{u^2(W)}{W^2} + u^2(\rho_a)\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right)^2 + \rho_a^2 \frac{u^2(\rho)}{\rho^4} \qquad (9.1\text{-}2)$$

For ρ_a and $u(\rho_a)$ (density of air) see Appendix A.

If V and $u(V)$ are known instead of ρ and $u(\rho)$, ρ may be approximated by W/V , and $u_{rel}(\rho)$ may be replaced by $u_{rel}(V)$.

9.2 Conventional value of mass

The conventional value of mass of the object is

$$m_c = W[1 + (\rho_a - \rho_0)(1/\rho - 1/\rho_c)] \qquad (9.2\text{-}1)$$

Neglecting terms of second and higher order, the relative standard uncertainty $u_{rel}(m_c)$ is given by

$$u_{rel}^2(m_c) = \frac{u^2(W)}{W^2} + u^2(\rho_a)\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right)^2 + (\rho_a - \rho_0)^2 \frac{u^2(\rho)}{\rho^4} \qquad (9.2\text{-}2)$$

The same comments as given to (9.1-2) apply.

[6]In the majority of cases, especially when the results are used for trade, the value W is used as the result of the weighing

10 REFERENCES

[1] JCGM 100:2008 (GUM) Evaluation of measurement data — Guide to the expression of uncertainty in measurement, September 2008

[2] OIML R 76: Non-automatic Weighing Instruments Part 1: Metrological Requirements - Tests, Edition 2006 (E)

[3] EN 45501: Metrological Aspects of Non-automatic Weighing Instruments, Edition 2015

[4] OIML R111, Weights of Classes E1, E2, F1, F2, M1, M1-2, M2, M2-3, M3, Edition 2004 (E)

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APPENDIX A: ADVICE FOR ESTIMATION OF AIR DENSITY

Note: In Appendix A, the symbols are T for temperature in K, and t for temperature in °C

A1 Formulae for the density of air

The most accurate formula to determine the density of moist air is the one recommended by the CIPM [11][7]. For the purposes of this guideline, less sophisticated formulae which render slightly less precise results are sufficient.

A1.1 Simplified version of CIPM-formula, exponential version

From OIML R111 [4], section E3

$$\rho_a = \frac{0{,}34848\,p - 0{,}009 RH\ exp(0{,}061t)}{273{,}15 + t} \qquad \text{(A1.1-1)}$$

with

ρ_a	air density in kg/m³
p	barometric pressure in hPa
RH	relative humidity of air in %
t	air temperature in °C

The relative uncertainty of this approximation formula is $u_{form} / \rho_a = 2{,}4 \times 10^{-4}$ under the following conditions of environment

600 hPa ≤ p ≤ 1 100 hPa

20 % ≤ RH ≤ 80 %

15 °C ≤ t ≤ 27 °C

Apart from the uncertainty u_{form}, the uncertainties of the estimates for p, RH and t determine the uncertainty of ρ_a (see section A3).

A1.2 Average air density

Where measurement of temperature and barometric pressure is not possible, the mean air density at the site can be calculated from the altitude above sea level, as recommended in [4]

$$\rho_a = \rho_0 \exp\left(-\frac{\rho_0}{p_0} g h_{SL}\right) \qquad \text{(A1.2-1)}$$

with p_0 = 1 013,25 hPa

ρ_0 = 1,200 kg/m³

g = 9,81 m/s²

h_{SL} = altitude above sea level in metre

This calculation for air density is intended for 20 °C and RH = 50%.

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[7] The relative uncertainty of the CIPM-2007 air density formula, without the uncertainties of the parameters is $u_{form} / \rho_a = 2.2 \times 10^{-5}$, the best relative uncertainty achievable, which includes the uncertainty contributions for temperature, humidity and pressure measurements, is about $u(\rho_a) / \rho_a = 8 \times 10^{-5}$. The recommended ranges of temperature and pressure over which the CIPM-2007 equation may be used are: 600 hPa ≤ p ≤1 100 hPa, 15 ºC ≤ t ≤ 27 ºC.

The relative uncertainty of this approximation formula is $u_{\text{form}} / \rho_{\text{a}} =$ 1,2 x10^{-2}.

A2 Variations of parameters constituting the air density

In order to evaluate the uncertainties associated to the estimates p, RH and t, some advice about their typical variations are given in the following chapter. This information may be used when environmental measurements are not going to be performed.

A2.1 Barometric pressure

At any given location, the variation is at most Δp = ±40 hPa about the average[8]. Within these limits, the distribution is not rectangular as extreme values do occur only once in several years. It has been found that the distribution is basically normal. Taking into account the typical atmospheric pressure variation it is realistic to assume a standard uncertainty

$$u(p) = 10 \text{ hPa} \qquad \text{(A2.1-1)}$$

The average barometric pressure $p(h_{\text{SL}})$(in hPa) can be evaluated according to the International Standard Atmosphere, and may be estimated from the altitude h_{SL} in metres above sea level of the location, using the relation

$$p(h_{\text{SL}}) = p_0 \exp(-h_{\text{SL}} \times 0{,}00012 \text{ m}^{-1}) \qquad \text{(A2.1-2)}$$

with p_0 = 1 013,25 hPa

A2.2 Temperature

The possible variation $\Delta T = T_{\text{max}} - T_{\text{min}}$ of the temperature at the place of use of the instrument may be estimated from information which is easy to obtain

limits stated by the client from his experience,
reading from suitable recording means,
setting of the control instrument, where the room is acclimatized or temperature stabilized,

in case of default, sound judgement should be applied, leading to – e.g.

17 °C ≤ t ≤ 27 °C for closed office or laboratory rooms with windows,
ΔT ≤ 5 K for closed rooms without windows in the centre of a building,
- 10 °C ≤ t ≤ + 30 °C or ΔT ≤ 40 K for open workshops or factory spaces.

As stated for the barometric pressure, a rectangular distribution is unlikely to occur for open workshops or factory spaces where the atmospheric temperature prevails. However, to avoid different assumptions for different room situations, the assumption of rectangular distribution is recommended, leading to

$$u(T) = \Delta T / \sqrt{12} \qquad \text{(A2.2-1)}$$

A2.3 Relative humidity

The possible variation $\Delta RH = RH_{\text{max}} - RH_{\text{min}}$ of the relative humidity at the place of use of the instrument may be estimated from information which is easy to obtain

limits stated by the client from his experience,

[8]Example: at Hannover, Germany, the difference between highest and lowest barometric pressures ever observed over 20 years was 77,1 hPa (Information from DWD, the German Meteorological Service).

reading from suitable recording means,
setting of the control instrument, where the room is acclimatized,

in case of default, sound judgement should be applied, leading, for example, to

30 % ≤ RH ≤ 80 % for closed office or laboratory rooms with windows,
ΔRH ≤ 30 % for closed rooms without windows in the centre of a building,
20 % ≤ RH ≤ 80 % for open workshops or factory spaces.

It should be kept in mind that
at RH < 40 % electrostatic effects may already influence the weighing result on high resolution instruments,
at RH > 60 % corrosion may begin to occur.

As has been said for the barometric pressure, a rectangular distribution is unlikely to occur for open workshops or factory spaces where the atmospheric relative humidity prevails. However, to avoid different assumptions for different room situations, the assumption of rectangular distribution is recommended, leading to

$$u(RH) = \Delta RH \big/ \sqrt{12} \qquad \text{(A2.3-1)}$$

A3 Uncertainty of air density

The relative standard uncertainty of the air density $u(\rho_\mathrm{a})/\rho_\mathrm{a}$ may be calculated by

$$\frac{u(\rho_\mathrm{a})}{\rho_\mathrm{a}} = \sqrt{\left(\frac{u_p(\rho_\mathrm{a})}{\rho_\mathrm{a}} \cdot u(p)\right)^2 + \left(\frac{u_T(\rho_\mathrm{a})}{\rho_\mathrm{a}} \cdot u(T)\right)^2 + \left(\frac{u_{RH}(\rho_\mathrm{a})}{\rho_\mathrm{a}} \cdot u(RH)\right)^2 + \left(\frac{u_\mathrm{form}(\rho_\mathrm{a})}{\rho_\mathrm{a}}\right)^2} \qquad \text{(A3-1)}$$

with the sensitivity coefficients (derived from the CIPM formula for air density)

$u_p(\rho_\mathrm{a})/\rho_\mathrm{a}$ = 1 × 10^{-5} Pa^{-1} for barometric pressure

$u_T(\rho_\mathrm{a})/\rho_\mathrm{a}$ = - 4 × 10^{-3} K^{-1} for air temperature

$u_{RH}(\rho_\mathrm{a})/\rho_\mathrm{a}$ = - 9 × 10^{-3} for relative humidity (the unit for RH in this case is 1, not %)

These sensitivity coefficients may also be used for equation (A1.1-1).

Equation (A3-1) can be approximated as (A3-2) based on the following assumptions:

- the standard uncertainty for pressure variation based on meteorological data, that show it is a normal distribution, is 10 hPa
- the maximum variation for humidity is 100 %.
- the maximum variation of temperature in the location is included as ΔT

$$\frac{u(\rho_\mathrm{a})}{\rho_\mathrm{a}} = \sqrt{1{,}07 \times 10^{-4} + 1{,}33 \times 10^{-6}\,\mathrm{K}^{-2} \Delta T^2} \qquad \text{(A3-2)}$$

Examples of standard uncertainty of air density, calculated for different parameters using the formula (A.1.1-1)

$u(p)$ /hPa	ΔT /K	ΔRH /%	$\frac{u_p(\rho_a)}{\rho_a}u(p)$	$\frac{u_T(\rho_a)}{\rho_a}u(T)$	$\frac{u_{RH}(\rho_a)}{\rho_a}u(RH)$	$\frac{u_{form}(\rho_a)}{\rho_a}$	$\frac{u(\rho_a)}{\rho_a}$
10	2	20	1×10^{-2}	$-2{,}31 \times 10^{-3}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$1{,}03 \times 10^{-2}$
10	2	100	1×10^{-2}	$-2{,}31 \times 10^{-3}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$1{,}06 \times 10^{-2}$
10	5	20	1×10^{-2}	$-5{,}77 \times 10^{-3}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$1{,}16 \times 10^{-2}$
10	5	100	1×10^{-2}	$-5{,}77 \times 10^{-3}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$1{,}18 \times 10^{-2}$
10	10	20	1×10^{-2}	$-1{,}15 \times 10^{-3}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$1{,}53 \times 10^{-2}$
10	10	100	1×10^{-2}	$-1{,}15 \times 10^{-3}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$1{,}55 \times 10^{-2}$
10	20	20	1×10^{-2}	$-2{,}31 \times 10^{-2}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$2{,}52 \times 10^{-2}$
10	20	100	1×10^{-2}	$-2{,}31 \times 10^{-2}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$2{,}53 \times 10^{-2}$
10	30	20	1×10^{-2}	$-3{,}46 \times 10^{-2}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$3{,}61 \times 10^{-2}$
10	30	100	1×10^{-2}	$-3{,}46 \times 10^{-2}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$3{,}61 \times 10^{-2}$
10	40	20	1×10^{-2}	$-4{,}62 \times 10^{-2}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$4{,}73 \times 10^{-2}$
10	40	100	1×10^{-2}	$-4{,}62 \times 10^{-2}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$4{,}73 \times 10^{-2}$
10	50	20	1×10^{-2}	$-5{,}77 \times 10^{-2}$	$-5{,}20 \times 10^{-4}$	$2{,}4\times10^{-4}$	$5{,}86 \times 10^{-2}$
10	50	100	1×10^{-2}	$-5{,}77 \times 10^{-2}$	$-2{,}60 \times 10^{-3}$	$2{,}4\times10^{-4}$	$5{,}87 \times 10^{-2}$

ΔT is the maximum variation of temperature and ΔRH is the maximum variation of humidity in the location of the balance.

APPENDIX B: COVERAGE FACTOR *k* FOR EXPANDED UNCERTAINTY OF MEASUREMENT

Note: in this Appendix the general symbol y is used for the result of measurement, not a particular quantity as an indication, an error, a mass of a weighed body etc.

B1 Objective

The coverage factor k shall in all cases be chosen such that the expanded uncertainty of measurement has a coverage probability of 95,45 %.

B2 Normal distribution and sufficient reliability

The value $k = 2$, corresponding to a 95,45% probability, applies where

a) a normal (Gaussian) distribution can be attributed to the error of indication,

and

b) the standard uncertainty $u(E)$ is of sufficient reliability (i.e. it has a sufficient number of degrees of freedom), see JCGM 100 [1].

Normal distribution may be assumed where several (i.e. $N \geq 3$) uncertainty components, each derived from “well-behaved” distributions (normal, rectangular or the like), contribute to $u(E)$ in comparable amounts.

Sufficient reliability is depending on the degrees of freedom. This criterion is met where no Type A contribution to $u(E)$ is based on less than 10 observations. A typical Type A contribution stems from repeatability. Consequently, if during a repeatability test a load is applied not less than 10 times, sufficient reliability can be assumed.

B3 Normal distribution, no sufficient reliability

Where a normal distribution can be attributed to the error of indication, but $u(E)$ is not sufficiently reliable, then the effective degrees of freedom υ_{eff} have to be determined using the Welch-Satterthwaite formula

$$\upsilon_{\text{eff}} = \frac{u^4(E)}{\sum_{i=1}^{N} \frac{u_i^4(E)}{\upsilon_i}} \qquad \text{(B3-1)}$$

where $u_i(E)$ are the contributions to the standard uncertainty as per (7.1.3-1a), and υ_i is the degrees of freedom of the standard uncertainty contribution $u_i(E)$. Based on υ_{eff} the applicable coverage factor k is read from the extended table of [1], Table G.2 or the underlying t-distribution described in [1], Annex C.3.8 may be used to determine the coverage factor k.

B4 Determining k for non-normal distributions

In any of the following cases, the expanded uncertainty is $U(y) = ku(y)$.

It may be obvious in a given situation that $u(y)$ contains one Type B uncertainty component $u_1(y)$ from a contribution whose distribution is not normal but, e.g., rectangular or triangular, which is significantly greater than all the remaining components. In such a case, $u(y)$ is split up in the (possibly dominant) part u_1 and u_R = square root of $\sum u_j^2$ with j ≥ 2, the combined standard uncertainty comprising the remaining contributions, see [1].

If u_R ≤ 0,3 u_1, then u_1 is considered to be "dominant" and the distribution of y is considered to be essentially identical with that of the dominant contribution.

The coverage factor is chosen according to the shape of distribution of the dominant component

for trapezoidal distribution with $\beta < 0{,}95$,

(β = edge parameter, ratio of smaller to larger edge of trapezoid)

$$k = \left\{1 - \sqrt{\left[0{,}05\left(1 - \beta^2\right)\right]}\right\} / \sqrt{\left[\left(1 + \beta^2\right)/6\right]} \qquad \text{(B4-1)}$$

for a rectangular distribution (β = 1), k = 1,65,

for a triangular distribution (β = 0), k = 1,90,

for U-shaped distribution, k = 1,41.

The dominant component may itself be composed of 2 dominant components $u_1(y)$, $u_2(y)$, e.g. 2 rectangular making up one trapezoid, in which case u_R will be determined from the remaining u_j with $j \geq 3$.

APPENDIX C: FORMULAE TO DESCRIBE ERRORS IN RELATION TO THE INDICATIONS

C1 Objective

This Appendix offers advice on how to derive, from the discrete values obtained at calibration and/or given in a calibration certificate, errors and associated uncertainties for any other reading R within the calibrated weighing range.

It is assumed that the calibration yields n sets of data $I_{\mathrm{N}j}$, E_j, U_j, or alternatively $m_{\mathrm{N}j}$, I_j, U_j, together with the coverage factor k and an indication of the distribution of E underlying k.

In any case, the nominal indication $I_{\mathrm{N}j}$ is considered to be $I_{\mathrm{N}j} = m_{\mathrm{N}j}$.

It is further assumed that for any $m_{\mathrm{N}j}$ the error E_j remains the same if I_j is replaced by $I_{\mathrm{N}j}$, it is therefore sufficient to look at the data $I_{\mathrm{N}j}$, E_j, u_j, and to omit the suffix N for simplicity.

C2 Functional relations

C2.1 Interpolation

There are several polynomial formulae for interpolation[9] between tabulated values and equidistant values which are relatively easy to employ. However, the test loads may not, in many cases, be equidistant, which leads to quite complicated interpolation formulae if applying a single formula to cover the whole weighing range.

Linear interpolation between two adjacent points may be performed by

$$E(R) = E_k + (R - I_k)(E_{k+1} - E_k)/(I_{k+1} - I_k) \quad \text{(C2.1-1)}$$

$$U(E(R)) = U_k + (R - I_k)(U_{k+1} - U_k)/(I_{k+1} - I_k) \quad \text{(C2.1-2)}$$

for a reading R with $I_k < R < I_{k+1}$. A higher order polynomial would be needed to estimate the possible interpolation error – this is not further elaborated.

C2.2 Approximation

Approximation should be performed by calculation or by algorithms based on the "minimum χ^2" approach, that is, the parameters of a function f are determined so that

$$\chi^2 = \sum p_j v_j^2 = \sum p_j \left(f(I_j) - E_j\right)^2 = \text{minimum} \quad \text{(C2.2-1)}$$

with

p_j = weighting factor (basically proportional to $1/u_j^2$),

v_j = residual,

f = approximation function containing n_{par} parameters to be determined,

$j = 1 \ldots n$,

---

[9]An interpolation formula is understood to yield exactly the given values between which interpolation takes place. An approximation formula will normally not yield the given values exactly.

n = number of test points.

From the observed chi-squared value $\chi^2{}_{\text{obs}}$, if the following condition is met [12]

$$\chi^2{}_{\text{obs}} \leq \upsilon \qquad \text{(C2.2-2a)}$$

with the degrees of freedom $\upsilon = n - n_{\text{par}}$, it is justified to assume the form of the model function $E(I) = f(I)$ to be mathematically consistent with the data underlying the approximation.

An alternative option for testing the goodness of the fit is to assume that the maximum value of the weighted differences will have to fulfil

$$\max\left(\frac{\left|f(I_j) - E_j\right|}{U(f(I_j))}\right) < 1 \qquad \text{(C2.2-2b)}$$

that is, the expanded uncertainty must include the residual for each point *j*. This condition is much more restrictive than equation (C2.2-2a).

C2.2.1 Approximation by polynomials

Approximation by a polynomial yields the general function

$$E(R) = f(R) = a_0 + a_1 R + a_2 R^2 + ... + a_{n_{\text{a}}} R^{n_{\text{a}}} \qquad \text{(C2.2-3)}$$

The degree n_{a} of the polynomial should be chosen such that $n_{\text{par}} = n_{\text{a}} + 1 \leq n/2$.

The calculation is best performed by matrix calculation.

Let	$\boldsymbol{X}_{(n \text{ x } n_{\text{par}})}$	be a matrix whose n rows are (1, I_j, I_j^2,..., $I_j^{n_{\text{a}}}$),
	$\boldsymbol{a}_{(n_{\text{par}} \text{ x } 1)}$	be a column vector whose components are the coefficients a_0, a_1, ... , $a_{n_{\text{a}}}$ to be determined of the approximation polynomial,
	$\boldsymbol{e}_{(n \text{ x } 1)}$	be a column vector whose components are the E_j,
	$\boldsymbol{U}(\boldsymbol{e})_{(n \text{ x } n)}$	be the variance-covariance matrix of $\boldsymbol{e}$.

$\boldsymbol{U}(\boldsymbol{e})$ is given by

$$\boldsymbol{U}(\boldsymbol{e}) = \boldsymbol{U}(m_{\text{ref}}) + \boldsymbol{U}(I_{\text{Cal}}) + \boldsymbol{U}(mod) \qquad \text{(C2.2-3a)}$$

where

$\boldsymbol{U}(m_{\text{ref}})$ is the covariance matrix associated with the reference values m_{ref}

(4.2.4-2). Considering reasonably high correlation among the referencevalues

$$\boldsymbol{U}(m_{\text{ref}}) = \boldsymbol{s}_{m_{\text{ref}}} \boldsymbol{s}_{m_{\text{ref}}}{}^{\text{T}} \qquad \text{(C2.2-3b)}$$

where $\boldsymbol{s}_{m_{\text{ref}}}$ is the column vector of the uncertainties $u(m_{\text{ref}})$ (equ. 7.1.2-14),

$\boldsymbol{U}(I_{\mathrm{Cal}})$ is a diagonal matrix whose elements are $u_{jj} = u^2(I_j)$,

$\boldsymbol{U}(mod)$ is an additional covariance matrix, which is given by

$$\boldsymbol{U}(mod) = s_{\mathrm{m}}{}^2 \boldsymbol{I} \qquad \text{(C2.2-3c)}$$

where ***I*** is the identity matrix and s_{m} is an uncertainty due to the model. This contribution is considered in order to take into account the model inadequacy.

Initially s_{m} is set to zero, if the χ^2 test (C2.2-2a) fails, s_{m} is enlarged in an iterative way, until the χ^2 test result is satisfied.

If $\boldsymbol{U}(I_{\mathrm{Cal}})$ is the dominant contribution, the covariances maybe be neglected and $\boldsymbol{U}(\boldsymbol{e})$ can be approximated to a diagonal matrix whose elements are

$$u_{jj} = u^2(E_j) + s_{\mathrm{m}}{}^2 \qquad \text{(C2.2-3d)}$$

The weighting matrix $\boldsymbol{P}$ is

$$\boldsymbol{P} = \boldsymbol{U}(\boldsymbol{e})^{-1} \qquad \text{(C2.2-4)}$$

and the coefficients a_0, a_1,… are found by solving the normal equations

$$\boldsymbol{X}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{X}\boldsymbol{a} - \boldsymbol{X}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{e} = \boldsymbol{0} \qquad \text{(C2.2-5)}$$

with the solution

$$\hat{\boldsymbol{a}} = \left(\boldsymbol{X}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{e} \qquad \text{(C2.2-6)}$$

The n residuals $v_j = f(I_j) - E_j$ are comprised in the vector

$$\boldsymbol{v} = \boldsymbol{X}\hat{\boldsymbol{a}} - \boldsymbol{e} \qquad \text{(C2.2-7)}$$

and χ^2_{obs} is obtained by

$$\chi^2_{\mathrm{obs}} = \boldsymbol{v}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{v} \qquad \text{(C2.2-8)}$$

Provided the condition of (C2.2-2) is met, the variances and covariances for the coefficients a_i are given by the matrix

$$\boldsymbol{U}(\hat{\boldsymbol{a}}) = \left(\boldsymbol{X}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{X}\right)^{-1} \qquad \text{(C2.2-9)}$$

Where the condition (C2.2-2) is not met, one of these procedures may be applied

- a: repeat the approximation with an approximating polynomial of higher degree n_{a}, as long as $n_{\mathrm{a}} + 1 \le n/2$,
- b: repeat the approximation after increasing $\boldsymbol{U}(mod)$.

The results of the approximation, $\hat{\boldsymbol{a}}$ and $\boldsymbol{U}(\hat{\boldsymbol{a}})$ may be used to determine the approximated errors and the associated uncertainties for the n points I_j.

The errors $E_{\mathrm{appr}j}$ are comprised in the vector

$$\boldsymbol{e}_{\mathrm{appr}} = \boldsymbol{X}\hat{\boldsymbol{a}} \tag{C2.2-10}$$

with the uncertainties given by

$$u^2\left(E_{\mathrm{appr}j}\right) = \mathrm{diag}\left(\boldsymbol{X}\boldsymbol{U}(\hat{\boldsymbol{a}})\boldsymbol{X}^{\mathsf{T}}\right). \tag{C2.2-11}$$

They also serve to determine the error and its associated uncertainty for any other indication – called a reading R to discriminate from the indications I_j – within the calibrated weighing range.

Let

$\boldsymbol{r}$ be a column vector whose elements are $\left(1, R, R^2, R^3, \ldots, R^{n_\mathrm{a}}\right)^{\mathsf{T}}$,

$\boldsymbol{r}'$ be a column vector whose elements are the derivatives $\left(0, 1, 2R, 3R^2, \ldots, n_\mathrm{a} R^{n_\mathrm{a}-1}\right)^{\mathsf{T}}$.

The error is

$$E_{\mathrm{appr}}(R) = \boldsymbol{r}^{\mathsf{T}}\hat{\boldsymbol{a}} \tag{C2.2-12}$$

and the uncertainty is given by

$$u^2\left(E_{\mathrm{appr}}\right) = \left(\boldsymbol{r}'^{\mathsf{T}}\hat{\boldsymbol{a}}\right)U(R)\left(\boldsymbol{r}'^{\mathsf{T}}\hat{\boldsymbol{a}}\right)^{\mathsf{T}} + \boldsymbol{r}^{\mathsf{T}}\boldsymbol{U}(\hat{\boldsymbol{a}})\boldsymbol{r} \tag{C2.2-13}$$

The first term on the right-hand side simplifies, as all 3 matrices are only one dimensional, to

$$\left(\boldsymbol{r}'^{\mathsf{T}}\hat{\boldsymbol{a}}\right)\boldsymbol{U}(R)\left(\boldsymbol{r}'^{\mathsf{T}}\hat{\boldsymbol{a}}\right)^{\mathsf{T}} = \left(a_1 + 2a_2R + 3a_3R^2 + \ldots + n_\mathrm{a}a_{n_\mathrm{a}}R^{n_\mathrm{a}-1}\right)^2 u^2(R) \tag{C2.2-14}$$

with $u^2(R) = d_0^2/12 + d_R^2/12 + s^2(R) + u_{\mathrm{rel}}^2(\delta R_{\mathrm{ecc}})R^2$ as per (7.1.1-12).

C2.2.2 Approximation by a straight line

Many modern electronic instruments are well designed, and corrected internally to achieve good linearity. Therefore errors mostly result from incorrect adjustment, and the error increases in proportion to R. For such instruments it may be appropriate to restrict the polynomial to a linear function, provided it is sufficient in view of condition (C2.2-2).

The standard solution is to apply (C2.2-3) with $n_\mathrm{a} = 1$

$$E(R) = f(R) = a_0 + a_1R \tag{C2.2-15}$$

One variation to this is to set $a_0 = 0$ and to determine only a_1. This can be justified by the fact that due to zero-setting – at least for increasing loads – the error $E(R = 0)$ is automatically zero

$$E(R) = f(R) = a_1R \tag{C2.2-16}$$

Another variation is to define the coefficient a ($= a_1$ in (C2.2-16)) as the mean of all relative errors $q_j = E_j / I_j$. This allows inclusion of errors of net indications after a tare balancing operation if these have been determined at calibration

$$a = \sum \left(E_j / I_j \right) / n \qquad \text{(C2.2-17)}$$

The calculations, except for the variation (C2.2-17), may be performed using the matrix formulae in C2.2.1.

Other possibilities are given hereafter.

C2.2.2.1 Linear regression as per (C2.2-15) may be performed by software.

Correspondence between results is typically

”intercept”	⇔	a_0
”slope”	⇔	a_1

However, simple pocket calculators may not be able to perform linear regression based on weighted error data, or linear regression with $a_0 = 0$.

C2.2.2.2 To facilitate programming the calculations by computer in non-matrix notation, the relevant formulae are presented hereafter.

If condition (C2.2-2a) is intended to be fulfilled, the method starts with the first linear regression using

$$p_j = 1 / u^2 \left(E_j \right) \qquad \text{(C2.2-18a)}$$

If (C2.2-2a) is not yet fulfilled, then the standard deviation of the fit can be determined as

$$std\,fit = \sqrt{\frac{\sum_j \left(f\left(I_j\right) - E_j \right)^2}{\left(n - n_{\mathrm{par}} \right)}} \qquad \text{(C2.2-18b)}$$

As a second step new weighting factors have to determined as

$$p'_j = 1 / \left(u^2 \left(E_j \right) + std\,fit^2 \right) \qquad \text{(C2.2-18c)}$$

With these new weighting factors a new linear regression has to be determined. Following this method, the linear regression fulfils condition (C2.2-2a).

If condition (C2.2-2b), which is more restrictive, is intended to be fulfilled, it is very likely that an additional uncertainty component, s_m, has to be included in (C2.2-18a. Initially $s_m$is set to zero, then s_m is enlarged in an iterative way until the condition in (C2.2-2b) is satisfied. A proposal to increase the step to enlarge s_m may be to consider 1/10 of the resolution of the instrument.

In the following expressions for simplicity, all indices ” j ” have been omitted from I , E , p .

a) linear regression for (C2.2-15)

$$a_0 = \frac{\sum pE \sum pI^2 - \sum pI \sum pIE}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \tag{C2.2-15a}$$

$$a_1 = \frac{\sum p \sum pIE - \sum pE \sum pI}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \tag{C2.2-15b}$$

$$\chi^2 = \sum p\left(a_0 + a_1 I - E\right)^2 \tag{C2.2-15c}$$

$$u^2\left(a_0\right) = \frac{\sum pI^2}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \tag{C2.2-15d}$$

$$u^2\left(a_1\right) = \frac{\sum p}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \tag{C2.2-15e}$$

$$\mathrm{cov}\left(a_0, a_1\right) = -\frac{\sum pI}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \tag{C2.2-15f}$$

(C2.2-15) applies for the approximated error of the reading R, and the uncertainty of the approximation $u\left(E_{\mathrm{appr}}\right)$ is given by

$$u^2\left(E_{\mathrm{appr}}\right) = a_1^2 u^2(R) + u^2(a_0) + R^2 u^2(a_1) + 2R\,\mathrm{cov}(a_0, a_1) \tag{C2.2-15g}$$

b) linear regression with a_0 = 0

$$a_1 = \sum pIE \Big/ \sum pI^2 \tag{C2.2-16a}$$

$$\chi^2 = \sum p\left(a_1 I - E\right)^2 \tag{C2.2-16b}$$

$$u^2\left(a_1\right) = 1 \Big/ \sum pI^2 \tag{C2.2-16c}$$

(C2.2-16) applies for the approximated error of the reading R, and the assigned uncertainty $u\left(E_{\mathrm{appr}}\right)$ is given by

$$u^2\left(E_{\mathrm{appr}}\right) = a_1^2 u^2(R) + R^2 u^2(a_1) \tag{C2.2-16d}$$

c) mean gradients

In this variant the uncertainties are $u\left(E_j / I_j\right) = u\left(E_j\right) / I_j$ and $p_j = I_j^2 / u^2\left(E_j\right)$.

$$a = \left(\sum pE/I\right) / \sum p \quad \text{(C2.2-17a)}$$

$$\chi^2 = \sum p(a - E/I)^2 \quad \text{(C2.2-17b)}$$

$$u^2(a) = 1/\sum p \quad \text{(C2.2-17c)}$$

(C2.2-17) applies for the approximated error of the reading *R* which may be also a net indication, and the uncertainty of the approximation $u(E_{\mathrm{appr}})$ is given by

$$u^2(E_{\mathrm{appr}}) = a^2 u^2(R) + R^2 u^2(a) \quad \text{(C2.2-17d)}$$

C3 Terms without relation to the readings

While terms that are not a function of the indication do not offer any estimated value for an error to be expected for a given reading in use, they may be helpful to derive the ”global uncertainty” mentioned in 7.5.2.

C3.1 Mean error

The mean of all errors is

$$E^0 = \bar{E} = \frac{1}{n}\sum_{j=1}^{n} E_j \quad \text{(C3.1-1)}$$

with the standard deviation

$$s(E) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(\bar{E} - E_j\right)^2} = u_{\mathrm{appr}} \quad \text{(C3.1-2)}$$

Note: the data point $I = 0$, $E = 0$ shall be included as I_1, E_1.

Where $\bar{E}$ is close to zero, only $s^2(E)$ may be added in (7.5.2-2a). In other cases, in particular where $|\bar{E}| \geq u(W)$, (7.5.2-3a) should be used, with $u(W)$ increased by $u_{\mathrm{appr}} = s(E)$.

C3.2 Maximum error

The ”maximum error” shall be understood as the largest absolute value of all errors

$$E_{\max} = |E_j|_{\max} \quad \text{(C3.2-1)}$$

C3.2.1 With $E^0 = E_{\max}$, (7.5.2-3a) would certainly describe a ”global uncertainty” which would cover any error in the weighing range with a higher coverage probability than 95 %. The advantage is that the formula is simple and straightforward.

C3.2.2 Assuming a rectangular distribution of all errors over the – fictitious! – range $\pm E_{\max}$, E^0 could be defined as the standard deviation of the errors

$$E^0 = E_{\max} / \sqrt{3} \quad \text{(C3.2-2)}$$

to be inserted into (7.5.2-2a).

APPENDIX D: SYMBOLS

Symbols that are used in more than one section of the main document are listed and explained hereafter.

Symbol	Definition
D	drift, variation of a value with time
E	error (of an indication)
I	indication of an instrument
I_{ref}	reference value of the indication of an instrument
K_T	sensitivity of the instrument to the temperature variation
L	load on an instrument
Max	maximum weighing capacity
Max_1	upper limit of the weighing range with the smallest scale interval
Max'	upper limit of specified weighing range, $Max' < Max$
Min	value of the load below which the weighing result may be subject to an excessive relative error (from [2] and [3])
Min'	lower limit of specified weighing range, $Min' > Min$
R	indication (reading) of an instrument not related to a test load
R_{min}	minimum weight
$R_{\mathrm{min,SF}}$	minimum weight for a safety factor >1
Req	user requirement for relative weighing accuracy
T	temperature (in K)
Tol	specified tolerance value
U	expanded uncertainty
U_{gl}	global expanded uncertainty
W	weighing result, weight in air
d	scale interval, the difference in mass between two consecutive indications of the indicating device
d_1	smallest scale interval
d_{T}	effective scale interval $< d$, used in calibration tests
g	local gravity acceleration
k	coverage factor
k_{s}	adjustment factor
m	mass of an object
m_{c}	conventional value of mass, preferably of a standard weight
m_{N}	nominal value of mass of a standard weight
m_{ref}	reference weight (“true value“) of a test load
mpe	maximum permissible error (of an indication, a standard weight etc.) in a given context
n	number of items, as indicated in each case
p	barometric pressure
s	standard deviation
t	temperature (in °C)
u	standard uncertainty

u_{rel}	standard uncertainty related to a base quantity
υ	number of degrees of freedom
ρ	density
ρ_0	reference density of air, ρ_0 = 1,2 kg/m³
ρ_a	air density
ρ_c	reference density of a standard weight, ρ_c = 8 000 kg/m³

Suffix	**related to**
B	air buoyancy (at calibration)
D	drift
L	at load
N	nominal value
St	standard (mass)
T	test
adj	adjustment
appr	approximation
buoy	air buoyancy (weighing result)
cal	calibration
conv	convection
corr	correction
dig	digitalisation
ecc	eccentric loading
gl	global, overall
i , j	numbering
instr	weighing instrument
max	maximum value from a given population
min	minimum value from a given population
proc	weighing procedure
ref	reference
rel	relative
rep	repeatability
s	actual at time of adjustment
sub	substitution load
tare	tare balancing operation
temp	temperature
time	time
0	zero, no-load

APPENDIX E: INFORMATION ON AIR BUOYANCY

This Appendix gives additional information to the air buoyancy correction treated in 7.1.2.2.

E1 Density of standard weights

Where the density ρ of a standard weight, and its standard uncertainty $u(\rho)$ are not known, the following values may be used for weights of R111 classes E_2 to M_2 (taken from [4], Table B7).

Alloy/material	Assumed density ρ in kg/m³	Standard uncertainty $u(\rho)$ in kg/m³
Nickel silver	8 600	85
Brass	8 400	85
stainless steel	7 950	70
carbon steel	7 700	100
iron	7 800	100
cast iron (white)	7 700	200
cast iron (grey)	7 100	300
aluminium	2 700	65

For weights with an adjustment cavity filled with a considerable amount of material of different density, [4] gives a formula to calculate the overall density of the weight.

E2 Air buoyancy for weights conforming to OIML R111

As quoted in a footnote to 7.1.2.2, OIML R111 requires the density of a standard weight to be within certain limits that are related to the maximum permissible error mpe and a specified variation of the air density. The mpe are proportional to the nominal value for weights of ≥ 100 g. This allows an estimate of the relative uncertainty $u_{\text{rel}}(\delta m_{\text{B}})$. The corresponding formulae (7.1.2-5c) for the case that the instrument is adjusted immediately before calibration and (7.1.2-5d) for the case when the instrument is not adjusted before calibration have been evaluated in Table E2.1, in relation to the accuracy classes E_2 to M_1.

For weights of $m_{\text{N}} \leq$ 50 g the mpe are tabled in R111, the relative value mpe/m_{N} is increasing with decreasing mass. For these weights, Table E2.1 contains the absolute standard uncertainties $u(\delta m_{\text{B}}) = u_{\text{rel}}(\delta m_{\text{B}})\, m_{\text{N}}$.

The values in Table E2.1 can be used for an estimate of the uncertainty contribution if air buoyancy is not corrected for.

Table E2.1: Standard uncertainty of air buoyancy correction for standard weights conforming to R 111

Calculated according to 7.1.2.2 for the case the instrument is adjusted immediately before calibration (7.1.2-5c), u_A and the case where the instrument is not adjusted before calibration (7.1.2-5d), u_B.

	Class E_2			Class F_1			Class F_2			Class M_1		
m_N in g	*mpe* in mg	u_A in mg	u_B in mg	*mpe* in mg	u_A in mg	u_B in mg	*mpe* in mg	u_A in mg	u_B in mg	*mpe* in mg	u_A in mg	u_B in mg
50	0,100	0,014	0,447	0,30	0,043	0,476	1,00	0,14	0,58	3,0	0,43	0,87
20	0,080	0,012	0,185	0,25	0,036	0,209	0,80	0,12	0,29	2,5	0,36	0,53
10	0,060	0,009	0,095	0,20	0,029	0,115	0,60	0,09	0,17	2,0	0,29	0,38
5	0,050	0,007	0,051	0,16	0,023	0,066	0,50	0,07	0,12	1,6	0,23	0,27
2	0,040	0,006	0,023	0,12	0,017	0,035	0,40	0,06	0,08	1,2	0,17	0,19
1	0,030	0,004	0,013	0,10	0,014	0,023	0,30	0,04	0,05	1,0	0,14	0,15
0,5	0,025	0,004	0,008	0,08	0,012	0,016	0,25	0,04	0,04	0,8	0,12	0,12
0,2	0,020	0,003	0,005	0,06	0,009	0,010	0,20	0,03	0,03	0,6	0,09	0,09
0,1	0,016	0,002	0,003	0,05	0,007	0,008	0,16	0,02	0,02	0,5	0,07	0,07

Relative *mpe* and relative standard uncertainties $u_{rel}(\delta m_B)$ in mg/kg for weights of 100 g and greater

	Class E_2			Class F_1			Class F_2			Class M_1		
	mpe/m_N mg/kg	$u_{rel\,A}$	$u_{rel\,B}$	mpe/m_N mg/kg	$u_{rel\,A}$	$u_{rel\,B}$	mpe/m_N mg/kg	$u_{rel\,A}$	$u_{rel\,B}$	mpe/m_N mg/kg	$u_{rel\,A}$	$u_{rel\,B}$
≥ 100	1,60	0,23	8,89	5,00	0,72	9,38	16,0	2,31	11,0	50,0	7,22	15,88

APPENDIX F: EFFECTS OF CONVECTION

In 4.2.3 the generation of an apparent change of mass Δm_{conv} by a difference in temperature ΔT between a standard weight and the surrounding air has been explained in principle. More detailed information is presented hereafter, to allow an assessment of situations in which the effect of convection should be considered in view of the uncertainty of calibration.

All calculations of values in the following tables are based on [7]. The relevant formulae, and parameters to be included, are not reproduced here. Only the main formulae, and essential conditions are referenced.

The problem treated here is quite complex, both in the underlying physics and in the evaluation of experimental results. The precision of the values presented hereafter should not be overestimated.

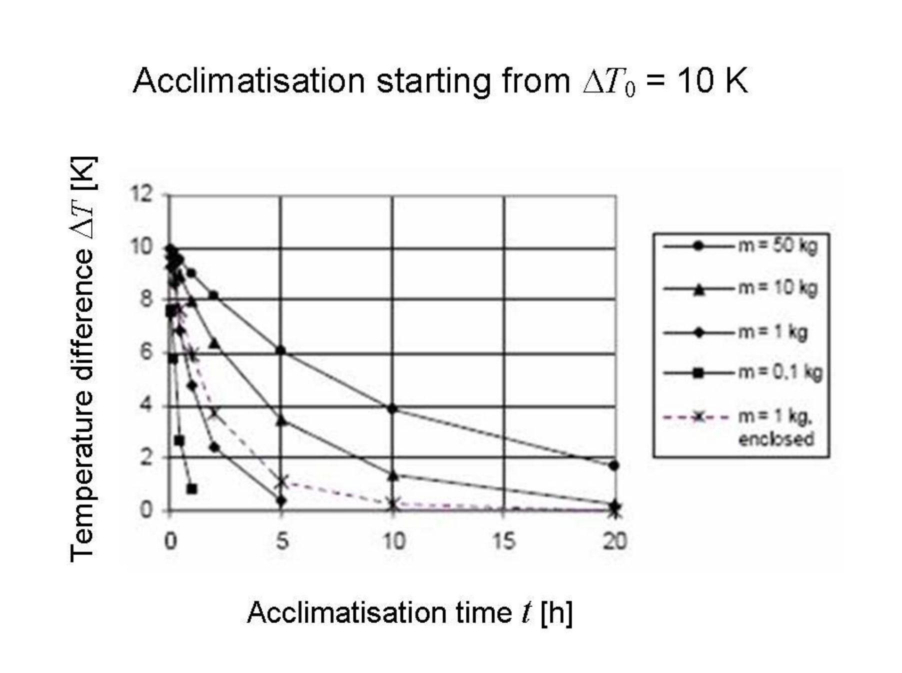
F1 Relation between temperature and time

An initial temperature difference ΔT_0 is reduced with time Δt by heat exchange between the weight and the surrounding air. The rate of heat exchange is fairly independent of the sign of ΔT_0, therefore warming up or cooling down of a weight occurs in similar time intervals.

Figure F1.1 gives some examples of the effect of acclimatisation. Starting from an initial temperature difference of 10 K, the actual ΔT after different acclimatisation times is shown for 4 different weights. The weights are supposed to rest on three fairly thin PVC columns in “free air”. In comparison, ΔT is also shown for a 1 kg weight resting on the same columns but enclosed in a bell jar which reduces the air flow of convection, so it takes about 1,5 times to 2 times as much time to achieve the same reduction of ΔT, as for the 1 kg piece without the jar.

References in [7]: formula (21), and parameters for cases 3b and 3c in Table 4.

Figure F1.1: Acclimatisation of standard weights



Tables F1.2 and F1.3 give acclimatisation times Δt for standard weights that may have to be waited if the temperature difference is to be reduced from a value ΔT_1 to a lower value ΔT_2. The conditions of heat exchange are the same as in Figure F1.1: Table F1.2 as for "m = 0,1 kg" to "m = 50 kg"; Table F1.3 as for "m = 1 kg enclosed".

Under actual conditions the waiting times may be shorter where a weight stands directly on a plane surface of a heat conducting support; they may be longer where a weight is partially enclosed in a weight case.

References in [7]: formula (26), and parameters for cases 3b, 3c in Table 4.

Table F1.2 Time intervals for reduction in steps of temperature differences
Weights standing on 3 thin PVC columns in free air

Acclimatisation time in min for ΔT to be reached from the next higher ΔT, case 3b							
	ΔT / K						
m/kg	20 K to 15 K	15 K to 10 K	10 K to 7 K	7 K to 5 K	5 K to 3 K	3 K to 2 K	2 K to 1 K
50	149,9	225,3	212,4	231,1	347,9	298,0	555,8
20	96,2	144,0	135,2	135,0	219,2	186,6	345,5
10	68,3	101,9	95,3	94,8	153,3	129,9	239,1
5	48,1	71,6	66,7	66,1	106,5	89,7	164,2
2	30,0	44,4	41,2	40,6	65,0	54,4	98,8
1	20,8	30,7	28,3	27,8	44,3	37,0	66,7
0,5	14,3	21,0	19,3	18,9	30,0	24,9	44,7
0,2	8,6	12,6	11,6	11,3	17,8	14,6	26,1
0,1	5,8	8,5	7,8	7,5	11,8	9,7	17,2
0,05	3,9	5,7	5,2	5,0	7,8	6,4	11,3
0,02	2,3	3,3	3,0	2,9	4,5	3,7	6,4
0,01	1,5	2,2	2,0	1,9	2,9	2,4	4,2

Examples for a 1 kg weight

to reduce ΔT from 20 K to 15 Kwill take 20,8 min,

to reduce ΔT from 15 K to 10 K will take 30,7 min,

to reduce ΔT from 10 K to 5 K will take 28,3 min + 27,8 min = 56,1 min.

Table F1.3 Time intervals for reduction in steps of temperature differences
Weights standing on 3 thin PVC columns, enclosed in a bell jar

Acclimatisation time in min for ΔT to be reached from the next higher ΔT, case 3c							
	$\Delta T\,/\,\mathrm{K}$						
m/kg	20 K to 15 K	15 K to 10 K	10 K to 7 K	7 K to 5 K	5 K to 3 K	3 K to 2 K	2 K to 1 K
50	154,2	235,9	226,9	232,1	388,7	342,7	664,1
20	103,8	158,6	152,4	155,6	260,2	228,9	442,2
10	76,8	117,2	112,4	114,7	191,5	168,1	324,0
5	56,7	86,4	82,8	84,3	140,5	123,1	236,5
2	37,8	57,5	54,9	55,8	92,8	81,0	155,0
1	27,7	42,1	40,1	40,7	67,5	58,8	112,0
0,5	20,2	30,7	29,2	29,6	48,9	42,4	80,5
0,2	13,3	20,1	19,1	19,2	31,7	27,3	51,6
0,1	9,6	14,5	13,7	13,8	22,6	19,5	36,6
0,05	6,9	10,4	9,8	9,9	16,1	13,8	25,7
0,02	4,4	6,7	6,3	6,2	10,2	8,6	16,0
0,01	3,2	4,7	4,4	4,4	7,1	6,0	11,1

F2 Change of the apparent mass

The air flow generated by a temperature difference ΔT is directed upwards where the weight is warmer, $\Delta T > 0$, than the surrounding air, and downwards where it is cooler $\Delta T < 0$. The air flow causes friction forces on the vertical surface of a weight, and pushing or pulling forces on its horizontal surfaces, resulting in a change Δm_{conv} of the apparent mass. The load receptor of the instrument is also contributing to the change, in a manner not yet fully investigated.

There is evidence from experiments that the absolute values of the change are generally smaller for $\Delta T < 0$ than for $\Delta T > 0$. It is therefore reasonable to calculate the mass changes for the absolute values of ΔT, using the parameters for $\Delta T > 0$.

Table F2.1 gives values for Δm_{conv} for standard weights, for the temperature differences ΔT appearing in Tables F1.2 and F1.3. They are based on experiments performed on a mass comparator with turning table for automatic exchange of weights inside a glass housing. The conditions prevailing at calibration of "normal" weighing instruments being different, the values in the table should be considered as estimates of the effects that may be expected at an actual calibration.

References in [7]: formula (34), and parameters for case 3d in Table 4

Table F2.1 Change in apparent mass Δm_{conv}

Change Δm_{conv} in mg of standard weights, for selected temperature differences ΔT								
	ΔT in K							
m in kg	20	15	10	7	5	3	2	1
50	113,23	87,06	60,23	43,65	32,27	20,47	14,30	7,79
20	49,23	38,00	26,43	19,25	14,30	9,14	6,42	3,53
10	26,43	20,47	14,30	10,45	7,79	5,01	3,53	1,96
5	14,30	11,10	7,79	5,72	4,28	2,76	1,96	1,09
2	6,42	5,01	3,53	2,61	1,96	1,27	0,91	0,51
1	3,53	2,76	1,96	1,45	1,09	0,72	0,51	0,29
0,5	1,96	1,54	1,09	0,81	0,61	0,40	0,29	0,17
0,2	0,91	0,72	0,51	0,38	0,29	0,19	0,14	0,08
0,1	0,51	0,40	0,29	0,22	0,17	0,11	0,08	0,05
0,05	0,29	0,23	0,17	0,12	0,09	0,06	0,05	0,03
0,02	0,14	0,11	0,08	0,06	0,05	0,03	0,02	0,01
0,01	0,08	0,06	0,05	0,03	0,03	0,02	0,01	0,01

The values in this table may be compared with the uncertainty of calibration, or with a given tolerance of the standard weights that are used for a calibration, in order to assess whether an actual ΔT value may produce a significant change of apparent mass.

As an example, Table F2.2 gives the temperature differences which are likely to produce, for weights conforming to R 111, values of Δm_{conv} not exceeding certain limits. The comparison is based on Table F2.1.

The limits considered are the maximum permissible errors, and $1/3$ thereof.

It appears that with these limits, the effect of convection is relevant only for weights of classes F_1 of OIML R111 or better.

Table F2.2 Temperature limits for specified Δm_{conv} values

ΔT_{A} = temperature difference for $\Delta m_{\text{conv}} \le mpe$

ΔT_{B} = temperature difference for $\Delta m_{\text{conv}} \le mpe/3$

Differences ΔT_{A} for $\Delta m_{\text{conv}} \le mpe$ and ΔT_{B} for $\Delta m_{\text{conv}} \le mpe/3$						
	Class E_2			Class F_1		
m_{N} in kg	mpe in mg	ΔT_{A} in K	ΔT_{B} in K	mpe in mg	ΔT_{A} in K	ΔT_{B} in K
50	75	12	4	250	>20	12
20	30	11	3	100	>20	11
10	15	10	3	50	>20	10
5	7,5	10	3	25	>20	10
2	3	9	1	10	>20	9
1	1,5	7	1	5	>20	7
0,5	0,75	6	1	2,5	>20	6
0,2	0,30	5	1	1,0	>20	5
0,1	0,15	4	1	0,50	>20	4
0,05	0,10	6	1	0,30	>20	6
0,02	0,08	10	2	0,25	>20	10
0,01	0,06	15	3	0,20	>20	15

APPENDIX G: MINIMUM WEIGHT

The minimum weight is the smallest sample quantity required for a weighment to just achieve a specified relative accuracy of weighing [13].

Consequently, when weighing a quantity representing minimum weight, $R_{\min}$, the relative measurement uncertainty of the weighing result equals the required relative weighing accuracy, *Req*, so that

$$\frac{U(R_{\min})}{R_{\min}} = Req \tag{G-1}$$

This leads to the following relation that describes minimum weight

$$R_{\min} = \frac{U(R_{\min})}{Req} \tag{G-2}$$

It is general practice that users define specific requirements for the performance of an instrument (User Requirement Specifications). Normally they define upper thresholds for measurement uncertainty values that are acceptable for a specific weighing application. Colloquially users refer to weighing process accuracy or weighing tolerance requirements. Very frequently users also have to follow regulations that stipulate the adherence to a specific measurement uncertainty requirement. Normally these requirements are indicated as a relative value, e.g. adherence to a measurement uncertainty of 0,1 %.

For weighing instruments, usually the global uncertainty is used to assess whether the instrument fulfils specific user requirements.

The global uncertainty is usually approximated by the linear equation (7.5.2-3e)

$$U_{gl}(W) \approx U(W=0) + \left\{ \begin{bmatrix} U(W=Max)- \\ U(W=0) \end{bmatrix} \Big/ Max \right\} R + |a_1| R = \alpha_{gl} + \beta_{gl} \cdot R \tag{G-3}$$

The relative global uncertainty thus is a hyperbolic function and is defined as

$$U_{\mathrm{gl,rel}}(W) = \frac{U_{\mathrm{gl}}(W)}{R} = \frac{\alpha_{\mathrm{gl}}}{R} + \beta_{\mathrm{gl}} \tag{G-4}$$

For a given accuracy requirement, *Req*, only weighings with

$$U_{\mathrm{gl,rel}}(W) \leq Req \tag{G-5}$$

fulfil the respective user requirement. Consequently only weighings with a reading of

$$R \geq \frac{\alpha_{\mathrm{gl}}}{Req - \beta_{\mathrm{gl}}} \tag{G-6}$$

have a relative measurement uncertainty smaller than the specific requirement set by the user and are thus acceptable. The limit value, i.e. the smallest weighing result that fulfils the user requirement is

$$R_{\min} = \frac{\alpha_{\mathrm{gl}}}{Req - \beta_{\mathrm{gl}}} \tag{G-7}$$

and is called “minimum weight”. Based on this value the user is able to define appropriate standard operating procedures that assure that the weighings he performs on the instrument comply with the minimum weight requirement, i.e. he only weighs quantities with higher mass than the minimum weight.

As measurement uncertainty in use may be difficult to estimate due to environmental factors such as high levels of vibration, draughts, influences induced by the operator, etc., or due to specific influences of the weighing application such as electrostatically charged samples, magnetic stirrers, etc., a safety factor is usually applied.

The safety factor *SF* is a number larger than one, by which the user requirement *Req* is divided. The objective is to ensure that the relative global measurement uncertainty is smaller than or equal to the user requirement *Req*, divided by the safety factor. This ensures that environmental effects or effects due to the specific weighing application that have an important effect on the measurement and thus might increase the measurement uncertainty of a weighing above a level estimated by the global uncertainty, still allow – with a high degree of insurance – that the user requirement *Req* is fulfilled.

$$U_{\mathrm{gl,rel}}(W) \leq Req\,/\,SF \tag{G-8}$$

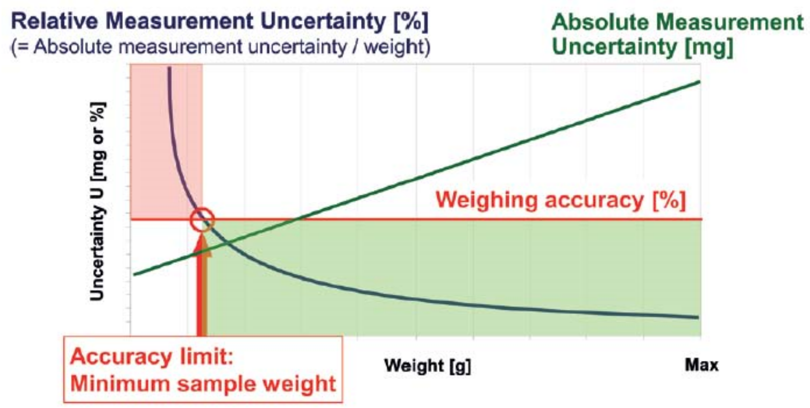
Consequently, the minimum weight based on the safety factor can be calculated as

$$R_{\mathrm{min,SF}} = \frac{\alpha_{\mathrm{gl}} \cdot SF}{Req - \beta_{\mathrm{gl}} \cdot SF} \tag{G-9}$$

The user is responsible for defining the safety factor depending on the degree to which environmental effects and the specific weighing application could influence the measurement uncertainty.

Note that the minimum weight refers to the net (sample) weight which is weighed on the instrument, i.e. the tare vessel mass must not be considered to fulfil the user requirement *Req*. Therefore, minimum weight is frequently called "minimum sample weight".

Figure G.1: Measurement uncertainty



Absolute (green line) and relative (blue line) measurement uncertainty of a weighing instrument. The accuracy limit of the instrument, the so-called minimum weight, is the intersection point between relative measurement uncertainty and the required weighing accuracy.

APPENDIX H: EXAMPLES

The examples presented in this Appendix demonstrate in different ways how the rules contained in this guideline may be applied correctly. They are not intended to indicate any preference for certain procedures as against others for which no example is presented.

Where a calibration laboratory wishes to proceed in full conformity to one of the examples, it may make reference to it in its quality manual and in any certificate issued.

Examples H1, H2 and H3 provide a basic approach for the determination of error and uncertainties in calibration. Example H4 provides a more sophisticated approach.

Note 1: The certificate should contain all the information presented in Hn.1, as far as known, and, as applicable, at least what is printed in bold figures in Hn.2 and Hn.3, with Hn = H1, H2…

Note 2: The values in the examples are indicated with more digits that may appear in a calibration certificate for illustrative purposes.

Note 3: For rectangular distributions infinite degrees of freedom are assumed.

H1 Instrument of 220 g capacity and scale interval 0,1 mg

Preliminary note:
The calibration of a laboratory balance is demonstrated. This example shows the complete standard procedure for the presentation of measurement results and the related uncertainties, as executed by most laboratories. An alternative method for the consideration of air buoyancy effects and convection effects is also presented as option 2 (in italic type).

<u>**First situation: Adjustment of sensitivity carried out independently of calibration**</u>

H1.1/AConditions specific for the calibration

Instrument:	**Electronic weighing instrument, description and identification**
Maximum Weighing Capacity *Max*/ Scale interval *d*	**220 g / 0,1 mg**
Temperature coefficient	K_T = 1,5 x 10 $^{-6}$/K (manufacturer's manual); only necessary for calculation of the uncertainty of a weighing result.
Built-in adjustment device	Acts automatically after switching-on the balance and when ΔT ≥ 3 K. Only necessary for calculation of uncertainty of a weighing result. Status: activated
Adjustment by calibrator	**Not adjusted immediately before calibration.**
Temperature during calibration	**21 °C measured at the beginning of calibration.**
Barometric pressure and humidity (*optional*)	*990 hPa, 50 % RH.*
Room conditions	Maximum temperature deviation 5 K (laboratory room without windows).If used for calculation of the buoyancy uncertainty as per formula 7.1.2-5e, it must be presented in the calibration certificate. Not relevant for the uncertainty of a weighing result, when built-in adjustment device is activated (ΔT ≥ 3 K). In this case the maximum temperature variation for the estimation of the uncertainty of a weighing result is 3 K.
Test loads/ acclimatization	**Standard weights, class E_2**, acclimatized to room temperature (*in option 2 a temperature difference of 2 K against room temperature is taken into account*).

H1.2/A Tests and results

Repeatability Requirements given in Chapter **5.1.** Indication at no load reset to zero where necessary; indications recorded.	**Test load 100 g (applied 5 times)**
	100,000 6 g
	100,000 3 g
	100,000 5 g
	100,000 4 g
	100,000 5 g
Standard deviation	***s* = 0,00011 g**

Eccentricity	**Position of the load**	**Test load 100 g**
Requirements given in Chapter **5.3.** Indication set to zero prior to test; load put in centre first then moved to the other positions.	Middle	100,000 6 g
	Front left	100,000 4 g
	Back left	100,000 5 g
	Back right	100,000 7 g
	Front right	100,000 5 g
Maximum deviation	$\left|\Delta I_{ecci}\right|_{max}$	**0,000 2 g**

Errors of indication:

General prerequisites: Requirements given in Chapter 5.2, weights distributed fairly evenly over the weighing range.
Test loads each applied once; discontinuous loading only upwards, indication at no load reset to zero if necessary.

<u>Option 1: Air densities unknown during adjustment and during calibration (i.e. no buoyancy correction applied to the error of indication values)</u>

Load m_{ref}	**Indication *I***	**Error of indication *E***
0,0000 g	0,000 0g	0,000 0 g
50,0000 g	50,000 4 g	0,000 4 g
99,9999 g	100,000 6 g	0,000 7 g
149,9999 g	150,000 9 g	0,001 0 g
220,0001 g	220,001 4 g	0,001 3 g

<u>*Option 2: Air density ρ_{as} unknown during adjustment and air density ρ_{acal} during calibration calculated according to the simplified CIPM formula (A1.1-1)*</u>

Measurement values used for calculation:
Barometric pressure p: *990 hPa*
Relative humidity RH: *50 %RH*
Temperature t: *21 °C*
Air density ρ_{acal}: *1,173 kg/m³*

Calculated buoyancy correction δm_B according to formula (4.2.4-4).

Numerical value used for calculation:
Density of the reference mass ρ_{cal}: *(7950 ± 70) kg/m³*
Buoyancy correction δm_B: *2,138 x 10^{-8} m_{ref}*

The calculated buoyancy correction δm_B of m_{ref} of load L following formula (4.2.4-4) is negligible as the relative resolution of the instrument is in the order of 10^{-6} and thus much larger than the buoyancy correction. The above table is effectual.

H1.3/A Errors and related uncertainties (budget of related uncertainties)

Conditions common to both options:

- The uncertainty for the zero position only results from the digitalisation d_0 and repeatability *s.*

- The eccentric loading is taken into account for the calibration according to (7.1.1-10).
- The conventional value of the test weights (class E_2) is taken into account for the calibration results. Therefore $U(\delta m_c) = U/k$ is calculated following formula (7.1.2-2).
- The drift of the weights has been statistically monitored and the factor k_D of formula (7.1.2-11) was chosen as 1,25.
- The degrees of freedom for the calculation of the coverage factor k are derived following appendix B3 and table G.2 of [1]. In the case of the example, the influence of the uncertainty of the repeatability test with 5 measurements is significant.
- The information about the relative uncertainty $U(E)_{rel} = u(E)/L$ is not mandatory, but helps to demonstrate the characteristics of the uncertainties.

Uncertainty budget for option 1 (no buoyancy correction applied to the error of indication values)

Additional condition:

The balance is not adjusted immediately before calibration. The procedure according to option 1 is applied, with no information about air density. Therefore formula (7.1.2-5d) is applied for the uncertainty due to air buoyancy. As an alternative in the table, formula (7.1.2-5e) was used, thereby assuming a temperature variation during use of 5 K.

Quantity or Influence	Load, indication and error in g Uncertainties in g					Formula
Load m_{ref} /g	**0,000 0**	**50,000 0**	**99,999 9**	**149,999 9**	**220,000 1**	
Indication *I* /g	**0,000 0**	**50,000 4**	**100,000 6**	**150,000 9**	**220,001 4**	
Error of indication *E* /g	**0,000 0**	**0,000 4**	**0,000 7**	**0,001 0**	**0,001 3**	7.1-1
Repeatability $u(\delta I_{rep})$/g	**0,000 114**					7.1.1-5
Resolution $u(\delta I_{dig0})$ /g	0,000 029					7.1.1-2a
Resolution $u(\delta I_{digL})$ /g	0,000 000	0,000 029				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$/g	0,000 000	0,000 029	0,000 058	0,000 087	0,000 127	7.1.1-10
Uncertainty of the indication $u(I)$/g	0,000 118	0,000 124	0,000 134	0,000 149	0,000 175	7.1.1-12
Test loads m_c /g	0,000 0	50,000 0	99,999 9	99,999 9 50,000 0	200,000 1 20,000 0	
Conventional mass $u(\delta m_c)$/g	0,000 000	0,000 015	0,000 025	0,000 040	0,000 062	7.1.2-2
Drift $u(\delta m_D)$/g	0,000 000	0,000 022	0,000 036	0,000 058	0,000 089	7.1.2-11
Buoyancy $u(\delta m_B)$/g	0,000 000	0,000 447	0,000 889	0,001 330	0,001 960	7.1.2-5d / Table E2.1
Convection $u(\delta m_{conv})$/g	Not relevant in this case (weights are acclimatized).					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$/g	0,000 000	0,000 448	0,000 890	0,001 332	0,001 963	7.1.2-14
Standard uncertainty of the error $u(E)$ /g	0,000 118	0,000 465	0,000 900	0,001 340	0,001 971	7.1.3-1a
ν_{eff} (degrees of freedom)	4	1104	15538	76345	357098	B3-1
***k*(95,45 %)**	**2,87**	**2,00**	**2,00**	**2,00**	**2,00**	[1]
***U*(*E*) = *ku*(*E*)/g**	**0,000 34**	**0,000 93**	**0,001 80**	**0,002 68**	**0,003 94**	7.3-1
$U_{rel}(E)$/%	----	0,001 86	0,001 80	0,001 79	0,001 79	
Alternative: Uncertainty due to buoyancy with formula (7.1.2-5e) instead of (7.1.2-5d), i.e. substituting the worst case approach with a value derived from the estimated room temperature variations of 5 K during use.						
Buoyancy $u(\delta m_B)$/g	0,000 000	0,000 103	0,000 201	0,000 304	0,000 446	7.1.2-5e
Uncertainty of the reference mass $u(m_{ref})$/g	0,000 000	0,000 107	0,000 205	0,000 312	0,000 459	7.1.2-14
Standard uncertainty of the error $u(E)$ /g	0,000 118	0,000 164	0,000 245	0,000 346	0,000 491	7.1.3-1a
ν_{eff} (degrees of freedom)	4	17	85	338	1377	B3-1
***k*(95,45 %)**	**2,87**	**2,16**	**2,03**	**2,01**	**2,00**	[1]
***U*(*E*) = *ku*(*E*)/g**	**0,000 34**	**0,000 35**	**0,000 50**	**0,000 69**	**0,000 98**	7.3-1
$U_{rel}(E)$/%	----	0,000 70	0,000 50	0,000 46	0,000 45	

It is seen in this example that the uncertainty of the reference mass is reduced significantly if an uncertainty contribution for buoyancy is taken into account that is based on the estimated room temperature changes during use rather than using the most conservative approach provided by (7.1.2-5d).

It would be acceptable to state in the certificate only the largest value of expanded uncertainty for all the reported errors: ***u*(*E*) = 0,003 94 g** (or alternatively **0,000 98 g**) based on ***k* = 2,00** accompanied by the statement that the coverage probability is at least 95 %.

The certificate shall give the advice to the user that the expanded uncertainty stated in the certificate is only applicable, when the error (E) is taken into account.

<u>*Uncertainty budget for option 2 (buoyancy correction applied to the error of indication values)*</u>

Additional condition:

The balance is not adjusted immediately before calibration. The procedure according to option 2 is applied, taking into account the determination of the air density and buoyancy correction. Therefore, formula (7.1.2-5a) is applied for the uncertainty due to air buoyancy.

Option 2 above has shown that the buoyancy correction δm_B is negligible as it is smaller than the relative resolution of the instrument, but the result of the calculation is nevertheless shown in the table below. Now, the uncertainty of the buoyancy correction $u(\delta m_B)$ is calculated using formula (7.1.2-5a). Note that the air density during adjustment (which occurred independently of the calibration) is unknown, so that the variation of air density over time is taken as an estimate for the uncertainty. Consequently, the uncertainty of the air density is derived based on assumptions for pressure, temperature and humidity variations which can occur at the installation site of the instrument.

Appendix A3 provides advice to estimate the uncertainty of the air density. The example uses the approximation of the uncertainty based on (A3-2) instead of the general equation (A3-1), i.e. with temperature being the only free parameter.

For a temperature variation of 5 K, the calculation with the approximation formula (A3-2) leads to a relative uncertainty of $u(\rho_a)/\rho_a$ = 1,18 × 10^{-2}, which, for an air density at calibration of ρ_a = 1,173 kg/m³, leads to an uncertainty $u(\rho_a)$= 0,014 kg/m³.The same result can be obtained if the exact formula for the uncertainty of the air density (A3-1) is taken.

The following numeric values are taken to calculate the relative uncertainty of the buoyancy correction, using formula (7.1.2-5a):

Air density ρ_{aCal}:	*(1,173 ± 0,014) kg/m³*
Density of the reference mass ρ_{Cal}:	*(7950 ± 70) kg/m³*

Formula (7.1.2-5a) leads to the relative uncertainty of the buoyancy correction of $u_{rel}(\delta m_B)$ = 3,203 × 10^{-8}

The relative uncertainty of the buoyancy correction is negligible as compared to the other contributions to the uncertainty of the reference mass but the result of the calculation is nevertheless shown in the table below.

This example has shown that the calculated correction of the error δm_B and the calculated relative uncertainty of the buoyancy correction $u(\delta m_B)$ are both negligible. This leads to an updated measurement uncertainty budget.

The uncertainty of convection effects due to non-acclimatized weights $u(\delta m_{conv})$ for a temperature difference of 2 K is shown. The rest of the uncertainty contributions are the same as in the table above and are not repeated in the table below.

Quantity or Influence	***Load, indication and error in g Uncertainties in g***					***Formula***
Load m_{ref} /g	***0,000 0***	***50,000 0***	***99,999 9***	***149,999 9***	***220,000 1***	
Correction δm_B /g	*0,000 0*	*0,000 001*	*0,000 002*	*0,000 003*	*0,000 005*	*4.2.4.3*
Indication I /g	***0,000 0***	***50,000 4***	***100,000 6***	***150,0009***	***220,001 4***	
Error of indication E /g	***0,000 0***	***0,000 4***	***0,000 7***	***0,001 0***	***0,001 3***	*7.1-1*
Buoyancy$u(\delta m_B)$/g	*0,000 0*	*0,00000 2*	*0,000 003*	*0,000 005*	*0,000 007*	*7.1.2-5a*
Convection$u(\delta m_{conv})$/g	*0,000 0*	*0,000 029*	*0,000 046*	*0,000 075*	*0,000 092*	*7.1.2-13 / Table F2.1*
Uncertainty of the reference mass$u(\boldsymbol{m}_{ref})$/g	*0,000 0*	*0,000 039*	*0,000 064*	*0,000 103*	*0,000 143*	*7.1.2-14*
Standard uncertainty of the error$u(E)$ /g	*0,000 118*	*0,000 130*	*0,000 149*	*0,000 181*	*0,000 226*	*7.1.3-1a*
ν_{eff}(degrees of freedom)	*4*	*6*	*11*	*25*	*62*	*B3-1*
k(95,45 %)	***2,87***	***2,52***	***2,25***	***2,11***	***2,05***	*[1]*
$U(E) = ku(E)$/g	***0,000 34***	***0,000 33***	***0,000 33***	***0,000 38***	***0,000 46***	*7.3-1*
$U_{rel}(E)$/%	*-----*	*0,000 66*	*0,000 33*	*0,000 25*	*0,000 21*	

It can be seen from this example that the contribution of buoyancy to the standard uncertainty is significant when the most conservative approach following formula (7.1.2-5d) is chosen.

If information about the temperature estimated room temperature variations during use is available and the uncertainty of the air buoyancy is calculated following formula (7.1.2-5e), the difference in the uncertainty of the error is less significant.

H1.4/A Uncertainty of a weighing result (for option 1)

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. The results must not be presented as part of the calibration certificate except for the approximated error of indication and the uncertainty of the approximated error which can form part of the certificate. Usually the information on the uncertainty of a weighing result is presented as an appendix to the calibration certificate or is otherwise shown if its contents are clearly separated from the calibration results.

Normal conditions of use of the instrument, as assumed, or as specified by the user may include:

- Built-in adjustment device available and activated ($\Delta T \geq 3$ K).Variation of room temperature $\Delta T = 5$ K.
- Tare balancing function operated.
- Loads not always centred carefully.

The uncertainty of a weighing result is derived using a linear approximation of the error of indication according to (C2.2-16).

The uncertainty of a weighing result is presented for option 1 only (no buoyancy correction applied to the error of indication values). The approximated error of indication per (C2.2-16) and the uncertainty of the approximated error of indication per (C2.2-16d) differ insignificantly between both options as the underlying weighting factors

$p_j = 1/u^2(E_j)$ differ in the order of a few parts per million, and the errors of indication are the same for both options (buoyancy correction smaller than the resolution of the instrument).

The designations *R* and *W* are introduced to differentiate from the weighing instrument indication *I* during calibration.

R: Reading when weighing a load on the calibrated instrument obtained after the calibration
W: Weighing result

Note that within the following table the reading *R* and all results are in g.

Quantity or Influence	Reading, weighing result and error in g Uncertainties in g or as relative value	Formula
Error of Indication $E_{appr}(R)$ for gross or net readings: Approximation by a straight line through zero	$E_{appr}(R) = 6{,}709 \times 10^{-6} R$	C2.2-16
Uncertainty of the approximated error of indication		
Standard uncertainty of the error $u(E_{appr})$	$u^2(E_{appr}) = 4{,}501 \times 10^{-11} u^2(R) + 1{,}543 \times 10^{-12} R^2$ [10]	C2.2-16d
Standard uncertainty of the error, neglecting the offset	$u(E_{appr}) = 1{,}242 \times 10^{-6} R$	
Uncertainties from environmental influences		
Temperature drift of sensitivity	$u_{rel}(\delta R_{temp}) = 1{,}299 \times 10^{-6}$	7.4.3-1
Buoyancy	$u_{rel}(\delta R_{buoy}) = 1{,}636 \times 10^{-6}$	7.4.3-4
Change in characteristics due to drift	Not relevant in this case (built-in adjustment activated and drift between calibrations negligible).	7.4.3-5
Uncertainties from the operation of the instrument		
Tare balancing operation	$u_{rel}(\delta R_{Tare}) = 1{,}072 \times 10^{-6}$	7.4.4 7.4.4-5
Creep, hysteresis (loading time)	Not relevant in this case (short loading time).	7.4.4-9a/b
Eccentric loading	$u_{rel}(\delta R_{ecc}) = 1{,}155 \times 10^{-6}$	7.4.4-10
Uncertainty of a weighing result		
Standard uncertainty, corrections to the readings $u(E_{appr})$ to be applied	$u(W) = \sqrt{(1{,}467 \times 10^{-8}\,g^2 + 8{,}390 \times 10^{-12} R^2)}$	7.4.5-1a 7.4.5-1b
Standard uncertainty, corrections to the readings $u(E_{appr})$ to be applied	$U(W) = 2\sqrt{(1{,}467 \times 10^{-8}\,g^2 + 8{,}390 \times 10^{-12} R^2)}$	7.5.-2b
Simplified to first order	$U(W) \approx 2{,}422 \times 10^{-4}\,g + 4{,}796 \times 10^{-6} R$	7.5.2-3d
Global uncertainty of a weighing result without correction to the readings		
$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$	$U_{gl}(W) \approx 2{,}422 \times 10^{-4}\,g + 1{,}150 \times 10^{-5} R$	7.5.2-3a 7.5.2-3e

[10] The first term is negligible as the uncertainty of the reading $u(R)$ is in the order of some g. Thus the first term is in the order of 10^{-7} g^2 while the second term represents values up to 15 g^2.

The condition regarding the observed chi-squared value following (C2.2-2a) was checked with positive result. The first linear regression is taking into account the weighing factors p'_j, equation (C2.2-18b).

Based on the global uncertainty, the minimum weight value for the instrument may be derived as per Appendix G.

Example:

Weighing tolerance requirement: 1 %

Safety factor: 3

The minimum weight according to formula (G-9), using the above equation for the global uncertainty in results is 0,072 9 g; i.e. the user needs to weigh a net quantity of material that exceeds 0,072 9 g in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1 % and a safety factor of 3 (equals a relative weighing tolerance of 0,33 %).

Second situation: Adjustment of sensitivity carried out immediately before calibration

H1.1/B Conditions specific for the calibration

Instrument:	**Electronic weighing instrument, description and identification**
Maximum Capacity *L*/Scale interval *d*	**220 g / 0,000 1 g**
Temperature coefficient	K_T = 1,5 x 10 $^{-6}$/K (manufacturer's manual) Only necessary for calculation of the uncertainty of a weighing result.
Built-in adjustment device	Acts automatically upon: after switch-on of the balance, and when $\Delta T \geq$ 3 K. Only necessary for calculation of uncertainty of a weighing result.Status: activated.
Adjustment by calibrator	**Adjusted immediately before calibration (built-in adjustment weights).**
Temperature during calibration	**21 °C measured at the beginning of calibration.**
Barometric pressure and humidity (optional)	*990 hPa, 50 % RH.*
Room conditions	Max. Temperature deviation 5 K (laboratory room without windows). Not relevant, when built-in adjustment device is activated ($\Delta T \geq$ 3 K). In this case the maximum temperature variation for the estimation of uncertainty of a weighing result is 3 K.
Test loads / acclimatization	**Standard weights, class E_2**, acclimatized to room temperature *(alternative a temperature difference of 2 K against room temperature is taken into account*).

H1.2/B Tests and results

<u>Option 1: Air densities unknown during adjustment /calibration (i.e. no buoyancy correction applied to the error of indication values)</u>

The repeatability test is omitted and the results from the first calibration are taken into account. Also the eccentricity test was omitted and the results from the first calibration are taken into account. This can be done as only the sensitivity of the balance was adjusted and no influence to the repeatability and the eccentricity test can be estimated. Air density not calculated.

Errors of indication Requirements given in Chapter 5.2, weights distributed fairly evenly	Test loads each applied once; discontinuous loading only upwards, indication at no load reset to zero where necessary. Indications recorded:	
Load m_{ref}	**Indication *I***	**Error of indication *E***
0,000 0 g	0,000 0 g	0,000 0 g
50,000 0 g	50,000 0 g	0,000 0 g
99,999 9 g	99,999 8 g	- 0,000 1 g
149,999 9 g	149,999 9 g	0,000 0 g
220,000 1 g	220,000 0 g	- 0,000 1 g

<u>*Option 2: Air density ρ_{as} during adjustment and air density ρ_{acal} during calibration are identical as an adjustment was carried out immediately before calibration.*</u>

The air density is calculated according to the simplified CIPM formula (A1.1-1)

Measurement values used for calculation:
Barometric pressure p: *990 hPa*
Relative humidity RH: *50 %*
Temperature t: *21 °C*
Density ρ_s and ρ_{Cal}: *(7950 ± 70) kg/m³*
Air density ρ_{aCal}: *1,173 kg/m³*

Calculated buoyancy correction δm_B according to formula (4.2.4-4).

Numerical value used for calculation:
Density of the reference mass ρ_{Cal}: *(7950 ± 70) kg/m³*
Buoyancy correction δm_B: *2,138 x 10^{-8} m_{ref}*

The calculated buoyancy correction δm_B of m_{ref} of Load L following formula (4.2.4-4) is negligible as the relative resolution of the instrument is in the order of 10^{-6} and thus much larger than the buoyancy correction. The above table is effectual.

H1.3/B Errors and related uncertainties (budget of related uncertainties)

Conditions:

- The uncertainty for the zero position only results in the digitalisation d_0 and repeatability *s.*
- The eccentric loading is taken into account for the calibration according to (7.1.1-10).
- The conventional mass of the test weights (class E_2) is taken into account for the calibration results. Therefore $U(\delta m_c) = U/k$ is calculated following formula 7.1.2-2.

- The drift of the weights was statistically monitored and the factor k_D of formula 7.1.2-11 was chosen as 1,25.
- The degrees of freedom for the calculation of the coverage factor k are derived following appendix B3 and table G.2 of [1]. In the case of the example, the influence of the uncertainty of the repeatability test with 5 measurements is significant.
- The information about the relative uncertainty $U(E)_{rel} = u(E)/m_{ref}$ is not mandatory, but helps to demonstrate the characteristics of the uncertainties.

<u>Uncertainty budget for option 1 (no buoyancy correction applied to the error of indication values)</u>

Additional condition:
The balance is adjusted immediately before the calibration and no information about air density at the time of calibration is available. Therefore, formula (7.1.2-5c) is relevant.

Quantity or Influence	Load, indication and error in g Uncertainties in g					Formula
Load m_{ref} /g	**0,000 0**	**50,000 0**	**99,999 9**	**149,999 9**	**220,000 1**	
Indication I / g	**0,000 0**	**50,000 0**	**99,999 8**	**149,999 9**	**220,000 0**	
Error of indication E /g	**0,000 0**	**0,000 0**	**-0,000 1**	**0,000 0**	**-0,000 1**	7.1-1
Repeatability $u(\delta I_{rep})$/g	**0,000 114**					7.1.1-5
Resolution $u(\delta I_{dig0})$ /g	0,000 029					7.1.1-2a
Resolution $u(\delta I_{digL})$ /g	0,000 0	0,000 029				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$/g	0,000 0	0,000 029	0,000 058	0,000 087	0,000 127	7.1.1-10
Uncertainty of the indication $u(I)$/g	0,000 118	0,000 124	0,000 134	0,000 149	0,000 175	7.1.1-12
Test loads m_c /g	0,000 0	50,000 0	99,999 9	99,999 9 50,000 0	200,000 1 20,000 0	
Conventional mass $u(\delta m_c)$/g	0,000 0	0,000 015	0,000 025	0,000 040	0,000 063	7.1.2-2
Drift $u(\delta m_D)$/g	0,000 0	0,000 022	0,000 036	0,000 058	0,000 090	7.1.2-10
Buoyancy $u(\delta m_B)$/g	0,000 000	0,000 014	0,000 022	0,000 036	0,000 055	7.1.2-5c / Table E2.1
Convection $u(\delta m_{conv})$/g	Not relevant in this case (weights are acclimatized)					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$/g	0,000 00	0,000 03	0,000 049	0,000 079	0,000 123	7.1.2-14
Standard uncertainty of the error $u(E)$ /g	0,000 118	0,000 128	0,000 143	0,000 169	0,000 214	7.1.3-1a
ν_{eff}(degrees of freedom)	4	6	9	19	49	B3-1
***k*(95,45 %)**	**2,87**	**2,52**	**2,32**	**2,14**	**2,06**	[1]
$U(E) = ku(E)$/g	**0,000 34**	**0,000 32**	**0,000 33**	**0,000 36**	**0,000 44**	7.3-1
$U_{rel}(E)$/%	----	0,000 64	0,000 33	0,000 24	0,000 20	

It would be acceptable to state in the certificate only the largest value of expanded uncertainty for all the reported errors: **$U(E)$= 0,000 44 g**, based on **k = 2,06** accompanied by the statement that the coverage probability is at least 95 %.The certificate shall give the advice to the user that the expanded uncertainty stated in the certificate is only applicable, when the Error (E) is taken into account.

Uncertainty budget for option 2 (buoyancy correction applied to the error of indication values)

Additional condition:

The balance is adjusted immediately before calibration. The procedure according to option 2 is applied, taking into account the determination of the air density and buoyancy correction. Therefore, formula (7.1.2-5a) is applied for the uncertainty due to air buoyancy.

As an adjustment has been carried out immediately before the calibration, the expected maximum values for pressure, temperature and humidity variations which can occur at the installation site of the instrument do not have to be taken into account in contrast to the scenario where the adjustment has been performed independent of the calibration. The only contributing factor to the standard uncertainty of the air density originates from the uncertainty of the measurement of the environmental parameters.

The following numeric values are taken to calculate the relative uncertainty of the buoyancy correction, using formula (7.1.2-5a):

Air density ρ_{aCal}:	*1,173 kg/m³*
Density of the reference mass ρ_{Cal}:	*(7950 ± 70) kg/m³*

Furthermore, the following uncertainties for temperature, pressure and humidity measurement are taken for calculating the relative uncertainty of the air density according to (A3-1):

$u(T)$ *= 0,2 K*
$u(p)$ *= 50 Pa*
$u(RH)$ *= 1%*

This leads to $\frac{u(\rho_a)}{\rho_a}$ *= 9,77 × 10^{-4}, and* $u(\rho_a)$ *= 0,00115 kg/m³.*

Formula (7.1.2-5a) leads to the relative uncertainty of the buoyancy correction of $u(\delta m_B)$ = 3,014 × 10^{-8}

As an alternative the additional uncertainty of convection effects due to non-acclimatized weights $u(\delta m_{conv})$ for a temperature difference of 2 K is shown.

Quantity or Influence	*Load, indication and error in g Uncertainties in g*					*Formula*
Load m_{ref} /g	*0,000 0*	*50,000 0*	*99,999 9*	*149,999 9*	*220,000 1*	
Correction δm_B /g	*0,000 0*	*0,000 001*	*0,000 002*	*0,000 003*	*0,000 005*	*4.2.4-4*
Indication I /g	*0,000 0*	*50,000 0*	*99,999 8*	*149,999 9*	*220,000 0*	
Error of indication E /g	*0,000 0*	*0,000 0*	*-0,000 1*	*0,000 0*	*-0,000 1*	
Buoyancy $u(\delta m_B)$/g	*0,000 0*	*0,000 001 5*	*0,000 003 0*	*0,000 004 5*	*0,000 006 6*	*7.1.2-5a*
Convection $u(\delta m_{conv})$/g	Not relevant in this case (weights are acclimatized).					
Uncertainty of the reference mass $u(m_{ref})$ /g	*0,000 000*	*0,000 026*	*0,000 044*	*0,000 066*	*0,000 110*	*7.1.2-14*
Standard uncertainty of the error $u(E)$ /g	*0,000 118*	*0,000 127*	*0,000 141*	*0,000 163*	*0,000 207*	*7.1.3-1a*
ν_{eff}(degrees of freedom)	*4*	*6*	*9*	*16*	*43*	*B3-1*
k(95,45 %)	***2,87***	***2,52***	***2,32***	***2,17***	***2,06***	*[1]*
U(E) = ku(E)/g	***0,000 34***	***0,000 32***	***0,000 33***	***0,000 35***	***0,000 43***	*7.3-1*
$U_{rel}(E)$/ %	*----*	*0,000 64*	*0,000 33*	*0,000 23*	*0,000 20*	
Alternative the additional uncertainty of convection effects due to non-acclimatized weights $u(\delta m_{conv})$ for a temperature difference of 2 K is shown.						
Convection $u(\delta m_{conv})$/g	*0,000 000*	*0,000 029*	*0,000 046*	*0,000 075*	*0,000 092*	*7.1.2-13*
Uncertainty of the reference mass $u(m_{ref})$ /g	*0,000 000*	*0,000 031*	*0,000 051*	*0,000 079*	*0,000 122*	*7.1.2-14*
Standard uncertainty of the error $u(E)$ / g	*0,000 118*	*0,000 128*	*0,000 144*	*0,000 168*	*0,000 214*	*7.1.3-1a*
ν_{eff} (degrees of freedom)	*4*	*6*	*10*	*19*	*49*	*B3-1*
k(95,45 %)	***2,87***	***2,52***	***2,28***	***2,14***	***2,06***	*[1]*
U(E) = ku(E)/g	***0,000 34***	***0,000 32***	***0,000 33***	***0,000 36***	***0,000 44***	*7.3-1*
$U_{rel}(E)$/%	*----*	*0,000 64*	*0,000 33*	*0,000 24*	*0,000 20*	

The expanded uncertainties of the error using option 1 and using the option 2 are almost identical as the uncertainty of the reference mass $u(m_{ref})$ is very small as compared to the uncertainty of the indication $u(I)$. In this example, the determination of pressure and humidity on site to determine the buoyancy correction and to minimize the uncertainty contribution due to buoyancy does not significantly improve the results of the calibration.

H1.4/B Uncertainty of a weighing result (for option 1)

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. The results must not be presented as part of the calibration certificate except for the approximated error of indication and the uncertainty of the approximated error which can form part of the certificate. Usually the information on the uncertainty of a weighing result is presented as an appendix to the calibration certificate or is otherwise shown if its contents are clearly separated from the calibration results.

Normal conditions of use of the instrument, as assumed, or as specified by the user may include:

- Built-in adjustment device available and activated ($\Delta T \geq 3$ K)
- Variation of room temperature $\Delta T = 5$ K
- Tare balancing function operated

- Loads not always centred carefully

The uncertainty of a weighing result is derived using a linear approximation of the error of indication according to (C2.2-16).

The uncertainty of a weighing result is presented for option 1 only (no buoyancy correction applied to the error of indication values). The approximated error of indication per (C2.2-16) and the uncertainty of the approximated error of indication per (C2.2-16d) differ insignificantly between both options as the underlying weighting factors $p_j = 1/u^2(E_j)$ differ in the order of a few per mil, and the errors of indication are the same for both options (buoyancy correction smaller than the resolution of the instrument).

The designations *R* and *W* are introduced to differentiate from the weighing instrument indication *I* during calibration.

R: Reading when weighing a load on the calibrated instrument obtained after the calibration
W: Weighing result

Note that within the following table the reading *R* and all results are in g.

Quantity or Influence	Reading, weighing result and error in kg Uncertainties in g or as relative value	Formula
Error of Indication $E_{appr}(R)$ for gross or net readings: Approximation by a straight line through zero	$E_{appr}(R) = -3{,}895 \times 10^{-7} R$	C2.2-16
Uncertainty of the approximated error of indication		
Standard uncertainty of the error $u(E_{appr})$	$u^2(E_{appr}) = 1{,}517 \times 10^{-13} u^2(R) + 4{,}015 \times 10^{-13} R^2$ [11]	C2.2-16d
Standard uncertainty of the error, neglecting the offset	$u(E_{appr}) = 6{,}337 \times 10^{-7} R$	
Uncertainties from environmental influences		
Temperature drift of sensitivity	$u_{rel}(\delta R_{temp}) = 1{,}299 \times 10^{-6}$	7.4.3-1
Buoyancy	$u_{rel}(\delta R_{buoy}) = 1{,}636 \times 10^{-6}$	7.4.3-4
Change in characteristics due to drift	Not relevant in this case (built-in adjustment activated and drift between calibrations negligible)	7.4.3-5
Uncertainties from the operation of the instrument		
Tare balancing operation	$u_{rel}(\delta R_{Tare}) = 5{,}774 \times 10^{-7}$	7.4.4-5
Creep, hysteresis (loading time)	Not relevant in this case (short loading time).	7.4.4-9a/b
Eccentric loading	$u_{rel}(\delta R_{ecc}) = 1{,}154 \times 10^{-6}$	7.4.4-10
Uncertainty of a weighing result		
Standard uncertainty, corrections to the readings $u(E_{appr})$ to be applied	$u(W) = \sqrt{(1{,}466 \times 10^{-8}\,\text{g}^2 + 6{,}433 \times 10^{-12} R^2)}$	7.4.5-1a 7.4.5-1b
Standard uncertainty, corrections to the readings $u(E_{appr})$ to be applied	$U(W) = 2\sqrt{(1{,}466 \times 10^{-8}\,\text{g}^2 + 6{,}433 \times 10^{-12} R^2)}$	7.5.1-2b
Simplified to first order	$U(W) \approx 2{,}422 \times 10^{-4}\,\text{g} + 4{,}090 \times 10^{-6} R$	7.5.2-3d
Global uncertainty of a weighing result without correction to the readings		
$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$	$U_{gl}(W) \approx 2{,}422 \times 10^{-4}\,\text{g} + 4{,}479 \times 10^{-6} R$	7.5.2-3a

The condition regarding the observed chi-squared value following (C2.2-2a) was checked with positive result. The first linear regression taking into account the weighing factors p'_j, equation (C2.2-18b).

Based on the global uncertainty, the minimum weight value for the instrument may be derived as per Appendix G.

Example:

Weighing tolerance requirement: 1%

Safety factor: 3

The minimum weight according to formula (G-9), using the above equation for the global

[11] The first term is negligible as the uncertainty of the reading $u(R)$ is in the order of some mg. Thus the first term is in the order of 10^{-11} mg² while the second term represents values up to 10^{-7} mg².

uncertainty results in 0,0727 g; i.e. the user needs to weigh a net quantity of material that exceeds 0,0727 g in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1 % and a safety factor of 3 (equals a relative weighing tolerance of 0,33%).

H2 Instrument of 60 kg capacity, multi-interval

Preliminary note:
The calibration of a multi-interval balance with scale intervals 2 g / 5 g / 10 g is demonstrated. This example shows the complete standard procedure for the presentation of measurement results and the related uncertainties as executed by most laboratories. An alternative method for the consideration of air buoyancy effects is also presented as option 2 (in italic type).

<u>First situation: adjustment of sensitivity carried out independently of calibration</u>

H2.1/A Conditions specific for the calibration

Instrument	**Electronic non-automatic weighing instrument, description and identification**
Upper limits of the intervals Max_i/Scale intervals d_i	**12 000 g / 2 g** **30 000 g / 5 g** **60 000 g / 10 g**
Sensitivity of the instrument to temperature variation	K_T= 2×10^{-6}/K (manufacturer's manual); only necessary for calculation of the uncertainty of a weighing result.
Built-in adjustment device	Acts automatically after switching on the balance, and when $\Delta T \geq$ 3 K; only necessary for calculation of uncertainty of a weighing result. Status: activated.
Adjustment by calibrator	**Not adjusted immediately before calibration.**
Temperature during calibration	**21 °C at the beginning of calibration** **23 °C at the end of the calibration.**
Barometric pressure and humidity (optional)	*990 hPa, 50 % RH.*
Room conditions	Maximum temperature variation during use 10 K (laboratory room with windows). If used for the buoyancy uncertainty as per formula 7.1.2-5e, it must be presented in the calibration certificate. Not relevant for the uncertainty of a weighing result, when built-in adjustment device is activated ($\Delta T \geq$ 3 K). In this case the maximum temperature variation for the estimation ofthe uncertainty of a weighing result is 3 K.
Test loads / Acclimatization	**Standard weights, class F_2**, acclimatized to room temperature.

H2.2/A Tests and results

Repeatability Requirements given in Chapter **5.1** Indication at no load reset to zero where necessary Repeatability test carried out in interval 1 and 2	**Test load 10 000 g** applied 5 times(standard deviation assumed constant over interval 1)	**Test load 25 000 g** applied 5 times(standard deviation assumed constant over interval 2 and 3)
	9 998 g	24 995 g
	10 000 g	25 000 g
	9 998 g	24 995 g
	10 000 g	24 995 g
	10 000 g	25 000 g
Standard deviation	***s* = 1,095 g**	***s* = 2,739 g**

Eccentricity Requirements given in Chapter **5.3** Indication set to zero prior to test; load put in centre first then moved to the other positions	**Position of the load**	**Test load 20 000 g**
	Centre	19 995 g
	Front left	19 995 g
	Back left	19 995 g
	Back right	19 990 g
	Front right	19 990 g
Maximum deviation	$\left\|\Delta I_{ecci}\right\|_{max}$	**5 g**

Errors of indication

General prerequisites: Requirements given in Chapter 5.2, weights distributed fairly evenly over the weighing range.

Test loads each applied once; discontinuous loading only upwards, indication at no load reset to zero if necessary.

<u>Option 1: Air density unknown during adjustment and during calibration (i.e. no buoyancy correction applied to the error of indication values)</u>

Requirements given in chapter **5.2**, weights distributed fairly evenly. Test loads each applied once; discontinuous loading only upwards; indication at no load reset to zero where necessary	**Load m_{ref} (m_N)**	**Indication *I***	**Error of indication *E***
	0 g	0 g	0 g
	10 000 g	10 000 g	0 g
	20 000 g	19 995 g	-5 g
	40 000 g	39 990 g	- 10 g
	60 000 g	59 990 g	- 10 g

Option 2: Air density ρ_{as} during adjustment unknown and air density ρ_{aCal} during calibration calculated according to the simplified CIPM formula (A1.1-1)

Measurement values used for calculation:

Barometric pressure p:	*990 hPa*
Relative humidity RH:	*50 %*
Temperature t:	*21 °C*
Air density ρ_{aCal}	*1,173 kg/m³*

Calculated buoyancy correction δm_B according to formula 4.2.4-4:

Numerical value used for calculation

Density of the reference mass ρ_{Cal} :	*(7950 ± 70) kg/m³*
Buoyancy correction δm_B:	*2,138 × $10^{-8} m_N$*

The calculated correction δm_B of the loads m_N following formula 4.2.4-4 is negligible as the relative resolution of the instrument is in the order of 10^{-4} and thus much larger than the buoyancy correction. The above table is effectual.

H2.3/A Errors and related uncertainties (budget of related uncertainties)

Conditions common to both options:

- The uncertainty of the error at zero only comprises the uncertainty of the no-load indication (scale interval $d_0 = d_1$= 2 g) and the repeatability *s.* The uncertainty of the indication at load is not taken into consideration at zero.
- The eccentric loading is taken into account for the calibration according to (7.1.1-10).
- The error of indication is derived using the nominal weight value as reference value, therefore the maximum permissible errors of the test weights are taken into account for deriving the uncertainty contribution due to the reference mass: $u(\delta m_c)$is calculated as $u(\delta m_c) = Tol/\sqrt{3}$ following formula (7.1.2-3).
- The average drift of the weights monitored over 2 recalibrations in two-yearly intervals was $|D| \leq mpe/2$. Therefore the uncertainty contribution due to the drift of the weights was set to $u(\delta m_D) = mpe/2\sqrt{3}$. This corresponds to a $k_D$factor of 1,5 (assuming the worst-case scenario of $U = mpe$ / 3).
- The weights are acclimatized with a residual temperature difference of 2 K to the ambient temperature.
- The degrees of freedom for the calculation of the coverage factor *k* are derived following appendix B3 and table G.2 of [1]. In the case of the example, the influence of the uncertainty of the repeatability test with 5 measurements is significant.
- The information about the relative uncertainty $U(E)_{rel} = U(E)/m_{ref}$ is not mandatory, but helps to demonstrate the characteristics of the uncertainties.

Uncertainty budget for option 1 (no buoyancy correction applied to the error of indication values)

Additional condition:
The balance is not adjusted immediately before calibration. The procedure according to option 1 is applied, with no information about air density. Therefore formula (7.1.2-5d) is applied for the uncertainty due to air buoyancy. As an alternative in the table, formula (7.1.2-5e) was used, thereby assuming a temperature variation during use of 10 K.

Quantity or Influence	Load, indication and error in g Uncertainties in g					Formula
Load $m_{ref}(m_N)$ /g	**0**	**10 000**	**20 000**	**40 000**	**60 000**	
Indication I /g	**0**	**10 000**	**19 995**	**39 990**	**59 990**	
Error of Indication E /g	**0**	**0**	**- 5**	**- 10**	**- 10**	7.1-1
Repeatability $u(\delta I_{rep})$ /g	**1,095**		**2,739**			7.1.1-5
Resolution $u(\delta I_{dig0})$ /g	0,577					7.1.1-2a
Resolution $u(\delta I_{digL})$ /g	0,000	0,577	1,443	2,887	2,887	7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /g	0,000	0,722	1,443	2,887	4,330	7.1.1-10
Uncertainty of the indication$u(I)$ /g	1,238	1,545	3,464	4,950	5,909	7.1.1-12
Test loads m_N /g	0	10 000	20 000	20 000 20 000	20 000 20 000 20 000	
Weights $u(\delta m_c)$ /g	0,000	0,092	0,173	0,346	0,554	7.1.2-3
Drift $u(\delta m_D)$ / g	0,000	0,046	0,087	0,173	0,277	7.1.2-11
Buoyancy $u(\delta m_B)$ /g	0,000	0,110	0,217	0,433	0,658	7.1.2-5d / Table E2.1
Convection $u(\delta m_{conv})$ /g	Not relevant in this case (only relevant for F_1 and better).					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$ /g	0,000	0,151	0,290	0,581	0,904	7.1.2-14
Standard uncertainty of the error $u(E)$ /g	1,238	1,552	3,476	4,984	5,978	7.1.3-1a
ν_{eff} (degrees of freedom)	6	16	10	43	90	B3-1
***k*(95,45 %)**	**2,52**	**2,17**	**2,28**	**2,06**	**2,05**	[1]
***U*(*E*) = *ku*(*E*)/g**	**3,120**	**3,369**	**7,926**	**10,266**	**12,254**	7.3-1
$U_{rel}(E)$/%	----	0,0337 %	0,0396 %	0,0257 %	0,0204 %	
Alternative: Uncertainty due to buoyancy with formula (7.1.2-5e) instead of (7.1.2-5d), i.e. substituting the worst case approach with a value derived from the estimated room temperature variations of 10 K during use.						
Buoyancy $u(\delta m_B)$ /g	*0,000*	*0,046*	*0,089*	*0,178*	*0,276*	*7.1.2-5e*
Uncertainty of the reference mass $u(\delta m_{ref})$ /g	*0,000*	*0,113*	*0,213*	*0,462*	*0,678*	*7.1.2-14*
Standard uncertainty of the error $u(E)$ /g	*1,238*	*1,549*	*3,471*	*4,968*	*5,948*	*7.1.3-1a*
ν_{eff} (degrees of freedom)	*6*	*16*	*10*	*43*	*88*	*B3-1*
k(95,45 %)	***2,52***	***2,17***	***2,28***	***2,06***	***2,05***	*[1]*
U(E) = ku(E)/g	***3,120***	***3,362***	***7,913***	***10,234***	***12,193***	*7.3-1*
$U_{rel}(E)$/%	**----**	*0,0336*	*0,0396*	*0,0256*	*0,0203*	

It is seen in this example that the uncertainty of the reference mass is reduced significantly if an uncertainty contribution for buoyancy is taken into account that is based on the estimated room temperature changes during use rather than using the most conservative approach provided by (7.1.2-5d). However, as the uncertainty of the reference mass is very small compared to the uncertainty of the indication, the standard uncertainty of the error is almost not affected.

*It would be acceptable to state in the certificate only the largest value of the expanded uncertainty for all the reported errors: **U(E)= 12,254 g**, based on **k = 2,05** accompanied by the statement that the coverage probability is at least 95 %.*

The certificate shall give the advice to the user that the expanded uncertainty stated in the certificate is only applicable, when the Error (E) is taken into account.

<u>*Uncertainty budget for option 2 (buoyancy correction applied to the error of indication values)*</u>

Additional condition:
The balance is not adjusted immediately before calibration. The procedure according to option 2 is applied, taking into account the determination of the air density and buoyancy correction. Therefore, formula (7.1.2-5a) is applied for the uncertainty due to air buoyancy.

Note that the air density during adjustment (which occurred independent of the calibration) is unknown, so that the variation of air density over time is taken as an estimate for the uncertainty. Consequently, the uncertainty of the air density is derived based on assumptions for pressure, temperature and humidity variations which can occur at the installation site of the instrument.

Appendix A3 provides advice to estimate the uncertainty of the air density. The example uses the approximation of the uncertainty based on (A3-2) instead of the general equation (A3-1), i.e. with temperature being the only free parameter.

For a temperature variation of 10 K, the calculation with the approximation formula (A3-2) leads to a relative uncertainty of $u(\rho_a)/\rho_a = 1{,}55 \times 10^{-2}$*, which, for an air density at calibration of* $\rho_a = 1{,}173$ *kg/m³, leads to an uncertainty* $u(\rho_a) = 0{,}018$ *kg/m³.*

The following numeric values are taken to calculate the relative uncertainty of the buoyancy correction, using formula (7.1.2-5a):

Air density ρ_{aCal}:	*(1,173 ± 0,018) kg/m³*
Density of the reference mass ρ_{Cal}:	*(7950 ± 70) kg/m³*

Formula (7.1.2-5a) leads to the relative uncertainty of the buoyancy correction of $u_{rel}(\delta m_B) = 3{,}334 \times 10^{-8}$

The relative uncertainty of the buoyancy correction is negligible as compared to the other contributions to the uncertainty of the reference mass.

This example has shown that the calculated correction of the error δm_B *and the calculated relative uncertainty of the buoyancy correction* $u_{rel}(\delta m_B)$ *are both negligible. This leads to an updated measurement uncertainty budget:*

Quantity or Influence	Load, indication and error in g Uncertainties in g					Formula
Load $m_{ref}(m_N)$/g	**0**	**10 000**	**20 000**	**40 000**	**60 000**	
Correction δm_B/g	**0**	**0**	**0**	**0**	**0**	4.2.4-4
Indication I /g	**0**	**10000**	**19 995**	**39 990**	**59 990**	
Error of Indication E/g	**0**	**0**	**- 5**	**- 10**	**- 10**	7.1-1
Repeatability $u(\delta I_{rep})$ /g	**1,095**		**2,739**			7.1.1-5
Resolution $u(\delta I_{dig0})$ /g	0,577					7.1.1-2a
Resolution $u(\delta I_{digL})$ /g	0,000	0,577	1,443	2,887	2,887	7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /g	0,000	0,722	1,443	2,887	4,330	7.1.1-10
Uncertainty of the indication $u(I)$ /g	1,238	1,545	3,464	4,950	5,909	7.1.1-12
Test loads m_N /g	0	10 000	20 000	20 000 20 000	20 000 20 000 20 000	
Weights $u(\delta m_c)$ /g	0,000	0,092	0,173	0,346	0,554	7.1.2-3
Drift $u(\delta m_D)$/ g	0,000	0,046	0,087	0,173	0,277	7.1.2-11
Buoyancy $u(\delta m_B)$ /g	0,000	0,000	0,001	0,001	0,002	7.1.2-5a
Convection $u(\delta m_{conv})$ /g	Not relevant in this case (only relevant for F_1 and better).					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$/g	0,000	0,103	0,194	0,387	0,620	7.1.2-14
Standard uncertainty of the error $u(E)$ /g	1,238	1,549	3,470	4,965	5,941	7.1.3-1a
ν_{eff} (degrees of freedom)	6	15	10	43	88	B3-1
k(95,45 %)	**2,52**	**2,17**	**2,28**	**2,06**	**2,05**	[1]
$U(E) = ku(E)$/g	**3,120**	**3,360**	**7,910**	**10,228**	**12,180**	7.3-1
$U_{rel}(E)$/%	**----**	0,0360	0,0396	0,0256	0,0203	

It can be seen from this example that the contribution of buoyancy to the standard uncertainty is insignificant. Furthermore, the standard uncertainties of the error using option 1 and option 2 are almost identical as the uncertainty of the reference mass $u(m_{ref})$ is very small as compared to the uncertainty of the indication $u(I)$. The determination of pressure and humidity on site in addition to the temperature measurement to correct for buoyancy and to minimize the associated uncertainty contribution does not significantly improve the results of the calibration.

H2.4/A Uncertainty of a weighing result (for option 1)

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. The results must not be presented as part of the calibration certificate except for the approximated error of indication and the uncertainty of the approximated error which can form part of the certificate. Usually the information on the uncertainty of a weighing result is presented as an appendix to the calibration certificate or is otherwise shown if its contents are clearly separated from the calibration results.

Normal conditions of use of the instrument, as assumed, or as specified by the user may include:

- Built-in adjustment device available and activated ($\Delta T \geq 3$ K)
- Variation of room temperature $\Delta T = 10$ K
- Tare balancing function operated

- Loads not always centred carefully

The uncertainty of a weighing result is derived using a linear approximation of the error of indication according to (C2.2-16).

The uncertainty of a weighing result is presented for option 1 only (no buoyancy correction applied to the error of indication values). The approximated error of indication per (C2.2-16) and the uncertainty of the approximated error of indication per (C2.2-16d) differ insignificantly between both options as the underlying weighting factors $p_j = 1/u^2(E_j)$ differ in the order of a few per mil, and the errors of indication are the same for both options (buoyancy correction smaller than the resolution of the instrument).

Buoyancy according to chapter 7.4.3.2 is not taken into account as the estimation of the uncertainty at calibration has shown that this influence is negligible.

The designations *R* and *W* are introduced to differentiate from the weighing instrument indication *I* during calibration.

R: Reading when weighing a load on the calibrated instrument obtained after the calibration
W: Weighing result

Note that within the following table the reading *R* and all results are in g.

Quantity or Influence		Reading, weighing result and error in g Uncertainties in g or as relative value	Formula
Error of Indication $E_{appr}(R)$ for gross or net readings: Approximation by a straight line through zero		$E_{appr}(R) = -1{,}717 \times 10^{-4} R$	C2.2-16
Uncertainty of the approximated error of indication			
Standard uncertainty of the error $u(E_{appr})$		$u^2(E_{appr}) = 2{,}950 \times 10^{-8} u^2(R) + 4{,}172 \times 10^{-9} R^2$ [12]	C2.2-16d
Standard uncertainty of the error, neglecting the intercept		$u(E_{appr}) = 6{,}459 \times 10^{-5} R$	
Uncertainties from environmental influences			
Temperature drift of sensitivity		$u_{rel}(\delta R_{temp}) = 1{,}732 \times 10^{-6}$	7.4.3-1
Buoyancy		Not relevant in this case.	7.4.3-2
Change in characteristics due to drift		Not relevant in this case (built-in adjustment activated and drift between calibrations negligible)	7.4.3-5
Uncertainties from the operation of the instrument			
Tare balancing operation		$u_{rel}(\delta R_{Tare}) = 1{,}444 \times 10^{-4}$	7.4.4-5
Creep, hysteresis (loading time)		Not relevant in this case (short loading time).	7.4.4-9a/b
Eccentric loading		$u_{rel}(\delta R_{ecc}) = 1{,}443 \times 10^{-4}$	7.4.4-10
Uncertainty of a weighing result, for partial weighing intervals (PWI)			
Standard uncertainty, corrections to the readings $u(E_{appr})$ to be applied	*PWI 1*	$u(W) = \sqrt{(1{,}867\,\text{g}^2 + 4{,}589 \times 10^{-8} R^2)}$	7.4.5-1b
	PWI 2	$u(W) = \sqrt{(9{,}917\,\text{g}^2 + 4{,}589 \times 10^{-8} R^2)}$	
	PWI 3	$u(W) = \sqrt{(16{,}167\,\text{g}^2 + 4{,}589 \times 10^{-8} R^2)}$	
Expanded uncertainty, corrections to the readings E_{appr} to be applied	*PWI 1*	$U(W) = 2\sqrt{(1{,}867\,\text{g}^2 + 4{,}589 \times 10^{-8} R^2)}$	7.5.1-2b
	PWI 2	$U(W) = 2\sqrt{(9{,}917\,\text{g}^2 + 4{,}589 \times 10^{-8} R^2)}$	
	PWI 3	$U(W) = 2\sqrt{(16{,}167\,\text{g}^2 + 4{,}589 \times 10^{-8} R^2)}$	
Simplified to first order	*PWI 1*	$U(W) \approx 2{,}733\,\text{g} + 2{,}574 \times 10^{-4} R$	7.5.2-3f
	PWI 2	$U(W) \approx 10{,}190\,\text{g} + 3{,}434 \times 10^{-4}(R - 12000\,\text{g})$	
	PWI 3	$U(W) \approx 20{,}311\,\text{g} + 3{,}923 \times 10^{-4}(R - 30000\,\text{g})$	
Global uncertainty of a weighing result without correction to the readings			
$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$	*PWI 1*	$U_{gl}(W) \approx 2{,}733\,\text{g} + 4{,}291 \times 10^{-4} R$	7.5.2-3a
	PWI 2	$U_{gl}(W) \approx 10{,}190\,\text{g} + 5{,}151 \times 10^{-4}(R - 12000\,\text{g})$	
	PWI 3	$U_{gl}(W) \approx 20{,}311\,\text{g} + 5{,}641 \times 10^{-4}(R - 30000\,\text{g})$	

The condition regarding the observed chi-squared value following (C2.2-2a) was checked with positive result. The linear regression was performed taking into account the weighing factors p'_j of equation (C2.2-18b).

[12] The first term is negligible as the uncertainty of the reading $u(R)$ is in the order of some g. Thus the first term is in the order of 10^{-7} g^2 while the second term represents values up to 15 g^2.

Based on the global uncertainty, the minimum weight value for the instrument may be derived as per Appendix G.

Example:

Weighing tolerance requirement: 1 %

Safety factor: 2

The minimum weight according to formula G-9, using the above equation for the global uncertainty in PWI 1 results in 598 g; i.e. the user needs to weigh a net quantity of material that exceeds 598 g in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1% and a safety factor of 2 (equals a relative weighing tolerance of 0,5%).

Second situation: adjustment of sensitivity carried out immediately before calibration

H2.1/B Conditions specific for the calibration

Instrument	**Electronic non-automatic weighing instrument, description and identification**
Upper limits of the intervals Max_i/Scale intervals d_i	**12 000 g / 2 g 30 000 g / 5 g 60 000 g / 10 g**
Sensitivity of the instrument to temperature variation	K_T= 2×10^{-6}/K (manufacturer's manual); only necessary for calculation of uncertainty of a weighing result.
Built-in adjustment device	Acts automatically after switching on the balance, and when $\Delta T \geq$ 3 K; only necessary for calculation of the uncertainty of a weighing result. Status: activated.
Adjustment by calibrator	**Adjusted immediately before calibration (built-in adjustment weights).**
Temperature during calibration	**23 °C at the beginning of calibration 24 °C at the end of the calibration**
Barometric pressure and humidity (optional)	*990 hPa, 50 % RH.*
Room conditions	Maximum temperature variation during use 10 K (room without windows).Not relevant, when built-in adjustment device is activated ($\Delta T \geq$ 3 K). In this case the maximum temperature variation for the estimation of uncertainty of a weighing result is 3 K.
Test loads / acclimatization	**Standard weights, class F_2**, acclimatized to room temperature.

H2.2/B Tests and results

Repeatability Requirements given in Chapter **5.1** Indication at no load reset to zero where necessary Repeatability test carried out in interval 1 and 2	**Test load 10 000 g** applied 5 times (standard deviation assumed constant over interval 1)	**Test load 25 000 g** applied 5 times (standard deviation assumed constant over interval 2 and 3)
	10 000 g	25 000 g
	10 000 g	25 000 g
	9 998 g	25000g
	10 000 g	24995g
	10 000 g	25 000 g
Standard deviation	***s* = 0,894 g**	***s* = 2,236 g**

Eccentricity Requirements given in Chapter **5.3** Indication set to zero prior to test; load put in centre first then moved to the other positions	**Position of the load**	**Test load 20 000 g**
	Centre	20 000g
	Front left	20 000g
	Back left	20 000g
	Back right	20 000g
	Front right	19 995g
Maximum deviation	$\left\|\Delta I_{\mathrm{ecc}i}\right\|_{\max}$	**5 g**

Errors of indication

General prerequisites:

Requirements given in Chapter 5.2, weights distributed fairly evenly over the weighing range.
Test loads each applied once; discontinuous loading only upwards, indication at no load reset to zero if necessary.

<u>Option 1: Air density unknown during adjustment / calibration (i.e. no buoyancy correction applied to the error of indication values).</u>

Requirements given in chapter **5.2**, weights distributed fairly evenly. Test loads each applied once; discontinuous loading only upwards; indication at no load reset to zero where necessary	**Load m_{ref} (m_{N})**	**Indication *I***	**Error of indication *E***
	0 g	0 g	0 g
	10 000 g	10 000 g	0 g
	20 000 g	20 000 g	0 g
	40 000 g	40 000 g	0 g
	60 000 g	60 000 g	0 g

Option 2: Air density ρ_{as} during adjustment and air density ρ_{aCal} during calibration are identical as an adjustment was carried out immediately before calibration

The air density is calculated according to the simplified CIPM formula (A1.1-1):

Measurement values used for calculation:	
Barometric pressure p:	*990 hPa*
Relative humidity RH:	*50 %*
Temperature t:	*23 °C*
Air density ρ_{aCal}:	*1,165 kg/m³*

Calculated buoyancy correction δm_B according to formula (4.2.4-4):

Numerical value used for calculation	
Density of the reference mass ρ_{Cal}:	*(7950 ± 70) kg/m³*
Buoyancy correction δm_B:	*$2,762 \times 10^{-8}\, m_N$*

The calculated correction δm_B of the loads m_N following formula (4.2.4-4) is negligible as the relative resolution of the instrument is in the order of 10^{-4} and thus much larger than the buoyancy correction. The above table is effectual.

H2.3/B Errors and related uncertainties (budget of related uncertainties)

Conditions common to both options:

- The uncertainty of the error at zero only comprises the uncertainty of the no-load indication (scale interval $d_0 = d_1$= 2 g) and the repeatability *s.* The uncertainty of the indication at load is not taken into consideration at zero.
- The eccentric loading is taken into account for the calibration according to (7.1.1-10).
- The error of indication is derived using the nominal weight value as reference value, therefore the maximum permissible errors of the test weights are taken into account for deriving the uncertainty contribution due to the reference mass: $u(\delta m_c)$is calculated as $u(\delta m_c) = Tol/\sqrt{3}$ following formula (7.1.2-3).
- The average drift of the weights monitored over 2 recalibrations in two-yearly intervals was $|D| \leq mpe/2$. Therefore the uncertainty contribution due to the drift of the weights was set to $u(\delta m_D) = mpe/2\sqrt{3}$. This corresponds to a $k_D$factor of 1.5 (assuming the worst-case scenario of $U = mpe$ / 3).
- The weights are acclimatized with a residual temperature difference of 2 K to the ambient temperature.
- The degrees of freedom for the calculation of the coverage factor *k* are derived following appendix B3 and table G.2 of [1]. In the case of the example, the influence of the uncertainty of the repeatability test with 5 measurements is significant.
- The information about the relative uncertainty $U(E)_{rel} = U(E)/m_{ref}$ is not mandatory, but helps to demonstrate the characteristics of the uncertainties.

Uncertainty budget for option 1 (no buoyancy correction applied to the error of indication values)

Additional condition:
The balance is adjusted immediately before calibration. The procedure according to option 1 is applied, with no information about air density. Therefore, formula (7.1.2-5c) is applied for the uncertainty due to air buoyancy.

Quantity or Influence	Load, indication and error in g Uncertainties in g					Formula
Load $m_{ref}(m_N)$ /g	**0**	**10 000**	**20 000**	**40 000**	**60 000**	
Indication I /g	**0**	**10 000**	**20 000**	**40 000**	**60 000**	
Error of Indication E /g	**0**	**0**	**0**	**0**	**0**	7.1-1
Repeatability $u(\delta I_{rep})$ /g	**0,894**		**2,236**			7.1.1-5
Resolution $u(\delta I_{dig0})$ /g	0,577					7.1.1-2a
Resolution $u(\delta I_{digL})$ /g	0,000	0,577	1,443	2,887	2,887	7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /g	0,000	0,722	1,443	2,887	4,330	7.1.1-10
Uncertainty of the indication$u(I)$ /g	1,065	1,410	3,082	4,690	5,694	7.1.1-12
Test loads m_N /g	0	10 000	20 000	20 000 20 000	20 000 20 000 20 000	
Weights $u(\delta m_c)$ /g	0,000	0,092	0,173	0,346	0,554	7.1.2-3
Drift $u(\delta m_D)$ /g	0,000	0,046	0,087	0,173	0,277	7.1.2-11
Buoyancy $u(\delta m_B)$ /g	0,000	0,023	0,043	0,087	0,139	7.1.2-5c
Convection $u(\delta m_{conv})$ /g	Not relevant in this case (only relevant for F_1 and better).					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$ /g	0,000	0,106	0,198	0,397	0,635	7.1.2-14
Standard uncertainty of the error $u(E)$ /g	1,065	1,414	3,089	4,707	5,739	7.1.3-1a
ν_{eff} (degrees of freedom)	8	25	14	78	172	B3-1
k(95,45 %)	**2,37**	**2,11**	**2,20**	**2,05**	**2,025**	[1],
$U(E) = ku(E)$/g	**2,523**	**2,983**	**6,795**	**9,650**	**11,601**	7.3-1
$U_{rel}(E)$/%	**----**	0,0298	0,0340	0,0241	0,0193	

It would be acceptable to state in the certificate only the largest value of the expanded uncertainty for all the reported errors: **$U(E)$ = 11,601 g**, based on **k = 2,025** accompanied by the statement that the coverage probability is at least 95%.

The certificate shall give the advice to the user that the expanded uncertainty stated in the certificate is only applicable, when the Error (E) is taken into account.

Uncertainty budget for option 2 (buoyancy correction applied to the error of indication values)

Additional condition:
The balance is adjusted immediately before calibration. The procedure according to option 2 is applied, taking into account the determination of the air density and buoyancy correction. Therefore formula (7.1.2-5a) is applied for the uncertainty due to air buoyancy.

As an adjustment has been carried out immediately before the calibration, the expected maximum values for pressure, temperature and humidity variations which can occur at the installation site of the instrument do not have to be taken into account – in contrast to the scenario where the adjustment has been performed independent of the calibration. The only contributing factor to the standard uncertainty of the air density originates from the uncertainty of the measurement of the environmental parameters.

The following numeric values are taken to calculate the relative uncertainty of the buoyancy correction, using formula (7.1.2-5a):

Air density ρ_{aCal}: 1,165 kg/m³

Density of the reference mass ρ_{Cal}: (7950 ± 70) kg/m^3

Furthermore, the following uncertainties for temperature, pressure and humidity measurement are taken for calculating the relative uncertainty of the air density according to (A3-1):

$u(T)$ = 0,2 K
$u(p)$ = 50 Pa
$u(RH)$ = 1%

This leads to $\frac{u(\rho_a)}{\rho_a} = 9{,}77 \times 10^{-4}$, *and* $u(\rho_a) = 0{,}00114$ *kg/m^3.*

Formula (7.1.2-5a) leads to the relative uncertainty of the buoyancy correction of $u_{rel}(\delta m_B) = 3{,}892 \times 10^{-8}$.

The relative uncertainty of the buoyancy correction is negligible as compared to the other contributions to the uncertainty of the reference mass.

This example has shown that the calculated correction of the error δm_B and the calculated relative uncertainty of the buoyancy correction $u_{rel}(\delta m_B)$ are both negligible. This leads to an updated measurement uncertainty budget:

Quantity or Influence	***Load, indication and error in g Uncertainties in g***					***Formula***
Load $m_{ref}(m_N)$/g	***0***	***10 000***	***20 000***	***40 000***	***60 000***	
Correction δm_B/g	***0***	***0***	***0***	***0***	***0***	*4.2.4-4*
Indication I /g	***0***	***10 000***	***20 000***	***40 000***	***60 000***	
Error of Indication E/g	***0***	***0***	***0***	***0***	***0***	*7.1-1*
Repeatability $u(\delta I_{rep})$/g	***0,894***		***2,236***			*7.1.1-5*
Resolution $u(\delta I_{dig0})$ /g	*0,577*					*7.1.1-2a*
Resolution $u(\delta I_{digL})$ /g	*0,000*	*0,577*	*1,443*	*2,887*	*2,887*	*7.1.1-3a*
Eccentricity $u(\delta I_{ecc})$ /g	*0,000*	*0,722*	*1,443*	*2,887*	*4,330*	*7.1.1-10*
Uncertainty of the indication $u(I)$ /g	*1,065*	*1,410*	*3,082*	*4,690*	*5,694*	*7.1.1-12*
Test loads m_N /g	*0*	*10 000*	*20 000*	*20 000 20 000*	*20 000 20 000 20 000*	
Weights $u(\delta m_c)$ /g	*0,000*	*0,092*	*0,173*	*0,346*	*0,554*	*7.1.2-3*
Drift $u(\delta m_D)$/g	*0,000*	*0,046*	*0,087*	*0,173*	*0,277*	*7.1.2-11*
Buoyancy $u(\delta m_B)$ /g	*0,000*	*0,000*	*0,001*	*0,001*	*0,002*	*7.1.2-5c*
Convection $u(\delta m_{conv})$ /g	*Not relevant in this case (only relevant for F_1 and better).*					*7.1.2-13*
Uncertainty of the reference mass $u(m_{ref})$/g	*0,000*	*0,103*	*0,194*	*0,387*	*0,620*	*7.1.2-14*
Standard uncertainty of the error $u(E)$ /g	*1,065*	*1,414*	*3,089*	*4,706*	*5,727*	*7.1.3-1a*
ν_{eff} (degrees of freedom)	*8*	*25*	*14*	*78*	*172*	*B3-1*
k(95,45 %)	***2,37***	***2,11***	***2,20***	***2,05***	***2,025***	*[1]*
$U(E) = ku(E)$/g	***2,523***	***2,983***	***6,794***	***9,648***	***11,598***	*7.3-1*
$U_{rel}(E)$/%	***----***	*0,0301*	*0,0340*	*0,0241*	*0,0193*	

The expanded uncertainties of the error using the standard procedure and using the option are almost identical as the uncertainty of the reference mass $u(m_{ref})$ is very small as compared to the uncertainty of the indication $u(I)$. In this example, the determination of pressure and humidity on site to determine the buoyancy correction and to minimize the uncertainty contribution due to buoyancy does not significantly improve the results of the calibration.

H2.4/B Uncertainty of a weighing result (for option 1)

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. The results must not be presented as part of the calibration certificate except for the approximated error of indication and the uncertainty of the approximated error which can form part of the certificate. Usually the information on the uncertainty of a weighing result is presented as an appendix to the calibration certificate or is otherwise shown if its contents are clearly separated from the calibration results.

Normal conditions of use of the instrument, as assumed, or as specified by the user may include:

- Built-in adjustment device available and activated ($\Delta T \geq 3$ K)
- Variation of room temperature $\Delta T = 10$ K
- Tare balancing function operated
- Loads not always centred carefully

The uncertainty of a weighing result is derived using a linear approximation of the error of indication according to (C2.2-16).

The uncertainty of a weighing result is presented for option 1 only (no buoyancy correction applied to the error of indication values). The approximated error of indication per (C2.2-16) and the uncertainty of the approximated error of indication per (C2.2-16d) differ insignificantly between both options as the underlying weighting factors $p_j = 1/u^2(E_j)$ differ in the order of a few per mil, and the errors of indication are the same for both options (buoyancy correction smaller than the resolution of the instrument).

The designations *R* and *W* are introduced to differentiate from the weighing instrument indication *I* during calibration.

R: Reading when weighing a load on the calibrated instrument obtained after the calibration
W: Weighing result

Note that within the following table the reading *R* and all results are in g.

<table>
<tr><th>Quantity or Influence</th><th colspan="2">Reading, weighing result and error in kg
Uncertainties in g or as relative value</th><th>Formula</th></tr>
<tr><td>Error of Indication $E_{appr}(R)$ for gross or net readings: Approximation by a straight line through zero</td><td colspan="2">$E_{appr}(R) = 0$</td><td>C2.2-16</td></tr>
<tr><td colspan="4">Uncertainty of the approximated error of indication</td></tr>
<tr><td>Standard uncertainty of the error $u(E_{appr})$</td><td colspan="2">$u^2(E_{appr}) = 0 \times u^2(R) + 3{,}651 \times 10^{-9} R^2$</td><td>C2.2-16d</td></tr>
<tr><td>Standard uncertainty of the error, neglecting the offset</td><td colspan="2">$u(E_{appr}) = 6{,}043 \times 10^{-5} R$</td><td></td></tr>
<tr><td colspan="4">Uncertainties from environmental influences</td></tr>
<tr><td>Temperature drift of sensitivity</td><td colspan="2">$u_{rel}(\delta R_{temp}) = 1{,}732 \times 10^{-6}$</td><td>7.4.3-1</td></tr>
<tr><td>Buoyancy</td><td colspan="2">Not relevant in this case.</td><td>7.4.3-2</td></tr>
<tr><td>Change in adjustment due to drift</td><td colspan="2">Not relevant in this case (built-in adjustment activated and drift between calibrations negligible)</td><td>7.4.3-5</td></tr>
<tr><td colspan="4">Uncertainties from the operation of the instrument</td></tr>
<tr><td>Tare balancing operation</td><td colspan="2">$u_{rel}(\delta R_{Tare}) = 0$</td><td>7.4.4-5</td></tr>
<tr><td>Creep, hysteresis (loading time)</td><td colspan="2">Not relevant in this case (short loading time).</td><td>7.4.4-9a/b</td></tr>
<tr><td>Eccentric loading</td><td colspan="2">$u_{rel}(\delta R_{ecc}) = 1{,}443 \times 10^{-4}$</td><td>7.4.4-10</td></tr>
<tr><td colspan="4">Uncertainty of a weighing result, for partial weighing intervals (PWI)</td></tr>
<tr><td rowspan="3">Standard uncertainty, corrections to the readings E_{appr} to be applied</td><td>PWI 1</td><td>$u(W) = \sqrt{(1{,}467\,g^2 + 2{,}449 \times 10^{-8} R^2)}$</td><td rowspan="3">7.4.5-1b</td></tr>
<tr><td>PWI 2</td><td>$u(W) = \sqrt{(7{,}417\,g^2 + 2{,}449 \times 10^{-8} R^2)}$</td></tr>
<tr><td>PWI 3</td><td>$u(W) = \sqrt{(13{,}667\,g^2 + 2{,}449 \times 10^{-8} R^2)}$</td></tr>
<tr><td rowspan="3">Expanded uncertainty, corrections to the readings E_{app} to be applied</td><td>PWI 1</td><td>$U(W) = 2\sqrt{(1{,}467\,g^2 + 2{,}449 \times 10^{-8} R^2)}$</td><td rowspan="3">7.5.1-2b</td></tr>
<tr><td>PWI 2</td><td>$U(W) = 2\sqrt{(7{,}417\,g^2 + 2{,}449 \times 10^{-8} R^2)}$</td></tr>
<tr><td>PWI 3</td><td>$U(W) = 2\sqrt{(13{,}667\,g^2 + 2{,}449 \times 10^{-8} R^2)}$</td></tr>
<tr><td rowspan="3">Simplified to first order</td><td>PWI 1</td><td>$U(W) \approx 2{,}422\,g + 1{,}706 \times 10^{-4} R$</td><td rowspan="3">7.5.2-3f</td></tr>
<tr><td>PWI 2</td><td>$U(W) \approx 6{,}616\,g + 2{,}355 \times 10^{-4} (R - 12000\,g)$</td></tr>
<tr><td>PWI 3</td><td>$U(W) \approx 11{,}951\,g + 2{,}744 \times 10^{-4} (R - 30000\,g)$</td></tr>
<tr><td colspan="4">Global uncertainty of a weighing result without correction to the readings</td></tr>
<tr><td rowspan="3">$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$</td><td>PWI 1</td><td>$U_{gl}(W) \approx 2{,}422\,g + 1{,}706 \times 10^{-4} R$</td><td rowspan="3">7.5.2-3a</td></tr>
<tr><td>PWI 2</td><td>$U_{gl}(W) \approx 6{,}616\,g + 2{,}355 \times 10^{-4} (R - 12000\,g)$</td></tr>
<tr><td>PWI 3</td><td>$U_{gl}(W) \approx 11{,}951\,g + 2{,}744 \times 10^{-4} (R - 30000\,g)$</td></tr>
</table>

The condition regarding the observed chi-squared value following (C2.2-2a) was checked with positive result. The linear regression was performed taking into account the weighing factors p'_j of equation (C2.2-18b).

Based on the global uncertainty, the minimum weight value for the instrument may be derived as per Appendix G.

Example:
Weighing tolerance requirement: 1 %

Safety factor: 2

The minimum weight according to formula G-9, using the above equation for the global uncertainty in PWI 1 results in 502 g; i.e. the user needs to weigh a net quantity of material that exceeds 502 g in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1% and a safety factor of 2 (equals a relative weighing tolerance of 0,5%).

H3 Instrument of 30 000 kg capacity, scale interval 10 kg

Preliminary note:
The calibration of a weighbridge for road vehicles is demonstrated. This example shows the complete standard procedure for the presentation of measurement results and the related uncertainties as executed by most laboratories.

Test loads should preferably consist only of standard weights that are traceable to the SI unit of mass.

This example shows the use of standard weights and substitution loads. The instrument under calibration is used as comparator to adjust the substitution load so that it brings about approximately the same indication as the corresponding load made up of standard weights.

<u>First situation: adjustment of sensitivity carried out independent of calibration</u>

(Instrument status: as it was found)

H3.1/A Conditions specific for the calibration

Instrument:	**Electronic non-automatic weighing instrument, description and identification**, with OIML R76 certificate of conformity or EN 45501 type approval but not verified
Maximum weighing capacity *Max* / scale interval *d*	**30 000 kg / 10 kg**
Load receptor	3 m wide, 10 m long, 4 points of support
Installation	Outside, in plain air, under shadow
Temperature coefficient	K_T = 2 × 10^{-6}/K (manufacturer's manual); only necessary for calculation of uncertainty of a weighing result.
Built-in adjustment device	Not provided.
Adjustment by calibrator	Not adjusted immediately before calibration.
Scale interval for testing	**Higher resolution (service mode), d_T= 1 kg**
Duration of tests	From 9h to 13h (this information could be useful in relation with possible effects of creep and hysteresis)
Temperature during calibration	**17°C at the beginning of the calibration** **20°C at the end of the calibration**
Barometric pressure and environmental conditions during calibration (optional)	*1 010 hPa ± 5 hPa ; no rain, no wind*
Test loads	Standard weights: • 10 parallelepiped standard weights, cast iron, 1 000 kg each, certified to class M_1 tolerance of *mpe* = 50 g (OIML R111 [4]) Substitution loads made up of steel or cast iron: • 2 steel containers filled with loose steel or cast iron, each weighing ≈ 2 000 kg; • 2 steel containers filled with loose steel or cast iron, each weighing ≈ 3 000 kg; • trailer to support the standard weights or the steel containers, weight adjusted to ≈ 10 000 kg; • small metallic pieces, used to adjust the substitution loads. Lifting and manoeuvring means for standard weights and substitution loads: • forklift, weight ≈ 4500 kg, capacity 6 000 kg to move standard weights and substitution loads; • vehicle with trailer and crane, lifting capacity 10 000 kg, to transport and to move standard weights and substitution loads.

H3.2/A Tests and results

Repeatability Requirements given in Chapter **5.1** Indication at no load reset to zero where necessary After unloading, no-load indications were between 0 and 2 kg	**Test load ≈ 10 420 kg:** Fork lift with 2 steel containers, moved on alternating from either long end of load receptor, load centred by eyesight	***Test load ≈ 24 160 kg:*** *Loaded vehicle moved on alternating from either long end of load receptor, load centred by eyesight (alternatively or additionally performed)*
	10 405 kg	*24 145 kg*
	10 414 kg	*24 160 kg*
	10 418 kg	*24 172 kg*
	10 412 kg	*24 152 kg*
	10 418 kg	*24 156 kg*
	10 425 kg	*24 159 kg*
Standard deviation	***s* = 6,74 kg**	***s = 9,03 kg***

Eccentricity Requirements given in Chapter **5.3** Indication set to zero prior to test; load put in centre first then moved to the other positions	**Position of the load**	**Test load ≈ 10 420 kg:** Fork lift with 2 steel containers
	Centre	10 420 kg
	Front left	10 407 kg
	Back left	10 435 kg
	Back right	10 433 kg
	Front right	10 413 kg
Maximum difference between center indication and the off-center indications (in the four corners)	$\lvert \Delta I_{ecci} \rvert_{max}$	**15 kg**

Eccentricity *(alternatively or additionally performed with rolling loads)* *Requirements given in Chapter* ***5.3*** *Indication set to zero prior to test and prior the change of direction;*	***Position of the load***	***Test load ≈ 24 160 kg:*** *heaviest and most concentrated available vehicle*
	Left	*24 160 kg*
	Centre	*24 157 kg*
	Right	*24 181 kg*
	(change direction) *Right*	*24 177 kg*
	Centre	*24 157 kg*
	Left	*24 162 kg*
Maximum difference between center indication and the two off-center indications *(along the longitudinal axis)*	$\lvert \Delta I_{ecci} \rvert_{max}$	***24 kg***

Errors of indication
Standard Procedure: Requirements given in chapter **5.2**, weights distributed fairly evenly.

Test loads built up by substitution, with 10 000 kg standard weights (10 weights × 1 000 kg) and 2 substitution loads L_{sub1} and L_{sub2} of approximately 10 000 kg each (the trailer and the sum of 4 containers). Test loads applied once; continuous loading only upwards. This may include creep and hysteresis effects in the results, but reduces the amount of loads to be moved on and off the load receptor.

Indications after removal of standard weights recorded but no correction applied; all loads arranged reasonably around centre of load receptor.

Indications recorded:

LOAD				
Standard weights m_N	**Substitution loads** L_{sub}	**Total test load** $L_T = m_N + L_{sub}$	**Indication** I	**Error of indication** E
0 kg	0 kg	0 kg	0 kg	0 kg
5 000 kg ½ m_{ref}	0 kg	5 000 kg	5 002 kg $I(½\ m_{ref})$	2 kg
10 000 kg m_{ref}	0 kg	10 000 kg	10 010 kg $I(m_{ref})$	10 kg
0 kg	10 000 kg L_{sub1}	10 000 kg	10 010 kg $I(L_{sub1})$	10 kg
5 000 kg ½ m_{ref}	10 000 kg L_{sub1}	15 000 kg	15 015 kg $I(½\ m_{ref} + L_{sub1})$	15 kg
10 000 kg m_{ref}	10 000 kg L_{sub1}	20 000 kg	20 018 kg $I(m_{ref} + L_{sub1})$	18 kg
0 kg	20 010 kg $L_{sub1}+L_{sub2}$	20 010 kg	20 028 kg $I(L_{sub1}+L_{sub2})$	18 kg
5 000 kg ½ m_{ref}	20 010 kg $L_{sub1}+L_{sub2}$	25 010 kg	25 035 kg $I(½\ m_{ref} + L_{sub1}+L_{sub2}$	25 kg
10 000 kg m_{ref}	20 010 kg $L_{sub1}+L_{sub2}$	30 010 kg	30 040 kg $I(m_{ref} + L_{sub1}+L_{sub2})$	30 kg
0 kg	0 kg	0 kg	4 kg	4 kg E_0

Air density ρ_{as} during adjustment is unknown and air density ρ_{aCal} is unknown.

No buoyancy correction is applied to the error of indication values. Using standard weights class M_1 the relative uncertainty for buoyancy effect is calculated according to (7.1.2-5d) is $1{,}6\times10^{-5}$ (since the instrument is not adjusted immediately before calibration). The uncertainty is small enough, so a more elaborate calculation of this uncertainty component based on actual data for air density is superfluous (the uncertainty of buoyancy is smaller than the scale interval of the high resolution mode d_T and is negligible).

The limit of density for class M_1 standard weights is established to be $\rho \geq 4\ 400$ kg m^{-3} [4]. This limit may be considered also for the substitution loads. In this case, the relative uncertainty estimated for the buoyancy effect of the substitution loads is the same as above (for standard weights) and is small enough; a more elaborate calculation of this uncertainty component based on actual data is superfluous.

Note: In the estimation of density for substitution loads, it is necessary to take into account any internal cavities, which are not open to the atmosphere (for example at tanks, reservoirs). It is necessary to estimate the density of such a load as a whole, not to suppose that it has the same density as the material from which it is built.

H3.3/A Errors and related uncertainties (budget of related uncertainties)

Conditions:

- The uncertainty of the error at zero only comprises the uncertainty of the no-load indication (scale interval d = 1 kg) and the repeatability s. The uncertainty of the indication at load is not taken into consideration at zero
- The eccentric loading is taken into account for the calibration according to (7.1.1-10) because it cannot be excluded during the error of indication test. If both eccentricity tests were performed, then the result with the largest relative value should be used.
- The error of indication is derived using the nominal weight value as reference value, therefore the maximum permissible errors of the test weights are taken into account for deriving the uncertainty contribution due to the reference mass: $u(\delta m_c)$ is calculated as $u(\delta m_c) = mpe/\sqrt{3}$ following formula (7.1.2-3). For each standard weight of 1000 kg $u(\delta m_c) = 50/\sqrt{3} \approx 29$ g.
- In the absence of information on drift, the value of D is chosen $D = mpe$. For each standard weight of 1000 kg mpe = ± 50 gand $u(\delta m_c) = 50/\sqrt{3} \approx 29$ g, following formula (7.1.2-11).
- The instrument is not adjusted immediately before calibration. The standard procedure is applied, with no information about air density. Therefore, formula (7.1.2-5d) is applied for the uncertainty due to air buoyancy.
- The load remains on the load receptor for a significant period of time during the calibration. Based on chapter 7.1.1 that states that additional uncertainty contributions might have to be taken into account, the creep and hysteresis effects in the results are calculated following formula (7.4.4-7) and included in the uncertainty of the indication.
- The weights are acclimatized with a residual temperature difference of 5 K to the ambient temperature. The effects of convection are not relevant (usually they are only relevant for weights of class F_1 or better).
- The degrees of freedom for the calculation of the coverage factor k are derived following appendix B3 and table G.2 of [1]. In the case of the example, the influence of the uncertainty of the repeatability test with 6 measurements is significant.
- The information about the relative uncertainty $U(E)_{rel} = U(E)/m_{ref}$ is not mandatory, but helps to demonstrate the characteristics of the uncertainties.

Quantity or Influence	Load, indication, error and uncertainties in kg					Formula
Total test load **$L_T = m_N + L_{subj}$/kg**	**0**	**5 000**	**10 000** m_{ref}	10 000*) L_{sub1}	**15 000**	
Indication *I* /kg	**0**	**5 002**	**10 010** $I(m_{ref})$	10 010 $I(L_{sub1})$	**15 015**	
Error of Indication *E* /kg	**0**	**2**	**10**	10 $\Delta I_1 = 0$	**15**	7.1-1
Repeatability $u(\delta I_{rep})$ /kg	6,74					7.1.1-5
Resolution $u(\delta I_{dig0})$ /kg	0,29					7.1.1-2a
Resolution $u(\delta I_{digL})$ /kg	0,00	0,29				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /kg	0,00	2,08	4,16	4,16	6,24	7.1.1-10
Creep / hysteresis $u_{rel}(\delta I_{time})$ /kg	0,00	0,38	0,77	0,77	1,16	7.4.4-7
Uncertainty of the indication $u(I)$/kg	6,75	7,08	7,97	7,97	9,27	7.1.1-12
Standard weights m_N/kg	**0**	**5 000**	**10 000**	0	**5 000**	
Uncertainty $u(\delta m_c)$ /kg	0,00	0,14	0,29	0,00	0,14	7.1.2-3
Drift $u(\delta m_D)$ /kg	0,00	0,14	0,29	0,00	0,14	7.1.2-11
Buoyancy $u(\delta m_B)$ /kg	0,00	0,08	0,16	0,00	0,08	7.1.2-5d
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$ /kg	0,00	0,22	0,44	0,00	0,22	7.1.2-14
Substitution loads L_{subj} /kg	**0**	**0**	**0**	10 000 $L_{sub1} = m_{ref} + \Delta I_1$	**10 000** L_{sub1}	
Uncertainty $u(L_{subj})$/kg	0,00	0,00	0,00	11,28	11,28	7.1.2-15b
Buoyancy $u(\delta m_B)$ /kg	0,00	0,00	0,00	0,16	0,16	7.1.2-5d
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case					
Uncertainty of substitution loads $u(L_{subj})$ /kg	0,00	0,00	0,00	11,28	11,28	7.1.2-15b 7.1.2-14
Standard uncertainty of the error $u(E)$ /kg	6,75	7,08	7,98	--------	14,60	7.1.3-1c
υ_{eff} (degrees of freedom)	5	6	9	--------	109	B3-1
***k*(95,45 %)**	**2,65**	**2,52**	**2,32**	--------	**2,02**	[1]
***U*(*E*) = *ku*(*E*)/kg**	**18**	**18**	**19**	--------	**29**	7.3-1
$U_{rel}(E)$ /%	----	0,36	0,19	--------	0,20	

(continue)

Quantity or Influence	Load, indication, error and uncertainties in kg				Formula
Total test load $L_T = m_N + L_{subj}$ **/kg**	**20 000** $m_{ref2}+L_{sub2}$	20 010*)	**25 010**	**30 010**	
Indication I **/kg**	**20 018** $I(m_{ref2}+L_{sub2})$	20 028 $I(L_{sub1}+L_{sub2})$	**25 035**	**30 040**	
Error of Indication E **/kg**	**18**	18 ΔI_2=10	**25**	**30**	7.1-1
Repeatability $u(\delta I_{rep})$ /kg	6,74				7.1.1-5
Resolution $u(\delta I_{dig0})$ /kg	0,29				7.1.1-2a
Resolution $u(\delta I_{digL})$ /kg	0,29				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /kg	8,32	8,32	10,40	12,48	7.1.1-10
Creep / hysteresis $u_{rel}(\delta I_{time})$ /kg	1,54	1,54	1,93	2,31	7.4.4-7
Uncertainty of the indication $u(I)$/kg	10,82	10,82	12,54	14,38	7.1.1-12
Standard weights m_N**/kg**	**10 000**	0	**5 000**	**10 000**	
Uncertainty $u(\delta m_c)$ /kg	0,29	0,00	0,14	0,29	7.1.2-3
Drift $u(\delta m_D)$ /kg	0,29	0,00	0,14	0,29	7.1.2-11
Buoyancy $u(\delta m_B)$ /kg	0,16	0,00	0,08	0,16	7.1.2-5d
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case				7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$ /kg	0,44	0,00	0,22	0,44	7.1.2-14
Substitution loads L_{subj} /kg	**10 000** L_{sub1}	20 010 $L_{sub1}+L_{sub2}=2m_{ref1}+\Delta I_2$	**20 010** $L_{sub1}+L_{sub2}$	**20 010** $L_{sub1}+L_{sub2}$	
Uncertainty $u(L_{subj})$/kg	11,28	19,02	19,02	19,02	7.1.2-15b
Buoyancy $u(\delta m_B)$ /kg	0,16	0,32	0,32	0,32	7.1.2-5d
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case				
Uncertainty of substitution loads $u(L_{subj})$ /kg	11,28	19,02	19,02	19,02	7.1.2-15b 7.1.2-14
Standard uncertainty of the error $u(E)$ /kg	15,64	--------	22,79	23,85	7.1.3-1c
υ_{eff} (degrees of freedom)	144	--------	653	783	B3-1
***k*(95,45 %)**	**2,02**	--------	**2,00**	**2,00**	[1]
$U(E) = ku(E)$**/kg**	**32**	--------	**46**	**48**	7.3-1
$U_{rel}(E)$ /%	0,16	--------	0,18	0,16	

*) The values written in this column (for the same total load value as in previous column, after substitution of standard weights with substitution loads) are not reported in the calibration certificate, but are used in next columns. In order to remember this, the bold font is not used in this column and the final 5 cells are empty.

It would be acceptable to state in the certificate only the largest value of the expanded uncertainty for all the reported errors: ***U*(*E*) = 48 kg**, based on ***k* = 2** accompanied by the statement that the coverage probability is at least 95 %.

The certificate shall give the advice to the user that the expanded uncertainty stated in the certificate is only applicable, when the Error (E) is taken into account.

H3.4/A Uncertainty of a weighing result

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. The results must not be presented as part of the calibration certificate, except for the approximated error of indication and the uncertainty of the approximated error which can form part of the certificate. Usually the information on the uncertainty of a weighing result is presented as an appendix to the calibration certificate or is otherwise shown if its contents are clearly separated from the calibration results.

The normal conditions of use of the instrument, as assumed, or as specified by the user may include:

Variation of temperature ΔT =40 K
Loads not always centred carefully
Tare balancing function operated
Loading times: normal, that is shorter than at calibration
Readings in normal resolution, d = 10 kg

The error of indication at 30 000 kg is 30 kg, and this value is taken for the change in adjustment due to drift.

The designations R and W are introduced to differentiate from the weighing instrument indication I during calibration.

- R: Reading when weighing a load on the calibrated instrument obtained after the calibration
- W: Weighing result

Note that within the following table the reading R and all results are in kg.

Quantity or Influence	Reading, weighing result and error in kg Uncertainties in g or as relative value	Formula
Error of Indication $E_{appr}(R)$ for gross or net readings: Approximation by a straight line through zero	$E_{appr}(R) = 9.379 \times 10^{-4} R$	C2.2-16
Uncertainty of the approximated error of indication		
Standard uncertainty of the error $u(E_{appr})$	$u(E_{appr}) = \sqrt{8{,}797 \times 10^{-7} u^2(R) + 1{,}316 \times 10^{-7} R^2}$	C2.2-16d
Standard uncertainty of the error, neglecting the offset	$u(E_{appr}) = 3{,}627 \times 10^{-4} R$	
Uncertainties from environmental influences		
Temperature drift of sensitivity	$u_{rel}(\delta R_{temp}) = \frac{2 \times 10^{-6} \times 40}{\sqrt{12}} = 2{,}309 \times 10^{-5}$	7.4.3-1
Buoyancy	Not relevant in this case.	7.4.3-3
Change in adjustment due to drift (change of *E*(*Max*) over 1 year = 30 kg)	$u_{rel}(\delta R_{adj}) = \lvert 30 \rvert / (30000\sqrt{3}) = 5{,}774 \times 10^{-4}$	7.4.3-6
Uncertainties from the operation of the instrument		
Tare balancing operation	$u_{rel}(\delta R_{Tare}) = 3{,}457 \times 10^{-4}$	7.4.4-5
Creep, hysteresis (loading time)	Not relevant in this case (short loading time).	7.4.4-7
Eccentric loading	$u_{rel}(\delta R_{ecc}) = 8{,}311 \times 10^{-4}$	7.4.4-10
Uncertainty of a weighing result		
Standard uncertainty, corrections to the readings E_{appr} to be applied	$u(W) = \sqrt{(62{,}133\ \text{kg}^2 + 1{,}276 \times 10^{-6} R^2)}$	7.4.5-1a 7.4.5-1b
Expanded uncertainty, corrections to the readings $E_{appr}$to be applied	$U(W) = 2\sqrt{(62{,}133\ \text{kg}^2 + 1{,}276 \times 10^{-6} R^2)}$	7.5.1-2b
Simplified to first order	$U(W) \approx 16\ \text{kg} + 1{,}79 \times 10^{-3} R$	7.5.2-3d
Global uncertainty of a weighing result without correction to the readings		
$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$	$U_{gl}(W) \approx 16\,\text{kg} + 2{,}73 \times 10^{-3} R$	7.5.2-3a

The condition regarding the observed chi-squared value following (C2.2-2a) was checked with positive result. The first linear regression taking into account the weighing factors p'_j, equation (C2.2-18b).

Based on the global uncertainty, the minimum weight value for the instrument may be derived as per Appendix G.

Example:

Weighing tolerance requirement: 1 %

Safety factor: 1

The minimum weight according to formula (G-9), using the above equation for the global uncertainty results in 2 169 kg; i.e. the user needs to weigh a net quantity of material that exceeds 2 169 kg in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1 % and a safety factor of 1.

If a safety factor is included, it might be chosen to be 2. Because of the large global uncertainty, a higher safety factor might not be able to be realised.

The minimum weight according to formula (G-9), using the above equation for the global uncertainty results in 6 950 kg; i.e. the user needs to weigh a net quantity of material that exceeds 6 950 kg in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1 % and a safety factor of 2 (equals a relative weighing tolerance of 0,50 %).

Second situation: adjustment of sensitivity carried out immediately before calibration

(Previously, repair and maintenance operations were performed on the instrument)

H3.1/B Conditions specific for the calibration

Instrument:	**Electronic non-automatic weighing instrument, description and identification**, with OIML R76 certificate of conformity or EN 45501 type approval but not verified
Maximum weighing capacity *Max* / scale interval *d*	**30 000 kg / 10 kg**
Load receptor	3 m wide, 10 m long, 4 points of support
Installation	Outside, in plain air, under shadow
Temperature coefficient	K_T = 2 × 10^{-6}/K (manufacturer's manual); only necessary for calculation of uncertainty of a weighing result.
Built-in adjustment device	Not provided.
Adjustment by calibrator	Adjusted immediately before calibration.
Scale interval for testing	**Higher resolution (service mode), d_T = 1 kg**
Duration of tests	From 14h to 18h
Temperature during calibration	**22°C at the beginning of the calibration** **18°C at the end of the calibration**
Barometric pressure during calibration	*1 010 hPa ± 5 hPa; no rain, no wind*
Test loads	Standards weights: • 10 parallelepiped standard weights, cast iron, 1 000 kg each, certified to class M_1 tolerance of *mpe* = 50 g (OIML R111 [4]) Substitution loads made up of steel or cast iron: • 2 steel containers filled with loose steel or cast iron, each weighing ≈ 2 000 kg; • 2 steel containers filled with loose steel or cast iron, each weighing ≈ 3 000 kg; • trailer to support the standard weights or the steel containers, weight adjusted to ≈ 10 000 kg; • small metallic pieces, used to adjust the substitution loads. Lifting and manoeuvring means for standard weights and substitution loads: • forklift, weight ≈ 4500 kg, capacity 6 000 kg to move standard weights and substitution loads; • vehicle with trailer and crane, lifting capacity 10 000 kg, to transport and to move standard weights and substitution loads.

H3.2/B Tests and results

Repeatability Requirements given in Chapter **5.1** Indication at no load reset to zero where necessary After unloading, no-load indications were between 0 and 2 kg	**Test load ≈ 10 420 kg:** Fork lift with 2 steel containers (or the empty trailer), moved on alternating from either long end of load receptor, load centred by eyesight	***Test load ≈ 24 160 kg:*** *Loaded vehicle (or loaded trailer), moved on alternating from either long end of load receptor, load centred by eyesight (alternatively or additionally performed)*
	10 415 kg	*24 155 kg*
	10 418 kg	*24 160 kg*
	10 422 kg	*24 162 kg*
	10 416 kg	*24 152 kg*
	10 422 kg	*24 156 kg*
	10 419 kg	*24 159 kg*
Standard deviation	**s = 2,94 kg**	***s = 3,67 kg***

Eccentricity Requirements given in Chapter **5.3** Indication set to zero prior to test; load put in centre first then moved to the other positions	**Position of the load**	**Test load ≈ 10 420 kg:** Fork lift with 2 steel containers
	Centre	10 420 kg
	Front left	10 417 kg
	Back left	10 423 kg
	Back right	10 425 kg
	Front right	10 425 kg
Maximum difference between center indication and the off-center indications (in the four corners)	$\lvert\Delta I_{ecci}\rvert_{max}$	**5 kg**

Eccentricity *(alternatively or additionally performed with rolling loads)* *Requirements given in Chapter **5.3*** *Indication set to zero prior to test and prior the change of direction;*	***Position of the load***	***Test load ≈ 24 160 kg:*** *heaviest and most concentrated available vehicle*
	Left	*24 151 kg*
	Centre	*24 160 kg*
	Right	*24 169 kg*
	(change direction) *Right*	*24 167 kg*
	Centre	*24 160 kg*
	Left	*24 150 kg*
Maximum difference between center indication and the two off-center indications *(along the longitudinal axis)*	$\lvert\Delta I_{ecci}\rvert_{max}$	***10 kg***

Errors of indication

Standard Procedure: Requirements given in chapter **5.2**, weights distributed fairly evenly.

Test loads built up by substitution, with 10 000 kg standard weights (10 weights × 1 000 kg) and 2 substitution loads L_{sub1} and L_{sub2} of approximately 10 000 kg each (the trailer and the sum of 4 containers). Test loads applied once; continuous loading only upwards. This may include creep and hysteresis effects in the results, but reduces the amount of loads to be moved on and off the load receptor.

Indications after removal of standard weights recorded but no correction applied; all loads arranged reasonably around centre of load receptor.

Indications recorded:

LOAD				
Standard weights m_N	**Substitution loads** L_{sub}	**Total test load** $L_T=m_N+L_{sub}$	**Indication** I	**Error of indication** E
0 kg	0 kg	0 kg	0 kg	0 kg
5 000 kg ½ m_{ref}	0 kg	5 000 kg	5 002 kg I(½ m_{ref})	2 kg
10 000 kg m_{ref}	0 kg	10 000 kg	10 005 kg I (m_{ref})	5 kg
0 kg	10 000 kg L_{sub1}	10 000 kg	10 005 kg $I(L_{sub1})$	5 kg
5 000 kg ½ m_{ref}	10 000 kg L_{sub1}	15 000 kg	15 007 kg I(½ $m_{ref}+L_{sub1}$)	7 kg
10 000 kg m_{ref}	10 000 kg L_{sub1}	20 000 kg	20 008 kg $I(m_{ref}+L_{sub1})$	8 kg
0 kg	20 010 kg $L_{sub1}+L_{sub2}$	20 010 kg	20 018 kg $I(L_{sub1}+L_{sub2})$	8 kg
5 000 kg ½ m_{ref}	20 010 kg $L_{sub1}+L_{sub2}$	25 010 kg	25 020 kg I(½ m_{ref} $L_{sub1}+L_{sub2}$)	10 kg
10 000 kg m_{ref}	20 010 kg $L_{sub1}+L_{sub2}$	30 010 kg	30 022 kg $I(m_{ref}+L_{sub1}+L_{sub2})$	12 kg
0 kg	0 kg	0 kg	4 kg	4 kg E_0

Air density ρ_{as} during adjustment is unknown and air density ρ_{aCal} is unknown.

No buoyancy correction is applied to the error of indication values. Using standard weights class M_1, the relative uncertainty for buoyancy effect is calculated according to (7.1.2-5c) and it is $7{,}2\times10^{-6}$ (since the instrument is adjusted immediately before calibration), The uncertainty is small enough; a more elaborate calculation of this uncertainty component based on actual data for air density is superfluous (the uncertainty of buoyancy is smaller than the scale interval of the high resolution mode d_T and is negligible).

The limit of density for class M_1 standard weights is established to be $\rho \geq 4\,400$ kg m^{-3} [4]. This limit may be considered also for the substitution loads. In this case, the relative uncertainty estimated for the buoyancy effect of the substitution loads is the same as above (for standard weights) and is small enough; a more elaborate calculation of this uncertainty component based on actual data is superfluous.

Note: In the estimation of density for substitution loads, it is necessary to take into account any internal cavities, which are not open to the atmosphere (for example at tanks, reservoirs). It is necessary to estimate the density of such a load as a whole, not to suppose that it has the same density as the material from which it is built.

H3.3/B Errors and related uncertainties (budget of related uncertainties)

Conditions:

- The uncertainty of the error at zero only comprises the uncertainty of the no-load indication (scale interval d = 1 kg) and the repeatability s. The uncertainty of the indication at load is not taken into consideration at zero.
- The eccentric loading is taken into account for the calibration according to (7.1.1-10) because it cannot be excluded during the error of indication test. If both eccentricity tests were performed, then the result with the largest relative value should be used.
- The error of indication is derived using the nominal weight value as reference value, therefore the maximum permissible errors of the test weights are taken into account for deriving the uncertainty contribution due to the reference mass: $u(\delta m_c)$ is calculated as $u(\delta m_c) = mpe/\sqrt{3}$ following formula (7.1.2-3). For each standard weight of 1000 kg $u(\delta m_c) = 50/\sqrt{3} \approx 29$ g.
- In the absence of information on drift, the value of D is chosen $D = mpe$. For each standard weight of 1000 kg $mpe = \pm\ 50$ gand $u(\delta m_c) = 50/\sqrt{3} \approx 29$ g, following formula (7.1.2-11).
- The instrument is adjusted immediately before calibration. The standard procedure is applied, with no information about air density. Therefore formula (7.1.2-5c) is applied for the uncertainty due to air buoyancy.
- The load remains on the load receptor for a significant period of time during the calibration. Based on chapter 7.1.1 that states that additional uncertainty contributions might have to be taken into account, the creep and hysteresis effects in the results are calculated following formula (7.4.4-7) and included in the uncertainty of the indication.
- The weights are acclimatized with a residual temperature difference of 5 K to the ambient temperature. The effects of convection are not relevant (usually they are only relevant for weights of class F1 or better).
- The degrees of freedom for the calculation of the coverage factor k are derived following appendix B3 and table G.2 of [1]. In the case of the example, the influence of the uncertainty of the repeatability test with 6 measurements is significant.
- The information about the relative uncertainty $U(E)_{rel} = U(E)/m_{ref}$ is not mandatory, but helps to demonstrate the characteristics of the uncertainties.

Quantity or Influence	Load, indication, error and uncertainties in kg					Formula
Total test load **$L_T = m_N+L_{subj}$/kg**	**0**	**5 000**	**10 000** m_{ref}	10 000*) L_{sub1}	**15 000**	
Indication I /kg	**0**	**5 002**	**10 005** I (m_{ref})	10 005 I (L_{sub1})	**15 007**	
Error of Indication E /kg	**0**	**2**	**5**	5 $\Delta I_1=0$	**7**	7.1-1
Repeatability $u(\delta I_{rep})$ /kg	2,94					7.1.1-5
Resolution $u(\delta I_{dig0})$ /kg	0,29					7.1.1-2a
Resolution $u(\delta I_{digL})$ /kg	0,00	0,29				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /kg	0,00	0,69	1,39	1,39	2,08	7.1.1-10
Creep / hysteresis $u_{rel}(\delta I_{time})$ /kg	0,00	0,39	0,77	0,77	1,16	7.4.4-7
Uncertainty of the indication $u(I)$/kg	2,96	3,08	3,37	3,37	3,81	7.1.1-12
Standard weights m_N/kg	**0**	**5 000**	**10 000**	0	**5 000**	
Uncertainty $u(\delta m_c)$ /kg	0,00	0,14	0,29	0,00	0,14	7.1.2-3
Drift $u(\delta m_D)$ /kg	0,00	0,14	0,29	0,00	0,14	7.1.2-11
Buoyancy $u(\delta m_B)$ /kg	0,00	0,04	0,07	0,00	0,04	7.1.2-5c
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$ /kg	0,00	0,21	0,42	0,00	0,21	7.1.2-14
Substitution loads L_{subj} /kg	**0**	**0**	**0**	10 000 $L_{sub1}=$ $m_{ref}+\Delta I_1$	**10 000** L_{sub1}	
Uncertainty $u(L_{subj})$/kg	0,00	0,00	0,00	4,78	4,78	7.1.2-15b
Buoyancy $u(\delta m_B)$ /kg	0,00	0,00	0,00	0,07	0,07	7.1.2-4
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case					
Uncertainty of substitution loads $u(L_{subj})$ /kg	0,00	0,00	0,00	4,78	4,78	7.1.2-15b 7.1.2.-4
Standard uncertainty of the error $u(E)$ /kg	2,96	3,08	3,39	--------	6,12	7.1.3-1c
υ_{eff} (degrees of freedom)	5	6	8	--------	93	B3-1
***k*(95,45 %)**	**2,65**	**2,52**	**2,32**	--------	**2,03**	[1]
$U(E) = ku(E)$/kg	**8**	**8**	**8**	--------	**12**	7.3-1
$U_{rel}(E)$ /%	----	0,16	0,08	--------	0,08	

(continue)

Quantity or Influence	Load, indication, error and uncertainties in kg				Formula
Total test load $L_T = m_N + L_{subj}$/kg	**20 000** $m_{ref2}+L_{sub2}$	20 010*)	**25 010**	**30 010**	
Indication *I* /kg	**20 008** $I(m_{ref2}+L_{sub2})$	20 018 $I(L_{sub1}+L_{sub2})$	**25 020**	**30 022**	
Error of Indication *E* /kg	**8**	8 ΔI_2=10	**10**	**12**	7.1-1
Repeatability $u(\delta I_{rep})$ /kg	2,94				7.1.1-5
Resolution $u(\delta I_{dig0})$ /kg	0,29				7.1.1-2a
Resolution $u(\delta I_{digL})$ /kg	0,29				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /kg	2,77	2,77	3,47	4,16	7.1.1-10
Creep / hysteresis $u_{rel}(\delta I_{time})$ /kg	1,54	1,54	1,93	2,31	7.4.4-7
Uncertainty of the indication $u(I)$/kg	4,34	4,34	4,95	5,61	7.1.1-12
Standard weights m_N/kg	**10 000**	0	**5 000**	**10 000**	
Uncertainty $u(\delta m_c)$ /kg	0,29	0,00	0,14	0,29	7.1.2-3
Drift $u(\delta m_D)$ /kg	0,29	0,00	0,14	0,29	7.1.2-11
Buoyancy $u(\delta m_B)$ /kg	0,07	0,00	0,04	0,07	7.1.2-5c
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case				7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$ /kg	0,42	0,00	0,21	0,42	7.1.2-14
Substitution loads L_{subj} /kg	**10 000** L_{sub1}	20 010 $L_{sub1}+L_{sub2}=2\,m_{ref1}+\Delta I_2$	**20 010** $L_{sub1}+L_{sub2}$	**20 010** $L_{sub1}+L_{sub2}$	
Uncertainty $u(L_{subj})$/kg	4,78	7,80	7,80	7,80	7.1.2-15a
Buoyancy $u(\delta m_B)$ /kg	0,07	0,14	0,14	0,14	7.1.2-5c
Convection $u(\delta m_{conv})$ /kg	Not relevant in this case				7.1.2-13
Uncertainty of substitution loads $u(L_{subj})$ /kg	4,78	7,80	7,80	7,80	7.1.2-15a 7.1.2-4
Standard uncertainty of the error $u(E)$ /kg	6,47	--------	9,24	9,62	7.1.3-1a
ν_{eff} (degrees of freedom)	117	--------	486	569	B3-1
***k*(95,45 %)**	**2,02**	--------	**2,01**	**2,00**	[1]
***U*(*E*) = *ku*(*E*)/kg**	**13**	--------	**19**	**19**	7.3-1
$U_{rel}(E)$ /%	0,06	--------	0,07	0,06	

*) The values written in this column (for the same total load value as in previous column, after substitution of standard weights with substitution loads) are not reported in the calibration certificate, but are used in next columns. In order to remember this, the bold font is not used in this column and the final 5 cells are empty.

It would be acceptable to state in the certificate only the largest value of the expanded uncertainty for all the reported errors: ***U*(*E*) = 19 kg**, based on ***k* = 2** accompanied by the statement that the coverage probability is at least 95 %.

The certificate shall give the advice to the user that the expanded uncertainty stated in the certificate is only applicable, when the Error (*E*) is taken into account.

H3.4/B Uncertainty of a weighing result

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. The results must not be presented as part of the calibration certificate, except for the approximated error of indication and the uncertainty of the approximated error which can form part of the certificate. Usually the information on the uncertainty of a weighing result is presented as an appendix to the calibration certificate or is otherwise shown if its contents are clearly separated from the calibration results.

The normal conditions of use of the instrument, as assumed, or as specified by the user may include:

Variation of temperature ΔT = 40 K
Loads not always centred carefully
Tare balancing function operated
Loading times: normal, that is shorter than at calibration
Readings in normal resolution, d = 10 kg

For the change in adjustment due to drift, the error of indication at 30 000 kg is assumed to be 15 kg. This is the mpe at initial verification, and taken as the instrument is in good condition after maintenance and repair.

The designations R and W are introduced to differentiate from the weighing instrument indication I during calibration.

R: Reading when weighing a load on the calibrated instrument obtained after the calibration
W: Weighing result

Note that within the following table the reading R and all results are in kg.

Quantity or Influence	Reading, weighing result and error in kg Uncertainties in g or as relative value	Formula
Error of Indication $E_{appr}(R)$ for gross or net readings: Approximation by a straight line through zero	$E_{appr}(R) = 4{,}280 \times 10^{-4} R$	C2.2-16
Uncertainty of the approximated error of indication		
Standard uncertainty of the error $u(E_{appr})$	$u^2(E_{appr}) = 1{,}832 \times 10^{-7} u^2(R) + 2{,}204 \times 10^{-8} R^2$	C2.2-16d
Standard uncertainty of the error, neglecting the offset	$u(E_{appr}) = 1{,}485 \times 10^{-4} R$	
Uncertainties from environmental influences		
Temperature drift of sensitivity	$u_{rel}(\delta R_{temp}) = \frac{2 \times 10^{-6} \times 40}{\sqrt{12}} = 2{,}309 \times 10^{-5}$	7.4.3-1
Buoyancy	Not relevant in this case.	7.4.3-2
Change in adjustment due to drift (change of *E*(*Max*) over 1 year = 15 kg)	$u_{rel}(\delta R_{adj}) = \|15\|/(30000\sqrt{3}) = 2{,}887 \times 10^{-4}$	7.4.3-6
Uncertainties from the operation of the instrument		
Tare balancing operation	$u_{rel}(\delta R_{Tare}) = 1{,}154 \times 10^{-4}$	7.4.4-5
Creep, hysteresis (loading time)	Not relevant in this case (short loading time).	7.4.4-7
Eccentric loading	$u_{rel}(\delta R_{ecc}) = 2{,}770 \times 10^{-4}$	7.4.4-10
Uncertainty of a weighing result		
Standard uncertainty, corrections to the readings E_{appr} to be applied	$u(W) = \sqrt{(25{,}333 \text{ kg}^2 + 1{,}960 \times 10^{-7} R^2)}$	7.4.5-1a 7.4.5-1b
Expanded uncertainty, corrections to the readings E_{appr} to be applied	$U(W) = 2\sqrt{(25{,}333 \text{ kg}^2 + 1{,}960 \times 10^{-7} R^2)}$	7.5.1-2b
Simplified to first order	$U(W) \approx 10{,}067 \text{ kg} + 6{,}113 \times 10^{-4} R$	7.5.2-3d
Global uncertainty of a weighing result without correction to the readings		
$U_{gl}(W) = U(W) + \|E_{appr}(R)\|$	$U_{gl}(W) \approx 10 \text{ kg} + 1{,}04 \times 10^{-3} R$	7.5.2-3a

The condition regarding the observed chi-squared value following (C2.2-2a) was checked with positive result. The first linear regression taking into account the weighing factors p'_j, equation (C2.2-18b).

Based on the global uncertainty, the minimum weight value for the instrument may be derived as per Appendix G.

Example:

Weighing tolerance requirement: 1 %

Safety factor: 1

The minimum weight according to formula (G-9), using the above equation for the global uncertainty results in 1 123 kg; i.e. the user needs to weigh a net quantity of material that exceeds 1 123 kg in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1 % and a safety factor of 1.

If a safety factor is included, it might be chosen to be 2. Because of the large global uncertainty, a higher safety factor might not be able to be realised.

The minimum weight according to formula (G-9), using the above equation for the global uncertainty results in 2 542 kg; i.e. the user needs to weigh a net quantity of material that exceeds 2 542 kg in order to achieve a relative (global) measurement uncertainty for a relative weighing tolerance requirement of 1 % and a safety factor of 2 (equals a relative weighing tolerance of 0,50 %).

H3.5 Further information to the example: Details of the substitution procedure (4.3.3)

It is highly recommended to let the substitution load indicate – as far as possible –

the same value as the standard load (as demonstrated for the indication of 10 005 kg for the second situation).

For this purpose, the substitution load can be adjusted by adding or removing small metallic parts until you get the same indication value (10 005 kg). The value of mass assigned to first substitution load is $L_{sub1} = m_N$ = 10 000 kg.

Note: Both m_N and m_{ref} can be used ($m_{ref} = m_N$).

In the same table, the situation when it was not possible to adjust the substitution load to achieve the indication value 20 008 kg is presented. The value of mass assigned to second substitution load is $L_{sub2} = m_N + I(L_{sub2}) - I(m_N)$ = 10 000 kg + 20 018 kg −20 008 kg = 10 010 kg, and the total substitution load L_{sub} is $L_{sub} = L_{sub1} + L_{sub2}$ = 20 010 kg.

H4 Determination of the error approximation function

Preliminary note:

In this example the main procedure for the determination of the coefficients of the calibration function and the evaluation of the related uncertainties as described in Appendix C is shown.

H4.1 Conditions specific for the calibration

Instrument	**Electronic weighing instrument**
Maximum Capacity *Max*/ Scale interval d	**400 g / 0,000 1 g**
Adjustment by calibrator	**Adjusted immediately before calibration (built-in adjustment weights).**
Room conditions	**Temperature 23 °C** **Air density ρ_{aCal}=1,090 kg/m³, $u(\rho_{aCal})$=0,004 kg/m³**
Test loads / acclimatization	**Standard weights, Class E_2**, acclimatized to room temperature: δm_{conv} =0; $u(\delta m_{conv})$=0.

H4.2 Tests and calibration results

Repeatability test performed at 200 g	$s(I)$ = 0,052 mg	7.1.1-5
Eccentricity test performed at 200 g	$\lvert \Delta I_{ecci} \rvert_{max}$= 0,10 mg $u_{rel}(I_{ecc})$= 0,000 144	7.1.1-11
Calibration method	The test loads applied increasing by steps with unloading between the separate steps. Number of test points n = 9. Number of cycles N = 3.	
Uncertainty due to the repeatability	$u(\delta I_{\text{rep}}) = s(I_j)/\sqrt{N}$ = 0,030 mg	7.1.1-6

H4.3 Errors and related uncertainties (budget of related uncertainties)

Conditions:

- The uncertainty of the error at zero comprises the uncertainty of the no-load indication and the repeatability.
- The eccentric loading is taken into account for the calibration according to (7.1.1-10)
- The error of indication is derived using the calibration value as reference value, the uncertainty contribution due to the reference mass is given by the Calibration Certificate $u(\delta m_c) = U/2$.
- In addition, also the air density at the time of calibration ρ_{a1} is known.
- The drift of the weights is estimated by subsequent recalibrations.

The results are:

m_N	m_c/g	$U(\delta m_c)$ / mg	$u(\delta m_D)$ / mg
50 g	50,000 006	0,030	0,005
100 g	99,999 987	0,050	0,010
200 g	200,000 013	0,090	0,015
200 g*	199,999 997	0,090	0,015

ρ_{Cal}=8000 kg/m^3, $u(\rho_{Cal})$=60 kg/m^3

Calibration carried out at an air density ρ_{a1}=1,045 kg/m^3.

From equation (4.2.4-4) $\delta m_B = 0$, therefore $m_{ref} = m_c$.

- The weights are acclimatized to the ambient temperature, the temperature variation during the balance calibration is negligible.
- The balance is adjusted immediately before calibration and air density at the calibration time is determined.
- The air buoyancy uncertainty is determined by

$$u^2(\delta m_{\text{B}}) = m_{\text{N}}^2 \; u_{\text{rel}}^2(\delta m_{\text{B}})$$

by (7.1.2-5b).

Note that in this example this contribution is negative, for this reason, the variance contribution instead of the uncertainty is given.

Loads from 0 g to 200 g

Quantity or Influence	Load and indication in g Error and uncertainties in mg					Formula
Load m_{ref}/g	**0**	**50,000 006**	**99,999 987**	**149,999 993**	**200,000 013**	
Indication *I* /g (mean value)	**0,000 000**	**50,000 067**	**100,000 100**	**150,000 233**	**200,000 267**	
Error of Indication *E* /mg	**0,000**	**0,061**	**0,113**	**0,240**	**0,254**	7.1-1
Repeatability *s* /mg	**0,030**					7.1.1-6
Resolution $u(\delta I_{dig0})$ /mg	0,029					7.1.1-2a
Resolution $u(\delta I_{digL})$/mg	0,000	0,029				7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /mg	0,000	0,007	0,014	0,022	0,029	7.1.1-10
Uncertainty of the indication$u(I)$ /mg	0,042	0,051	0,053	0,055	0,058	7.1.1-12
Test loads m_N/g	0	50	100	100 50	200	
Weights $u(\delta m_c)$ /mg	0,000	0,015	0,025	0,040	0,045	7.1.2-3
Drift $u(\delta m_D)$ /mg	0,000	0,005	0,010	0,015	0,015	7.1.2-11
Buoyancy$u^2(\delta m_B)$/mg^2	0,000	$-4{,}83\times10^{-5}$	$-1{,}93\times10^{-4}$	$-4{,}35\times10^{-4}$	$-7{,}73\times10^{-4}$	7.1.2-5b
Convection $u(\delta m_{conv})$/mg	Not relevant in this case.					7.1.2-13
Uncertainty of the reference mass $u(m_{ref})$/ mg	0,000	0,014	0,023	0,037	0,038	7.1.2-14
Standard uncertainty of the error $u(E)$ / mg	0,042	0,053	0,058	0,067	0,070	7.1.3-1a

Loads from 250 g to 400 g

Quantity or Influence	Load and indication in g Error and uncertainties in mg				Formula
Load m_{ref} (m_N) /g	**250,000 019**	**300,000 000**	**350,000 006**	**400,000 010**	
Indication I/g	**250,000 100**	**300,000 200**	**350,000 267**	**400,000 400**	
Error of Indication E /mg	**0,081**	**0,200**	**0,261**	**0,390**	7.1-1
Repeatability s /mg	**0,030**				7.1.1-6
Resolution $u(\delta I_{dig0})$ /mg	0,029				7.1.1-2a
Resolution $u(\delta I_{digL})$/mg	0,000	0,029			7.1.1-3a
Eccentricity $u(\delta I_{ecc})$ /mg	0,036	0,043	0,051	0,058	7.1.1-10
Uncertainty of the indication$u(I)$ /mg	0,062	0,067	0,072	0,077	7.1.1-12
Test loadsm_N/g	50 200	100 200	50 100 200	200 200 *	
Weights $u(\delta m_c)$ /mg	0,060	0,070	0,085	0,090	7.1.2-3
Drift $u(\delta m_c)$/mg	0,020	0,025	0,030	0,030	7.1.2-11
Buoyancy$u^2(\delta m_B)$/mg^2	$-1{,}21\times10^{-3}$	$-1{,}74\times10^{-3}$	$-2{,}37\times10^{-3}$	$-3{,}09\times10^{-3}$	7.1.2-5b
Convection $u(\delta m_{conv})$ /mg	Not relevant in this case.				7.1.2-13
Uncertainty of the reference mass$u(m_{ref})$ /mg	0,053	0,062	0,076	0,077	7.1.2-14
Standard uncertainty of the error $u(E)$ /mg	0,082	0,091	0,104	0,109	7.1.3-1a

From the calibration results the calibration function $E = f(I)$ is determined.

As an example the linear regression model $E = a_1 I$ is considered.

The coefficientsa_1 is determined by equation C2.2-6.

Table H4.1 shows the matrix ***X*** and the vector ***e***. The relevant covariance matrix ***U***(***e***) is given in Table H4.4, which is determined by (C2.2-3a).

Table H4.2 shows the covariance matrices ***U***(m_{ref}), which is determined by (C2.2-3b), where the column vector $\mathbf{s}_{m_{ref}}$ is given by the uncertainties of the reference mass $u(m_{ref})$.

Table H4.3 shows the covariance matrix ***U***(I_{cal}) which is a diagonal matrix having on the diagonal the square values of ***U***(I_{cal}).

At the first step no contribution is considered for ***U***(*mod*) (s_m = 0).

As the number of test points is n = 9 and the number of parameters is n_{par} = 1, the degrees of freedom are $\upsilon = n - n_{par} = 8$.

Table H4.1: Matrix *X* and vector *e*

X/g	***e***/mg
0	0,000
50,000 067	0,061
100,000 100	0,213
150,000 233	0,274
200,000 267	0,254
250,000 100	0,181
300,000 200	0,200
350,000 267	0,261
400,000 400	0,390

Table H4.2: Covariance matrix $U(m_{\text{ref}})$

0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	$2{,}017\times10^{-4}$	$3{,}274\times10^{-4}$	$5{,}294\times10^{-4}$	$5{,}457\times10^{-4}$	$7{,}503\times10^{-4}$	$8{,}736\times10^{-4}$	$1{,}077\times10^{-3}$	$1{,}091\times10^{-3}$
0,000	$3{,}274\times10^{-4}$	$5{,}316\times10^{-4}$	$8{,}596\times10^{-4}$	$8{,}860\times10^{-4}$	$1{,}218\times10^{-3}$	$1{,}418\times10^{-3}$	$1{,}749\times10^{-3}$	$1{,}772\times10^{-3}$
0,000	$5{,}294\times10^{-4}$	$8{,}596\times10^{-4}$	$1{,}390\times10^{-3}$	$1{,}433\times10^{-3}$	$1{,}970\times10^{-3}$	$2{,}294\times10^{-3}$	$2{,}829\times10^{-3}$	$2{,}865\times10^{-3}$
0,000	$5{,}457\times10^{-4}$	$8{,}860\times10^{-4}$	$1{,}433\times10^{-3}$	$1{,}477\times10^{-3}$	$2{,}030\times10^{-3}$	$2{,}364\times10^{-3}$	$2{,}915\times10^{-3}$	$2{,}953\times10^{-3}$
0,000	$7{,}503\times10^{-4}$	$1{,}218\times10^{-3}$	$1{,}970\times10^{-3}$	$2{,}030\times10^{-3}$	$2{,}792\times10^{-3}$	$3{,}250\times10^{-3}$	$4{,}009\times10^{-3}$	$4{,}060\times10^{-3}$
0,000	$8{,}736\times10^{-4}$	$1{,}418\times10^{-3}$	$2{,}294\times10^{-3}$	$2{,}364\times10^{-3}$	$3{,}250\times10^{-3}$	$3{,}785\times10^{-3}$	$4{,}668\times10^{-3}$	$4{,}728\times10^{-3}$
0,000	$1{,}077\times10^{-4}$	$1{,}749\times10^{-3}$	$2{,}829\times10^{-3}$	$2{,}915\times10^{-3}$	$4{,}009\times10^{-3}$	$4{,}668\times10^{-3}$	$5{,}756\times10^{-3}$	$5{,}831\times10^{-3}$
0,000	$1{,}091\times10^{-4}$	$1{,}772\times10^{-3}$	$2{,}865\times10^{-3}$	$2{,}953\times10^{-3}$	$4{,}060\times10^{-3}$	$4{,}728\times10^{-3}$	$5{,}831\times10^{-3}$	$5{,}906\times10^{-3}$

Table H4.3: Covariance matrix $U(I_{\text{Cal}})$

$1{,}735\times10^{-3}$	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	$2{,}620\times10^{-3}$	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	$2{,}776\times10^{-3}$	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	$3{,}037\times10^{-3}$	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	$3{,}401\times10^{-3}$	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	$3{,}870\times10^{-3}$	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	$4{,}443\times10^{-3}$	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	$5{,}120\times10^{-3}$	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	$5{,}901\times10^{-3}$

Table H4.4: Covariance matrix $\boldsymbol{U}(\boldsymbol{e})$ with s_m = 0

$1{,}735\times10^{-3}$	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	$2{,}822\times10^{-3}$	$3{,}274\times10^{-4}$	$5{,}294\times10^{-4}$	$5{,}457\times10^{-4}$	$7{,}503\times10^{-4}$	$8{,}736\times10^{-4}$	$1{,}077\times10^{-3}$	$1{,}091\times10^{-3}$
0,000	$3{,}274\times10^{-4}$	$3{,}308\times10^{-3}$	$8{,}596\times10^{-4}$	$8{,}860\times10^{-4}$	$1{,}218\times10^{-3}$	$1{,}418\times10^{-3}$	$1{,}749\times10^{-3}$	$1{,}772\times10^{-3}$
0,000	$5{,}294\times10^{-4}$	$8{,}596\times10^{-4}$	$4{,}427\times10^{-3}$	$1{,}433\times10^{-3}$	$1{,}970\times10^{-3}$	$2{,}294\times10^{-3}$	$2{,}829\times10^{-3}$	$2{,}865\times10^{-3}$
0,000	$5{,}457\times10^{-4}$	$8{,}860\times10^{-4}$	$1{,}433\times10^{-3}$	$4{,}878\times10^{-3}$	$2{,}030\times10^{-3}$	$2{,}364\times10^{-3}$	$2{,}915\times10^{-3}$	$2{,}953\times10^{-3}$
0,000	$7{,}503\times10^{-4}$	$1{,}218\times10^{-3}$	$1{,}970\times10^{-3}$	$2{,}030\times10^{-3}$	$6{,}662\times10^{-3}$	$3{,}250\times10^{-3}$	$4{,}009\times10^{-3}$	$4{,}060\times10^{-3}$
0,000	$8{,}736\times10^{-4}$	$1{,}418\times10^{-3}$	$2{,}294\times10^{-3}$	$2{,}364\times10^{-3}$	$3{,}250\times10^{-3}$	$8{,}228\times10^{-3}$	$4{,}668\times10^{-3}$	$4{,}728\times10^{-3}$
0,000	$1{,}077\times10^{-3}$	$1{,}749\times10^{-3}$	$2{,}829\times10^{-3}$	$2{,}915\times10^{-3}$	$4{,}009\times10^{-3}$	$4{,}668\times10^{-3}$	$1{,}088\times10^{-2}$	$5{,}831\times10^{-3}$
0,000	$1{,}091\times10^{-3}$	$1{,}772\times10^{-3}$	$2{,}865\times10^{-3}$	$2{,}953\times10^{-3}$	$4{,}060\times10^{-3}$	$4{,}728\times10^{-3}$	$5{,}831\times10^{-3}$	$1{,}181\times10^{-2}$

H4.4 Results

Applying (C2.2-6) and (C2.2-9), the results are

$$a_1 = 0{,}00083 \text{ mg/g}$$

The covariance matrix $U(\hat{a})$ is

$5{,}109\times10^{-8}$ (mg/g)^2

from which

$$u(a_1) = 0{,}00023 \text{ mg/g}$$

From (C2.2-8)

$$\chi^2_{\text{obs}} = 12{,}5$$

As in this case the χ^2test (C2.2-2a) fails, an uncertainty contribution s_m is added.

Considering s_m = 0,05 mg, the correspondent covariance matrix $\boldsymbol{U}(mod)$ is given by a diagonal matrix 9x9 having $s_m^2 = 0{,}05^2$ on the diagonal. Table H4.5 shows the correspondent covariance matrix $\boldsymbol{U}(\boldsymbol{e})$.

Table H4.5: Covariance matrix $\boldsymbol{U}(\boldsymbol{e})$ evaluated with s_m =0,05 mg

$4{,}235\times10^{-3}$	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	$5{,}322\times10^{-3}$3	$3{,}274\times10^{-4}$	$5{,}294\times10^{-4}$	$5{,}457\times10^{-4}$	$7{,}503\times10^{-4}$	$8{,}736\times10^{-4}$	$1{,}077\times10^{-3}$	$1{,}091\times10^{-3}$
0,000	$3{,}274\times10^{-4}$	5,808E-03	$8{,}596\times10^{-4}$	$8{,}860\times10^{-4}$	$1{,}218\times10^{-3}$	$1{,}418\times10^{-3}$	$1{,}749\times10^{-3}$	$1{,}772\times10^{-3}$
0,000	$5{,}294\times10^{-4}$	$8{,}596\times10^{-4}$	$6{,}927\times10^{-3}$	$1{,}433\times10^{-3}$	$1{,}970\times10^{-3}$	$2{,}294\times10^{-3}$	$2{,}829\times10^{-3}$	$2{,}865\times10^{-3}$
0,000	$5{,}457\times10^{-4}$	$8{,}860\times10^{-4}$	$1{,}433\times10^{-3}$	$7{,}378\times10^{-3}$	$2{,}030\times10^{-3}$	$2{,}364\times10^{-3}$	$2{,}915\times10^{-3}$	$2{,}953\times10^{-3}$
0,000	$7{,}503\times10^{-4}$	$1{,}218\times10^{-3}$	$1{,}970\times10^{-3}$	$2{,}030\times10^{-3}$	$9{,}162\times10^{-3}$	$3{,}250\times10^{-3}$	$4{,}009\times10^{-3}$	$4{,}060\times10^{-3}$
0,000	$8{,}736\times10^{-4}$	$1{,}418\times10^{-3}$	$2{,}294\times10^{-3}$	$2{,}364\times10^{-3}$	$3{,}250\times10^{-3}$	$1{,}073\times10^{-2}$	$4{,}668\times10^{-3}$	$4{,}728\times10^{-3}$
0,000	$1{,}077\times10^{-3}$	$1{,}749\times10^{-3}$	$2{,}829\times10^{-3}$	$2{,}915\times10^{-3}$	$4{,}009\times10^{-3}$	$4{,}668\times10^{-3}$	$1{,}338\times10^{-2}$	$5{,}831\times10^{-3}$
0,000	$1{,}091\times10^{-3}$	$1{,}772\times10^{-3}$	$2{,}865\times10^{-3}$	$2{,}953\times10^{-3}$	$4{,}060\times10^{-3}$	$4{,}728\times10^{-3}$	$5{,}831\times10^{-3}$	$1{,}431\times10^{-2}$

The new results are

$$a_1 = 0{,}00084 \text{ mg/g}$$

The covariance matrix is

$5{,}637\times10^{-8}$ (mg/g)^2

from which

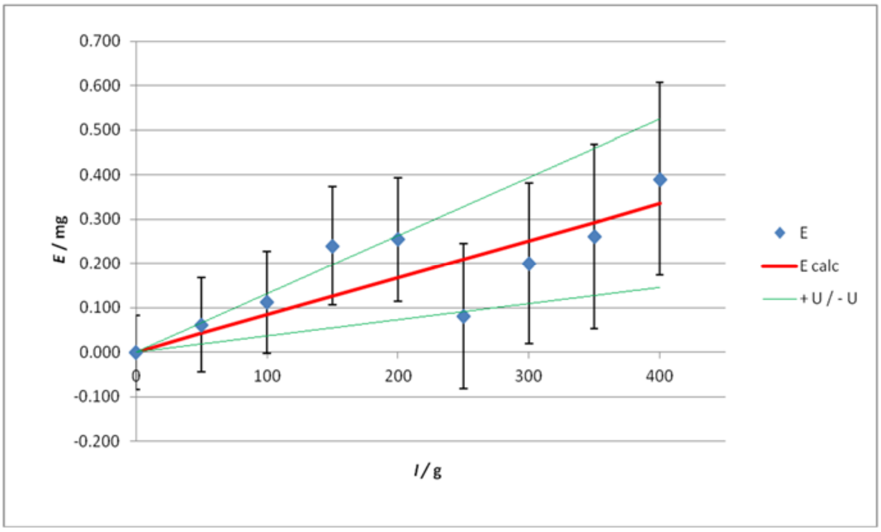
$$u(a_1) = 0{,}00024 \text{ mg/g}$$

and

$$\chi^2_{\text{obs}} = 7{,}3$$

In this case, the χ^2 test (C2.2-2a) passes. The plot of the result is shown in Figure H4-1.

Figure H4-1: Measured errors of indication *E* and the linear fitting function with the associated uncertainty bands



The residuals and the uncertainties associated with the calibration points are calculated by (C2.2-7) and (C2.2-11) respectively, and are shown in Table H4.6.

Table H4.6: Calculated error, residuals and uncertainties associated to the calibration points

I /g	E /mg	E_{appr} /mg	Residual ν /mg	$u(E_{appr})$ /mg	$U(E_{appr})$ /mg	Residual Test (C2.2-2b)
0	0,000	0,000	0,000	0,000	0,000	YES
50,000 067	0,061	0,042	-0,019	0,012	0,024	YES
100,000 200	0,113	0,084	-0,029	0,024	0,047	YES
150,000 267	0,240	0,126	-0,114	0,036	0,071	NO
200,000 267	0,254	0,168	-0,086	0,047	0,095	YES
250,000 200	0,081	0,210	0,129	0,059	0,119	NO
300,000 200	0,200	0,252	0,052	0,071	0,142	YES
350,000 267	0,261	0,293	0,032	0,083	0,166	YES
400,000 400	0,390	0,335	-0,055	0,095	0,190	YES

If the alternative method given with (C2.2-2b) is followed, which is much more restrictive, the residual test fails in two points according to Table H4.6.

In order to obtain the goodness of the fit according to the condition (C2.2-2b), it is necessary to consider a contribution s_m = 0,25 mg and therefore a new matrix $\boldsymbol{U}(\boldsymbol{e})$ is calculated, which is given in Table H4.7.

Table H4.7: Covariance matrix $U(e)$ evaluated with s_m =0,25 mg

$6,423\times10^{-2}$	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	$6,532\times10^{-2}$	$3,274\times10^{-4}$	$5,294\times10^{-4}$	$5,457\times10^{-4}$	$7,503\times10^{-4}$	$8,736\times10^{-4}$	$1,077\times10^{-3}$	$1,091\times10^{-3}$
0,000	$3,274\times10^{-4}$	$6,581\times10^{-2}$	$8,596\times10^{-4}$	$8,860\times10^{-4}$	$1,218\times10^{-3}$	$1,418\times10^{-3}$	$1,749\times10^{-3}$	$1,772\times10^{-3}$
0,000	$5,294\times10^{-4}$	$8,596\times10^{-4}$	$6,693\times10^{-2}$	$1,433\times10^{-3}$	$1,970\times10^{-3}$	$2,294\times10^{-3}$	$2,829\times10^{-3}$	$2,865\times10^{-3}$
0,000	$5,457\times10^{-4}$	$8,860\times10^{-4}$	$1,433\times10^{-3}$	$6,738\times10^{-2}$	$2,030\times10^{-3}$	$2,364\times10^{-3}$	$2,915\times10^{-3}$	$2,953\times10^{-3}$
0,000	$7,503\times10^{-4}$	$1,218\times10^{-3}$	$1,970\times10^{-3}$	$2,030\times10^{-3}$	$6,916\times10^{-2}$	$3,250\times10^{-3}$	$4,009\times10^{-3}$	$4,060\times10^{-3}$
0,000	$8,736\times10^{-4}$	$1,418\times10^{-3}$	$2,294\times10^{-3}$	$2,364\times10^{-3}$	$3,250\times10^{-3}$	$7,073\times10^{-2}$	$4,668\times10^{-3}$	$4,728\times10^{-3}$
0,000	$1,077\times10^{-3}$	$1,749\times10^{-3}$	$2,829\times10^{-3}$	$2,915\times10^{-3}$	$4,009\times10^{-3}$	$4,668\times10^{-3}$	$7,338\times10^{-2}$	$5,831\times10^{-3}$
0,000	$1,091\times10^{-3}$	$1,772\times10^{-3}$	$2,865\times10^{-3}$	$2,953\times10^{-3}$	$4,060\times10^{-3}$	$4,728\times10^{-3}$	$5,831\times10^{-3}$	$7,431\times10^{-2}$

With this approach the result is

a_1= 0,00084 mg/g

The covariance matrixis

$1,745\times10^{-7}$ (mg/g)^2

Therefore

$u(a_1)$= 0,00042 mg/g

The plot of the results is shown in Figure H4-2. The calculated residuals and the uncertainties associated to the calibration points are shown in Table H4.8.

Figure H4-2: Measured errors of indication *E* and the linear fitting function with the associated uncertainty bands

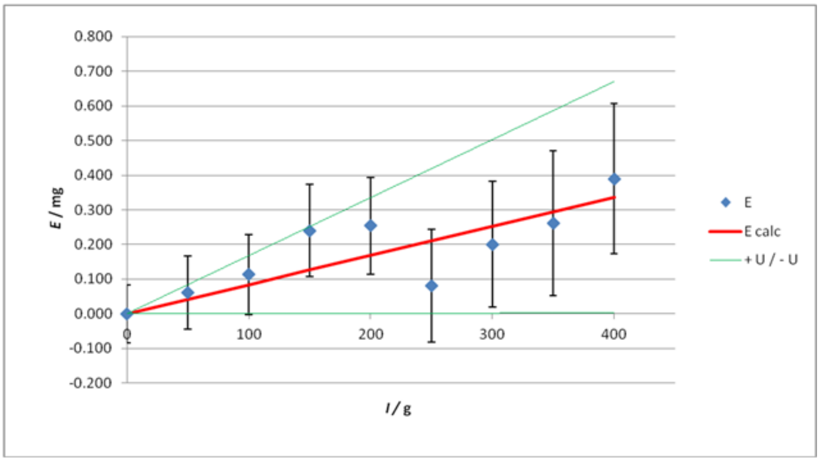


Table H4.8: Calculated error, residuals and uncertainties associated to the calibration points

I /g	*E* /mg	E_{appr} /mg	Residual ν /mg	$u(E_{appr})$ /mg	$U(E_{appr})$ /mg	Residual Test
0	0,000	0,000	0,000	0,000	0,000	YES
50,000 067	0,061	0,042	-0,019	0,021	0,042	YES
100,000 200	0,213	0,084	-0,029	0,042	0,084	YES
150,000 267	0,274	0,126	-0,114	0,063	0,125	YES
200,000 267	0,254	0,168	-0,086	0,084	0,167	YES
250,000 200	0,181	0,210	0,129	0,104	0,209	YES
300,000 200	0,200	0,252	0,052	0,125	0,251	YES
350,000 267	0,261	0,294	0,033	0,146	0,292	YES
400,000 400	0,390	0,336	-0,054	0,167	0,334	YES

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