



**VoteTrustUSA**  
National Coalition for Election Integrity



## **E-Voter Education Project**

# **Random Auditing of E-Voting Systems: How Much is Enough?**

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*This is the second in a series of presentations which will explain the risks and benefits of electronic voting (e-voting) systems to candidates, elected officials and the public at large.*

**Abstract** – *E-voting systems with a means of independent verification, such as a voter-verified paper record or ballot, can be audited to ensure that all votes on the audited systems were counted as cast. While a random sample comprising a small percentage of thousands of voting systems may be adequate to confirm the outcome of all but the closest statewide races, we will show herein using the example of a Congressional District (CD), that small-percentage audits are inadequate to verify the outcome of all but the most decisive races (landslides). To be assured of finding a discrepancy in the vote tallies that would trigger additional auditing or a full hand recount of a race for the US House of Representatives or smaller local races, a much larger percentage of systems must be audited than commonly suggested. Each race must be considered a separate auditing process taking into account the vote margin, the number of precincts in which the race appeared on the ballot and the possibility of miscounts concentrated in a relatively small number of large precincts. To reduce the potential for miscounts, additional checks and balances such as pre-election auditing of ballot definition programming are necessary and will be the subject of a future report.*

**Acknowledgement** – *Dr. Mark Lindeman and Ellen Theisen generously provided numerous valuable insights and suggestions for improvement (many of which were actually implemented).*

## Background

Concerns about proprietary software, frequent hardware failures<sup>1</sup>, configuration errors<sup>2</sup> and the potential for deliberate exploitation of e-voting systems, which include both Direct Recording Electronic (DRE, e.g., touch screen, pushbutton) and optically scanned paper ballot (OSPB) systems, have led computer scientists, legislators and election integrity activists to advocate mandatory manual random auditing (MMRA) of a small percentage of voting system tallies as a means of checking end-to-end vote counting integrity. Such audits can be conducted by selecting a random sample of precincts and comparing the system tallies to hand-to-eye counts of the voter-verified paper ballots or records in the same precincts. (Presently, DRE voting systems that do not produce such paper

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<sup>1</sup> Howard Stanislevic, VoteTrustUSA E-Voter Education Project, DRE Reliability: Failure by Design?, [http://www.votetrustusa.org/pdfs/DRE\\_Reliability.pdf](http://www.votetrustusa.org/pdfs/DRE_Reliability.pdf)

<sup>2</sup> Ellen Theisen, Ballot Definition Files, No Review Is Provided for a Key Component of Voting System Software, <http://www.votersunite.org/info/BallotProgramming.pdf>

records cannot be independently audited.) If a discrepancy is found in the initial audit, additional auditing can be conducted to determine if other precincts have been corrupted, which could change the outcome of a race. But if an initial audit were found to be “clean”, i.e., without discrepancies, there may be no obvious basis for additional investigation, thereby allowing miscounts that could change an electoral outcome to go undiscovered. Determining the probability of such an occurrence and adjusting the auditing percentage accordingly for each race should therefore be a high priority.

***N.B.*** – *While some laws or regulations may require the random selection of voting machines or systems rather than precincts, to simplify the following discussion we will use the terms “precinct” and “within-precinct miscount” (WPM) until later on in this work. We will also limit our discussion to 2-candidate races.*

### **Random Sampling Without Replacement**

One method of auditing the vote count is to select precincts randomly using a uniform distribution in which all precincts have an equal chance of being selected for auditing. Once a precinct has been selected, it cannot be selected again for a given race, so the pool of remaining precincts to be sampled is reduced by one. The initial number of precincts to be sampled can be stated as a percentage of the total number of precincts within a state or other jurisdiction by law or regulation. Typically, the initial sample size is from 1% to 5% of the total precincts in the pool.

The method of auditing described above may be referred to as random sampling without replacement. It relies on a statistical technique known as the hypergeometric distribution, nicely illustrated by Dr. Charles Stanton<sup>3</sup> with an accompanying Java applet at the following URL:

<http://www.math.csusb.edu/faculty/stanton/probstat/lotto.html>

Stanton describes the hypergeometric distribution as follows:

There are three parameters:  $N$ , the total number of objects (precincts);  $R$ , the number of objects of the first type (in our case, corrupt precincts); and  $k$ , the number of objects to be chosen (the precincts to be audited). The probability function  $f(x)$  is:

$f(x) = C(R,x) * C(N-R, k-x) / C(N,k)$  for  $x = \max(0, k+R-N) .. \min(R,k)$   
where  $C(n,k)$  is the binomial coefficient.

This is analogous to picking lottery numbers, but the “winners” in this case are the corrupt precincts detected by the audit.

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<sup>3</sup> <http://www.math.csusb.edu/faculty/stanton/>

Ellen Theisen has written a paper<sup>4</sup> with an accompanying Excel spreadsheet<sup>5</sup> to calculate the odds of finding one or two of a potential series of corrupt precincts using the hypergeometric distribution. Dopp, et al, have done similar work, also taking within-precinct vote shift percentage into account.<sup>6</sup> Also see Neff's work comparing Congressional to statewide races in California.<sup>7</sup> We are not aware of any published work that takes precinct size into account so we will do so herein.

For a given auditing percentage (e.g., 2%), a larger total number of precincts will result in a larger set of precincts to be audited. This means that for a given percentage of sampling, there will be more chances to find a corrupt precinct within a large set of precincts than within a small one.

For any given race, the greater the total number of precincts:

- the more corrupt precincts will be required to change the outcome;
- the more attempts to find the corrupt precincts will be allowed;
- and the greater the chances of finding a corrupt precinct will be.

All this makes it more likely to find a corrupt precinct in a large race than a small race. Therefore, in most states, the outcome of a statewide race (e.g., Governor or US Senator) is actually easier to confirm with a small-percentage audit than a race for the US House of Representatives. (The exceptions are the states with only one Congressional District.)

### **Practical Auditing Considerations**

The precise method of conducting audits unannounced (or maintaining the chain of custody to prevent tampering if they are announced) is beyond the scope of this work. Also, it is assumed that reconciling the number of poll book signatures with the number of ballots cast at each precinct is already standard procedure to rule out ballot stuffing, loss or destruction. The Brennan Center's report, "The Machinery of Democracy", suggests some ways of ensuring transparent random selection, which we would encourage.<sup>8</sup> A transparent method of comparing precinct totals to central tabulator totals for the same precincts should also be

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<sup>4</sup> Theisen, Auditing Election Equipment — The Real Scoop!,  
<http://www.votersunite.org/info/auditingissues.pdf>

<sup>5</sup> Theisen, <http://www.votersunite.org/info/AuditEffectivenessCalculator.xls>

<sup>6</sup> Kathy Dopp, et al, How Can Independent Paper Audits Detect and Correct Vote Miscounts?,  
[http://electionarchive.org/ucvAnalysis/US/paper-audits/Paper\\_Audits.pdf](http://electionarchive.org/ucvAnalysis/US/paper-audits/Paper_Audits.pdf)

<sup>7</sup> C. Andrew Neff, Election Confidence, A Comparison of Methodologies and Their Relative Effectiveness at Achieving It, <http://www.votehere.net/papers/ElectionConfidence.pdf>

<sup>8</sup> The Brennan Center for Justice, The Machinery of Democracy, Appendix F,  
<http://brennancenter.org/programs/downloads/SecurityFull7-3Reduced.pdf>

implemented as proposed by Jover.<sup>9</sup> The random audits discussed in this work are therefore designed to detect within-precinct miscounts of legally cast ballots.

When conducting an actual audit, the first thing we must consider is that due to the way districts are drawn, each race on the ballot may appear in some precincts in a jurisdiction but not in others or may extend to one or more adjacent jurisdictions. Any audit that selects precincts in which the race is not on the ballot is irrelevant to the race in question, so we must consider each race as a *separate auditing process*, regardless of jurisdictional boundaries.

An audit is only effective if it can rule out miscounts large enough to affect the outcome of the race in question. Once the relevant precincts have been identified, some of which may actually be in neighboring jurisdictions, we must decide what percentage of the precincts needs to be audited to confirm the outcome of the race. This is not trivial. If we don't audit enough precincts, a significant miscount may elude detection; if we wish to audit more precincts than necessary to confirm the outcome, we may incur unnecessary expense or may not even be permitted to conduct the audit. If a race is close enough (typically a margin of 0.25% to 0.5%), a full hand count might be obtainable without an audit. So we are primarily interested in auditing those races that are close enough to be reversed by undetected miscounts, but not necessarily close enough to trigger a full manual recount.

We won't know in advance how any potential miscount will be distributed among the corrupt precincts. There may be a lot of precincts with a few miscounted votes, a small number of precincts with many miscounted votes, or something in between. We should have no preconceptions. This is why the initial audit must be conducted randomly with each precinct having an equal chance of being selected for auditing. Other methods, such as proportional sampling, are possible but are more difficult to implement in practice.

Since only races with no opposing candidates are decided by 100% of the vote, we will also need to specify the maximum percentage of total votes that could be switched from one candidate to another within a single precinct without arousing suspicion. We will call this parameter the *within-precinct miscount* (WPM).

To determine how much auditing is enough to find a miscount, it's necessary to do some reverse engineering. The greater the difference between the vote shares of the winning and losing candidates (the margin), the more corrupt precincts it would take to reverse the reported outcome. The minimum number of votes to reverse the outcome is half the margin plus one vote (switched from the actual winning candidate to the actual loser). This is the size of the discrepancy we must ultimately find or rule out.

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<sup>9</sup> Juan M. Jover, Ph.D., *Democracy At Risk: The 2004 Election in Ohio*, Section VIII: Transparent Aggregation of Voting Results Using the Internet, <http://a9.g.akamai.net/7/9/8082/v001/www.democrats.org/pdfs/ohvrreport/section08.pdf>

With a WPM of 20%, on average it would take  $1 / \text{WPM}$ , or five precincts to swing the vote by the equivalent of a whole precinct. We can multiply this value by half the margin to obtain the percentage of precincts that could change the outcome:  $5 \times 0.5 = 2.5$  times the margin. E.g., a race with a margin of 5% can be reversed by  $2.5 \times 5\%$  or 12.5% of its precincts, each precinct having a 20% vote shift. However, this assumes that every precinct has the same number of votes, which is highly unlikely in practice.

If the miscounts are concentrated in larger precincts but we implicitly assume that each precinct has only the *average* number of votes (i.e., the total votes in the race divided by the total number of precincts) there is a risk that we may sample too few precincts. For the audit to be effective, we must address this problem by taking precinct size into account.

For the initial audit, we will need to calculate the probability that the first in a series of corrupt precincts will be found, thereby triggering additional auditing. Using random sampling without replacement, the odds of finding a corrupt precinct can be calculated by taking into account:

- the total number of precincts in which the race appeared on the ballot;
- the percentage of the above precincts to be audited;
- the hypothetical number of corrupt precincts we are trying to detect, which we will refer to as  $P_C$ .

The last parameter above ( $P_C$ ) deserves special attention. We will never know the actual percentage of corrupt precincts in advance, but fortunately we won't have to. We will only need to select the minimum value of  $P_C$  that could be large enough to reverse the reported outcome of the race in question. If there are fewer than  $P_C$  corrupt precincts, we may not find a single one of them with our audit, but we will have audited enough such that any undiscovered miscounts would not be enough to change the outcome.

To be certain that the value of  $P_C$  is small enough, we can assume that the total number of switched votes in each corrupt precinct is equal to the *highest* number possible, resulting in the *smallest* percentage of corrupt precincts that could change the outcome. We can do so by assuming that *all* the miscounted votes in the race will be cast in the precincts with the *highest* vote counts, while actually auditing precincts randomly in case the miscounts are elsewhere.

In other words, we will keep the *distribution* of the sampling uniform and completely random because we won't know *a priori* what the distribution of the miscounts will be, but in determining *the size* of the audit, we will use the worst-case scenario in which *all* the miscounts occur in the *largest* precincts. To do

this, we can sort the precincts by their reported vote counts, add these up to obtain the *smallest* number of precincts that could reverse the outcome of the race (i.e., 2.5 times the margin), and determine the size of the random audit based on this worst-case scenario.

Thus, by setting the following parameters, we can now design an effective audit for each race:

- the acceptable degree of certainty that the race was decided correctly (100% certainty would require a full recount; 99% is feasible with an audit and we can make some other conservative assumptions, such as minimizing the value of  $P_C$ , to get closer to absolute certainty);
- the margin (difference in vote share between the two leading candidates);
- WPM: A WPM of >20% would completely reverse a 60%-40% precinct or change the winner in a 70%-30% precinct, arousing suspicion even without an audit. So a 20% WPM seems an acceptable limit. (We provide further support for this argument below using the Edison-Mitofsky 2004 National Exit Poll precincts.);
- $P_C$ : The minimum number of corrupt precincts that could reverse the outcome of the race (the smallest number of whole precincts with an aggregate vote count equal to at least 2.5 x the margin).

## Graphical Analysis

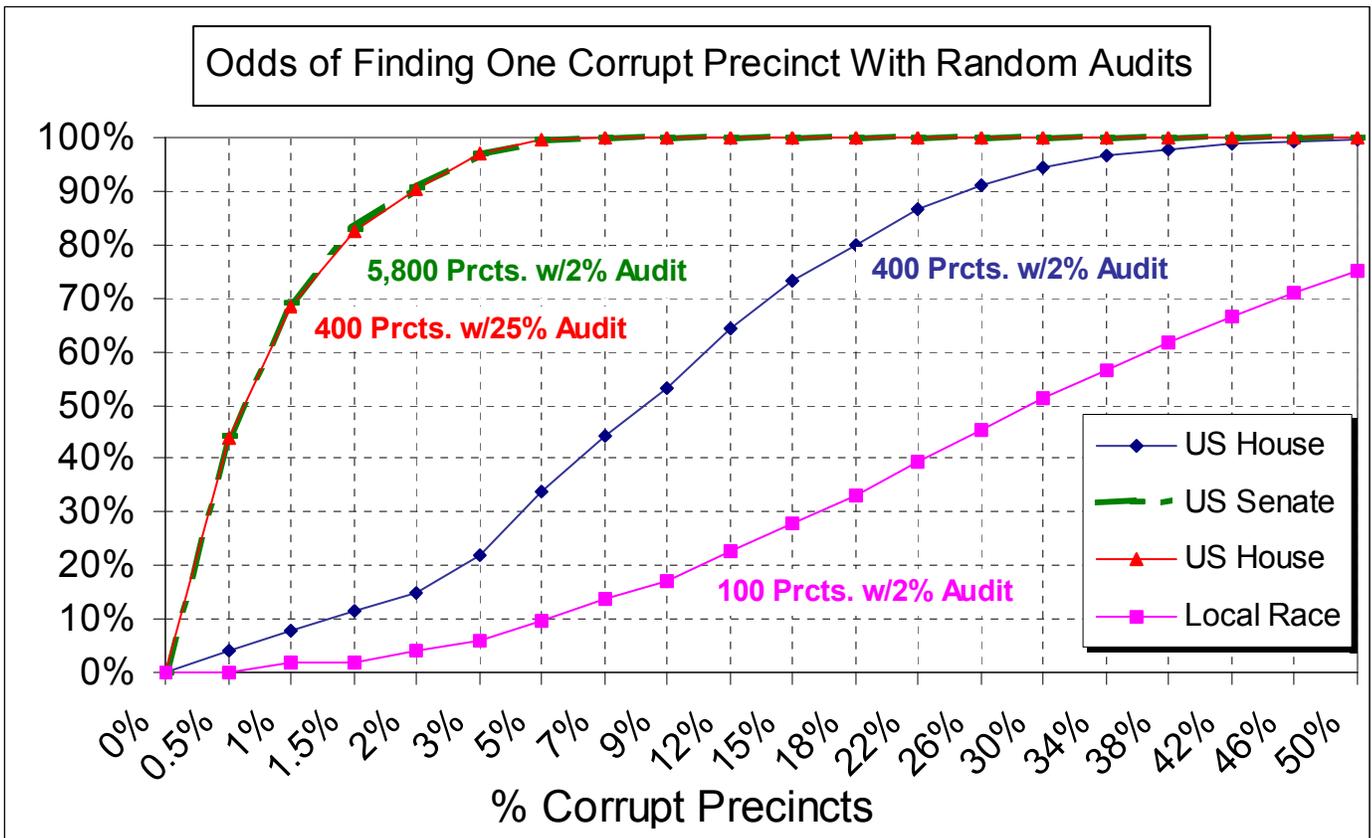
To better visualize the small-percentage auditing problem, it is helpful to consider a few graphical examples.

### 1. Number of precincts

For a hypothetical set of corrupt precincts ranging in size from 0.5% to 50%, the following graph illustrates the odds of detecting the first in the set of such precincts in:

- a US Senate race with 5,800 precincts and a 2% random audit (116 precincts);
- a US House race with 400 precincts and a 2% random audit (8 precincts);
- a US House race with 400 precincts and a 25% random audit (100 precincts);

- a local or state legislative race with 100 precincts and a 2% random audit (2 precincts).



Clearly, with only a 2% audit, Congressional and local candidates are at a disadvantage when it comes to detection of miscounts. To achieve the same level of confidence enjoyed by their Senatorial counterparts, US House candidates would need 25% audits (100 precincts per race) as shown. A state or local legislative race would require a whopping 68% audit (68 precincts, not shown). Outcomes of close local races are therefore nearly impossible to verify without a full recount. Looking at it another way, with a 2% audit it's possible for the outcome of a local race with as much as a 20% margin to be reversed (with 50% of the precincts corrupted) without detection.

## 2. Within-Precinct Miscount

We have defined the term *within-precinct miscount* (WPM) as the maximum possible vote shift from one candidate to another as the result of error or fraud. In their recent report, "The Machinery of Democracy", the Brennan Center for Justice assumed a maximum vote shift of 5% per county, 7.5% per polling place

and 15% per voting machine.<sup>10</sup> Since a precinct can comprise one or more machines and a polling place can comprise one or more precincts, we decided to use vote shift per-precinct as the basic unit for this metric.

The higher the WPM, the fewer corrupt precincts it would take to change an outcome and the larger the audit would have to be to detect them. By examining the shift in George W. Bush's vote share between the 2000 and 2004 presidential elections in the 1,250 Edison-Mitofsky National Exit Poll precincts,<sup>11</sup> we have found that only nine of those precincts (0.72%) exhibited more than a 20% vote shift (24% mean absolute value) which would result in at most a 0.35% change in the overall vote margin. A race close enough to be affected by such a small undetected change in the margin should probably be subject to a full hand count regardless of the outcome of an audit, but it is encouraging that the assumption of a 20% WPM can result in an audit with the potential to detect such a small discrepancy. A 10% audit of a statewide race with 5,800 precincts and a 0.35% margin, assuming a 20% WPM, would have >99% probability of finding a corrupt precinct.

This indicates that the 20% limit may be a realistic assumption for the WPM. Reducing this value to 15% would reduce the audit's sample size but would also allow a larger undetected shift in the margin. Increasing the WPM beyond 20% would be unnecessary since a full hand count should be obtainable for a race close enough to be affected by so few undetected corrupt precincts.

To obtain the percentage of race-wide vote shift possible from the above graph, the "% Corrupt Precincts" values can be multiplied by the WPM of 20%, resulting in a range of 0.1% to 10% shifts of the race-wide tally from one candidate to the other. Such shifts could reverse the outcome of races with margins of 0.2% to 20% according to the following conditional:

$$\text{if } (A\% - B\%) / 2 < (P_C\% \times W\%) \text{ then FALSE}$$

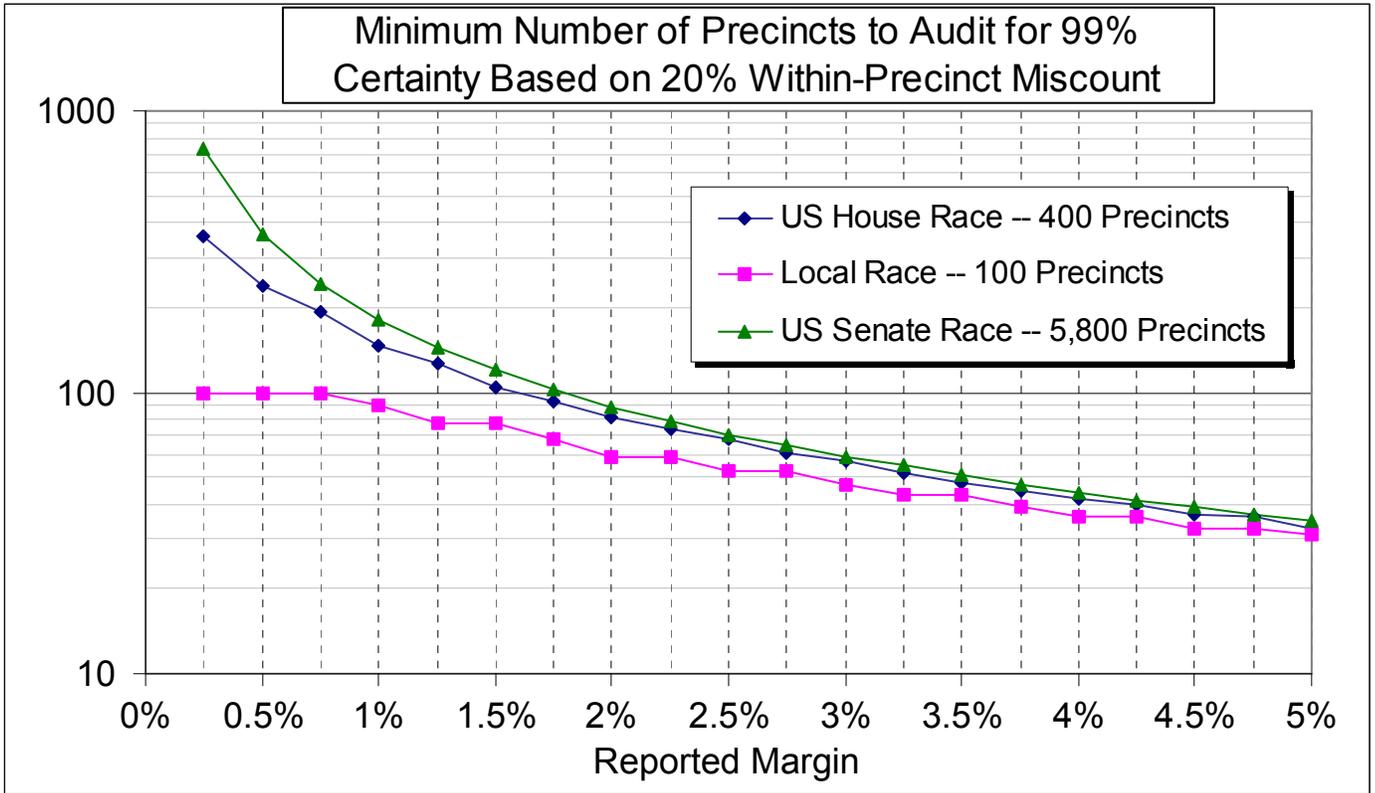
where A% is the "winning" candidate's initial reported vote share; B% is the "losing" candidate's initial reported vote share; P<sub>C</sub>% is the percentage of corrupt precincts; W% is the WPM which we assume to be 20%; FALSE is a reversed outcome of the race due to error or fraud.

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<sup>10</sup> The Brennan Center for Justice, Op. Cit., Pg.22,  
<http://brennancenter.org/programs/downloads/SecurityFull7-3Reduced.pdf>

<sup>11</sup> Data presented at the American Statistical Association, Philadelphia Fall Meeting, Oct. 14, 2005,  
<http://inside.bard.edu/~lindeman/slide2-large.jpg>

Using the 20% WPM, the following graph shows the number of precincts that would have to be audited to be 99% certain of finding one corrupt precinct, given that there were enough corrupt precincts to change the outcome of the race in question with margins of 0.25% to 5%:

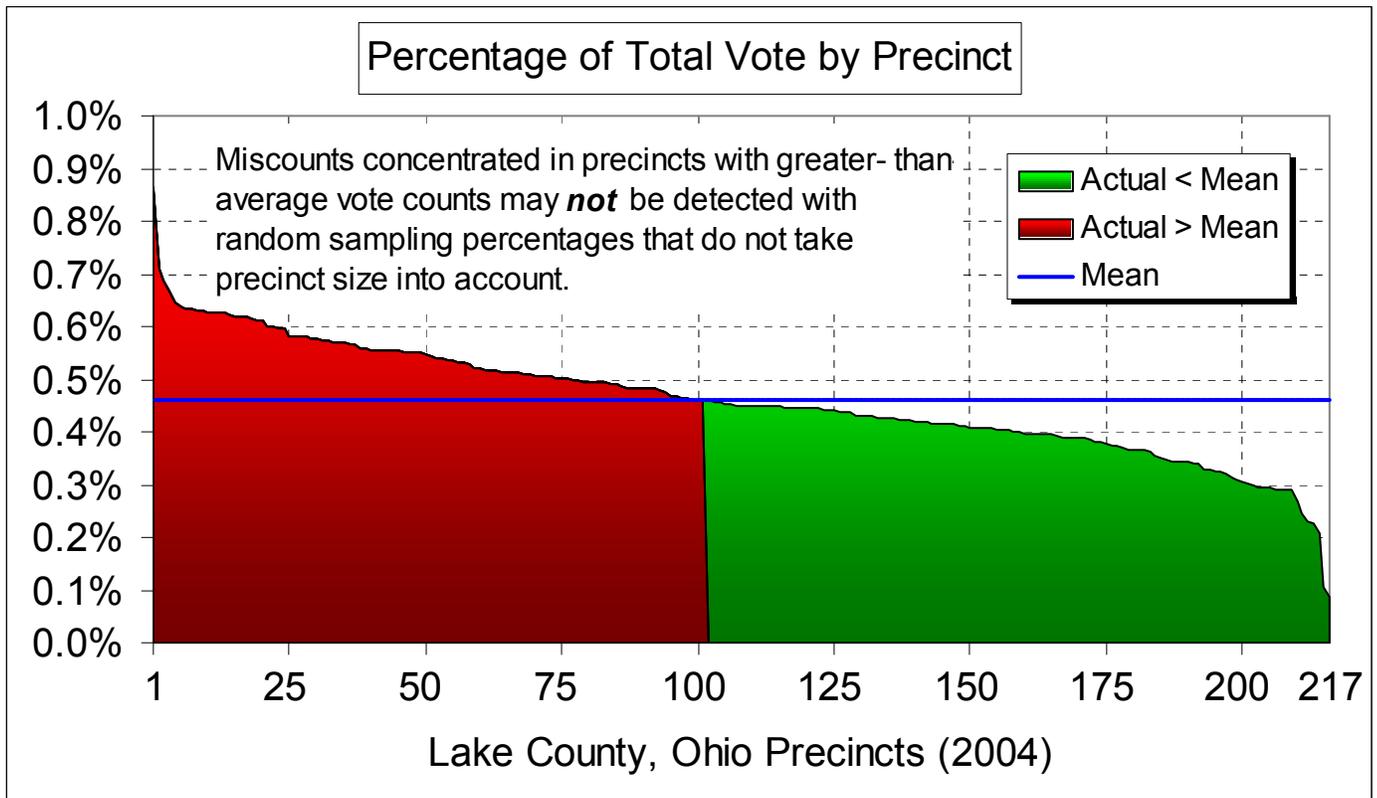


A log scale is used on the y-axis to show a greater range of precincts. When the margin reaches 5%, each race requires approximately the same sample size to achieve 99% certainty: 33 +/- 2 precincts. The required sample size differs more for races with narrower margins. For instance, with a 1-point margin, the local race requires a sample of 90 precincts (almost every precinct!), while the Senate race requires 182 precincts (only 3.1% of precincts). Also note that a random statewide sample would *not* contain the same precincts as a sample of comparable size drawn from a single Congressional or local district. So while precincts randomly selected for the statewide audit may be included in audits of smaller races, there would most likely have to be additional precinct samples for each Congressional and local race, depending on their individual precinct counts and margins.

Using the above methodology (which does not even take precinct size into account), in the 2004 general election for the US House of Representatives there were 57 races with margins of less than 17.4% requiring more than a 2% audit, 14 races (with margins of < 8.1%) requiring more than a 5% audit and 7 races (with margins of < 4.2%) requiring more than a 10% audit.

### 3. Concentration of Miscounts in Larger Precincts

The above analyses assume that all precincts in the jurisdiction have the same number of votes, which would not be the case in practice. For example, the following graph of precinct vote proportions from Lake County, Ohio from the 2004 general election shows a typical distribution of vote percentage by precinct within a jurisdiction:



We can see from the above graph that miscounts concentrated in precincts with above average vote counts would be less likely to be detected by a random audit with a sample size based on the implicit assumption that all precincts have the same (mean) number of votes. One way to address this problem is to over-sample so as to be certain that enough large precincts are examined, while still maintaining uniform random selection. Precincts can be sorted by their reported vote counts, and sample sizes can be selected by determining the absolute minimum number of precincts needed to reverse the outcome. This gives us the most conservative value of  $P_C$  possible by assuming that all miscounts occur in the largest precincts, while sampling all precincts randomly in case the miscounts are elsewhere. This methodology will be included in our auditing protocol presented later herein.

#### 4. Precincts vs. Voting Systems

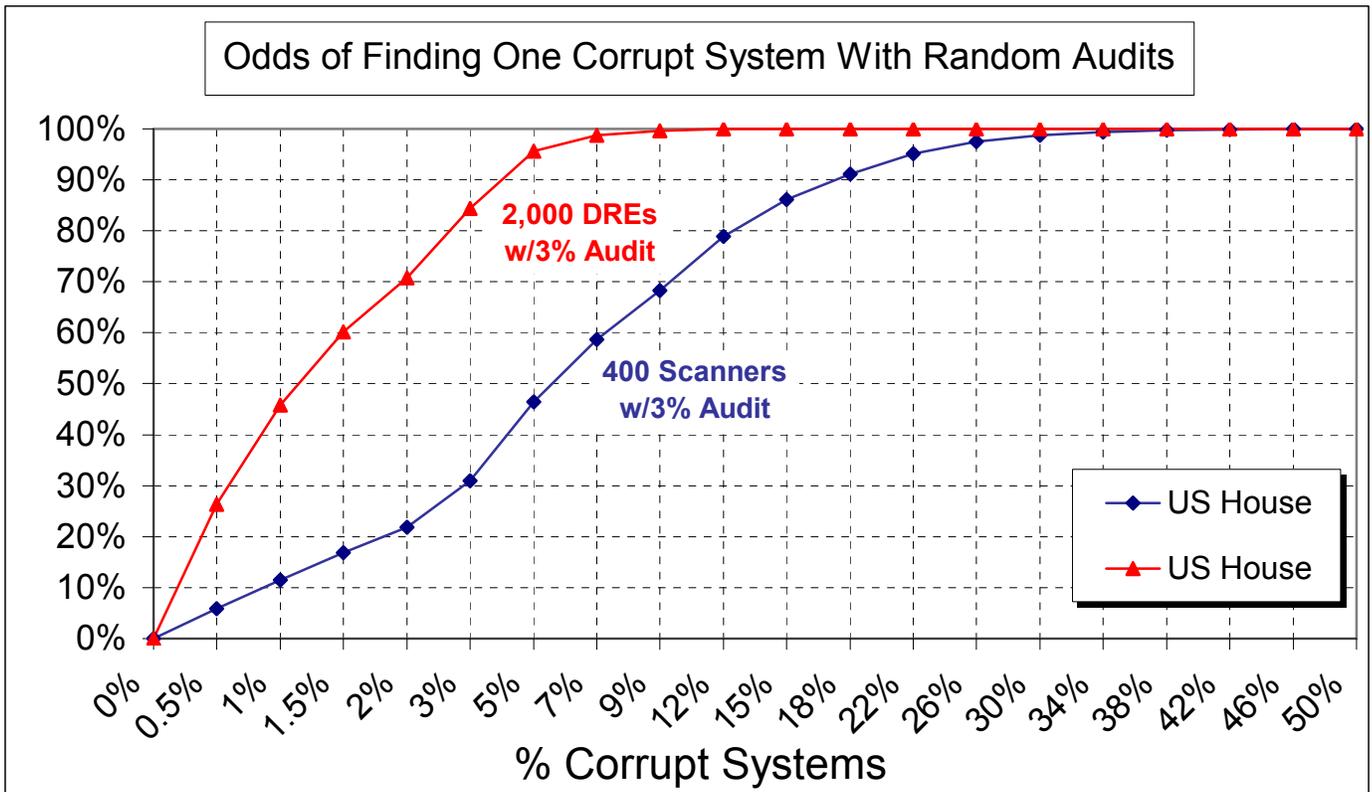
In some states such as New York, laws or regulations are written to audit individual voting systems rather than precincts. While a precinct may comprise more than one voting system or a fraction of a voting system, the voting system sampling method actually selects a percentage of individual DREs or optical scanners, regardless of their precinct locations. The voting system method also removes the ambiguity associated with auditing of absentee or early voting systems, which may not be precinct-based.

It is worth noting that because precinct-based optical scanners can handle many more ballots than DREs can, a population of optical scanners would generally be smaller than a population of DREs serving a comparable jurisdiction. This has an effect on the confidence level of audits because, all things being equal, the ability to audit a larger number of voting systems results in increased odds of finding a corrupt system.

While DREs have numerous disadvantages when compared to optical scanners, such as the potential for poor reliability to disenfranchise voters, higher costs, higher complexity, difficulty of testing via the user interface, etc., based purely on the ability to detect fraud through random auditing, DREs may actually be advantageous. This is especially true if auditing rules are written to take advantage of the greater sampling potential inherent in a larger number of smaller e-voting systems, i.e., the system method as opposed to the precinct method. The system method avoids the clustering effect of auditing by precinct and may also result in auditing a greater number of ballot styles with a given sampling percentage.

Consider the effect that having five times as many DREs as scanners from which to choose randomly would have on the statistical power of an audit of a US House race. Instead of 400 precincts per Congressional district with one optical scanner each, such a race might be run on 2,000 DREs. While it's hard to imagine how this could compensate for the fact that DREs are almost impossible to test thoroughly on Election Day, some mitigation of this risk could theoretically be achieved through the statistical power of post-election auditing.

The following graph compares a 3% audit of a US House race run on 400 optical scanners vs. 2,000 DREs:



Running the race on thousands of smaller systems results in a greater probability for the audit to detect fraud. With 5% corrupt systems, the odds of finding one with the scanner audit are 46.4%; with the DRE audit, 95.6%. However, if DRE VVPATs are harder to count by hand than paper ballots, this advantage may not be realized.

Whichever method is used, there need to be provisions in the law or regulations for auditing beyond the initial random sample percentage whenever the outcome cannot be confirmed by the initial audit or there are any unexplained discrepancies. Anything less is simply *trusting* the voting system.

It is worth noting that the Help America Vote Act mandates a maximum error rate for both DREs and optical scanners of no more than one in 500,000 votes, so there should be virtually *no* unexplainable discrepancies.

Given the above considerations, we can develop a few hypothetical examples.

## A Bicameral Audit

**Example 1 -- Michigan US Senate Race:** The state of Michigan has about 5,800 precincts. Two candidates for US Senate split the vote 51% to 49%. If we multiply the 2% margin by 2.5, we see that on average, 5% or 290 of the 5,800 precincts would be required to reverse the outcome. An audit of 1.6% (93 precincts) would have a 99.19% chance of finding a corrupt precinct.

However, by sorting all the precincts in the state by their reported vote counts, we find that 5% of the votes in this race were actually cast in only 3.6% of the precincts -- the 209 largest ones. This represents the highest concentration of miscounts for which we must search -- the smallest number of precincts that could change the outcome of the race -- the parameter we call  $P_C$ . Using the hypergeometric distribution, we find that if there are 209 corrupt precincts, a 2.2% audit will have a 99.14% chance of finding one. To confirm the outcome of this race, we will therefore audit 2.2% of the precincts in each county or a total of at least 128 precincts statewide instead of just the 93 precincts indicated above when precinct size was *not* taken into account.

**Example 2 -- US House Race:** Now let's compare the above to a race for US House of Representatives with the same 2% margin: Instead of 5,800 precincts, in the average Congressional District we would have only about 400 precincts serving about 400,000 registered voters. If we multiply the 2% margin by 2.5, we see that on average, 5%, or 20 of the 400 precincts would be required to reverse the outcome. By sorting all the precincts in the district, we find that 5% of the votes in this race were actually cast in only 3.75% of the precincts -- the 15 largest ones. This represents the highest concentration of miscounts for which we must search -- the smallest number of precincts that could change the outcome of the race or,  $P_C$ . Using the hypergeometric distribution, we find that if there are 15 corrupt precincts, a 26% audit (104 precincts) will have a 99.01% chance of finding one. To confirm the outcome of this race, we will therefore audit 104 precincts in this Congressional district. Had we not taken precinct size into account, we would have only audited 21% or 84 precincts, potentially allowing miscounts concentrated in larger precincts to go undetected.

A random audit of 2.2% of these precincts, which was sufficient for the statewide race, would examine only 9 precincts in this Congressional district. If 3.75% of these precincts were corrupt, there would only be a 29.36% chance of finding one, which is not likely to trigger an additional audit. To reach the 99% level of certainty in this race required a random audit of 26%, or 104 precincts.

Note that the only significant difference between the two races in the above examples was the number of precincts in each; the smaller number of precincts in the House race made the 2.2% audit almost useless.

## Our Random Auditing Protocol

Taking all of the above parameters, analyses and examples into account, we have developed the following random auditing protocol.

**If there are enough corrupt precincts (or machines) to change the outcome of the race in question, the following auditing protocol will find at least one:**

- 1) Randomly audit an initial percentage of precincts (or voting systems) in each county (or equivalent jurisdiction) if required by law or regulation. If DREs are in use, pick individual machines rather than whole precincts as this will result in a better chance of finding miscounts than picking whole precincts. Compare hand-to-eye counts of all paper ballots or records to the corresponding electronic machine or scanner totals for each race. These totals should be exactly the same (within one in 500,000 votes per HAVA standards) after any explainable discrepancies are taken into account. (Some examples of explainable discrepancies are: stray marks on optical scan ballots; DRE VVPAT printers jamming or running out of paper.) The following additional steps should be taken for each race to be audited.
- 2) Calculate the margin (reported vote difference between the two leading candidates).
- 3) Multiply the margin from Step 2 by 2.5. We will use this number of votes to calculate the minimum number of corrupt precincts (or machines) that could alter the outcome of the race -- assuming that in *each precinct (or machine)*, 20% of the votes have been switched from the reported loser to the reported winner, which could erase as much as a 40% margin.
- 4) Sort all precincts (or machines) in the race by the total votes cast in the race in each precinct (or machine). Count the number of precincts (or machines) needed (starting with the one with the largest vote count) to equal or exceed the number of votes from Step 3 above (i.e., 2.5 times the margin). This gives us the minimum number of corrupt precincts (or machines) that could reverse the outcome of the race.
- 5) Use the hypergeometric distribution to find the sample size needed to have 99% confidence of sampling at least one corrupt precinct (or machine), given the number of precincts (or machines) from Step 4 above and the total number of precincts (or machines) in the race (use this spreadsheet to find the appropriate sample size:  
<http://www.votersunite.org/info/AuditEffectivenessCalculator.xls>).
- 6) Calculate the percentage of total precincts (or machines) applicable to

this race represented by the result from Step 5.

7) If the percentage from Step 1 equals or exceeds the percentage from Step 6, go to Step 8. If the percentage from Step 1 is less than the percentage from Step 6, add precincts (or machines) randomly to at least equal the percentage from Step 6 for each race (always round up to obtain a whole number of precincts (or machines) in each jurisdiction). Audit the percentage of precincts (or machines) from Step 6 randomly in each county (or equivalent jurisdiction).

8) In addition, candidates should be allowed to audit a few precincts (or machines) of their choice, which they may consider to be suspect based on historical election data, pre-election polls, reports of malfunctions, etc., particularly if there are enough of these to affect the outcome of the race.

9) Any unexplained discrepancies between manual and electronic tallies must trigger additional auditing or full recounts. A reiteration of the auditing process starting with Step 4 above using the remaining unaudited precincts (or machines) should be used to determine if there are additional corrupt precincts (or machines). Any time a corrupt precinct or machine is found, this step should be repeated until the outcome is certain, up to and including a full manual recount.

## **Conclusion**

Auditing protocols proposed and implemented at the federal and state levels that rely on small-percentage random sampling without replacement are unlikely to detect miscounts sufficient to change the outcome of Congressional or smaller local races, even if such races initially appear to be decided by relatively wide margins. In states with small populations that do not have thousands of precincts or voting systems, this problem exists even for statewide races.

This does not mean that such legislation or auditing should be discouraged, but rather, that the limitations of such audits must be acknowledged by those who promote them so as not to engender a false sense of security.

Routine methods of error and fraud detection must be developed and employed to supplement small-percentage audits, particularly when there is no obvious trigger for additional auditing or full recounts. We think the above protocol is a good example.

We will discuss other comprehensive methods in a future report, but the protocol proposed herein will detect miscounts sufficient to change the outcome of almost any race and is generally much more efficient than a full recount, which may be difficult and/or expensive to obtain. Full recounts should of course be conducted to verify the outcome of any race too close to be confirmed by an audit.