# Stochastic Modeling of Coaxial-Connector Repeatability Errors*

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***Abstract*—We propose a new description of connector repeatability errors for coaxial one-port devices. Our approach is based on a stochastic model constructed as a lumped-element equivalent circuit with randomly varying frequency-independent parameters. We represent statistical properties of these parameters with a covariance matrix which is estimated from a small number of repeated measurements (typically 16) of a one-port device under test. We illustrate our approach by modeling connector repeatability errors for 1.85 mm coaxial offset shorts. These results show that our model is capable of reproducing the complicated frequency-dependent behavior of connector repeatability errors for coaxial one-port devices with typically only two or three random parameters.**

***Index Terms*—coaxial connector interface, connector repeatability errors, stochastic modeling**

## I. Introduction

CONNECTOR repeatability errors are the primary source of random errors in power and scattering parameter measurements in the coaxial environment [1–5]. Much theoretical and experimental work has been done to characterize the statistical properties of connector repeatability errors and thus better predict their impact on measurement accuracy [1–12]. In this work, we develop a stochastic model for connector repeatability errors of coaxial one-port devices that builds on the measurement-based approaches of [1, 2, 11]. We construct this model with the use of a lumped-element equivalent circuit, based on the model of [11], and by allowing the frequency-independent parameters of this circuit to randomly vary. Consequently, we can reproduce the complicated frequency-dependent behavior of the connector repeatability errors by means of a small set of frequency-independent random parameters and a corresponding set of fixed functions which capture the frequency dependence of these errors. We describe the statistical properties of these frequency-independent random parameters with a covariance matrix which is estimated from repeated reflection coefficient measurements of the one-port device under test. We further apply principal component analysis (see [13]) to reduce the dimensionality of this covariance matrix while capturing the most important error mechanisms. As a result, we are able to accurately reproduce the connector repeatability errors observed in measurements of one-port devices with typically only two or three randomly varying parameters.

## II. Model

Connector repeatability errors often exhibit a "ripple-type" frequency-dependent behavior [1, 4, 6, 8, 10]. Reference [1] attempts to quantitatively describe this behavior. It presents a simple model for connector repeatability errors based on the connector-interface equivalent circuit of [2]. This equivalent circuit is formed by a series inductance, a series resistance, and a shunt capacitance inserted between the device under test (DUT) and the vector network analyzer (VNA). The elements of this circuit represent the effects responsible for the connector repeatability errors, such as the misalignment of the center and outer conductors (the shunt capacitance) and the variation of the joint impedance (the series inductance and the series resistance). Reference [1] proposes then a simple procedure for the identification of the model parameters, based on repeated measurement of a highly reflective one-port device.

The stochastic model for connector repeatability errors as used here generalizes the approach of [1, 2] and is based on the general model for VNA random errors of [11]. This model stems from the observation that in many cases a single discontinuity cannot explain the observed ripple behavior in connector repeatability errors. This can be justified by the fact that some of the discontinuities responsible for the changes of connector interface electrical parameters are physically removed from the connector joint plane. Examples are bending of the center conductor fingers or flexing of beads that support the center conductor due to variation of the mechanical strain applied to the center conductor.

Our model, following that of [11], uses an arbitrary number of discontinuities located at different distances from the connector joint plane. This model is schematically shown in Figure 1. It describes an error due to imperfect connector repeatability with a set of $N+1$ small perturbations. These perturbations are inserted at different fixed distances $d_n$, for $n = 0, \ldots, N$ into the linear two-

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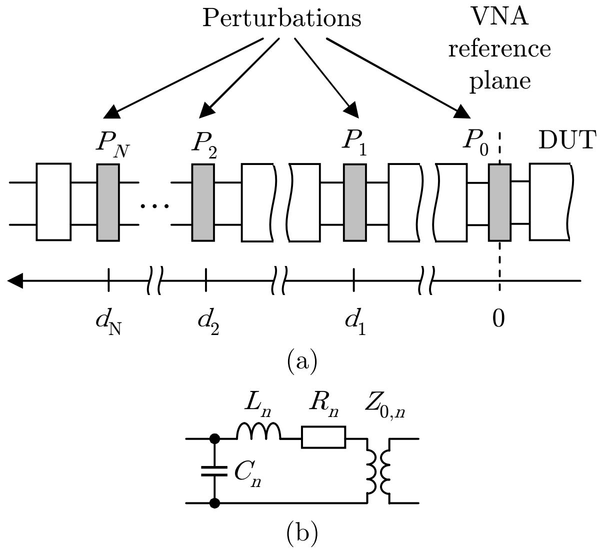



Figure 1. Schematic of the model for connector repeatability errors: (a) overview, (b) single perturbation.

port network representing the connector interface. For the perturbation $P_0$ which occurs at the connector joint we assume $d_0 = 0$.

We describe each of the perturbations with a lumped-element circuit shown in Figure 1b [11]. Compared to [1, 2], our circuit has an additional transformer which accounts for changes of the characteristic impedance. Such changes result, for example, from misalignment of outer and inner conductors, eccentricity of the inner conductor, or variation of the connector socket diameter. We further account for the changes in the skin-depth effect in the conductors by including additional components in the series inductance and resistance. Finally, we account also for the changes of DC resistance by adding an additional component to the series resistance.

In order to determine the joint contribution of the perturbations $P_n$, for $n = 0, \ldots, N$ to the error in corrected DUT $S$-parameter measurements, we assume that the perturbations are small which allows us to neglect multiple reflections and superimpose contributions of all of the perturbations. Consequently, at the frequency $f_k$, we write the connector repeatability error in the reflection coefficient measurement as a linear combination [11]:

$$\Delta\Gamma_k = \mathbf{w}\big(\mathbf{d}, \mathring{\Gamma}_k, f_k\big)^T \mathbf{p}, \tag{1}$$

where $\mathring{\Gamma}_k$ is the true value of the reflection coefficient of the one-port device under test at the frequency $f_k$, $\mathbf{d}$ is a vector of perturbation distances, $\mathbf{p}$ is a vector comprised of the frequency-independent parameters of the lumped elements in the perturbations $P_n$, $\mathbf{w}\big(\mathbf{d}, \mathring{\Gamma}_k, f_k\big)$ is a fixed function capturing the frequency dependence of the connector repeatability error, and superscript $T$ denotes the transpose. The function $\mathbf{w}\big(\mathbf{d}, \mathring{\Gamma}_k, f_k\big)$ is determined based on the structure of the model in Figure 1.

## III. Statistical properties of the model parameters

The electrical parameters which undergo random variations when reconnecting the interface are captured in the vector $\mathbf{p}$. This vector contains parameters of the lumped elements in the perturbations $P_n$. Since the parameters in the vector $\mathbf{p}$ describe the changes, we assume that $\mathrm{E}\left(\mathbf{p}\right) = 0$, where $\mathrm{E}\left(\cdot\right)$ denotes the expectation value operator [13]. We further characterize the statistical properties of the vector $\mathbf{p}$ with the covariance matrix

$$\mathbf{\Sigma_p} = \mathrm{E}\left(\mathbf{p}\mathbf{p}^T\right). \tag{2}$$

Under the assumption that the probability distribution function of the vector $\mathbf{p}$ is normal, the covariance matrix $\mathbf{\Sigma_p}$ constitutes a complete description of the statistical properties of the vector $\mathbf{p}$ [13].

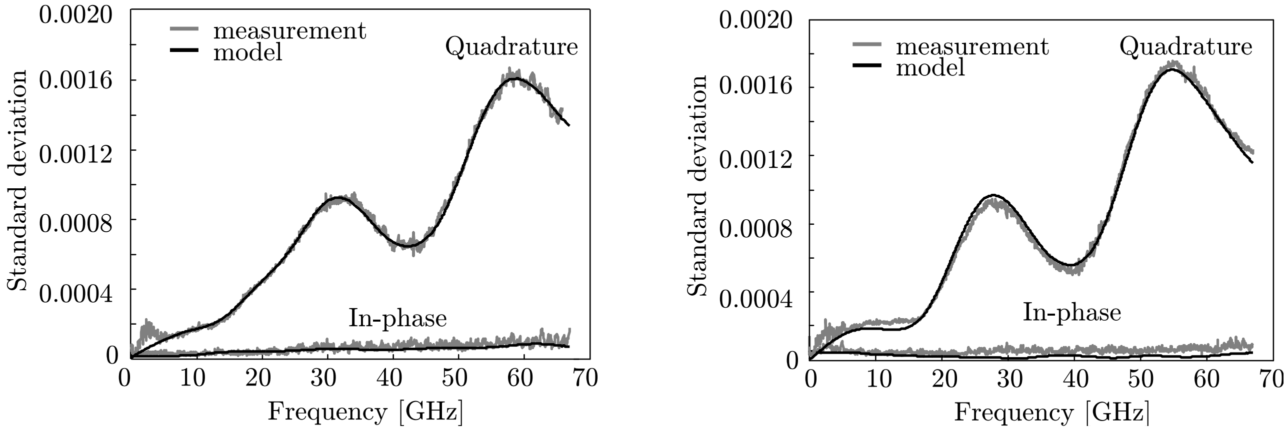
The assumption of normally-distributed parameters in the vector $\mathbf{p}$ can easily be justified from the physical point of view. Parameters in the vector $\mathbf{p}$ represent the impact of mechanical changes of the connector interface in terms of some equivalent electrical parameters. Since these changes are small, parameters in the vector $\mathbf{p}$ are linear combinations of changes in some geometrical parameters of the connector interface, such as displacements of the conductors, bending of the socket fingers, or flexing of the center conductor beads. We can reasonably assume that all of those mechanical changes are normally distributed; thus the vector $\mathbf{p}$ also has a normal probability distribution function.

## IV. Estimation of the covariance matrix of the model parameters

We estimate the covariance matrix of the model parameters $\mathbf{p}$ based on repeated corrected VNA measurements of the one-port device under test. Before each measurement, we reconnect the device under test. For a set of $M$ such measurements we estimate at each frequency the true value of the reflection coefficient $\mathring{\Gamma}_k$ as the mean value of all of the measurements, and then estimate the deviation of each measurement from the mean. We describe the frequency dependence of each of these deviations with the model (1) and then, in the nonlinear least-squares estimation procedure, we determine the estimates $\hat{d}_n$, for $n = 1, \ldots, N$, of the perturbation locations, and the estimates $\hat{\mathbf{p}}_m$, for $m = 1 \ldots, M$ of the lumped-element parameters for each connection. We then estimate the covariance matrix of the model parameters as the sample covariance of the estimates $\hat{\mathbf{p}}_m$, that is [13]

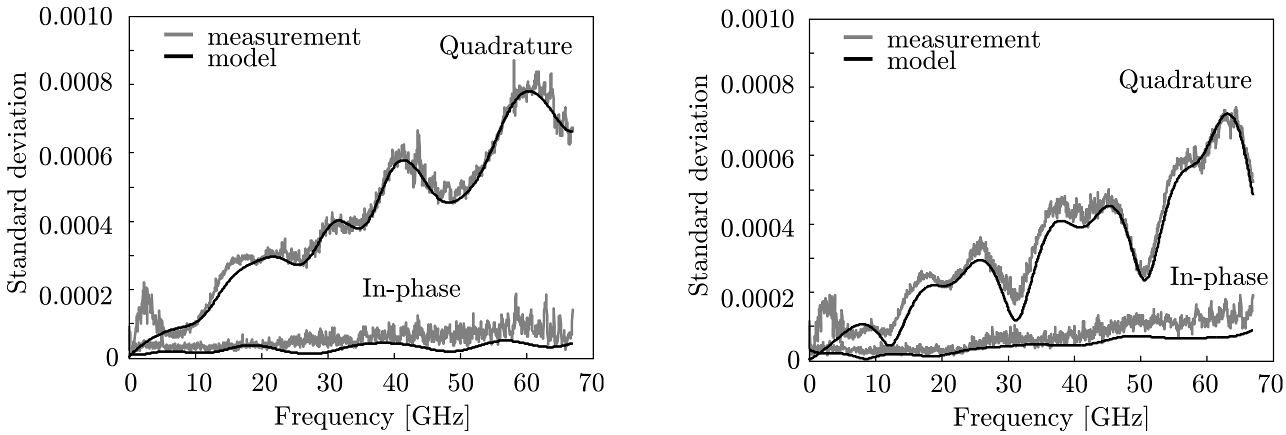
$$\hat{\mathbf{\Sigma}}_\mathbf{p} = \frac{1}{M-1} \sum_{m=1}^{M} \hat{\mathbf{p}}_m \hat{\mathbf{p}}_m^T. \tag{3}$$

We further reduce the dimensionality of this covariance matrix with the use of the principal component analysis [13]. We chose the number of principal components so as to capture 99 % of the total variance captured in the original




(a) (b)

Figure 2. In-phase and quadrature component of the standard deviation of 16 repeated reflection coefficient measurements of a 5.4 mm long, 1.85 mm coaxial offset short: (a) female, (b) male; measurement (gray) and stochastic model prediction (black).




(a) (b)

Figure 3. In-phase and quadrature component of the standard deviation of 16 repeated reflection coefficient measurements of a 7.6 mm long, 1.85 mm coaxial offset short: (a) female, (b) male; measurement (gray) and stochastic model prediction (black).

matrix (3). This results typically in two or three principal components.

## V. Experiments

We illustrate our approach by modeling connector repeatability errors for 5.4 mm long and 7.6 mm long coaxial offset-shorts in the 1.85 mm standard. Figures 2 and 3 show the in-phase and quadrature component (see [14]) of the standard deviation of 16 repeated measurements of the reflection coefficient of the two shorts along with the standard deviation predicted from our stochastic model. In both cases, the model we used employed three perturbations. We further used two principal components in the case of the 5.4 mm long short and three principal components in the case of the 7.6 mm long short. The agreement between the stochastic model prediction and measurements for the quadrature (phase) errors is very good except for a small discrepancy in the frequency range below 4 GHz. This discrepancy is caused by increased test-set drift in the frequency range below 4 GHz, phenomenon we also noticed in other experiments. The agreement for the in-phase (magnitude) errors is not as good. However, the in-phase errors are much smaller and, therefore, less important than the quadrature errors. Consequently, our stochastic model is capable of adequately representing the connector repeatability errors as observed in the measurements of the two shorts.

## VI. Conclusions

We proposed a new description for the connector repeatability errors in coaxial one-port devices. Our approach employs a stochastic model which is constructed as a small set of frequency-independent random variables (typically two or three) and a corresponding set of fixed frequency-dependent functions. We represent statistical properties of the frequency-independent parameters with a covariance matrix which is estimated from a small number (typically 16) of repeated measurements of the one-port device under test. We demonstrated our approach by modeling connector repeatability errors for 1.85 mm coaxial offset shorts with different lengths. We showed that our model is capable of reproducing the complicated frequency-dependent behavior of connector repeatability errors in the measurements of the two offset shorts.

The stochastic model for the connector repeatability errors we described has found an application in the covariance-based uncertainty analysis for VNA measurements [15]. This uncertainty analysis accounts for the statistical correlations between VNA measurement uncertainties at different frequencies. These correlations are important in many applications where VNA $S$-parameter measurements are used, such as uncertainty analysis of calibrated time-domain measurements or device modeling [15]. As the proposed stochastic model describes the connector repeatability errors in terms of some frequency independent error mechanisms, it is capable of capturing the statistical correlations between the connector repeatability errors at different frequencies.

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