# Newtonian Constant of Gravitation workshop
# NIST
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Clive Speake (presented by TJQ)

- Improvements to BIPM scheme
- Proposed way forward

# Possible improvements to BIPM Scheme (ccs,tjq)

- Terry has summarised the key points of the BIPM determinations.

- Increase in ratio of signal to random noise is important to ensure that the experiment can be repeated under different conditions a reasonable number of times in a reasonable time scale. However, we are looking for effects that are around 50 times the typical claimed uncertainties. ***So we should concentrate on bias effects***.

- The **autocollimator** had a large uncertainty but was made to cancel in the final result.

- For the BIPM results the sources of bias that were not cross-checked in the Cavendish and Servo methods were: possible effects from the **source masses** and also the complexity of the calculation of the **gravitational coupling** between the source masses and the torsion disc and its appendages (the moment of inertia was checked by experiment).

| Quantity | Fractional uncertainty ppm |
|---|---|
| Test masses $\delta m/m$ (correlated) | 1 |
| Source masses $\delta M/M$ (correlated) | 1 |
| Test mass type A servo (correlated) | 17 |
| Test mass type A Cavendish (correlated) | 8 |
| Source mass type A for both Servo and Cavendish (uncorrelated) | 12 |
| Servo type B uncertainty for source and test masses $\delta\alpha_s$ | 4 |
| Cavendish type B uncertainty for source and test masses $\delta\alpha_c$ | -3 |
| Servo type A uncertainty for 0.1 K temperature change $\delta\alpha_{sT}$ | -2 |
| Cavendish type A uncertainty for 0.1 K temperature change $\delta\alpha_{cT}$ | -7 |
| Angle measurement $\delta\Delta\phi/\Delta\phi$ (anti-correlated) | 47 |
| Capacitance calibration $\delta\Delta C/\Delta C$ | 6 |
| Voltage calibration 2 $\delta V/V$ | 12 |
| Timing error $2\Delta T_0/T_0$ | 0.5 |
| Moment of inertia of torsion disc | 13 |
| Anelasticity $\delta k/k_r$ | 6 |
| Uncertainty in mean servo torque $\delta\tau_s/\tau$ | 30 |
| Uncertainty in mean Cavendish torque $\delta\tau_c/\tau$ | 19 |
| Net uncertainty on servo value $\sigma_s$ | 61 |
| Net uncertainty on Cavendish value $\sigma_c$ | 54 |
| Covariance $\kappa$ | -2080 |
| Correlation coefficient | -0.63 |

- Type B uncertainties are related to uncertainties in calibration.
- The most significant Type B uncertainty arises from the calibration of the angle. The anti-correlation of this in the two methods enables us to reduce this significantly.
- **The sensitivity and linearity of the angular measurements required for the angular deflection in Cavendish method and the calibration of the electrostatic torque transducer in the Servo determination could be improved using interferometric techniques.**
- **If there is a bias in the final result derived from both experiments, most likely this bias is not included in this table!!**

# Improvement to BIPM torsion-strip balance design?

$$\Gamma_{ll} \approx GMm \cdot \frac{l(2l-1)!!}{(2l-2)!!} \cdot \frac{r^l}{R^{l+1}} (\sin\theta)^l \sin l\phi \qquad r < R$$

$$\Gamma_{44} = GMm \cdot 35 \frac{r^4}{R^5} (\sin\theta)^4 \sin 4\phi$$

$$\Gamma_{33} = GMm \cdot \frac{135}{8} \cdot \frac{r^3}{R^4} (\sin\theta)^3 \sin 3\phi$$

- Signal proportional to $\frac{r^l}{R^{l+1}}$ , $l$ = 4 currently with 4-fold symmetry.
- Gravity gradient coupling proportional to $\frac{1}{R^{l+2}}$
- Type B dimensional uncertainties scale as $\frac{d\Gamma}{\Gamma} = \frac{lr}{r} - \frac{(l+1)R}{R}$
- Type A dimensional uncertainties scale as $\frac{d\Gamma}{\Gamma} = \sqrt{\left(\frac{ldr}{r}\right)^2 + \left(\frac{(l+1)dR}{R}\right)^2}$
- Except for gravity gradients, it is a good idea to look at, say $l$=3.
- With $l = 2$ we can make use of the source and test mass scheme of Gundlach and Merkowitz (*GM*). However this dominantly quadrupole design of test mass *can* couple residual oscillation of the simple pendulum mode into the measurement.

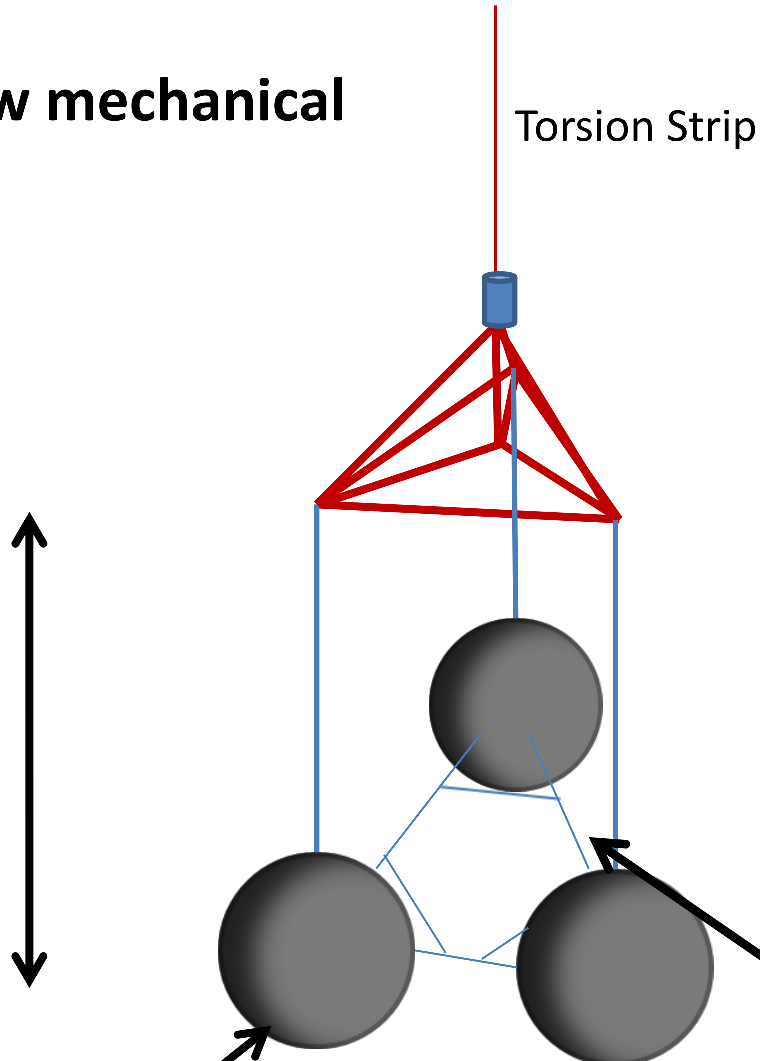
For example: if we keep the same dimensions (***r*** = 120 mm and ***R*** = 213 mm) and mass values ***M*** = 12 kg and ***m*** = 1.2 kg., we find

For $l$ = 4 torque $\approx 1.6\times10^{-8}$ Nm

For $l$ = 3 torque $\approx 1.4\times10^{-8}$ Nm *!*

Using 3 test masses of 1.2 kg would reduce the load on the torsion strip and this would not be optimum for the current geometry of the strip. So decisions need to be made.

# Possible new mechanical design




Lightweight ‘marionette’ support structure for test masses. Torque reduced by ($\sin\theta$)^3 or ~0.1 for 2:1 ratio in L:r

Light low thermal expansion structure to maintain positions of masses. Note also that the temperature sensitivity is much less for the strip than the round section fibre (see PTRS paper).

It would be a good idea to use single crystal masses

# Ways forward: suggestion (ccs)

- Two phases in establishing reliable value for G.
  - Establish an agreement between two/three laboratories using essentially **the same method**. This would establish a working relationship between the labs and check basic metrology and agreement in methodology. This result could, however, be subject to a common bias. Maybe repeat the Luther Towler experiment at NIST and HUST, for example.
  - Use two/three **substantially different** methods for determining G in two or three different laboratories, for example JILA and BIPM experiments and perhaps in due course by atomic interferometry.
- In each case the two/three groups work together to understand each others experiment and ultimately obtain an agreement on a value of *G* and its uncertainty.