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The dual free swinging simple pendulum approach for Big G determination

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Main features and targets

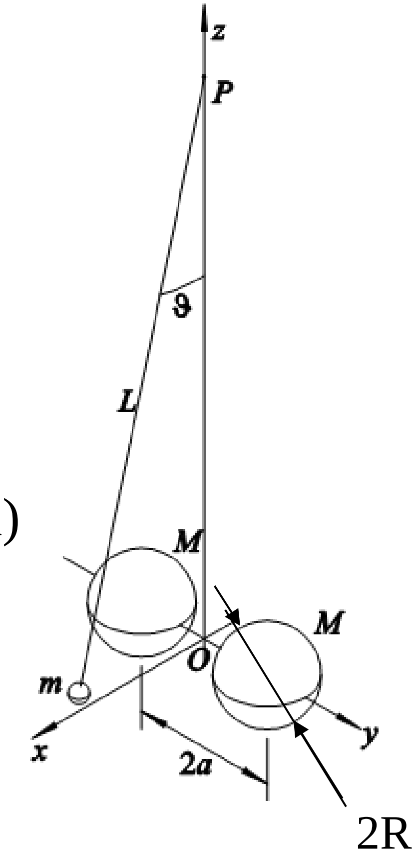
- Rejection of seismic noise
- Dual pendulum concept
- Better than 10^{-6} resolution
- 10^{-5} accuracy

Pilot experiment (2000-2005)

Based on

$$\nu_M = \nu_0 \sqrt{\left(1 + 2\,\frac{a_M}{a_g}\right)} \longrightarrow \frac{\Delta\nu}{\nu} = \frac{a_M}{a_g}$$

to better than 10^{-6}



- 5mm diameter BK7 bob
- 2 converging 0.9m Kevlar fibers (for degeneration removal)
- 3 mm diameter suspensions
- 30 mm diameter Au active masses
- Electrostatic shields
- Split PD optical detection with ns resolution
- 10^{-6} Torr vacuum

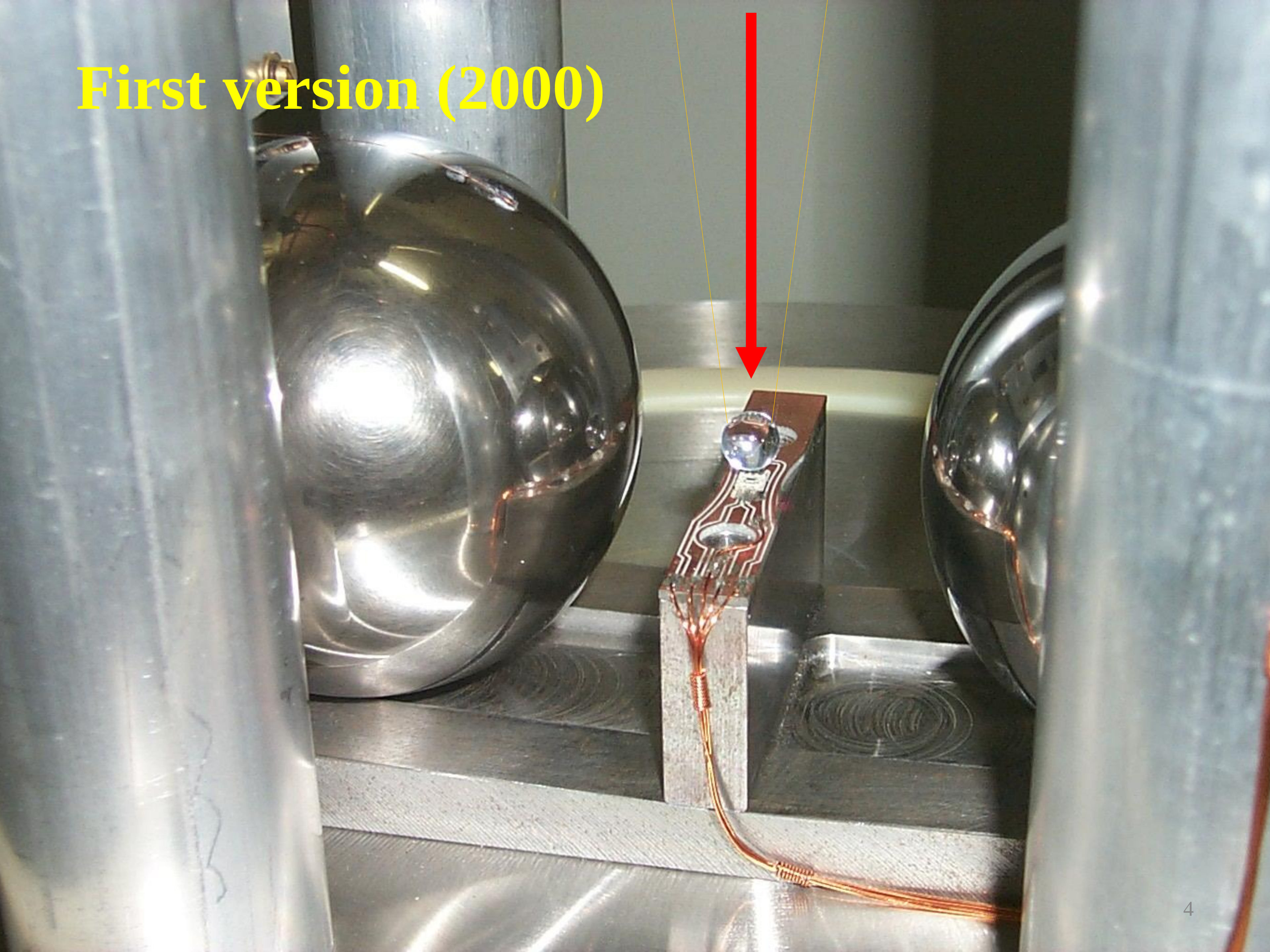
[1] A. De Marchi, M. Ortolano, F. Periale, and E. Rubiola, "The dynamic free pendulum method for *G* measurement," in *Proc. of 5th Symposium on Frequency Standards and Metrology*, Woods Hole, MA, Oct. 1995, p. 369.

[2] A. De Marchi, M. Ortolano, M. Berutto, and F. Periale, "Simple pendulum experiment for the determination of the gravitational constant *G*: progress report," in *Proc. of 6th Symposium on Frequency Standards and Metrology*, Fife, Scotland, Sept. 2001, pp. 538–540.

[3] M. Berutto, M. Ortolano, A. Mura, F. Periale, and A. De Marchi, "Toward the determination of *G* with a simple pendulum," *IEEE Journal Instr. and Meas.*, vol. 56, no. 2, pp. 249–252, 2007.

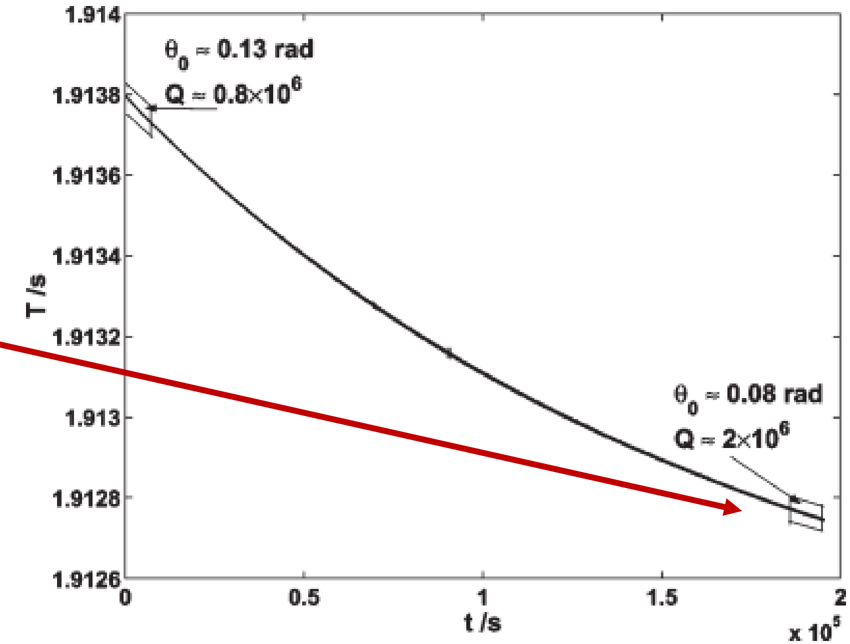
[4] M. Berutto, M. Ortolano, A. De Marchi, "The Period of a Free-Swinging Pendulum in Adiabatic and Non-Adiabatic Gravitational Potential Variations", Metrologia **46**, 119 (2009)

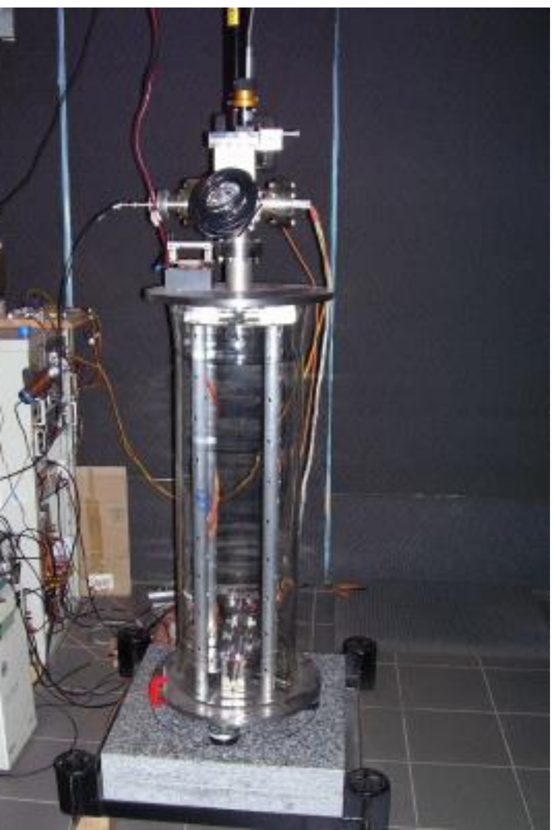
First version (2000)

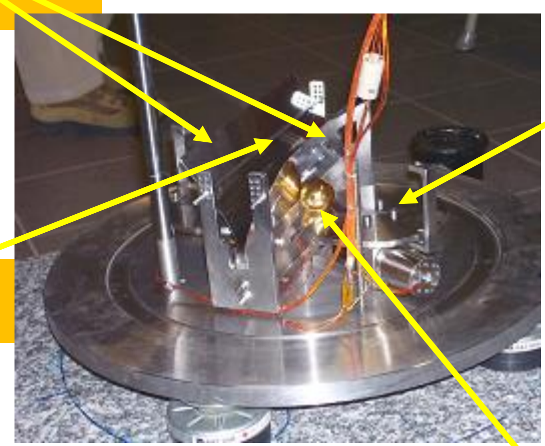


Results (2005)

- Measured Q up to $2{\cdot}10^{-6}$
- Months of total measurement time
- Repeatability 10^{-3} (accuracy?)
- Resolution 10^{-2}
  (limited by seismic angle noise)

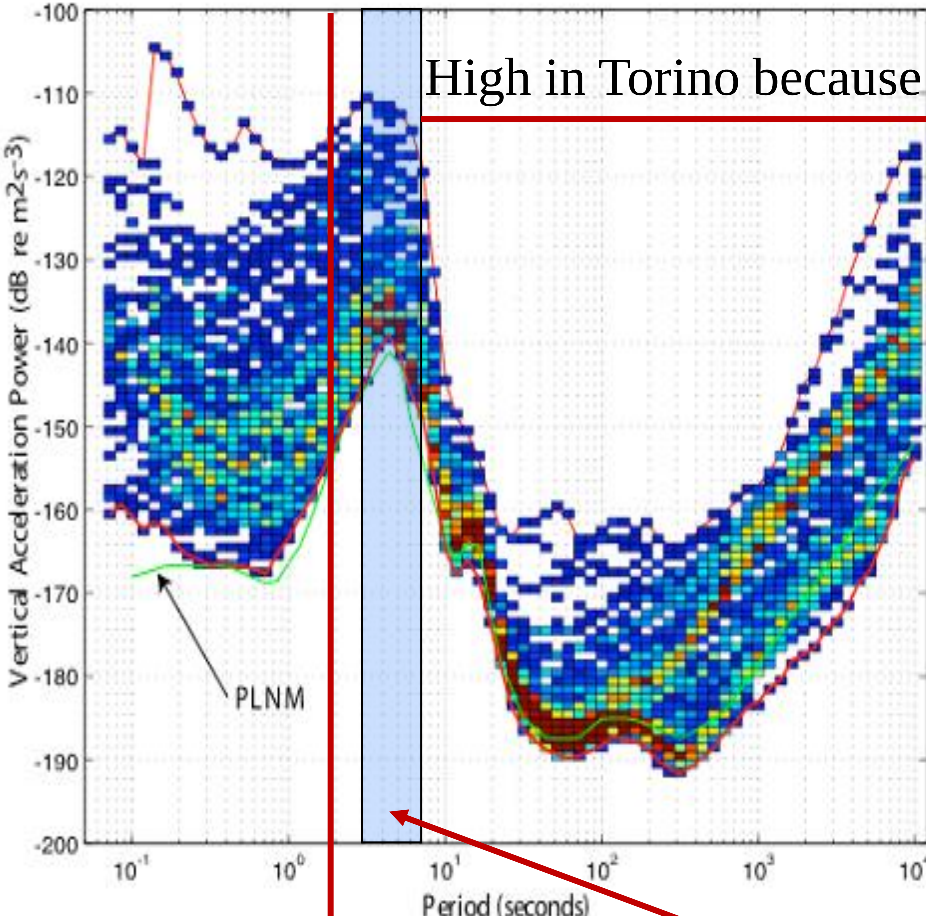






Rayleigh waves

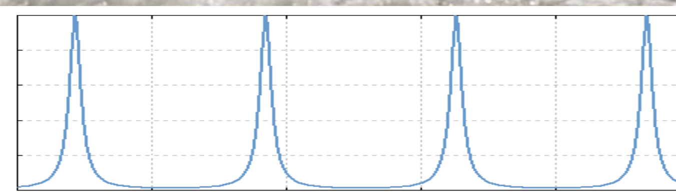
$\theta_{rms} = z_{rms}\omega/u_R$ if waves are sinusoidal



High in Torino because of alluvial ground (slow u_R < 300 m/s)

θ_{rms} = 10 n rad

That's why
…we overlooked them at first



0.9 m long pendulum

Waves beating the coasts

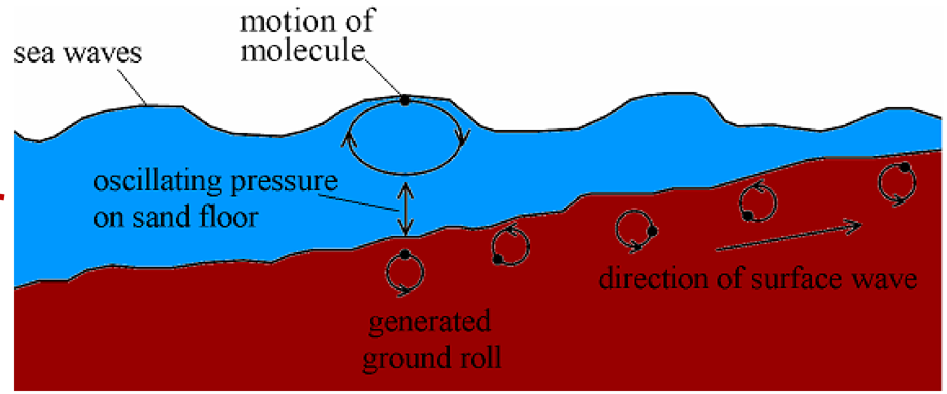
If so, the slope can be x100 or so

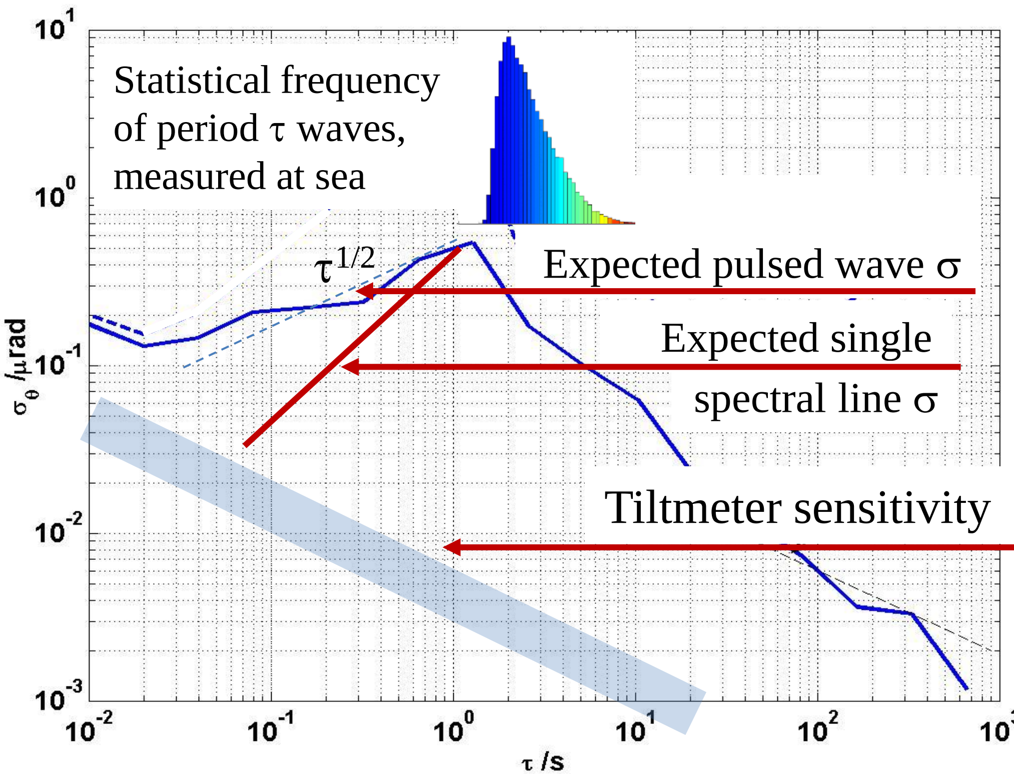
then $\theta_{rms}L/v$ = 0.3-0.5 μs_{rms}
which is more similar to what we have

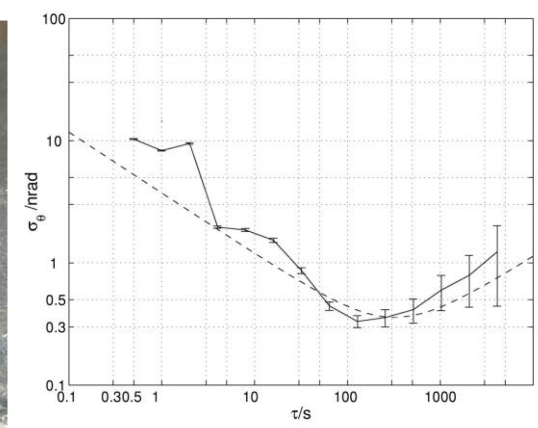
How are Rayleigh waves excited?

The official story

Sinusoidal wave (single spectral line)

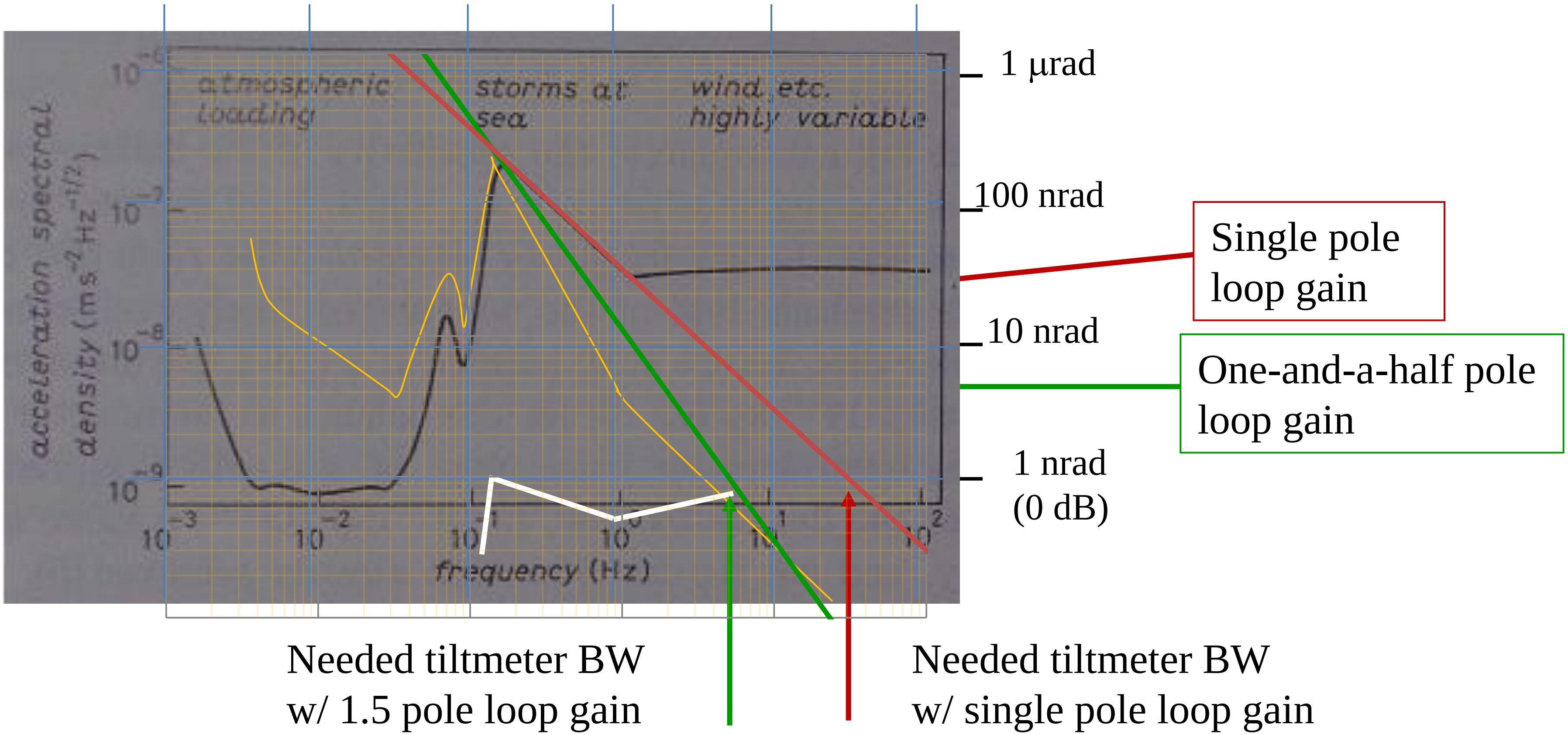






Home built 200 prad tiltmeter

Angular attitude control



feasible

very hard to make

Work in progress with Peltier driven thermal expansion motors

The dual pendulum concept

The goal is common-moding Rayleigh wave related seismic angle noise

•Use two pendulums oscillating in the same plane with rational T ratio (e.g. 21/20)

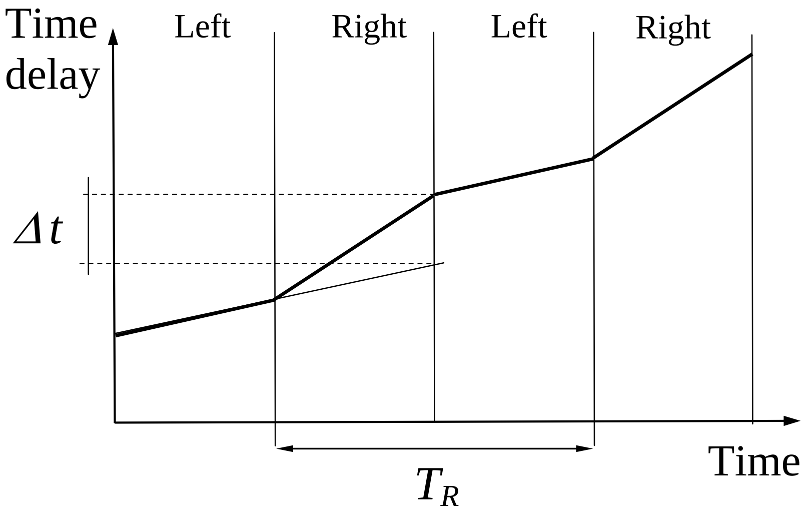
•Measure the time delay when both pass the detector at the same time (each 20T of slower)

•Change positions of the active masses every N such events ($T_R/2$ = N 20T = 1000s)

•Angle noise goes common mode

•$\Delta t/T_R$ yields directly $\Delta\nu/\nu$

•N values regression yields uncertainty $/N^{3/2}$



Type A uncertainty

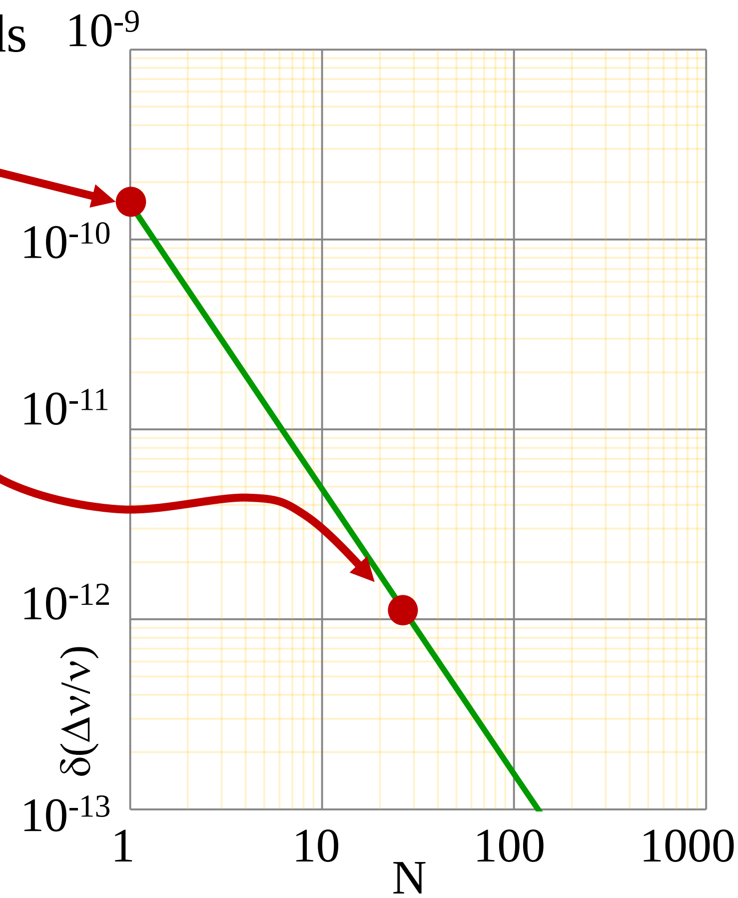
Free from angle noise, timing should be dominated by 1-3 ns detection noise

…say $\delta\Delta t$ = 6 ns counting start and stop both ends

$\delta(\Delta\nu/\nu) = 6\text{ns} / 40\text{s} = 1.5 \cdot 10^{-10}$ in $T_R/2$

With 25 measurements (1000 s), $1.2 \cdot 10^{-12}$

Then active masses are moved and same procedure applied. The obtained difference is the desired result, proportional to G



With Type A uncertainty

$$\frac{\delta\nu}{\nu} = \frac{1.7 \cdot 10^{-12}}{3 \cdot 10^{-7}} = 6 \cdot 10^{-6}$$

In one cycle T_R of 2000 s

averaging 36 cycles (20 hours) yields $1 \cdot 10^{-6}$ uncertainty

High Q is crucial in order to

- Avoid frequency locking / pulling between the two pendulums

$$\left(\frac{\Delta\nu}{\nu}\right)_{pull}\frac{\nu_1-\nu_2}{\nu}=\left(\frac{\xi}{2Q}\right)^2$$

e.g. $10^{-12}\,5\cdot10^{-2} < 10^{-4}/4Q^2 \longrightarrow Q>3\cdot10^4$

- Filter mechanical noise
  - ✓Structure vibrations
  - ✓Brownian motion $\longrightarrow \sigma_y(\tau)=\frac{1}{\vartheta_p\sqrt{2\pi\tau}}\sqrt{\frac{kT}{mgL\nu Q}}$ $< 10^{-12}$ in 1 s (10^{-5} on G)
  - ✓Thermal noise in fibers $< 10^{-13}$ in 1 s (10^{-6} on G)
  - ✓Seismic

- Obliterate flicker noise

- Operate in free ring down mode
  - ✓Long time constant (2 years for $Q>10^8$)
  - ✓Constant oscillation amplitude $\longrightarrow$ no frequency drift
  - ✓No feedback noise injection

- Help guaranteeing experiment modelization, and ultimately **ACCURACY**

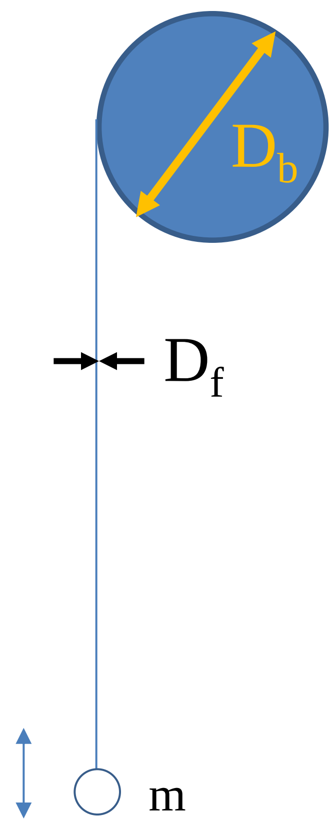
Q limitations

- Friction on residual air (${10^{-7}}$ Torr for $Q > 10^8$)

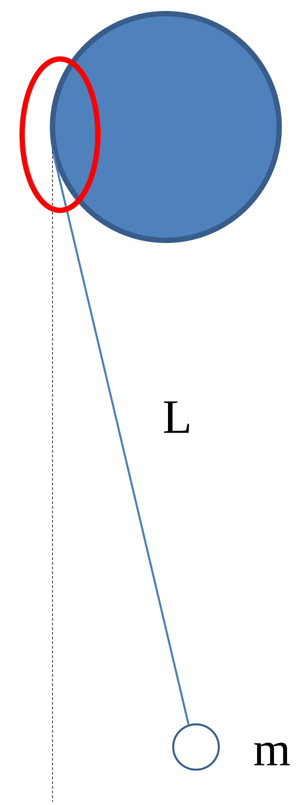
- Joule effect in conducting fibers
  - For for $Q > 10^8$ must be $P_d < 10$fW (1μJ stored energy)
  - $P_d = 2V^2/r$ for both fibres cutting Earth's $B_\perp$ with speed v
  - Must be $r > 2V^2/P_d = 2\,(LvB_\perp)^2/P_d = 70\ \Omega$

- Mechanical losses in fibers
  - Stretching
  - Bending at the suspensions

Loss mechanisms in the two fibers



Fiber bending
(period T)



$$\frac{Q_s}{Q_{mat}} = \frac{16}{9\sqrt{\varepsilon_0}\,\theta_0^2}$$

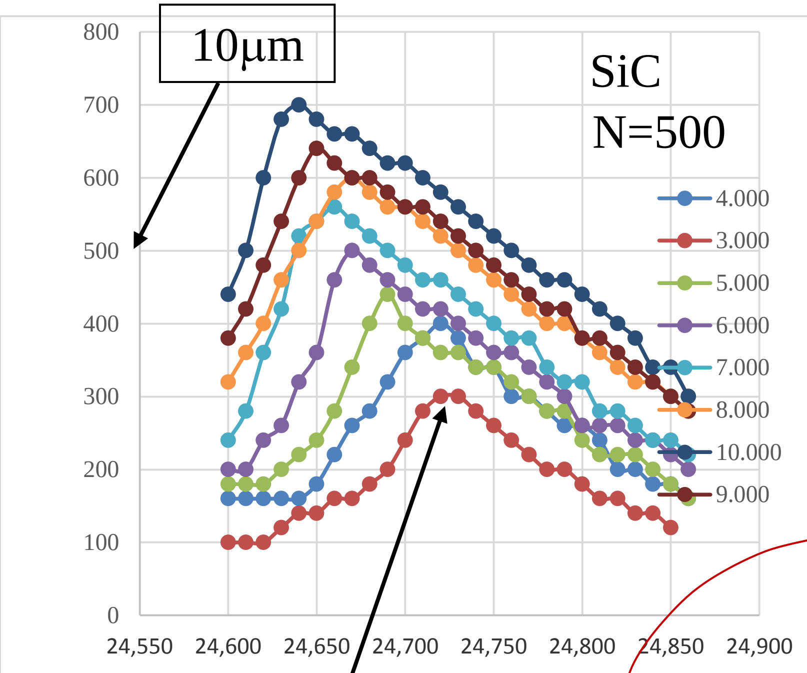
$$\frac{Q_b}{Q_{mat}} = \left(\frac{D_b\, L}{D_f^2}\,\varepsilon_0\,\theta_0\right)^{3/2}$$

Fiber stretching, period T/2

(Centrifugal force and stretching component of g
0.5 μN @ θ_0=0.02 rad)

$$\varepsilon_0 = \frac{m\,g}{2A\,E} \quad \text{with} \quad A = \pi D_f^2/4$$

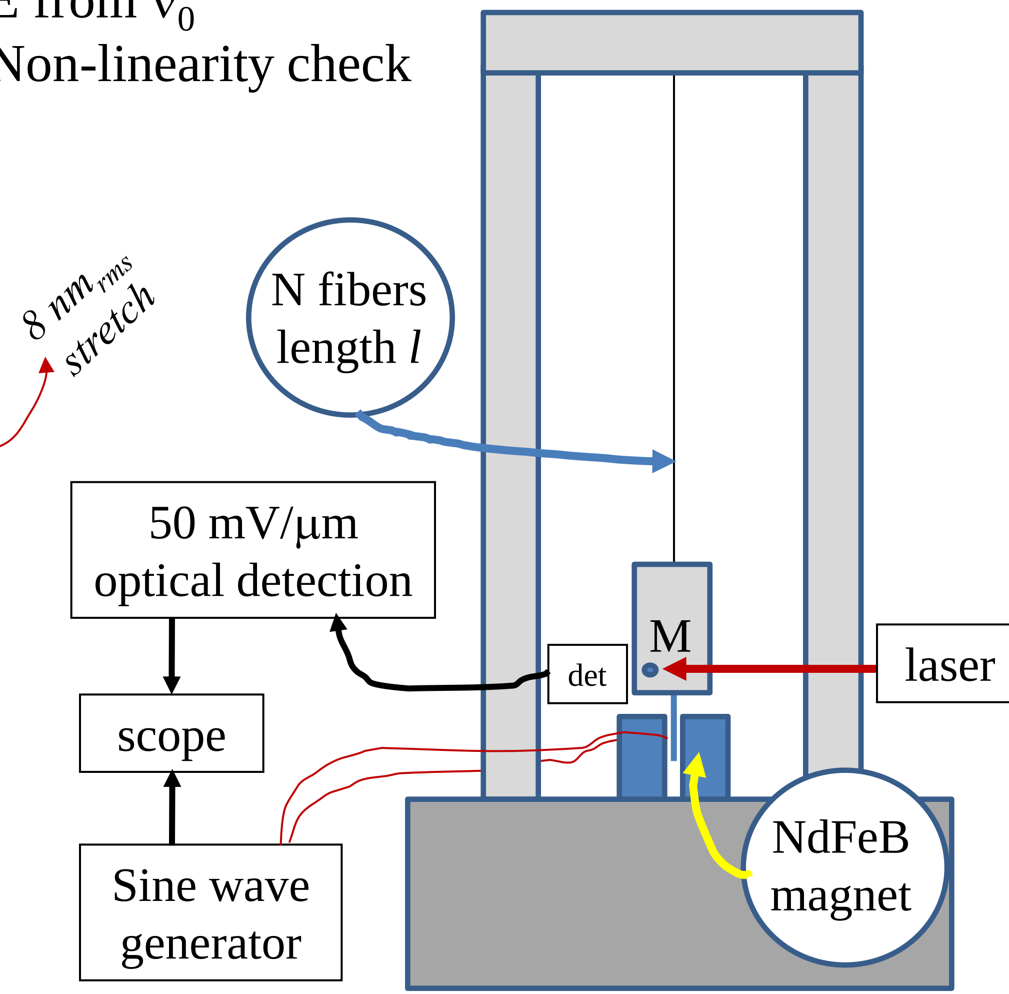
Measurement of fiber characteristics



10 MPa$_p$ (vs.5 kPa$_p$ in operation)

	E(GPa)	Q_{mat}	D_f
Kevlar 29	50	120	12
Carbon	240	1000	7,5
SiC	420	250	12

- Q_{mat} from resonance width
- E from ν_0
- Non-linearity check



Total Q prediction from loss in fibers

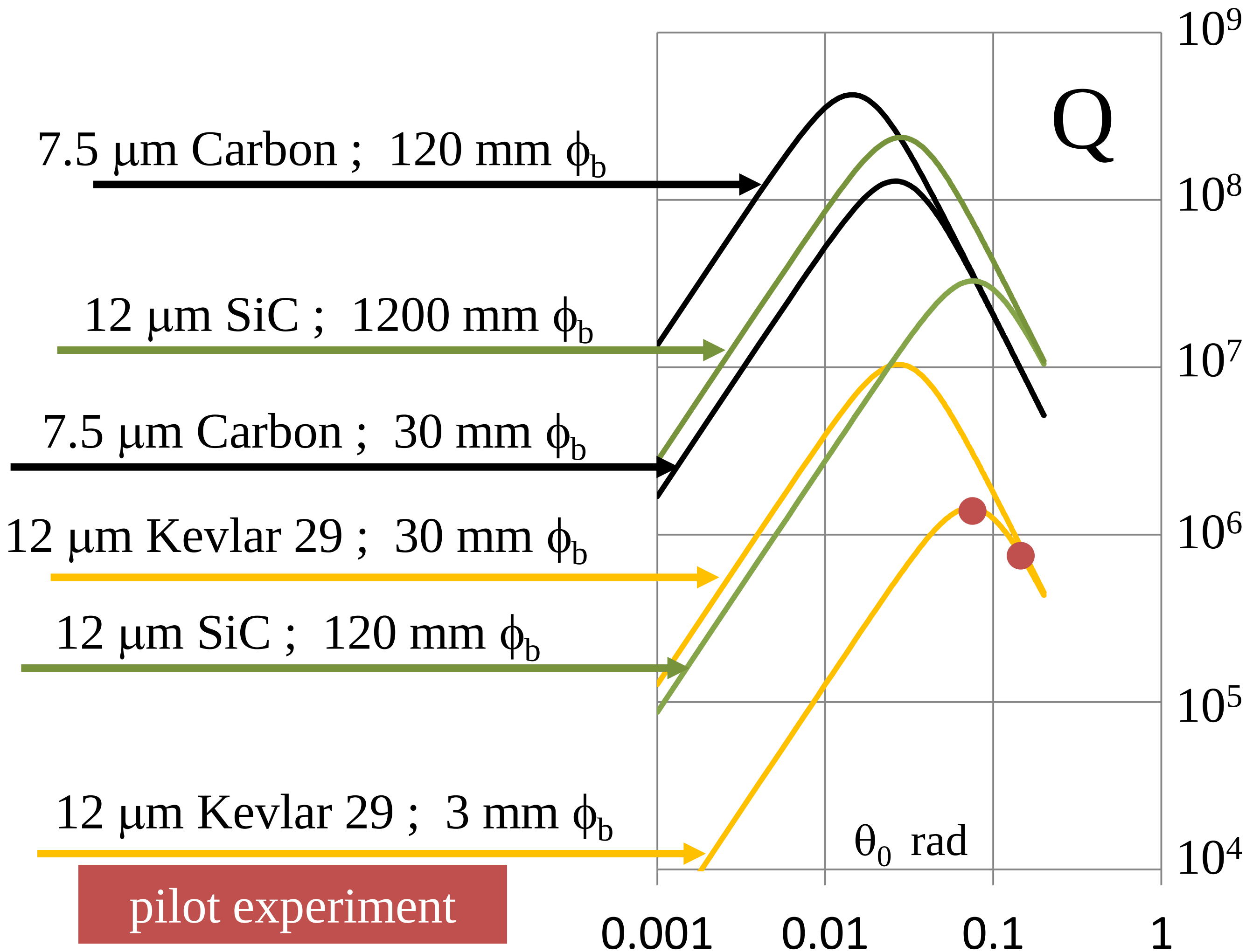
- Fiber and suspension diameters as indicated
- L = 0.9 m
- m = 0.16 g

L=1.2 m

x4/3

L=1.5 m

x5/3



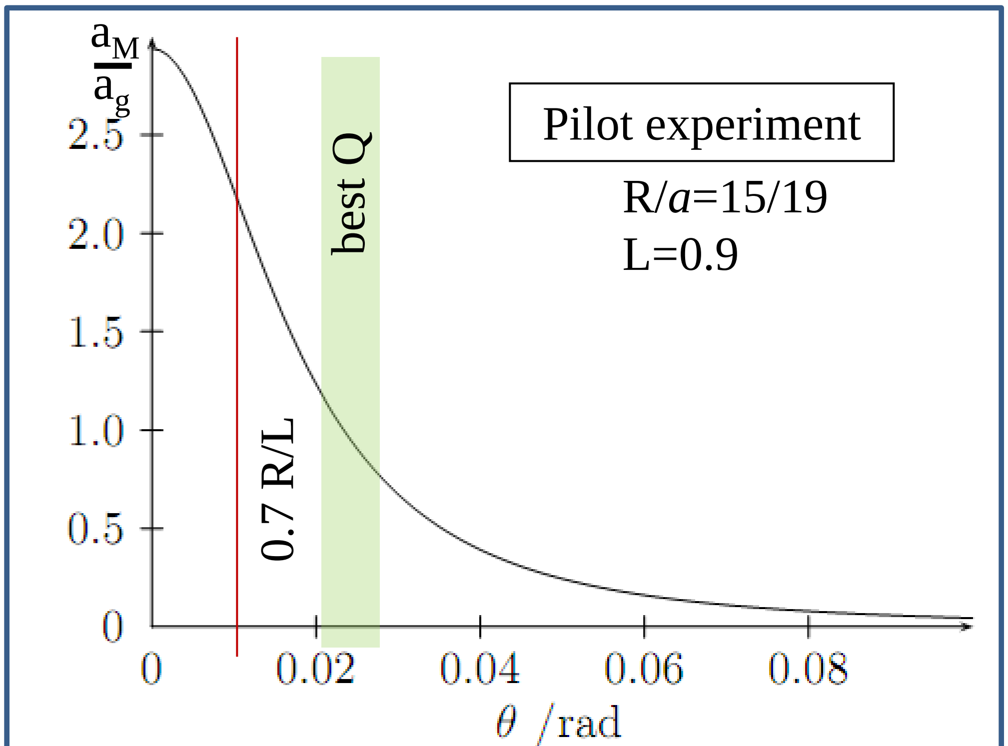
Experimental points

The amplitude dependence problem

$$\frac{a_M}{a_g} = \frac{\rho}{\rho_E}\frac{L}{R_E}\left(\frac{R}{a}\right)^3 \frac{1}{[1+(x/a)^2]^{3/2}} = \frac{\Delta\nu}{\nu}$$

true only for constant a_M/a_g

…but a_M/a_g depends strongly on θ

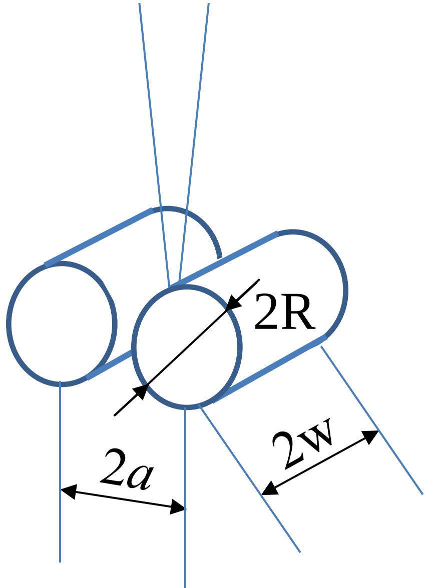


- Reduces the size of the effect or
- Misses best Q conditions
- Makes it difficult to extrapolate to small oscillations
- Complicates the connection between a_M/a_g and $\Delta\nu/\nu$

But there is a solution: cylinders

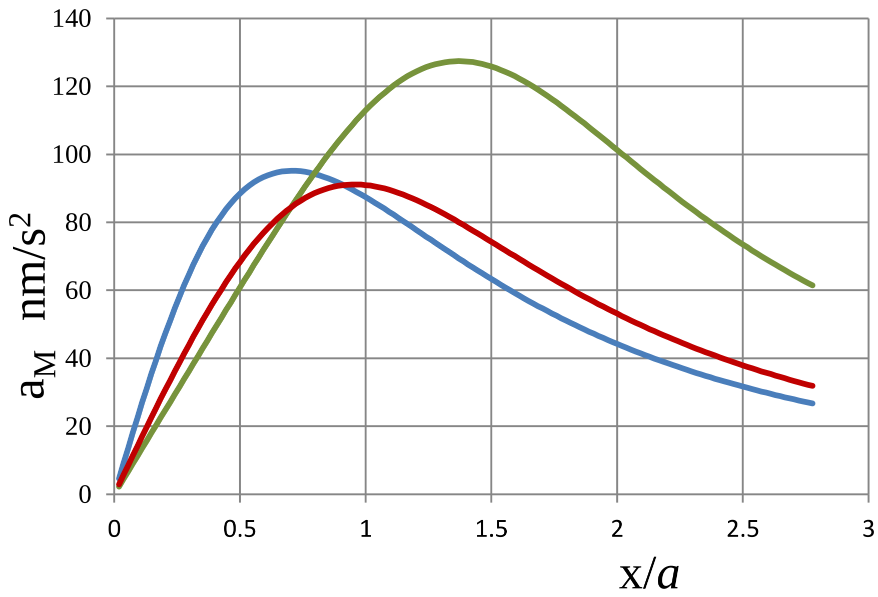
$$\frac{a_M}{a_g} = \frac{3}{4}\,\frac{\rho}{\rho_E}\,\frac{L}{R_E}\left(\frac{R}{a}\right)^2 \left\{ \frac{1}{\sqrt{1+[(w-x)/a]^2}} - \frac{1}{\sqrt{1+[(w+x)/a]^2}} \right\} \frac{a}{x}$$

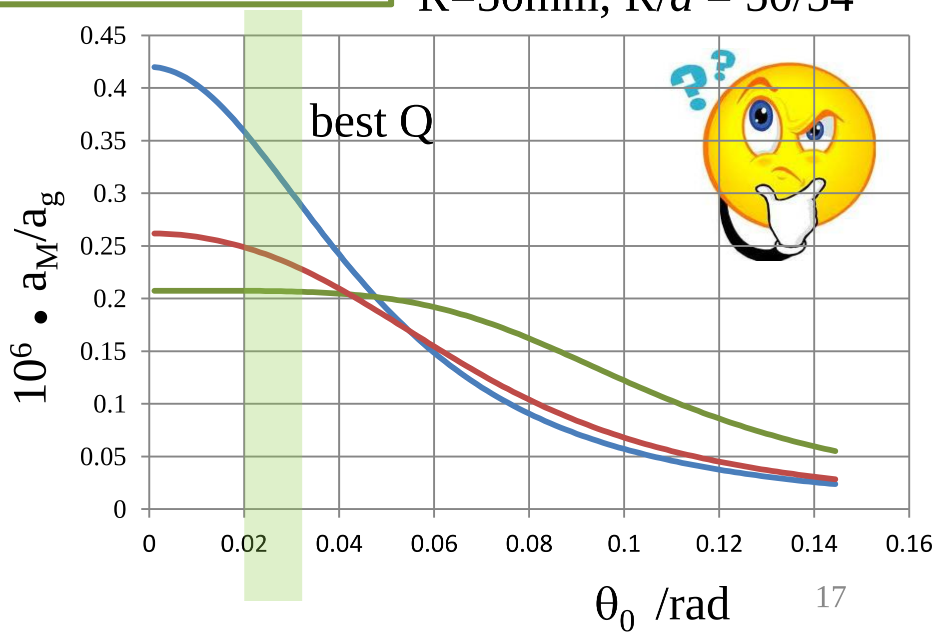
$$\lim_{x \to 0} \{\,\} = \frac{2(w/a)}{[1+(w/a)^2]^{3/2}}\,\frac{x}{a}$$



spheres | cylinders ; w/a= 0.71 | cylinders ; w/a= 1.25

active masses in W
R=50mm; R/a = 50/54





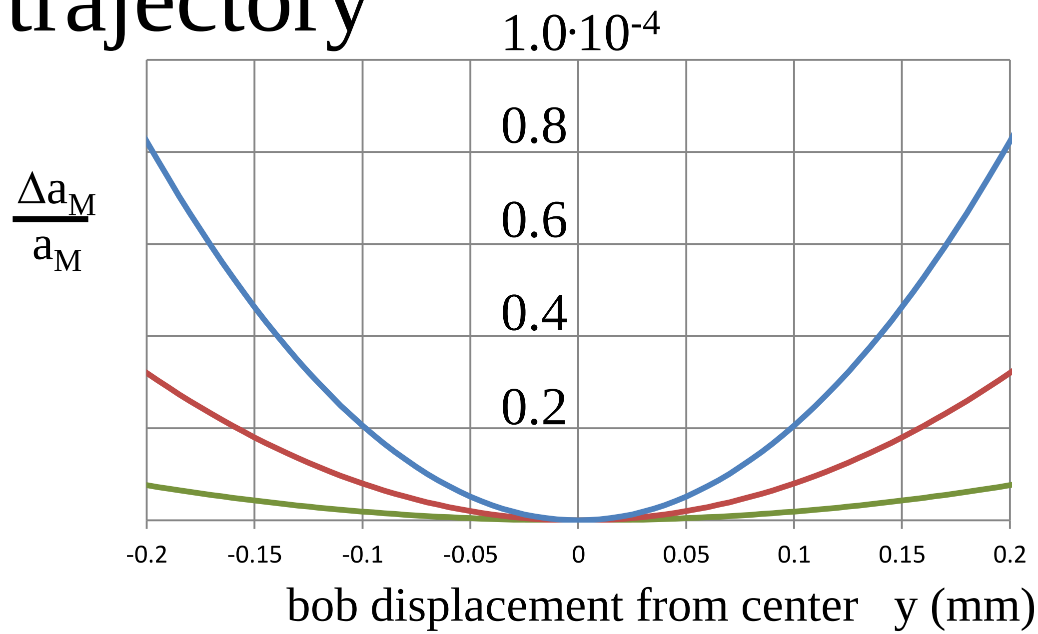
Sensitivity to bob trajectory

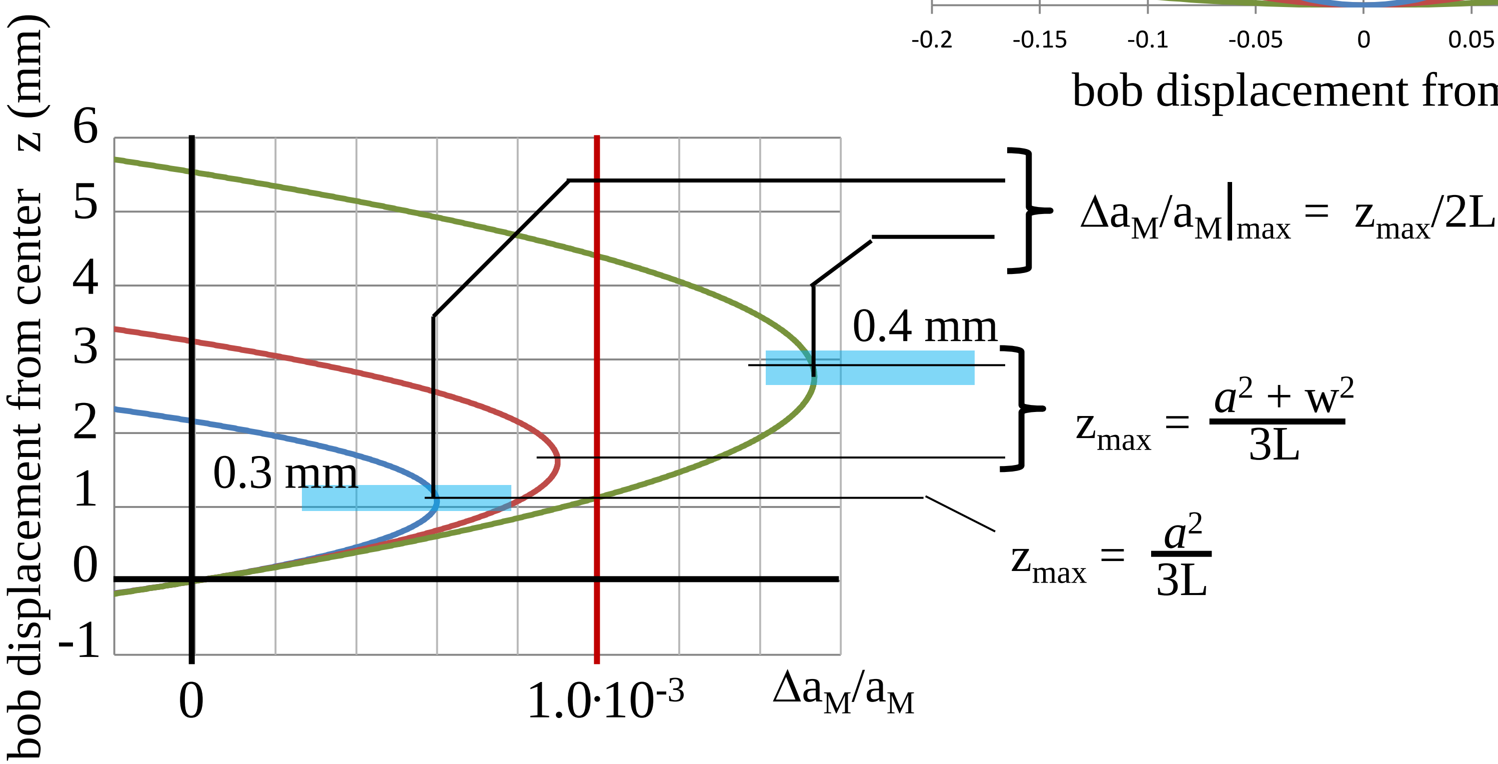
spheres

cylinders ; w/a = 0.71

cylinders ; w/a = 1.25

active masses in W
R=50mm; R/a = 50/54





From frequency shift to big G

- Substitute $\rho_E R_E$ with $(^3/_{4\pi})(^g/_G)$ in the $\Delta\nu/\nu$ asymptotic formula and

Half the gap between active masses

- Extract G

$$\boxed{G = \frac{2\pi\nu^2}{\rho} K \frac{\Delta\nu}{\nu}} \quad \text{with} \quad K = \frac{R}{w}\left\{\left(1+\frac{b}{R}\right)^2 + \left(\frac{w}{R}\right)^2\right\}^{3/2}$$

for cylindrical active masses case

- Introduce the asymptotic value of $\Delta\nu/\nu$ for small oscillations (experimental)
- Introduce the asymptotic value of ν for small oscillations (experimental)

Type B uncertainty contributions

- Geometrical factor K

$\delta b < 100$ nm for $10^{-6}/2$

gauge blocks!

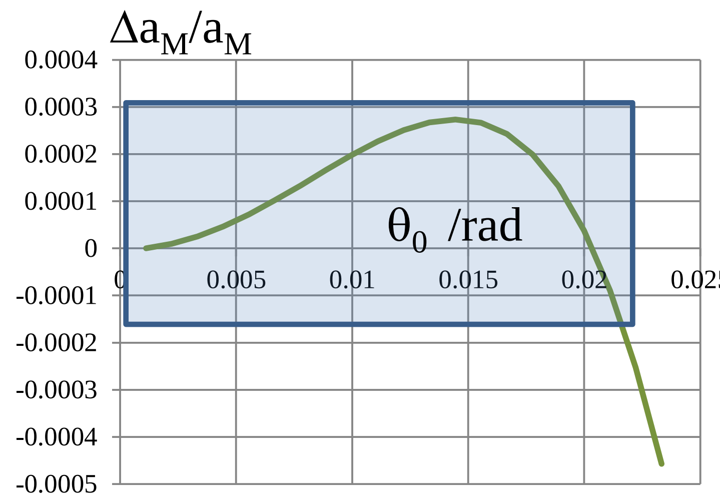
- Active masses density ρ known to $<10^{-5}$?
- Relationship between $\Delta\nu/\nu$ and a_M/a_g

Relationship between $\Delta\nu/\nu$ and a_M/a_g

- θ dependence of a_M

$$\left(\frac{\Delta\nu}{\nu}\Big/\frac{\overline{a_M}}{a_g}\right) - 1 < 3\times10^{-5}$$

The model can estimate certainly better than 10%



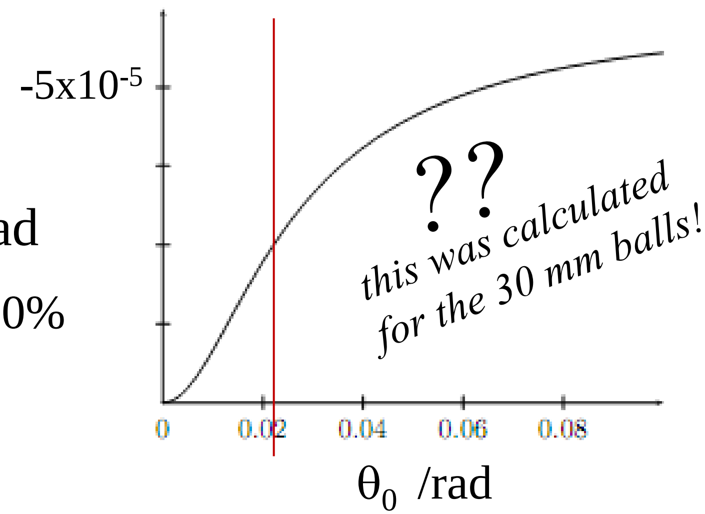
- Vertical displacement shift

As shown: 0.4 mm tolerance for 0.8×10^{-5}

- Adiabatic shift $\dfrac{\Delta\nu}{\nu} = -5\times10^{-5}$

at the chosen amplitude of 0.02 rad

modeling can estimate it certainly better than 10%



- Non-isochronism

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \cdots\right)$$ Must be modified for suspension shape

Tentative accuracy budget projection

cylindrical active masses in W
(w/*a* = 1.25; R=50mm; R/*a* = 50/54)

L = 0.9 m; 7.5 μm Carbon fibers
Suspensions diameter 30 mm ϕ_b

effect	bias	uncertainty	notes
θ dependence of a_M	$<3 \times 10^{-5}$	$<10^{-6}$	Optimization of w/*a*
Shift at bob's trajectory vertical position	1.44×10^{-3}	$< 10^{-7}$	300 nm uncertainty in *a* and w
uncertainty in bob's trajectory vertical position	0	2×10^{-6}	0.2 mm tolerance interval
bob's trajectory horizontal position	0	1.7×10^{-6}	0.2 mm tolerance interval
adiabaticity	-2.5×10^{-5}	2×10^{-6}	0.02 rad peak swing amplitude
non isochronism	2.5×10^{-5}	$< 10^{-6}$	
gap width	0	5×10^{-6}	100 nm gap uncertainty
active masses dimensions (diameter, length)	0	3×10^{-6}	300 nm uncertainty
active masses density	0	5×10^{-6}	??
Total Type B uncertainty		8.5×10^{-6}	
Total Type A uncertainty		$< 3 \times 10^{-7}$	