NBSIR 77-1317

Calibration of High-Voltage Pulse Measurement Systems Based on the Kerr Effect

---

Robert E. Hebner, Jr. and Martin Misakian

Electricity Division
Institute for Basic Standards
National Bureau of Standards

September 16, 1977

Final Report on

Sandia Corporation Order No. 02-8994
Sandia Technical Consultant: S. R. Booker, 9532
Sandia Laboratories Buyer: John G. Boyes

Prepared for
Sandia Corporation
Bldg. 894, Kirtland AFB
East Albuquerque, New Mexico 87115

Executive Summary

High voltage pulse measurements have been performed using systems based on the electro-optic Kerr effect for a number of years. In certain applications, these systems permit state-of-the-art measurements (uncertainties ≈ ±1%). Because the precision of the measurement can be significantly better than the accuracy, an investigation of techniques to improve the calibration of the system--and, thus, to advance the state of the art in pulse measurement--was undertaken.

The investigation focused on two areas. One was the experimental determination of correction factors which would account for differences in environmental factors between the calibration of the system and its use. These measurements yielded accurate corrections for variations in temperature and quantitative evidence of the magnitude of the wavelength dependence of a Kerr system's response.

The second was further study into the feasibility of calibrating the Kerr system at a number of discrete frequencies and using this calibration for pulse measurement. This approach was not completely successful, but did suggest more feasible alternatives. It was concluded that electrohydrodynamic effects in nitrobenzene introduce errors which are sufficiently large that no significant increase in accuracy is possible. Preliminary investigations into another fluid, however, suggest that the problems may be minimized by a judicious choice of the electro-optic material. It should also be emphasized that electrohydrodynamic phenomena are only evident under the application of a steady-state voltage. They, therefore, introduce errors during single frequency calibration but do not degrade the performance of the system under pulsed voltage.

TABLE OF CONTENTS

I. INTRODUCTION

High voltage pulse measurements with uncertainties of order 1% have become routine using the electro-optic Kerr effect. Because the precision of the measurements is more than an order of magnitude greater than the accuracy, it should be possible to improve the accuracy of the measurement.

This report discusses the limitations on calibration techniques and Kerr cell operation, both of which contribute to the measurement uncertainty. It is anticipated that the fundamental limitations are sufficiently well defined by this report that it provides guidance in the selection of specific research and development activities to improve the accuracy of pulse voltage measurement using the Kerr effect.

To accomplish a calibration of high-voltage pulse measuring systems, two techniques are recognized.[1] One is a comparison technique while the other requires a more fundamental analysis of error sources. The comparison technique is not suitable for this application. The primary reason is that there is no known (standard) system which has better accuracy than a Kerr system. For example, consider the most common measuring system, a voltage divider-oscilloscope combination. The calibration of this system could consist of

- -- determination of scale factors of oscilloscope;
- -- determination of the positional dependence of oscilloscope scale factors;
- -- measurement of system's step (frequency) response;
- -- low voltage determination of divider ratio;
- -- test for a voltage coefficient of the divider.

Uncertainties in these determinations limit the best high voltage measurement systems to uncertainties of about 1% (with much larger uncertainties being common). As a consequence, Kerr system accuracy cannot be improved by comparison with a pulse divider system. It is, therefore, necessary to perform a more fundamental calibration.

To identify the factors which influence the accuracy of Kerr system measurements, it is helpful to note that the uncertainty in voltage measurement can be expressed[2] as

$$\frac{\Delta V}{V} = \pm \frac{\Delta V_m}{V_m} \pm \frac{\Delta\phi}{2\phi} \quad . \tag{1}$$

In this equation, V_m is the cell constant and ϕ is the optical phase shift. The remainder of this report is an analysis and a quantification of these error terms. In Section II, the definition of the cell constant and errors in its determination are discussed. The errors associated with phase shift measurements are discussed in Section III.

II. FACTORS WHICH INFLUENCE THE CELL CONSTANT

The phase shift, ϕ, between two orthogonally polarized components of a linearly polarized light beam, upon passage through the Kerr system, is given by the expression

$$\phi = 2\pi \int_0^L BE^2 d\ell \quad . \tag{2}$$

In Eq. 2, B is the Kerr coefficient of the optically active material, E is the applied electric field, and ℓ is a spatial coordinate in the direction of light beam propagation. For voltage measurement it is assumed that Eq. 2 can be rewritten

$$\phi = \pi \, (V/V_m)^2 \tag{3}$$

where the cell constant, V_m, is defined as

$$V_m = d\ (2B\ell')^{-\frac{1}{2}} \quad . \tag{4}$$

In Eq. 4, the assumption is made that the Kerr cell is a parallel-plate structure with d being the spacing between the plates. In addition, it is assumed that any electric field non-uniformity can be accounted for by the effective plate length ℓ'.

Calibration of a Kerr cell consists of a determination of V_m. The values of d, B, and ℓ' are not independently known with sufficient accuracy to permit V_m to be calculated using Eq. 4. The technique which is used is to apply a known voltage to a Kerr cell, measure the phase shift, and using Eq. 3, calculate V_m. This value of V_m is then used to measure unknown voltages. As pointed out above, if pulsed voltages are used in the determination of V_m, no improvement in accuracy is possible. Attempts have been made, therefore, to calibrate the cell at a number of discrete frequencies--where improved accuracy is possible--and to apply this calibration to the extent permitted by the frequency response of the Kerr system.

The technique is valid to the extent that differences between experimental conditions during calibration and during measurement do not change the value of V_m. The remainder of this section will be devoted to a discussion of those factors which could modify the value of the cell constant, V_m. As suggested by Eq. 4, these factors will be considered in terms of their impact on the plate spacing, the Kerr coefficient, and the effective length of the parallel plates.

II.A FACTORS WHICH INFLUENCE THE PLATE SPACING

II.A.1 TEMPERATURE

In evaluating the effects of temperature on plate spacing, it is convenient to take the partial derivative of Eq. 4, with respect to temperature. This yields

$$\frac{1}{V_m}\frac{\partial V_m}{\partial T} = \frac{1}{d}\frac{\partial d}{\partial T} - \frac{1}{2B}\frac{\partial B}{\partial T} - \frac{1}{2\ell'}\frac{\partial \ell'}{\partial T} \quad . \tag{5}$$

The term containing the spacing, d, is in the form of a coefficient of linear expansion. Because the Kerr cell is constructed of glass and metal, it would be expected that

$$\frac{1}{d}\frac{\partial d}{\partial T} \approx 10^{-5}/K \quad . \tag{6}$$

Because the cells are used near room temperature, and because an inaccuracy of 0.1% in the determination of V_m is acceptable, the variation of the plate spacing with temperature is negligible. The second term in Eq. 5 is discussed in Section II.B.3 and the third in Section II.C.2.

No other reasonable, non-catastrophic variation in experimental conditions has been identified which could yield a significant variation in the plate spacing.

II.B FACTORS WHICH INFLUENCE THE KERR COEFFICIENT

II.B.1 GENERAL

It is widely recognized that the understanding of liquid structure is not sufficiently advanced to permit the calculation of the Kerr coefficient in nitrobenzene with the necessary accuracy to be useful in pulse measurement. The classical expression for the Kerr coefficient of a polar fluid is valuable in that it provides a means of identifying those parameters which may cause a variation in the Kerr coefficient. This expression is of the form[3]

$$B = \pi\rho_0\,(n^2 + 2)^2\,(\varepsilon + 2)^2\,(\theta_1 + \theta_2)/27\,n\lambda, \tag{7}$$

where ρ_0 is the number density of the molecules, n is the index of refraction of the light beam at a wavelength λ, ε is the dielectric constant of the material, and θ_1 and θ_2 represent expressions involving the temperature of the fluid, the static and optical polarizabilities and the dipole moment of the molecules.

The variations which have been investigated are the dependence of the Kerr coefficient on the wavelength of the incident light, on the frequency of the applied voltage, and upon the temperature of the material. These will be discussed in detail in the following sections.

II.B.2 WAVELENGTH

From Eq. 7, it can be seen that the Kerr coefficient, and thus the cell constant, has an explicit dependence on the wavelength. As shown in Fig. 1, measurements of the wavelength dependence have been made at a number of laboratories, including NBS. The data exhibit approximately a λ^{-1} variation.

In addition, more careful examination of the NBS data indicates dispersion in the index of refraction may also be significant. Fig. 2 shows a plot of the product $B\lambda$ vs λ. This drawing reflects the fact that $B\lambda$ may change by as much as 7% over the wavelength range from about 475 - 625 nm.

Elementary analysis[4] indicates that some variation should occur. There is strong absorption in nitrobenzene for wavelengths shorter than about 400 nm.[5] From Kramers-Kronig relationships, a wavelength dependent absorption should lead to a wavelength dependence in the index of refraction. From Eq. 7, therefore, it is reasonable to expect a wavelength dependence of $B\lambda$.

The significance of this variation on Kerr system calibration is twofold. From Fig. 2, it can be inferred that over visible wavelengths

$$\frac{1}{(B\lambda)} \frac{\partial(B\lambda)}{\partial\lambda} \sim -5 \times 10^{-4}/\text{nm} . \qquad (8)$$

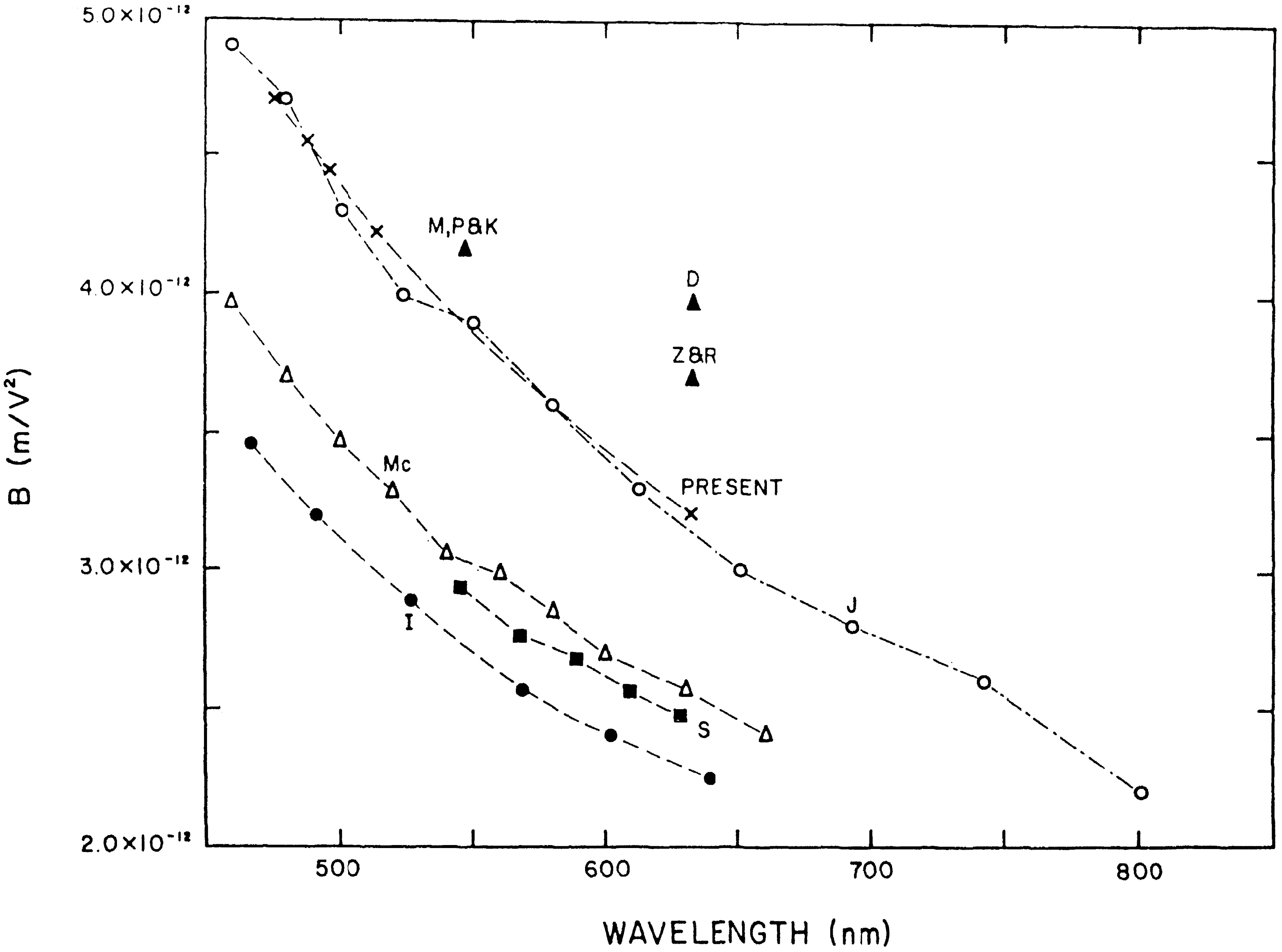


Fig. 1 Wavelength dependence of the Kerr coefficient of nitrobenzene. A comparison[22] of data from a variety of investigators indicates that although there is substantial disagreement as to the magnitude of the Kerr coefficient, there is general agreement as to its variation with wavelength.

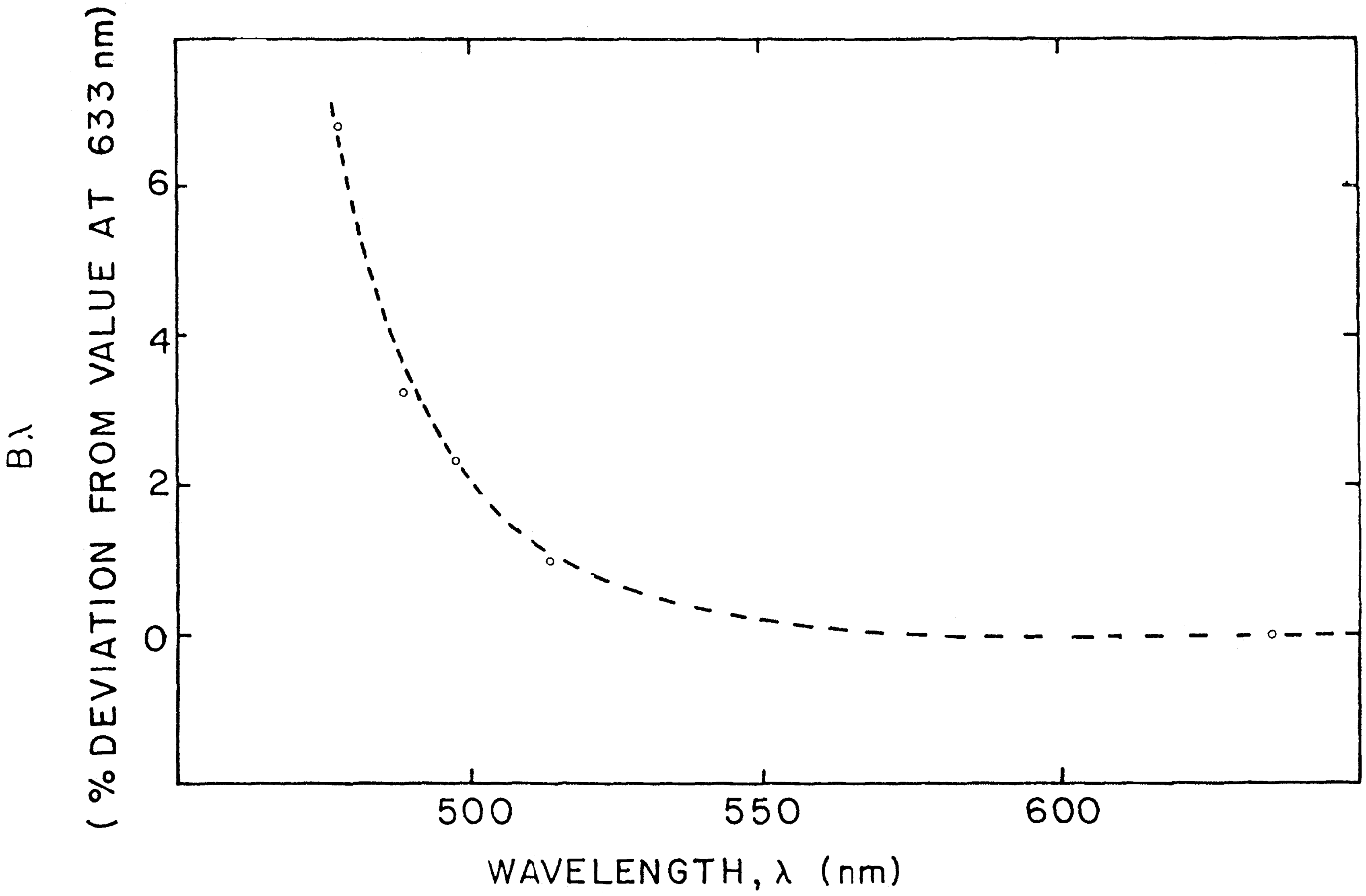


Fig. 2 Variation of the product of the Kerr coefficient and the wavelength with a change in wavelength. The open circles represent measured values. The imprecision of the measurement is less than 1%. These data emphasize the fact that the product Bλ is wavelength dependent.

If the light source used for Kerr effect measurement is a laser, the linewidth is sufficiently narrow that dispersion effects are negligible. If, however, the cell constant is determined at one wavelength, there is no way to accurately calculate the cell constant at a significantly different wavelength. This is to say, reliable data concerning the dispersion of the Kerr effect over a wide range of wavelengths are not available. This is not generally a problem as the use of a variety of lasers is not common practice.

II.B.3 TEMPERATURE

The temperature dependence of the Kerr coefficient of nitrobenzene has been redetermined during this contract period. Previous measurements[6] have indicated that over the range of normal room temperatures

$$\frac{1}{B}\frac{\partial B}{\partial T} \approx 10^{-2}/K \ . \tag{9}$$

For a nitrobenzene-filled Kerr cell, therefore, the term (Eq. 5) containing the Kerr coefficient is the only significant contribution to the temperature dependence of the cell constant. This is a fortunate situation because this variation is a fundamental property of the nitrobenzene and not an "accidental" property which is unique to each Kerr cell. As a practical matter, this means that given the temperature dependence of the Kerr coefficient of nitrobenzene, the cell constant of any nitrobenzene-filled Kerr cell can be corrected for temperature changes. From Eqs. 4 and 5, this simple relationship between the values of the cell constants at two temperatures T_1 and T_2 is

$$V_m(T_1) = [B(T_2)/B(T_1)]^{\frac{1}{2}} \ V_m(T_2) \ . \tag{10}$$

The temperature dependence of B is determined by measuring the cell constant of a given cell as a function of temperature and noting that V_m^{-2} is proportional to B; a plot of V_m^{-2} versus temperature indicates the relative change in B with temperature. The absolute variation in B(T) can be obtained by normalizing the data at a specific temperature to a previously measured value of B. This approach has been employed previously[6] and is used in the present set of measurements.

It should be noted in Eq. 10 that the only dependence on the Kerr coefficient is its ratio at two different temperatures. Therefore, any errors introduced by the normalization do not affect the accuracy of the correction of the value of the cell constant.

The glass cell used to determine the temperature dependence of B was cylindrical, approximately 7 cm long and 2 cm in diameter. The parallel-plate electrodes were about 5.8 cm long, 1.3 cm wide, and spaced about 0.3 cm apart. A calibrated copper-constantan thermocouple was inserted into a hollow electrode support to measure the nitrobenzene temperature. The cell with its electrical connections was totally immersed in an oil bath which could be cooled or heated over a temperature range of approximately 280 K to 340 K. Temperature measurements in the oil bath immediately adjacent to the cell, using a second thermocouple (chromel-alumel), revealed no temperature gradients in excess of 0.1 K between the interior of the cell and any of the selected points shown in Fig. 3. The nitrobenzene temperature measurements are believed to be accurate to within ±0.1 K; this represents a substantial improvement over previous work[6,7] (±0.5% of the measured value).

At elevated temperatures, measurements of V_m were made by first heating the oil bath; cell constant data were then obtained as the bath cooled. An analogous procedure was followed to obtain results below room temperature. Data were

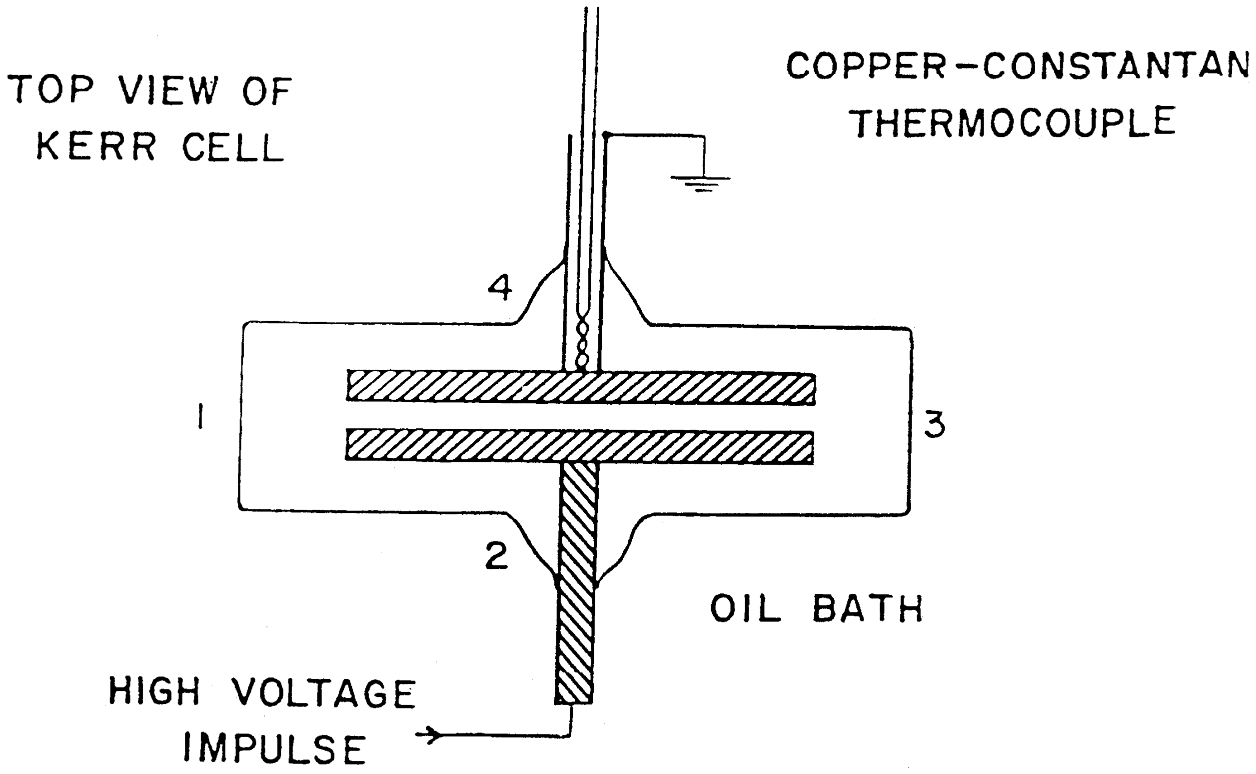


Fig. 3 Apparatus used in the determination of the temperature dependence of the Kerr coefficient of nitrobenzene. To insure that the thermocouple in the grounded electrode was measuring the temperature of the interelectrode nitrobenzene, the Kerr coefficient was not measured until temperature measurements at 1, 2, 3, and 4 agreed with measurements inside the grounded electrode to within 0.1 K.

collected from 285 K to 337 K.

Redundant systems of voltage measurement were used to determine the cell constant V_m. The peak value of the applied high voltage pulses measured with a standard Kerr cell at room temperature and a calibrated resistor divider were averaged to determine a single value. Differences between the two measurements were typically near 0.5%. A helium-neon laser (632.8 nm) was used as the light source.

Experimental evidence[6,8] as well as a theoretical[9] model using a classical statistical mechanical approach suggest that the temperature dependence of the electro-optic Kerr coefficient has the functional form

$$B(T) = \alpha_0 + \alpha_1/T + \alpha_2/T^2, \tag{11}$$

where the α's are constants. This relationship has been adequate to describe the behavior of B for nitrobenzene in earlier NBS studies and is employed again in the present work.

Fig. 4 shows the experimental results normalized to the previously measured value, $B = 3.26 \times 10^{-12} \pm 6\%\ \mathrm{mV}^{-2}$, at 295.6 K and 632.8 nm. The curve in Fig. 4 is a least squares fit to the data. This fit yields the following values for the constants in Eq. 11:

$$\begin{aligned} \alpha_0 &= 6.556 \times 10^{-12}\ \mathrm{mV}^{-2} \\ \alpha_1 &= -5.610 \times 10^{-9}\ \mathrm{K\ mV}^{-2} \\ \alpha_2 &= 1.379 \times 10^{-6}\ \mathrm{K}^2\ \mathrm{mV}^{-2} \quad . \end{aligned} \tag{12}$$

The residual standard deviation for the fit is 9.423×10^{-15}.

Although the above values are different by factors of 2.3 to 4 from earlier results over a more limited temperature range, the calculated values of B over a common temperature interval agree to within 2.8%. Because of improvement in experimental technique, the more recent values are considered to be more reliable.

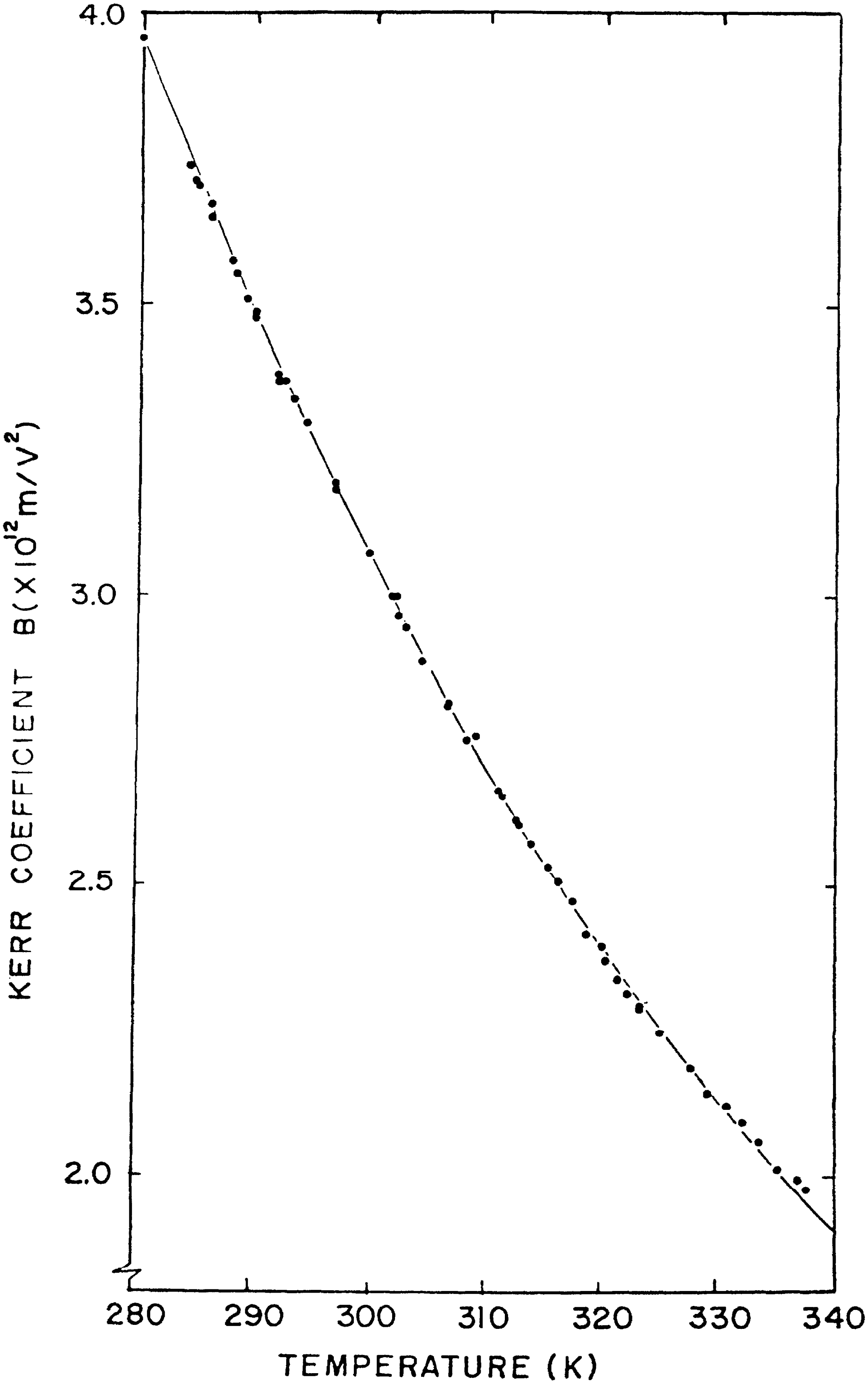


Fig. 4 Plot of the temperature dependence of the Kerr coefficient of nitrobenzene. The solid circles represent data points. The measurement imprecision is of order the dot size. The magnitude of the Kerr coefficient was determined by normalization to previous data[6] at one point. The reference

As has been noted previously, the above constants are appropriate for interpolating over the temperature range of the data (285 K to 337 K); extrapolation beyond this range may introduce substantial error.

II.B.4 FREQUENCY

From Eq. 7, it can be inferred that any dependence of the Kerr coefficient on the frequency of the applied voltage is due to the frequency dependence of the dielectric constant, ε. Measurement of the frequency dependence[10] of ε shows a flat response for frequencies below about 10^9 Hz, Fig. 5. This frequency independence served as one of the early justifications for the development of Kerr systems for the measurement of high voltage pulses. It permits calibration of a system at one or a number of discrete frequencies and subsequent use of this calibration for pulse measurements. This is appealing because, in general, single frequency, high voltage measurements are more accurate than high voltage pulse measurements by one or more orders of magnitude.

A direct method to verify that the cell constant is indeed frequency independent is to apply high voltages over the frequency range of interest and measure the cell constant. An attempt was previously made to perform such a measurement[2] and the results are summarized in Fig. 6.

These results are promising but not definitive because of the limited number of frequencies over which data were taken. The fundamental limitation to the extension of these data is the electrohydrodynamic phenomena for a certain range of low frequencies which result from the presence of space charge in nitrobenzene. These phenomena are discussed in more detail in Section II.C.3.b.

The present status, therefore, is that frequency independence cannot be fully verified because electrohydrodynamic effects at low frequencies prevent measurement of the cell constant. These effects, however, only complicate calibration; they have not caused an identified error in the measurement of microsecond duration, high voltage pulses.

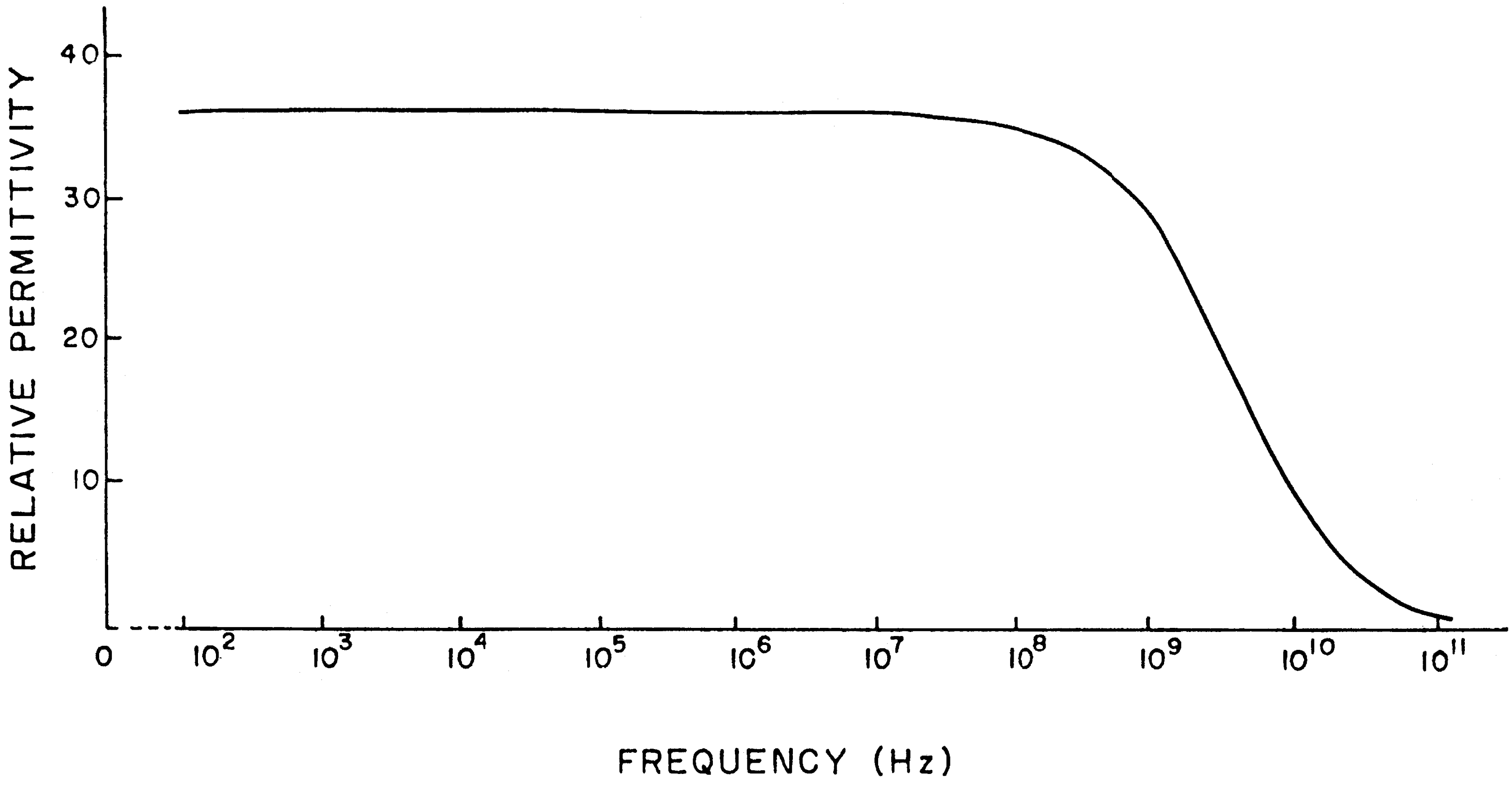


Fig. 5 Variation of the relative permittivity of nitrobenzene with the frequency of the applied voltage.[10] The frequency independence for frequencies below about 10^8 Hz is exploited for accurate pulse measurement.

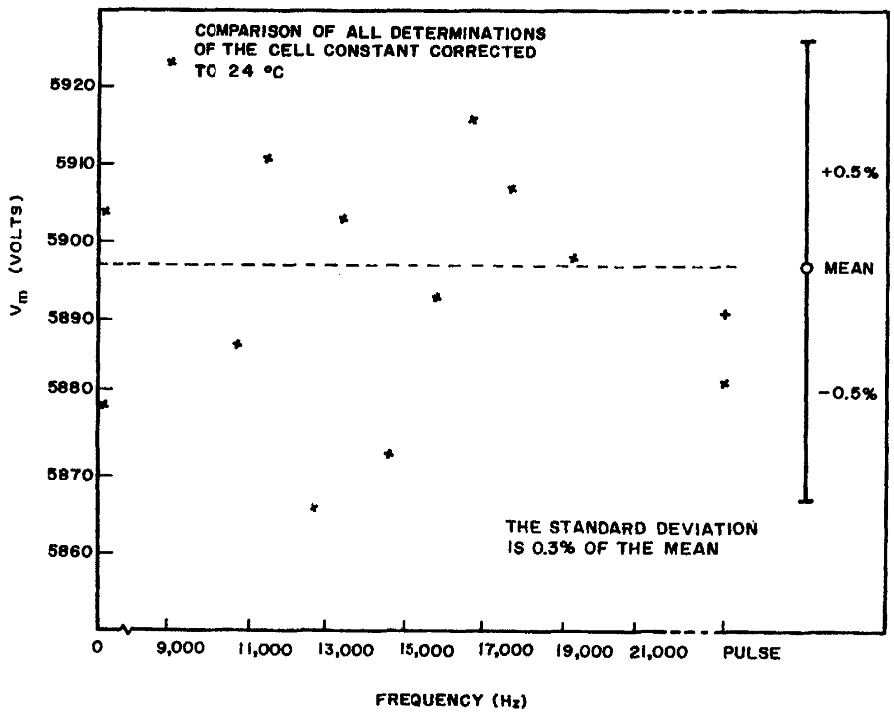


Fig. 6 Determination of the cell constant of a given cell at a variety of frequencies.[2] These results suggest that more accurate calibrations of Kerr systems are feasible.

II.C FACTORS WHICH INFLUENCE THE EFFECTIVE LENGTH

II.C.1 GENERAL

Care must be taken to distinguish between two different phenomena which influence the value of the effective length. Recall from Eqs. 1-3, the effective length, ℓ', is defined

$$\phi = 2\pi \int_0^L BE^2 d\ell = 2\pi B\ell' (V/d)^2 \quad . \tag{13}$$

The first phenomenon is geometrical in nature.[11] The Kerr cells used in this study are parallel-plate capacitors. The light beam passes between the plates, not only through the uniform field area but also through the fringing fields at the ends of the plates. The effect of this is shown in Fig. 7. It should be noted that in Fig. 7 the experimentally derived electric field distribution is asymmetric about the midplane. This is attributed to the perturbing effects of nearby grounded objects.

The second phenomenon which influences ℓ' can be more significant and is more difficult to discuss rigorously. To understand this effect it should be noted that in Eq. 13, the electric field has been replaced by the voltage divided by the spacing. If space charge exists, the field distribution is not uniform. So the effective length must also account for the field non-uniformity caused by space charge. An example of such distortion is shown in Fig. 8.

So the value of the effective length is a function of position with the positional dependence influenced both by cell geometry and space charge. For pulse measurements, space charge is negligible and compensation for variations due to geometry is made by confining the light beam to a single path. However, these effects become troublesome when uncertainties less than 1% are required.

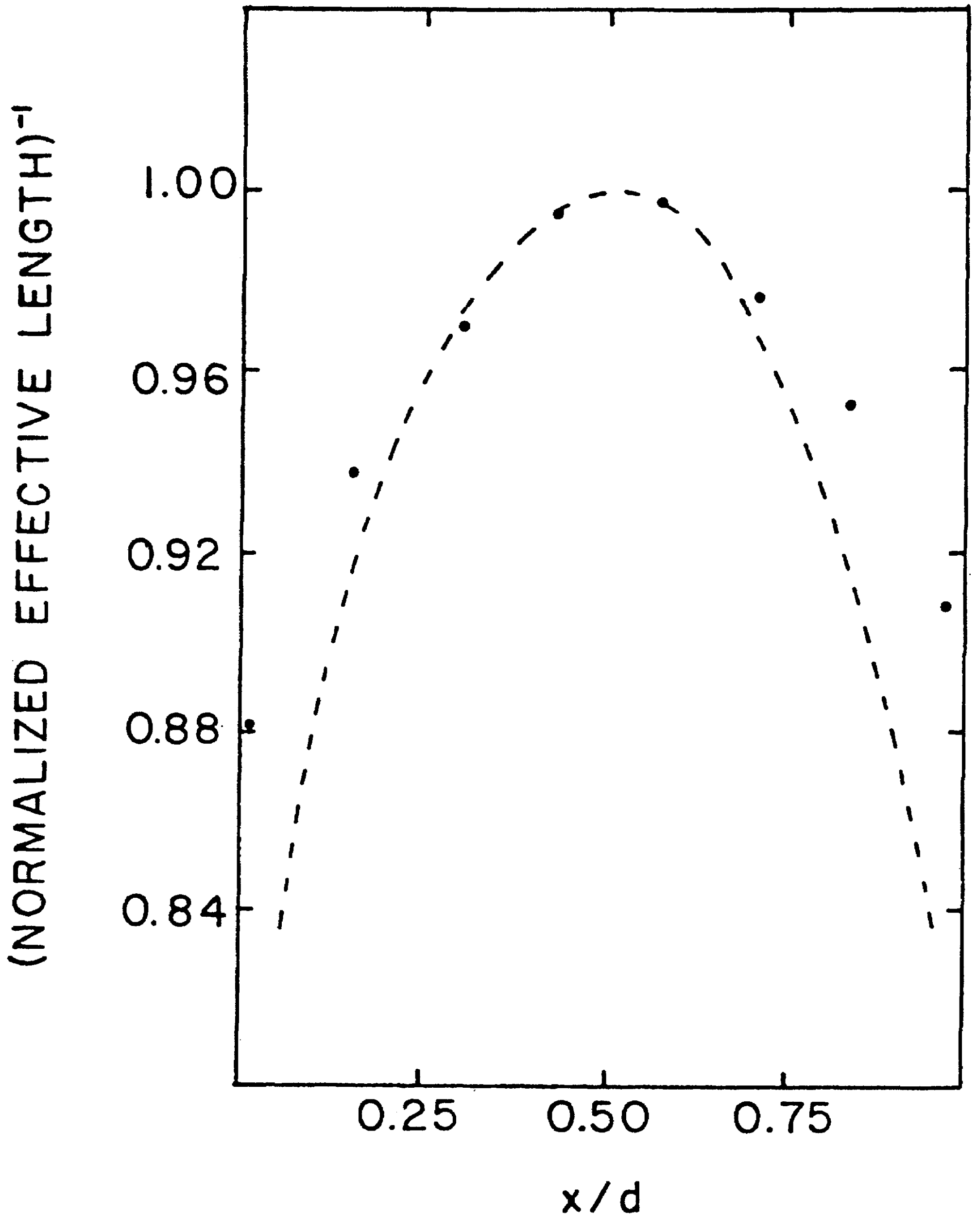


Fig. 7 Effect of fringing field. This plot shows calculated (dashed lines) and measured (solid dots) values of $\ell'_{x/d} = 0.5/\ell'_{x/d}$, where x is a variable along a central path between the electrodes and d is the plate spacing. It can be seen that the effective length for a parallel plate system is shortest for a point one half way (x/d = 0.5) between the electrodes. The calculated curve is symmetrical about the center while the measured points are assymmetrical due to nearby grounded objects.

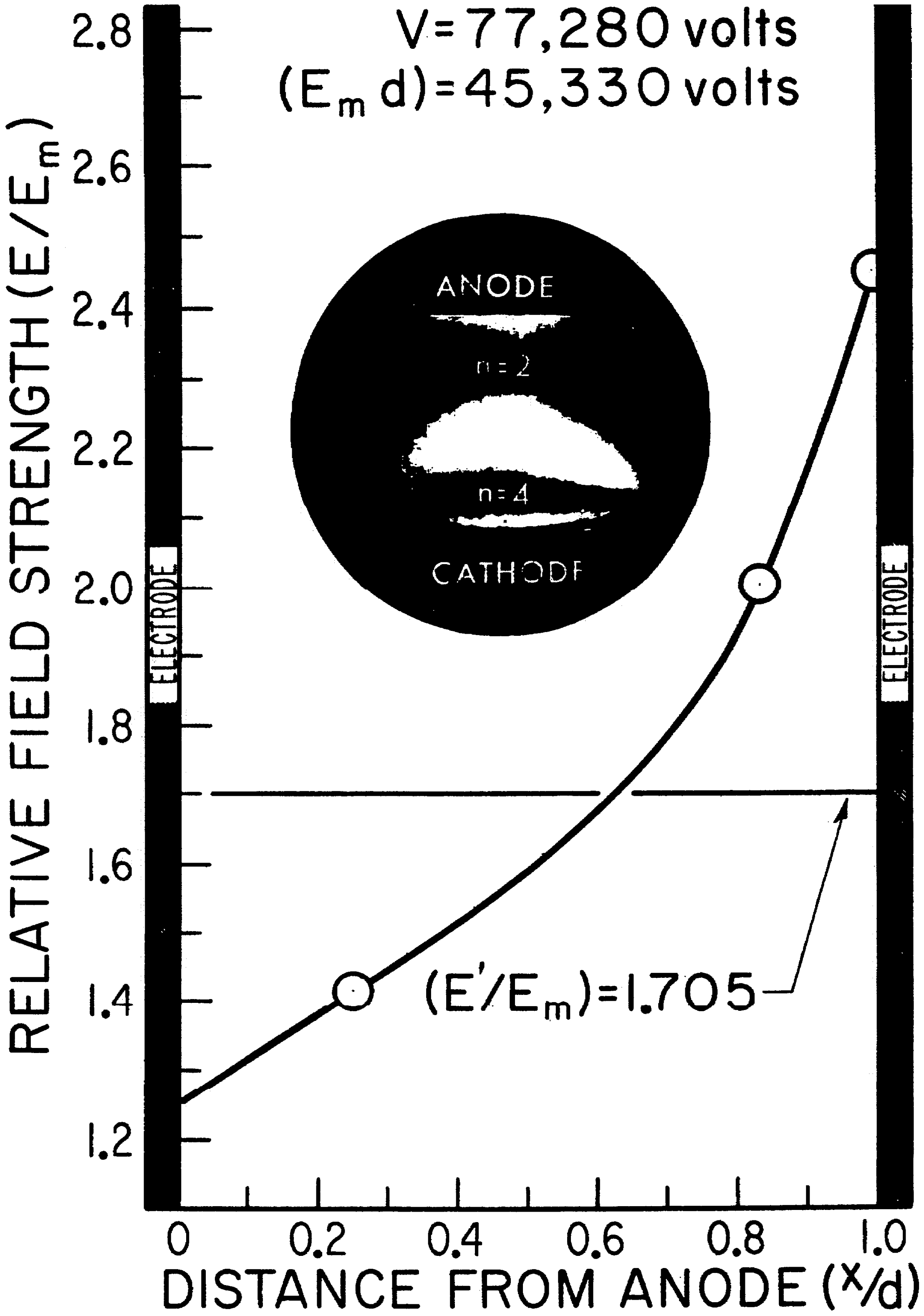


Fig. 8 Effect of space charge. Under direct voltage the effect of fringing fields is dominated by the effect of space charge. The quantity $E_m d$ is V_m as defined in Eq. 4.

II.C.2 TEMPERATURE

The effect of temperature on the geometric contribution to the effective length is negligible. For square-ended electrodes of thickness γ, spacing d and length ℓ and for a central path,[12]

$$\ell' = \ell + (d/\pi)\ [1 + \gamma/d \ \ln\ (1 + d/\gamma)]\ . \qquad (14)$$

It should be noted that recent work by Thacher[13] has extended this type of analysis to include non-central paths and has found the same general dependence on spacing and length.

Because ℓ' is only a function of geometry,

$$\frac{1}{\ell'}\frac{\partial \ell'}{\partial T} \approx 10^{-5}/K \quad . \qquad (15)$$

From Eq. 5, it can be seen that this term does not make a significant contribution to the variation in the cell constant.

In addition to the direct effect on geometric length, temperature variations can also change the value of the effective length by changing the space charge distribution. The effect of a temperature change is to modify the mobility of the charge carriers in the fluid. Theoretical consideration of simplified physical models indicates that the electric field distribution does depend on the mobility. Experimental confirmation has been obtained using a mixture of pentachlorobiphenyl and trichlorobenzene. As the temperature of the mixture was increased from 0 °C to 60 °C, the central field strength, under direct voltage, decreased by more than 20%. Of course, the applied voltage remained constant.

In nitrobenzene-filled Kerr cells, space charge has been found to be negligible if the voltage is applied for times shorter than about 100 μs. Under pulse conditions, therefore, no space charge distortion exists. Under steady-state conditions--for example, during a single frequency calibration--space charge distortion does exist. Because of the finite resistivity of the nitrobenzene, there can be significant self-heating during steady-state voltage application. This can lead to the temperature dependent change in the field distortion discussed above. In extreme cases, electrical breakdown could result.

II.C.3 FREQUENCY

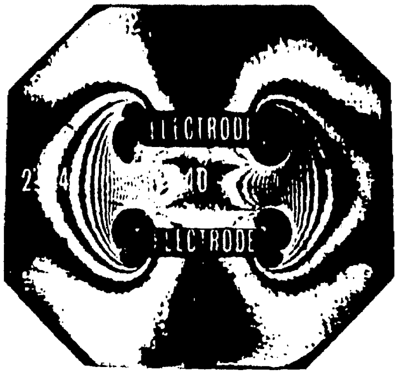
II.C.3.a GENERAL

From Eq. 14, it would be expected that the geometric contribution to the effective length would be frequency independent. Data such as that shown in Fig. 6 confirm this expectation.

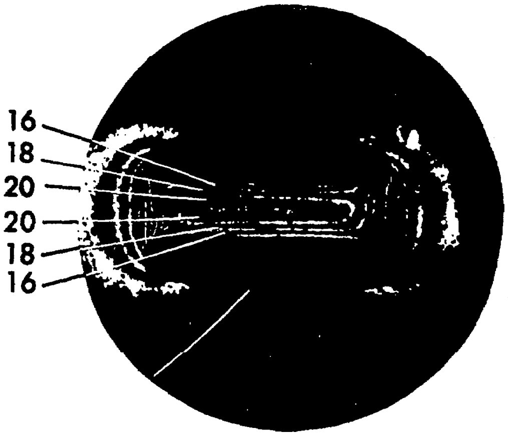
The effect of frequency on the space charge distribution, however, is significant. Three main sources of space charge production have long been recognized. These are molecular-level processes in the bulk (e.g., dissociation, chemical interactions, etc.), molecular-level processes at the interface between the liquid and the electrode and suspended particles in the fluid. Once the charge carriers have been produced, their trajectory depends on the applied voltage and presumably on the coupling to the neutral fluid. This electrohydrodynamic phenomenon is in turn dependent on the geometry of the cell. In the present state of understanding, space charge dynamics cannot be calculated with precision. An experimental measurement of the space charge distortion is, therefore, necessary. These measurements have been performed in a variety of liquids under direct and selected low frequency alternating voltages. This report will focus on results obtained in nitrobenzene, but measurements taken using a chlorinated biphenyl will also be discussed to emphasize the role of the fluid in space charge distortion.

II.C.3.b MEASUREMENTS IN NITROBENZENE

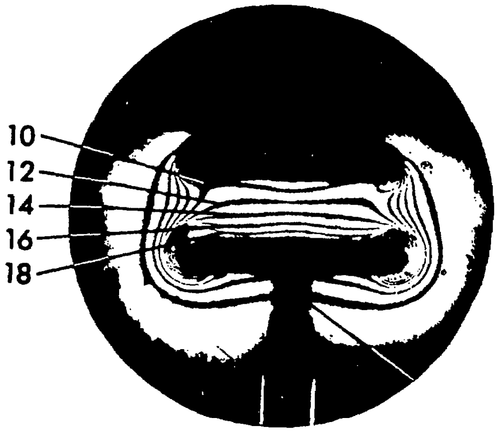
The basic behavior of nitrobenzene is summarized in Figs. 9, 10, and 11. The first point, which is emphasized by Fig. 9, is that the electric field distribution



PULSED VOLTAGE



ALTERNATING VOLTAGE (60 Hz)



DIRECT VOLTAGE

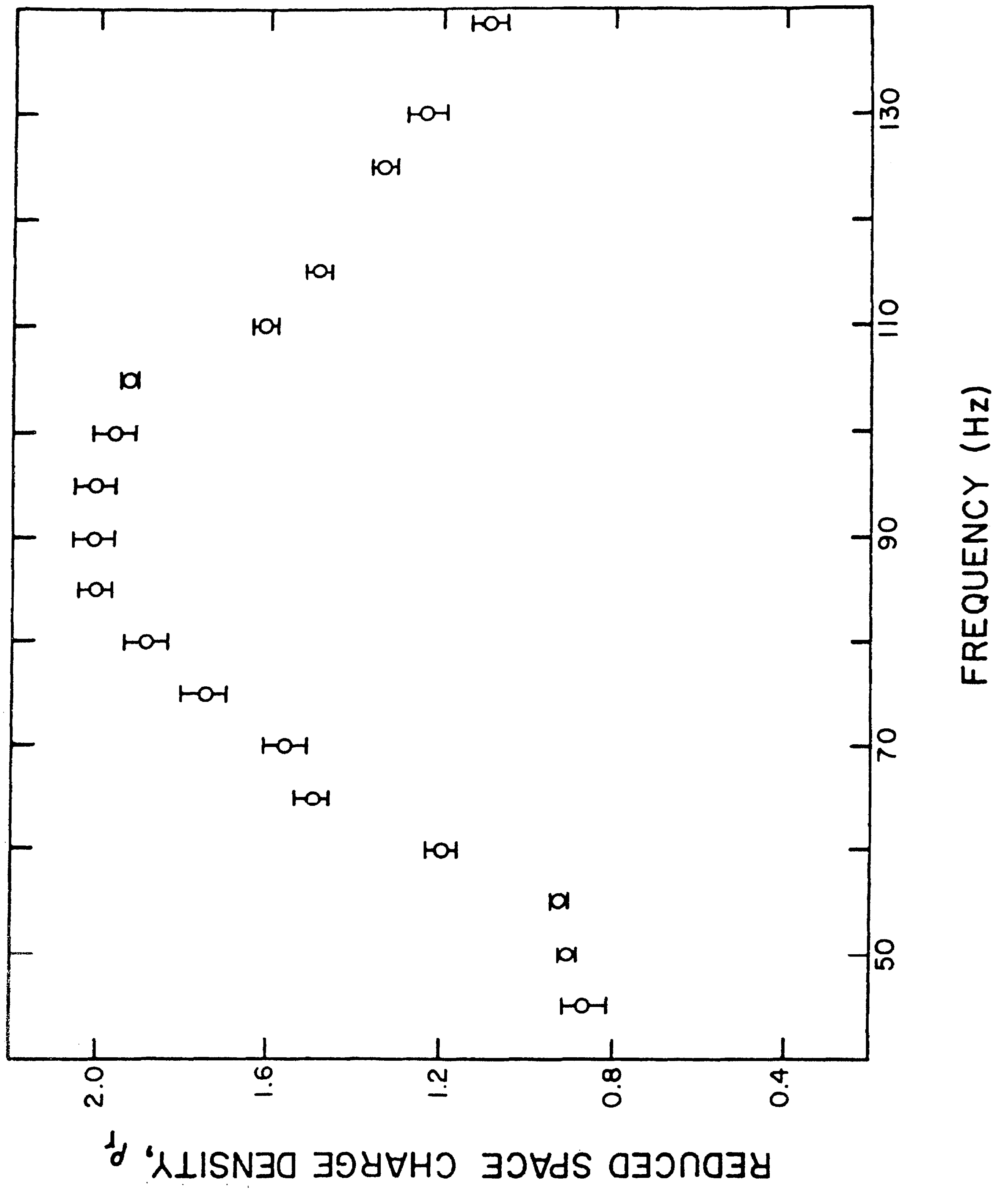
Fig. 9 Kerr response under pulsed, alternating and direct voltage. The differences among the three photographs demonstrate the frequency dependence of the Kerr effect. The numbers of the fringes correspond to integral values of $(V/V_m)^2$.

or, equivalently, the space charge distribution, does, indeed, depend on the frequency of the applied voltage. Under pulsed voltage the field between the plates is calculable from electrostatics.[13] The absence of interelectrode fringes in the central region indicates a uniform field. The other two photos at dc and 60 Hz indicate interelectrode fringes with very different distributions. This implies that the space charge distribution is frequency dependent. This dependence is shown more quantitatively in Fig. 10. In this figure, it can be seen that for the voltage levels and geometry used in this study,[14] the space charge density has a broad maximum in the frequency range between 60 and 90 Hz. The reduction in the space charge density as the frequency is increased above 60 Hz suggests that the effect of space charge may be minimized at somewhat higher frequencies. At higher frequencies, however, a different phenomenon dominates[2] as is shown in Fig. 11. From these photographs, it is apparent that the optical transmission of the cell is profoundly affected by high voltage in the audio frequency range. Within the purification limits attainable with the NBS nitrobenzene purification system (resistivity $\gtrsim 10^9\ \Omega \cdot \mathrm{m}$) this behavior could not be substantially modified by cell purification.

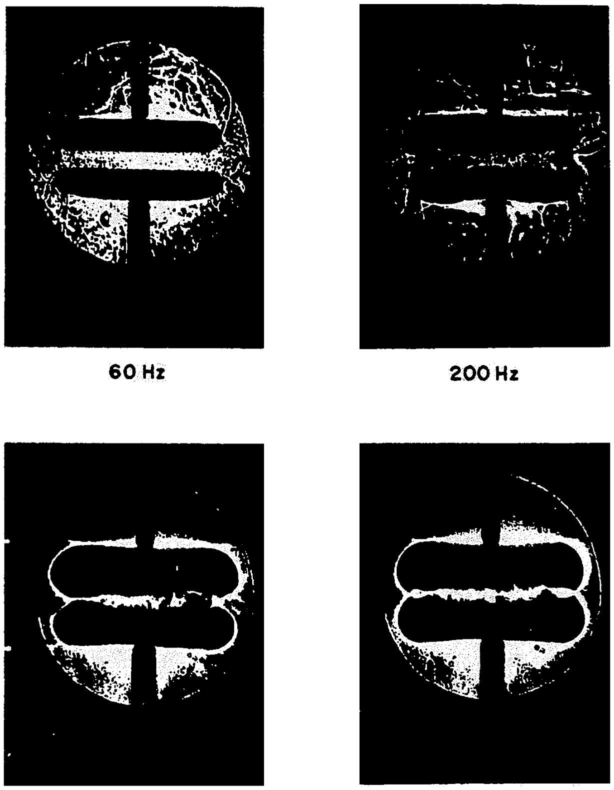
For this reason, further related studies on the time evolution of space charge were undertaken using a chlorinated biphenyl[15] as the Kerr fluid. This liquid was selected because it has a relatively large Kerr coefficient, was commercially available, and because of its widespread use as a high voltage insulating liquid. Since the study was initiated, polychlorinated biphenyls were identified as being environmentally harmful and further Kerr work was terminated at NBS. The completed study does, however, provide further information concerning space charge dynamics in a fluid other than nitrobenzene.

II.C.3.c MEASUREMENTS IN CHLORINATED BIPHENYLS

The measurements of space charge dynamics in a chlorinated biphenyl were


REDUCED SPACE CHARGE DENSITY, ρ_r
2.0
1.6
1.2
0.8
0.4
50
70
90
110
130
FREQUENCY (Hz)

OPTICAL TRANSMITTANCE OF KERR CELL AT SELECTED FREQUENCIES



60 Hz 200 Hz

770 Hz 912 Hz

Fig. 11 Optical transmittance of a Kerr cell at selected frequencies. At frequencies above a few hundred hertz, the optical distortion is sufficiently severe that no Kerr measurements are possible.

made using a "pulsed" 60 Hz voltage source. Specifically, the power source was capable of providing a preselected, integral number of half-cycles of a 60 Hz waveform at high voltage. These measurements are summarized in Fig. 12.

The filtration of the fluid was accomplished in the same way that nitrobenzene is filtered. Specifically, the liquid was drawn under vacuum through an approximately 28-cm-long column of activated alumina and micrometer-sized filters with a mechanical pump. The double filtration referred to in Fig. 12 consisted of drawing the same sample of fluid through the same column twice. Measurements were made in a tetrafluoroethylene cell having Rogowski profile electrodes.

From Fig. 12, it can be inferred that the space charge is due to contaminants which can be removed by filtration. The upper photographs also show the temporal evolution of the space charge and thus the electric field distribution. It can be seen that the field is uniform until about the fifth half-cycle; the distortion comes to equilibrium during the interval from about the fifth to eleventh half-cycle and remains stable after about the eleventh half-cycle.

It is interesting to compare this temporal behavior with related experimental studies. Dakin, Studniarz, and Hummert[16] have investigated the breakdown strength of transformer oil as a function of the number of cycles of 60 Hz voltage which was applied. They found the breakdown strength to be higher and less variable if fewer cycles are applied. They observed that there was a definite relationship between the duration of voltage application and breakdown strength and postulated that this relationship was due to space charge. The present study demonstrates the analogous temporal dependence for the space charge distortion.

In addition, previous work in nitrobenzene[14] had shown that at 60 Hz the space charge did not respond to the instantaneous value of the applied voltage. In this study, it is also apparent that the time necessary for the space charge to respond to a change in the applied voltage is long compared to a cycle of a 60 Hz waveform.

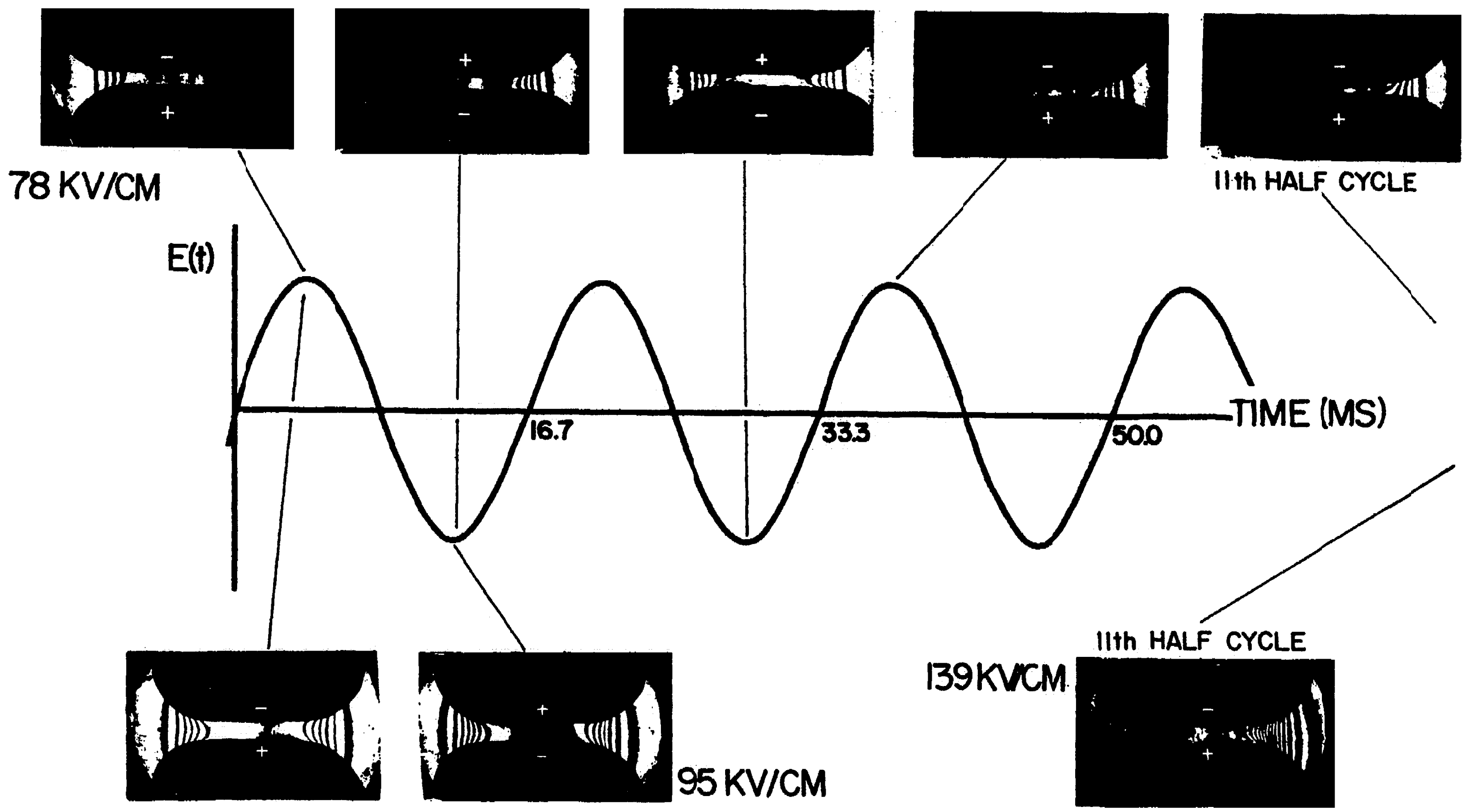


Fig. 12 Kerr response in chlorinated biphenyls. The upper sequence of photos shows the evolution of the electric field distribution during the first few cycles of a 60 Hz waveform after a single filtration cycle. Interelectrode field distortion due to space charge is negligible until about the fifth half cycle. The lower photos show that after double filtration, space charge remains negligible for arbitrarily long times.

With a second filtration, the net space charge density in the chlorinated biphenyl becomes negligibly small. This indicates, of course, that the space charge is not an intrinsic property of the fluid.

These preliminary results suggest that in a fluid other than nitrobenzene single frequency measurements may be possible. Because of the preliminary success in this fluid coupled with the lack of success in reducing the space charge density to an acceptable level in nitrobenzene, investigation of alternative fluids to nitrobenzene appears to be a reasonable goal for future research.

It should be acknowledged, however, that the gradients in the index of refraction which yield the transmittance shown in Fig. 11 may not result solely from the space charge density. The volume force can be expressed as the sum of two terms,[17]

$$\vec{F}_v = \rho\vec{E} + \tfrac{1}{2}\varepsilon_o \rho_m \nabla \left(E^2 \frac{dk}{d\rho_m}\right) , \tag{16}$$

where ρ is the charge density, ρ_m the mass density, E the electric field, ε_o permittivity of vacuum, and k the relative permittivity of the fluid. The second term takes into account the interaction between the field and liquid structure. It has not been demonstrated that this second term is negligible.

III. PHASE SHIFT MEASUREMENT

The second fundamental contribution to error in the measurement of voltage is the error in phase shift measurement. In most previous work the Kerr system was designed so that this error was sufficiently small to be negligible. Minimization of this error source has been possible because of the nonlinearity of the Kerr response. The conventional detection scheme uses a polarizer to convert the phase shift (Eq. 3) between the components of the light beam to a variation in transmitted intensity. If the incident light is polarized at an angle of 45° to the field direction and the second polarizer is oriented so that its transmission axis is orthogonal to that of the first, the phase shift is related to the transmittance by the expression

$$I = I_m \sin^2 (\phi/2) \quad . \tag{17}$$

In Eq. 17, I is the optical transmittance of the system. Combining Eqs. 17 and 2 yields

$$I = I_m \sin^2 [\pi/2 \; (V/V_m)^2] \quad . \tag{18}$$

An analysis of this equation shows that for $V \gtrsim 5 \; V_m$, a 1% change in the applied voltage will cause nearly a 100% change in the transmittance of the cell.

As a consequence, accurate voltage measurements can be performed with very modest requirements on the measurement of the relative transmittance, I/I_m. Typical systems have resolution of 0.1-0.2 I/I_m. So in a well designed system, the voltage resolution is typically of order 0.1% of the applied voltage.[18]

The possible change from nitrobenzene as the Kerr fluid and the general need to perform measurements in a variety of materials with smaller Kerr coefficients have generated the desire to perform accurate measurements in the voltage range $V < 5 \; V_m$. This requires a more accurate determination of the relative transmittance of the system, or, more fundamentally, of the optical phase shift.

A technique which has been recommended as a way to attain better accuracy than is obtainable using crossed polarizers has been discussed in the literature.[18] This approach replaces the second polarizer with an interferometer as shown in Fig. 13. The operational principles have been discussed previously. For this

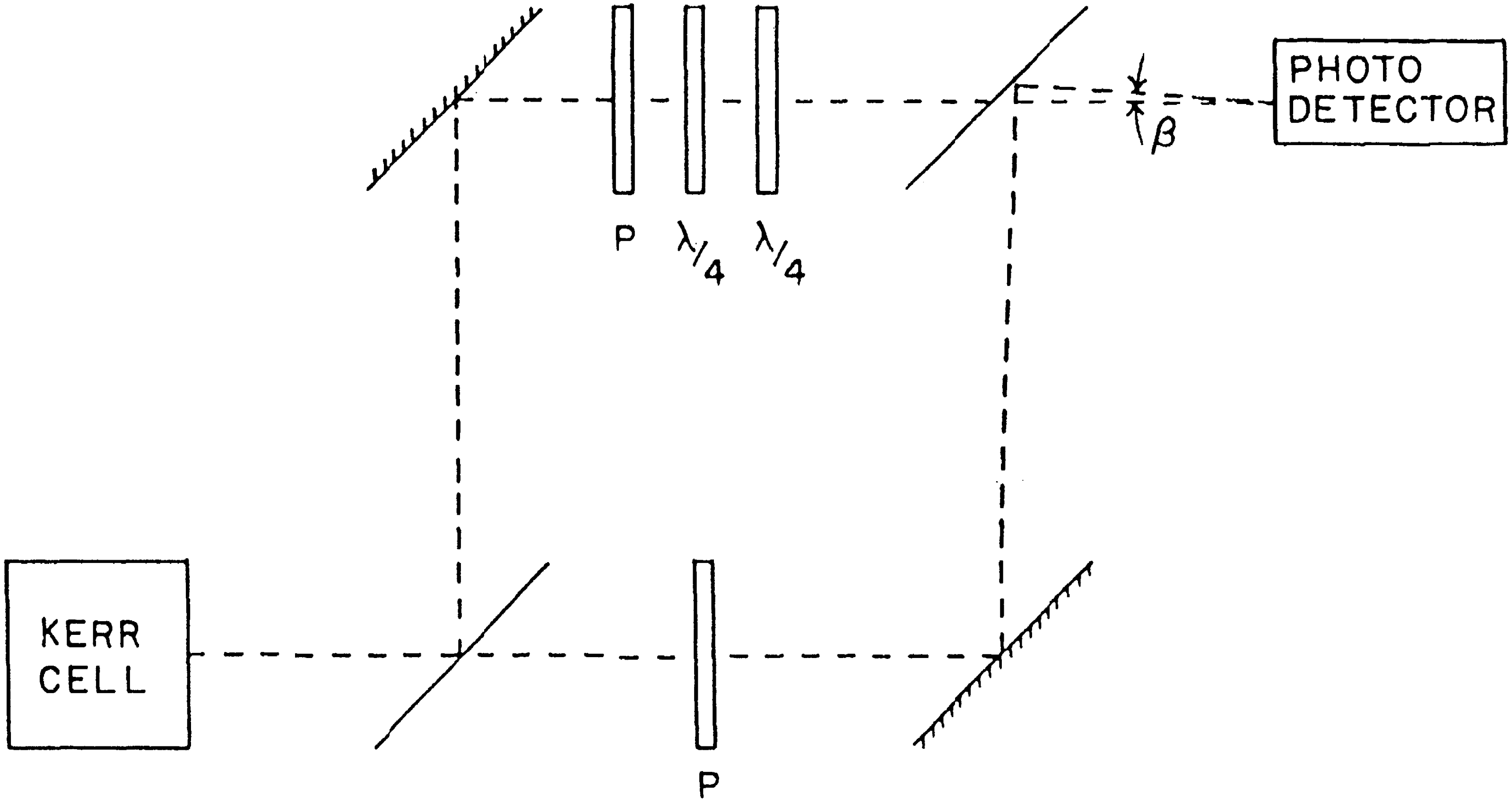


Fig. 13 Interferometric detection system. The interferometer instead of a single polarizer is used between the Kerr cell and the photodetector.

device, the transmitted intensity is given by the relationship[20]

$$I = I_m \cos^2 \left(\frac{\delta + \phi}{2}\right) \tag{19}$$

where ϕ, I, and I_m have the same meanings as above and δ, the phase shift introduced by the interferometer, is given by

$$\delta = \frac{2\pi}{\lambda} \cos \beta \ . \qquad (20$$

The angle β is defined in Fig. 13.

Eq. 19 predicts that in the absence of an electric field ($\phi = 0$), a conventional interferometric fringe pattern results. For $\phi \neq 0$, the fringes are shifted or, equivalently, the intensity at single point on the detector is changed.

It should be emphasized that interferometric detection offers no inherent increase in sensitivity when compared to a crossed polarizer detection scheme. This point can be demonstrated by considering the variation of the intensity at one point on the detection plane as the electric field is varied. It is assumed that the point is on a dark fringe when the electric field is equal to zero. This implies

$$\delta = (2n + 1)\ \pi,\ n = 0,1,2,.... \tag{21}$$

Combining Eqs. 19 and 21 yields

$$I = I_m \sin^2 (\phi/2) \tag{22}$$

which is identical to Eq. 17 which describes the transmittance of a crossed polarizer system.

So an interferometric detection scheme can be conceived as a crossed polarizer detector with the initial ($E = 0$) transmittance a function of position. The approach does offer advantages in the situation where $V \overset{<}{\sim} V_m$. In this case, the background or interference fringe pattern makes the determination of I_m convenient whereas it is impossible with a crossed polarizer detector.

There are additional detection techniques[21] applicable to the measurement of very small phase shifts that may be valuable in applications for which $V < 0.5\ V_m$. The conclusions of this discussion are summarized in Table 1.

Table 1. Range Of Voltages For Which Various Detection Schemes May Have Value

Relative Voltage, V/V_m	Detection Method
> 5	Crossed Polarizers
$0.5 \sim 5$	Crossed Polarizers Interferometer
<0.5	Interferometer Small Phase Shift Techniques

It should be emphasized that factors of cost and convenience, as well as the possibility of accepting reduced accuracy can provide justification for using a less than ideal detection method.

IV. ACKNOWLEDGMENTS

The authors would like to thank W. A. Bagley for help in the experimental phase of this investigation and J. C. Palla for assistance in the preparation of the manuscript.

V. REFERENCES

1. IEEE Standard, IEEE 4 "High Voltage Test Techniques," in press.

2. R. E. Hebner, "Calibration of Kerr Systems Used to Measure High Voltage Pulses," NBS Report, NBSIR 75-774, 1975.

3. C. G. Le Fevre and R.J.W. Le Fevre, "The Kerr Effect. Its Measurement and Applications in Chemistry," Rev. Pure Appl. Chem., vol. 5, pp. 261-318, 1955.

4. E. Frederico and C. Houssier, "Electric Dichroism and Electric Birefringence," Oxford: Clarendon Press, 1973, pp. 1-59.

5. D. C. Wunsch and A. Erteza, "Kerr Cell Measuring System for High Voltage Pulses," Rev. Sci. Instrum., vol. 35, pp. 816-820, 1964.

6. R. E. Hebner, E. C. Cassidy, and R. J. Sojka, "Development and Analysis of Techniques for Calibration of Kerr Cell Pulse-Voltage Measuring Systems VIII," National Bureau of Standards Report, NBSIR 74-564, 1974.

7. R. E. Hebner, E. C. Cassidy, and R. J. Sojka, "Kerr Coefficients of Nitrobenze and Water," NBS Report, NBSIR 74-544, 1974.

8. F. Gabler and P. Sokob, "Die Temperaturabhanzigkeit Des Kerreffektes Am Nitrobenzol," Zeitschr. Tech. Physik, vol. 6, pp. 197-201, 1936.

9. A. D. Buckingham and R. E. Raab, "A Molecular Theory of the Electro-optic Kerr Effect in Liquids," J. Chem. Soc., pp. 2341-2351, 1957.

10. G. L. Clark, "Dielectric Properties of Nitrobenzene in the Region of Anomalous Dispersion," J. Chem. Phys., vol. 25, pp. 125-219, 1956.

11. E. C. Cassidy, W. E. Anderson, and S. R. Booker, "Recent Refinements and Developments in Kerr System Electrical Measurement Techniques," IEEE Trans. Instrum. Meas., vol. IM-21, pp. 504-510, 1972.

12. L. Chaumont, "Recherches Experimentales sur le Phenomene Electro-optique de Kerr et sur les Methods Servant a l'Etude de la Lumiere Polarisee Elliptiquement," Ann. de Phys., vol. 5, pp. 17-78, 1916.

13. P. D. Thacher, "Optical Effects of Fringing Fields in Kerr Cells," IEEE Trans. Elec. Insul., vol. EI-11, pp. 40-50, 1976.

14. E. C. Cassidy, et. al., "Kerr Effect Studies of an Insulating Liquid Under Varied High Voltage Conditions," IEEE Trans. Elec. Insul., vol. EI-9, pp. 43-56, 1974.

15. M. Misakian and R. E. Hebner, "Kerr Coefficients of Polychlorinated Biphenyls and Chlorinated Naphthalene," J. Appl. Phys., vol. 47, pp. 4052-4055, 1976.

16. T. W. Dakin, S. A. Studniarz, and G. T. Hummert, "Dielectric Breakdown of Insulation Materials on the Front of Cycle and a Small Number of Cycles of 60 Hz Voltage," 1972 Annual Report Conference on Elec. Insul. and Dielectric Phenomena, National Academy of Sciences, Washington, D.C., 1973.

17. W.K.H. Panofsky and M. Phillips, "Classical Electricity and Magnetism," 2nd Edition, Addison-Wesley: Reading, MA, pp. 107-111, 1962.

18. R. E. Hebner and S. R. Booker, "A Portable Kerr System for the Measurement of High Voltage Pulses," in Proc. IEEE Southeastcon, vol. 1, pp. 3A-1-1 to 3A-1-5, 1975.

19. J. E. Thompson, M. Kristiansen, and M. O. Hagler, "Optical Measurement of High Electric and Magnetic Fields," IEEE Trans. Instrum. Meas., vol. IM-25, pp. 1-7, 1976.

20. J. E. Thompson, "Optical Measurements of High Electric and Magnetic Fields," thesis submitted to the Department of Electrical Engineering, Texas Tech University, Lubbock, Texas 79409.

21. See, for example, C. E. Hill and H. House, "The Problems in Using the Kerr Electro-optic Effect to Measure the Field Distribution in Non-polar Liquids," in Phenomenes De Conduction Dans Les Liquids Isolants, National Center for Scientific Research: Paris, pp. 465-481, 1968.

22. E. C. Cassidy, et. al., "Development and Analysis of Techniques for Calibration of Kerr Cell Pulse-Voltage Measuring Systems VII," National Bureau of Standards Report, NBSIR 73-403, 1973.