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PROCEDURES FOR THE CALIBRATION OF VOLUMETRIC TEST MEASURES

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I. INTRODUCTION

The calibration of a volumetric vessel requires the determination of three calibration constants:

1. The <u>containment volume</u>, V_C, which is the volume of water required to fill the vessel at 60°F.

2. The <u>delivery volume</u>, V_D, which is the volume of water which may be poured from the vessel at 60°F under specified conditions. V_D differs significantly from V_C.

3. The <u>neck constant</u>, K, which relates the true volume of neck to the value observed on the neck scale.

There are two methods for obtaining values of these test constants, the choice being based upon apparatus and equipment availability. The <u>volumetric transfer</u> requires previously calibrated vessels and is essentially a counting procedure in which the volume of the test vessel is determined from knowledge of the number of "dumps" of the standard vessel required to fill the test vessel. When previously calibrated standard volumetric vessels are not available, the <u>gravimetric method</u> may be used. This requires a balance upon which the mass of water required to fill the test vessel is measured. From measured values of mass and knowledge of water density, the required volumes may be calculated.

II. SUGGESTED CLEANING AND INSPECTION PROCEDURES

The vessels are prepared for calibration as follows:

1. Unpack each measure carefully, making special note of any damage that could have occurred to the vessel during shipment.

2. Inspect the vessel carefully to be sure that it is free of dents, rust, and broken gage glass.

3. Biodegradeable detergents are used to clean the interior of the vessels. When necessary, solvents are used prior to cleaning with detergents. Care is exercised in cleaning epoxy ¬coated vessels with solvents as some solvents may cause a reaction which would damage the coating.

4. Each vessel should be filled with water and allowed to stand for several minutes. During this time, an examination is made for leaks. After this inspection is made, the vessel is rinsed with distilled water, then with alcohol; it is then allowed to dry.

5. Smaller vessels, whose level is determined by the plane of the supporting base, should rest on a level bed plate and be checked for firm seating (that is, 3 point support or an adequately flat base). A vessel which rocks will not give a true meniscus reading during the calibration.

6. An information, or summary sheet, is prepared giving the following information. (See sample summary sheet, figure 1.)

   a. Identify the vessel or vessels to be tested by owner and test number.

   b. List all the vessels, their maker number, identification number, assumed scale divisions and assumed coefficient of expansion.

   c. List all other information necessary to make complete record of the test.

III. EQUIPMENT REQUIRED FOR GRAVIMETRIC CALIBRATIONS

The vast majority of work of this type involves measurement of volumes ranging from 5 gallons to 100 gallons, and this paper is addressed to workers operating within this range whose requirements are satisfied by an uncertainty of .01%. For volumes larger or smaller than this, although the general methods are the same, specialized techniques and apparatus are generally called for. It is necessary to either have calibrated volumetric standards scattered conveniently across the range of volumes to be calibrated or a balance of appropriate capacity and reproducibility.

The following equipment is necessary for the gravimetric method of calibration.

1. Balance or load cell as appropriate to the work level (with a standard deviation of 500mg or better).

2. Weights (calibrated to about 1:50,000) as appropriate to the work level.

3. Thermometer (calibrated to 0.1°C).

4. Barometer or barograph (calibrated to 2mm of mercury).

5. Hygrometer (calibrated to 4%).

6. Ball Bearings for neck calibration (2 1/2 inch, grade 50).

7. Bed plate (with adjustable legs).

8. Machinist level.

9. Storage tank for water.

10. Mixer (mixing water).

11. Stopwatch.

12. Assorted lengths of pipes.

13. Assorted hand tools.

IV. Gravimetric Calibration Procedure

The general technique is to determine the mass of water required to fill a vessel and the mass of water clinging to the inside walls of the vessel after draining in the specified manner. The differences between these masses, together with knowledge of water density permits one to calculate V_C and V_D. Although any type weighing procedure of appropriate precision may be used in this work, the general weighing technique called "double substitution" is the mode best adapted to this work. The data sheets illustrated in figures 2, 3 and 4 are designed for use with a free-swinging balance.

Prior to testing, a cap of some kind is placed over the neck opening; this acts as a lid to prevent evaporation of moisture from the vessel. This cap remains on the vessel during the entire weighing procedure.

STEP I - (See data sheet, figure 2.)

Weigh the vessel empty:

Each "double substitution" requires four observations: O_1, O_2, O_3, and O_4 as shown in figure 2.

O_1 Place weights on the pan nominally equal to the mass of the vessel and cap. Record the masses of the standard weights used, M_{s1}. For sets of standard weights having duplicate weights, record which weight is used so that the proper true mass correction is selected from the correction table. Turning points are observed on the balance and recorded on the data sheet.

O_2 The weights are removed and the empty vessel and cap are placed on the pan. Read and record the turning points.

O_3 A sensitivity weight, W_s, is added. Read and record the turning points.

O_4 The vessel is removed and the standard weights are returned to the pan along with the sensitivity weight. Read and record these turning points.

Environmental conditions are recorded before and after each "double substitution". Record T_1, B_1, and H_1, the air temperature, barometric pressure, and relative humidity respectively on the data sheet in the appropriate spaces. The date, time and description of the work, as well as the observer should also be recorded.

STEP II - (See data sheet, figure 3.)

Weigh the vessel full of water:

From the water storage tank, fill the vessel with water to any point on the neck scale, δ_1. This water should be stored for a day or so after distillation, this allows the water to equilibrate with ambient conditions. After filling the vessel, the water should be mixed thoroughly in order to remove most of the thermal

gradients which affect the observations leading to the calculated volume. This provides the added benefit of allowing any trapped air bubbles to rise to the surface.

Place the vessel on the bed plate and record the water temperature, T_w. Making sure that the vessel is level, agitate the water in the gage tube and read the meniscus. Record on the data sheet the meniscus scale reading, water temperature, and mix time. After thoroughly drying all water from the exterior, balance the vessel against the standard weights following the procedure as described in Step I. Record ambient conditions T_2, B_2, H_2 and standard weights M_{s2}.

STEP III - (See data sheet, figure 4.)

Weigh the vessel after draining:

Drain the water from the vessel, holding it in the drainage position for a specified interval (usually 10-30 seconds) after cessation of the main flow. Reweigh the vessel, recording M_{s3}, and ambient conditions T_3, B_3, H_3, and drain time.

V. NECK CALIBRATION (See data sheet, figure 5.)

In use, the vessel is not always filled to the same scale reading, δ_1, used in calibration but to any point δ_n. The volume difference between filling to δ_1 (in calibration) and δ_n (in use) may be obtained by neck scale calibration. This is accomplished by filling the vessel to near the bottom of its scale range, δ_A, and inserting in the vessel precision spheres of known diameter, d, which increase the water level to δ_B, δ_C, etc.

Essentially, this experiment consists of (effectively) adding water volume in known increments and recording the change in scale indications from δ_A to δ_B to δ_C, etc. This data permits graphic or analytical evaluation of the scale divisions, δ's, for a value of K where

$$K = \frac{\pi d^3}{(\delta_A - \delta_B)6}$$

The precision spheres are replaced by known volumes of water for vessels with neck diameters greater than four inches.

In performing a neck calibration, it should be evident that the vessel must be level to insure a true neck calibration.

VI. GRAVIMETRIC CALIBRATION CALCULATION

Each of the three double substitution weighings, described above, provide a numerical value of the difference between the known and unknown loads. This difference in mass units is calculated from the observed turning points and the value of the sensitivity weight described on the data sheets. It is referred to as an "A".

Associated with each "A" is a value for air density which is calculated from the formula [1]

$$\rho_a = \frac{464.56B - H(0.085594T^2 - 1.8504T + 34.47)}{(T + 273.16)1,000,000}$$

where

ρ_a = Density of air in grams/cm^3

B = Pressure in mm of Hg

H = Relative humidity in percent

T = Temperature of air in degrees Centigrade.

This calculation is also indicated on the raw data sheet.

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[1] Bowman, H. A., and Schoonover, R. M., NBS Journal of Research, C. Engineering and Instrumentation, Vol. 71C, Page 182.

One may now write three equations describing the three substitution weighings using the three numerical values of "A", A_1, A_2 and A_3, and a value of air density associated with each "A" ρ_1, ρ_2 and ρ_3.

$$A_1 = M_j - \rho_1 V_j - M_{s1} + \rho_1 V_{s1} + \text{measurement error} \qquad [1]$$

where M_j = Mass of the vessel

V_j = Metallic volume of the vessel

V_{s1} = Total volume of mass standards

$$A_2 = M_j - \rho_2 V_j + M_w - \rho_2 V_w - M_{s2} + \rho_2 V_{s2} + \text{measurement error} \qquad [2]$$

where V_w = Volume of water contained

M_w = Mass of the water

$$A_3 = M_j - \rho_3 V_j + M_{Rw} - \rho_3 V_{Rw} - M_{s3} + \rho_3 V_{s3} + \text{measurement error} \qquad [3]$$

where M_{Rw} = Mass of water retained

V_{Rw} = Volume of water retained

The measurement error term in the three foregoing equations will be discussed in detail in a future issue of OVERLAP. It is not included in subsequent equations below inasmuch as it does not enter into the calculation of a single calibration test.

The mass of water, M_w, may be calculated from the difference between equations 1 and 2.

$$M_w = \frac{A_2 - A_1 + M_{s2} - M_{s1} + \rho_1 V_{s1} - \rho_2 V_{s2}}{1 - \frac{\rho_2}{\rho_w}}$$

Detailed examination of the algebra leading to the above equation for M_w will show the term $V_j(\rho_2 - \rho_1)$. This term is believed to be acceptably close to zero under ordinary variability in air density so it is ignored.

Using the observed water temperature, T_w, we obtain a value of water density, ρ_w, by interpolation in table 1. The value of V_w, the contained volume of water in the vessel up to neck reading δ_j, may be written

$$V_w = \frac{A_2 - A_1 + M_{s2} - M_{s1} + \rho_1 V_{s1} - \rho_2 V_{s2}}{\rho_w - \rho_2} \qquad [4]$$

This value for V_w which is also the containment volume V_1 of the vessel is valid only for the observed water temperature T_w. By convention, vessel volumes are always referenced to a temperature of $60\,^\circ F$. For the conversion from the centigrade scale to fahrenheit scale use the formula $^\circ F = 9/5\,^\circ C + 32$. To compute a change in known volume, V_1 at temperature T, to a volume V_t at a reference temperature $60\,^\circ F$ the following formula is used:

$$V_t = V_1 [1 + \alpha (60 - T)] \qquad [5]$$

where α = the cubical temperature coefficient of expansion/$^\circ F$ of the vessel

The volume of the residual water, V_{Rw}, is obtained from the difference between equations 1 and 3.

$$V_{Rw} = \frac{A_3 - A_1 + M_{s3} - M_{s1} - \rho_3 V_{s3} + \rho_1 V_{s1}}{\rho_w - \rho_3} \qquad [6]$$

The delivery volume, V_D, of the vessel when drained as specified after filling to neck graduation δ_i, will be

$$V_D = V_w - V_{Rw} \qquad [7]$$

The calculated results, V_w and V_{Rw}, of equations 4 and 6 are in cubic centimeters if the weights are in grams, and air and water density in grams per cubic centimeter.

The volume delivered, V_D of equation 7, is from the fill point δ_i and can be expressed to a reference point δ_0 by the following modification:

$$V_D = V_w - V_{Rw} - (\delta_i - \delta_0)K \qquad [8]$$

By custom δ_0 is zero on the graduated scale and equation 8 can be simplified as follows:

$$V_D = V_w - V_{Rw} - K\delta_i \qquad [9]$$

δ's are not always in the same units as V_w and V_{Rw} and may require further conversion.

This completes the gravimetric calibration of the unknown vessel, and at this point it is appropriate to introduce a check standard measurement using the newly calibrated vessel as a reference. Assuming prior knowledge of the containment volume of the check standard, the correctness of the delivery volume, equation 9, may be quickly verified by pouring water from the vessel into the check standard.

The use of the check standard test is a routine procedure in many measurement laboratories and is highly recommended by NBS. It is desirable to maintain a permanent record of the measurements on each check standard because the spread of data among such measurements is descriptive of the process performance and the degree to which traceability to NBS may be demonstrated. The advantages and analysis of check standard data is discussed in the next issue of OVERLAP.

(See appendix for sample calculation for equations 4-8).

VII. Volumetric Calibration Procedure

A second procedure is available to calibrate volumetric vessels. This method is called the "volumetric transfer" method. It is essentially performed by transferring a known volume of water from a standard vessel into a vessel of unknown volume and calculating either a contained or delivered volume of the unknown vessel. This procedure is usually performed above the 10 gallons level.

Because of the various sizes of vessels, handling becomes a problem. The ideal situation would be to have a platform elevator and a large storage tank for water. Assuming the ideal situation, the "transfer" operation can be performed in the following manner.

(1) An inspection and cleaning process is performed on the unknown vessel, as described in section II, noting any defects such as leaking gage tubes, leaky valves and leaky seams.

(2) Select a standard vessel, U, of known delivery volume at a specified reference temperature and scale graduation and a known thermal coefficient of expansion, α, (cubic). The standard is placed on the platform elevator, along with an attached selected section of pipe, raised to a height appropriate for the test.

(3) Place the unknown vessel, Z, below the standard vessel. Level the unknown vessel either by the attached or built-in levels or by placing a level across the top of the open neck. It is important to use the same leveling procedure when using the unknown vessel as a known vessel at a later date. Attach a piece of pipe from the drain valve of the vessel to a water drain.

(4) A data sheet (see figure 6) is filled out, giving information pertaining to the unknown vessel being calibrated.

(5) If the vessel is to be calibrated for capacity, the internal surfaces of the unknown must be dry; for delivery, both the standard and the unknown must be wet. They are wetted by filling the standard and transferring the water into the unknown with a thirty second drain time after cessation of the main flow. Drain the unknown in a similar manner. This process is only for wetting the surfaces.

(6) Level the standard vessel. Fill the standard vessel, U, to any point on the neck scale, L_a, with water from the storage tank. Record the water temperature, T_A, on the data sheet. Agitate the water in the gage tube to get a uniform meniscus. Read the meniscus scale reading and record the reading on the data sheet.

(7)   Drain the standard in such a manner so as not
        to lose any water being transferred into the
        unknown. In some cases it is advisable to
        place a finger over the opening of the gage
        tube of the unknown to prevent water from
        escaping due to pressure. Time the drain for a
        specified interval (usually 30 seconds after
        cessation of the main flow.)

(8)   With the unknown standing in a level position,
        agitate the water level and read the meniscus
        scale reading. Record the neck reading, L_q and
        the water temperature (t_q) of the unknown
        vessels on the data sheet.

(9)   In situations where a smaller standard vessel
        is used to calibrate a larger unknown vessel,
        each step (5 thru 8) described above is
        repeated the appropriate number of times, each
        time draining the standard and the unknown for
        the specified time.

(10)   After one complete calibration or "run" is
        finished, a second run may be performed. (This
        second run serves as a check on the first run.)
        Drain the unknown for the specified time and
        repeat steps 5 thru 8.

(11)   A neck calibration, as described in section V
        is performed. On vessels containing double
        scale plates, denote whether one scale plate or
        both scale plates have been calibrated.

        VII. Data Reduction for Volumetric Calibration

By convention vessel volumes are always referenced to a
temperature of 60°F. If only one transfer of the standard vessel
U is required to complete the calibration of an unknown vessel Z,

$$\frac{M}{\rho_A} = U_{60}[1 + \alpha(t_A - 60)]$$

and

$$\frac{M}{\rho_Q} = Z_{60}[1 + \beta(t_Q - 60)]$$

where ρ_A = water density of the standard

ρ_Q = water density of the unknown

M = the same mass of water transferred from the standard vessel U_{60} into the unknown vessel Z_{60}

α = cubical coefficient of expansion/°F of the standard, U

β = cubical coefficient of expansion/°F of the unknown, Z.

Since the two masses of water are the same,

$$Z_{60}\rho_Q[1 + \beta(t_Q-60)] = U_{60}\rho_A[1 + \alpha(t_A-60)]$$

This expression can be solved for Z_{60}, which is the volume of interest. This can be either a containment or delivery volume.

$$Z_{60} = \frac{U_{60}\rho_A[1+\alpha(t_A-60)]}{\rho_Q[1+\beta(t_Q-60)]} \qquad [10]$$

The standard vessel, U_{60}, has a known volume up to the zero graduation on the neck scale, L_0. The difference in volume, Δ_A, between L_0 and L_A may be calculated by knowledge of the neck calibration constant. The numerical value of Δ_A is used in the calculation of Z_{60}.

$$Z_{60} = \frac{\rho_A[U_{60}[1+\alpha(t_A-60)]+\Delta_A]}{\rho_Q[1+\beta(t_Q-60)]} \qquad [11]$$

If a standard vessel, U, is used N times to calibrate an unknown vessel Z, equation 11 can be modified for N number of transfers as follows:

$$Z_{60} = \frac{\rho_A[U_{60}[1+\alpha(t_A-60)]+\Delta_A]+\rho_B[U_{60}[1+\alpha(t_B-60)]+\Delta_B]+\ldots+\rho_N[U_{60}[1+\alpha(t_N-60)]+\Delta_N]}{\rho_Q[1+\alpha(t_Q-60)]} \qquad [12]$$

The correctness of Z_{60} may now be tested by use of a check standard as previously mentioned.

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USCOMM-NBS-DC

VOLUMETRIC CALIBRATION SUMMARY SHEET

Test performed for:

XYZ Corporation   Test Number   123456

Items	Maker's Number	Scale Div.	Property Number	Coefficient of Expansion Assumed
5 Gallon	3	1 in^3	39-B	.0000265/°F
50 Gallon	5	1 in^3	39-C	.0000265

Maker: Doe Can Company   Previous Test No.   99571

Remarks:

_____

_____

_____

_____

Date Received   6-1-72   Date Shipped   7-15-72

Figure 1

Turning Points
from Balance Scale

A_1			Left	Right	Equilibrium Points
Mass Standards	$2kg_1$ $50g$				
	$2kg_2$ $20g_1$		9.2	11.9	
	$500g$ $5g$				
	$100g$		9.2		$0_1 = 10.55$
Empty Vessel	5 gallon + cap			13.0	
			10.0	13.0	$0_2 = 11.50$
Empty Vessel + Sensitivity Weight	5 gallon + cap		9.6	11.0	
			9.7		
	$+W_s$ (500mg)				$0_3 = 10.325$
Mass Standards + Sensitivity Weight	Stds			10.4	
			8.0	10.4	
	$+W_s$ (500mg)				$0_4 = 9.2$

$$A = \left(\frac{0_2 - 0_1 + 0_3 - 0_4}{2}\right)\left(\frac{W_s}{0_3 - 0_2}\right)$$

$$A_1 = \frac{2.075}{2} \times \frac{.500}{-1.175} =$$

$$A_1 = \underline{-0.441489}$$

	Begin	End	Corr	Corr Mean
Air Temp. °C	25.6	25.7	0	25.65
Barometer mm of Hg	751.04	750.90	0.35	751.32
R. H. Reading %	22.1	22.0	13.05	35.1
Calculated Air Density p_1	.00116			

Figure 2

Turning Points
from Balance Scale

A_2			Left	Right	Equilibrium Points
Mass Standards	20kg 500g 2kg$_1$ 50g 1kg 2g$_1$ 2g$_2$			12.7	
			10.3	12.7	
					$0_1 = 11.50$
Full Vessel	5 gallon + cap		10.8	13.0	
			10.9		
					$0_2 = 11.93$
Full Vessel + Sensitivity Weight	5 gallon + cap + W_s (500mg)		9.7	11.9	
			9.7		
					$0_3 = 10.8$
Mass Standards + Sensitivity Weight	Stds +W_s (500mg)			11.4	
			8.3	11.3	
					$0_4 = 9.83$

$$A = \left(\frac{0_2 - 0_1 + 0_3 - 0_4}{2}\right)\left(\frac{W_s}{0_3 - 0_2}\right)$$

$$A_2 = \frac{-0.265306}{}$$

Scale Reading	+1.0 div.				
		Begin	End	Corr	Corr Mean
Air Temp °C		25.8	25.9	0	25.85
Barometer mm of Hg		750.78	750.70	0.35	751.09
R. H. Reading %		22.0	22.2	13.0	35.1
H_2O Temp	w 24.835				
Mix Time	5 min.			Calculated Air Density ρ_2 .00116	

Figure 3

		Observer	A B	Test Number	123456	Balance	C.

Observer_____A B_____   Test Number_____123456_____   Balance_____C._____

Item_____5 gallon vessel (graduated neck)_____   Sheet Number__3 of 3__

Date_____6-15-72_____   Time__Begin 1020 End 1049__   Sensitivity Wt. __500mg__

Turning Points
from Balance Scale

A_3			Left	Right	Equilibrium Points
Mass Standards	$2kg_1$ 50g			11.7	
	$2kg_2$ 20g		9.8	11.7	
	500g 10g				$O_1 = 10.75$
	100g 5g				
Drained Vessel	5 gallon + cap				
			9.9	12.0	
			10.0		$O_2 = 10.98$
Drained Vessel + Sensitivity Weight	5 gallon + cap + W_s (500mg)			11.4	
			8.1	11.3	$O_3 = 9.73$
Mass Standards + Sensitivity Weight	Stds +W_s (500mg)		7.8	10.4	
					$O_4 = 9.1$

$$A = \left(\frac{O_2 - O_1 + O_3 - O_4}{2}\right)\left(\frac{W_s}{O_3 - O_2}\right)$$

$$A_3 = \underline{-0.17}$$

	Begin	End	Corr	Corr Mean
Air Temp °C	25.9	25.9	0	25.9
Barometer mm of hg	750.68	750.68	0.35	751.03
R. H. Reading %	22.2	22.2	13.0	35.2

Drain Time __10 sec.__   Calculated Air Density ρ_3 __.00116__

Figure 4

NECK CALIBRATION

Test No.: __123456__   Date: __6-15-72__

Item: __5 Gallon__   Observer: __JFH__

SCALE READING		VOLUME ADDED	DIFF IN SCALE READING
1. -10.5	δ_A	0	
2. -6.4	δ_B	1. 4.19 in^3	1. 4.1
3. -2.2	δ_C	2. 4.19 in$^{\cdot}$	2. 4.2
4. +2.2		3. 4.19 in^3	3. 4.4
5. +6.2		4. 4.19 in^3	4. 4.0
6. +10.3		5. 4.19 in^3	5. 3.9
TOTAL SCALE READING		TOTAL VOLUME ADDED	

Figure 5

10.5
10.3
——
20.8   divisions = 20.95 in^3

1   division = 20.95 in^3/20.8

1   division = 1.01 in^3

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VOLUME TRANSFER DATA SHEET

Nominal Value: 50 Gallon	Type of Leveling: Attached	Date: 06-12-73

Assumed Coefficient of Expansion: 0.0000265   Material: Stainless Steel
Scale Division: 1 in^3

Neck Diameter: 3 3/4"   Type of Value: ball   Water used: Stored

Identification Number: 39-C   To Contain
To Deliver   Standard

Run	Temperature	Meniscus Scale Reading	Nominal Value	Vol from Zero @ 60°F in Gallons
I	21.103°C	-12.0	50	50.0048
II	20.848	-8.0	50	50.0048

UNKNOWN

	Temperature	Scale Reading		
I	21.341	-12.0		
II	20.875	-7.9		

Figure 6

°C	0	1	2	3	4	5	6	7	8	9
0	.9998426	.9998485	.9998544	.9998603	.9998662	.9998721	.9998779	.9998838	.9998897	.9998956
1	.9999015	.9999056	.9999098	.9999139	.9999180	.9999222	.9999263	.9999304	.9999346	.9999387
2	.9999429	.9999453	.9999478	.9999502	.9999526	.9999551	.9999575	.9999599	.9999623	.9999648
3	.9999672	.9999680	.9999687	.9999695	.9999703	.9999711	.9999719	.9999727	.9999734	.9999742
4	.9999750	.9999742	.9999734	.9999725	.9999717	.9999709	.9999701	.9999693	.9999684	.9999676
5	.9999668	.9999644	.9999620	.9999597	.9999573	.9999549	.9999525	.9999502	.9999478	.9999454
6	.9999430	.9999391	.9999353	.9999314	.9999275	.9999237	.9999198	.9999159	.9999121	.9999082
7	.9999043	.9998990	.9998936	.9998883	.9998830	.9998776	.9998723	.9998669	.9998616	.9998563
8	.9998509	.9998441	.9998374	.9998306	.9998239	.9998172	.9998104	.9998037	.9997969	.9997902
9	.9997834	.9997753	.9997672	.9997590	.9997509	.9997428	.9997346	.9997265	.9997184	.9997102
10	.9997021	.9996926	.9996831	.9996737	.9996642	.9996548	.9996453	.9996358	.9996264	.9996169
11	.9996074	.9995966	.9995859	.9995751	.9995643	.9995535	.9995427	.9995319	.9995212	.9995104
12	.9994996	.9994876	.9994755	.9994635	.9994514	.9994394	.9994274	.9994153	.9994033	.9993912
13	.9993792	.9993659	.9993527	.9993394	.9993261	.9993128	.9992995	.9992862	.9992729	.9992597
14	.9992464	.9992319	.9992174	.9992030	.9991885	.9991740	.9991595	.9991450	.9991306	.9991161
15	.9991016	.9990859	.9990703	.9990546	.9990389	.9990233	.9990076	.9989920	.9989763	.9989606
16	.9989450	.9989282	.9989114	.9988946	.9988777	.9988609	.9988441	.9988273	.9988105	.9987937
17	.9987769	.9987590	.9987410	.9987231	.9987052	.9986872	.9986693	.9986514	.9986334	.9986155
18	.9985976	.9985786	.9985595	.9985405	.9985215	.9985024	.9984834	.9984644	.9984453	.9984263
19	.9984073	.9983872	.9983671	.9983470	.9983269	.9983068	.9982867	.9982666	.9982465	.9982264
20	.9982063	.9981852	.9981640	.9981429	.9981217	.9981006	.9980794	.9980583	.9980371	.9980160
21	.9979948	.9979726	.9979504	.9979283	.9979061	.9978839	.9978617	.9978395	.9978174	.9977952
22	.9977730	.9977498	.9977267	.9977035	.9976803	.9976571	.9976339	.9976108	.9975876	.9975644
23	.9975412	.9975170	.9974929	.9974687	.9974445	.9974203	.9973962	.9973720	.9973478	.9973236
24	.9972994	.9972742	.9972491	.9972239	.9971988	.9971736	.9971485	.9971233	.9970982	.9970730
25	.9970479	.9970218	.9969957	.9969696	.9969434	.9969173	.9968912	.9968651	.9968390	.9968128
26	.9967867	.9967597	.9967326	.9967056	.9966785	.9966515	.9966244	.9965974	.9965703	.9965433
27	.9965162	.9964882	.9964603	.9964323	.9964043	.9963764	.9963484	.9963204	.9962925	.9962645
28	.9962365	.9962076	.9961788	.9961499	.9961210	.9960922	.9960633	.9960344	.9960056	.9959767
29	.9959478	.9959180	.9958883	.9958585	.9958287	.9957990	.9957692	.9957395	.9957097	.9956799
30	.9956502	.9956196	.9955890	.9955584	.9955277	.9954971	.9954665	.9954359	.9954053	.9953746
31	.9953440	.9953125	.9952811	.9952496	.9952181	.9951866	.9951551	.9951237	.9950922	.9950607
32	.9950292	.9949969	.9949646	.9949323	.9949000	.9948676	.9948353	.9948030	.9947707	.9947383
33	.9947060	.9946729	.9946397	.9946066	.9945734	.9945403	.9945071	.9944740	.9944408	.9944077
34	.9943745	.9943406	.9943066	.9942726	.9942387	.9942047	.9941707	.9941368	.9941028	.9940689
35	.9940349	.9940001	.9939654	.9939306	.9938958	.9938611	.9938263	.9937915	.9937567	.9937220
36	.9936872	.9936516	.9936161	.9935805	.9935449	.9935094	.9934738	.9934383	.9934027	.9933671
37	.9933316	.9932953	.9932589	.9932226	.9931863	.9931500	.9931136	.9930773	.9930410	.9930046
38	.9929683	.9929312	.9928941	.9928570	.9928199	.9927828	.9927457	.9927086	.9926715	.9926344
39	.9925973	.9925594	.9925216	.9924837	.9924459	.9924080	.9923702	.9923323	.9922944	.9922566
40	.9922136	.9921754	.9921371	.9920987	.9920602	.9920217	.9919831	.9919444	.9919057	.9918668

TABLE 1

Density of Air Free Water in g/cm³ as a Function of the Celsius Temperature Scale Based on the Work by H. Wagenbreth and W. Blanke, PTB-Mitteilungen 6-71.

APPENDIX

Sample Calculation: for Gravimetric Calibration

From the sample data sheets, the following necessary data is
required for the calculation of the test measure volume

    1. numerical value for "A" (grams)

    2. air density (gram/cm^3)

    3. corrected mass values (grams)

    4. volume for mass standards at test temperature (cm^3)

	Empty Vessel	Full Vessel	Drained Vessel
"A"	-.441489	-.265306	-.17
Air Den.	.00116	.00116	.00116
Std.	4675	23554	4685
Vol. Stds.	556.67	2804.89	557.87

    5. coefficient of expansion (Cubical/°F)   = .0000265

    6. corrected water temperature   = 24.835°C

        corresponding water density   = .997094 grams/cm^3

    7. scale reading + 1.0 in^3 (must be converted to gallons)

Substituting numerical values in equation 4

$$V_w = \frac{-.265306-(-.441489)+23554-4675+0.00116(556.67)-0.00116(2804.89)}{0.997094-0.00116}$$

$$V_w = \frac{18876.56825}{.995934}$$

$$V_w = 18953.63372 \ cm^3$$

To convert from cm^3 to gallons multiply by .00026417 gal/cm^3

$$V_w = 18953.6337 \times .00026417$$

$$V_w = 5.00698 \ gallons$$

From equation 5 the volume at 60°F (conventional reference temperature) is determined.

$$V_{60} = 5.00698 \; [1 + .0000265 \; (-16.70)]$$

$$V_{60} = 5.00476$$

Substituting numerical values in equation 6:

$$V_{Rw} = \frac{-.17-(.441489)+4685-4675-0.00116(557.87)+0.00116(556.67)}{.997094 - 0.00116}$$

$$= \frac{10.270097}{.995934}$$

$$= 10.312025 \; cm^3$$

$$= 10.312025 \; X \; .00026417 = .002724 \; gallon$$

For a delivery volume for equation 7

$$5.00698 - .00272 = 5.00426 \text{ at test temperature.}$$

$$V_{60} = 5.00204 \text{ gallons}$$

Scale reading $= +1.0 \; in^3 = +.00432$ gallon

Volume of water delivered from the scale 0 point at 60°F on the graduated neck is 4.9977 gallons.