Nobel Lecture: Superposition, entanglement, and raising Schrödinger's cat[*]

David J. Wineland

National Institute of Standards and Technology, 325 Broadway, Boulder, Colorado 80305, USA

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I. INTRODUCTION

Experimental control of quantum systems has been pursued widely since the invention of quantum mechanics. In the first part of the 20th century, atomic physics helped provide a test bed for quantum mechanics through studies of atoms' internal energy differences and their interaction with radiation. The advent of spectrally pure, tunable radiation sources such as microwave oscillators and lasers dramatically improved these studies by enabling the coherent control of atoms' internal states to deterministically prepare superposition states, as, for example, in the Ramsey method (Ramsey, 1990). More recently this control has been extended to the external (motional) states of atoms. Laser cooling and other refrigeration techniques have provided the initial states for a number of interesting studies, such as Bose-Einstein condensation. Similarly, control of the quantum states of artificial atoms in the context of condensed-matter systems is achieved in many laboratories throughout the world. To give proper recognition to all of these works would be a daunting task; therefore, I will restrict these notes to experiments on quantum control of internal and external states of trapped atomic ions.

The precise manipulation of any system requires low-noise controls and isolation of the system from its environment. Of course the controls can be regarded as part of the environment, so we mean that the system must be isolated from the uncontrolled or noisy parts of the environment. A simple example of quantum control comes from nuclear magnetic resonance, where the spins of a macroscopic ensemble of protons in the state $|\downarrow\rangle$ (spin antiparallel to an applied magnetic field) can be deterministically placed in a superposition state $\alpha|\downarrow\rangle + \beta|\uparrow\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$) by application of a resonant rf field for a specified duration. Although the ensemble is macroscopic, in this example each spin is independent of the others and behaves as an individual quantum system.

But already in 1935, Erwin Schrödinger (Schrödinger, 1935) realized that, in principle, quantum mechanics should apply to a macroscopic system in a more complex way, which could then lead to bizarre consequences. In his specific example, the system is composed of a single radioactive particle and a cat placed together with a mechanism such that if the particle decays, poison is released, which kills the cat. Quantum mechanically we represent the quantum states of the radioactive particle as undecayed $= |\uparrow\rangle$ or decayed $= |\downarrow\rangle$, and live and dead states of the cat as $|L\rangle$ and $|D\rangle$. If the system is initialized in the state represented by the wave function $|\uparrow\rangle|L\rangle$, then after a duration equal to the half life of the particle, quantum mechanics says the system evolves to a superposition state where the cat is alive and dead simultaneously, expressed by the superposition wave function

$$\Psi = \frac{1}{\sqrt{2}}[|\uparrow\rangle|L\rangle + |\downarrow\rangle|D\rangle]. \tag{1}$$

Schrödinger dubbed this an entangled state because the state of the particle is correlated with the state of the cat. That is, upon measurement, if the particle is observed to be undecayed, one can say with certainty that the cat is alive, and visa versa. But before measurement, the particle and cat exist in both states. This extrapolation of quantum mechanics from individual quantum systems to the macroscopic world bothered Schrödinger (and a lot of other people). As one way out of the dilemma, in 1952, Schrödinger (Schrödinger, 1952b) wrote

> "...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences ...."

But of course these days, this argument doesn't hold and we can in fact experiment with individual or small numbers of quantum systems, deterministically preparing superpositions and entangled superpositions. Our control is best when we deal with very small numbers of particles, which enables us to realize many of the gedanken experiments that provided the basis for discussions between Schrödinger and the other founders of quantum mechanics. And, we can also make small analogs of Schrödinger's cat, which are by no means

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[*]The 2012 Nobel Prize for Physics was shared by Serge Haroche and David J. Wineland. These papers are the text of the address given in conjunction with the award.

macroscopic but have the same basic attributes. So far, it appears that our inability to make macroscopic "cats" is due just to technical, not fundamental, limitations. Admittedly, these technical limitations are formidable, but one can be optimistic about increasing the size of these states as technology continues to improve.

This contribution is based on the lecture I gave at the Nobel ceremonies in 2012. It is mostly a story about our group at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, whose combined efforts were responsible for some of the contributions to the field of trapped-ion quantum control. It will be a somewhat personal tour, giving my perspective of the development of the field, while trying to acknowledge some of the important contributions of others. For me, the story started when I was a graduate student.

II. SOME EARLY STEPS TOWARD QUANTUM CONTROL

From 1965 to 1970, I was a graduate student in Norman Ramsey's group at Harvard. Norman, with his close colleague Dan Kleppner and student Mark Goldenberg, had recently invented and demonstrated the first hydrogen masers (Goldenberg, Kleppner, and Ramsey, 1960; Kleppner, Goldenberg, and Ramsey, 1962). As part of this program, Norman wanted to make precise measurements of the hyperfine frequencies of all three isotopes of hydrogen, so I chose to work on deuterium. The experiment was relatively straightforward, complicated a bit by the relatively long wavelength (~ 92 cm) of deuterium's hyperfine transition relative to that of hydrogen (~ 21 cm) (Wineland and Ramsey, 1972). Most importantly, this experiment taught me to pay close attention to, and control as best as possible, all environmental effects that would shift the measured transition frequency from that found for an isolated atom. In addition to enjoying the detective work involved in this, I also became hooked on the aesthetics of long coherence times of superposition states (~ 1 s in the masers), and their importance in atomic clocks. Norman received the 1989 Nobel Prize in physics for his invention of the separated-fields method in spectroscopy and development of the hydrogen maser (Ramsey, 1990).

During my time as a graduate student, I also read about and was intrigued by the experiments of Hans Dehmelt and his colleagues Norval Fortson, Fouad Major, and Hans Schuessler at the University of Washington. The trapping of ions at high vacuum presented some nice advantages for precision spectroscopy, including the elimination of the first-order Doppler shifts and relatively small collision shifts. The Washington group made high-resolution measurements of the $^3\text{He}^+$ hyperfine transition, which has internal structure analogous to hydrogen, by storing the ions in an rf (Paul) trap. One challenge was that detection by optical pumping was (and still is) not feasible because of the short wavelengths required. Therefore, in a heroic set of experiments, state preparation was accomplished through charge exchange with a polarized Cs beam that passed through the ions. Detection was accomplished through a charge-transfer process ($^3\text{He}^+ + \text{Cs} \rightarrow {}^3\text{He} + \text{Cs}^+$) that depended on the internal state of $^3\text{He}^+$, followed by detection of the depleted $^3\text{He}^+$ ion number by observing the ions' induced currents in the trap

electrodes (Fortson, Major, and Dehmelt, 1966; Schuessler, Fortson, and Dehmelt, 1969).

Although these experiments were what first attracted me to ion trapping, my postdoctoral research with Dehmelt, starting in the fall of 1970, was focused on experiments where collections of electrons were confined in a Penning trap for a precise measurement of the electron's magnetic moment or g factor. These experiments were started by Dehmelt's graduate student, Fred Walls, who later became a colleague at the National Bureau of Standards. After a while, it became clear that systematic effects would be much better controlled if the experiment could be performed on single electrons. Therefore, the first task was to isolate a single trapped electron. This was accomplished by first loading a small number of electrons into the trap and driving their nearly harmonic motion (~ 60 MHz) along the magnetic field direction. This motion could be detected by observing the currents induced in the electrodes (proportional to the number of electrons). By adjusting the strength of the drive to a critical level, occasionally one of the electrons would gain enough energy to strike a trap electrode and be lost. Steps in the induced current level could then be used to determine when one electron was confined in the trap (Wineland, Ekstrom, and Dehmelt, 1973). Subsequent experiments on single electrons by Robert Van Dyck, Paul Schwinberg, and Dehmelt were used to make precision measurements of the electron's g factor (Van Dyck, Schwinberg, and Dehmelt, 1977; Dehmelt, 1990). For this and the development of the ion-trapping technique, Dehmelt and Wolfgang Paul shared the Nobel Prize in 1989, along with Ramsey.

The modes of motion for a single charged particle in a Penning trap include one circular mode about the trap axis called the magnetron mode. For the electron g-factor experiments, it was desirable to locate the electron as close to the trap axis as possible by reducing the amplitude of this mode. This could be accomplished with a form of "sideband cooling" (Wineland and Dehmelt, 1975a, 1976) as demonstrated by Van Dyck, Schwinberg, and Dehmelt (1978). Around this time, I was also stimulated by the papers of Arthur Ashkin (Ashkin, 1970a, 1970b) on the possibilities of radiation pressure from lasers affecting the motion of atoms. In analogy with the electron sideband cooling, Dehmelt and I came up with a scheme for cooling trapped-ion motion with laser beams (Wineland and Dehmelt, 1975b) (see below). The cooling could also be explained in terms of velocity-dependent radiation pressure as in a concurrent proposal by Ted Hänsch and Art Schawlow (Hänsch and Schawlow, 1975). We didn't anticipate all of the uses of laser cooling at the time, but it was clear that it would be important for high-resolution spectroscopy of trapped ions. For example, the largest systematic uncertainty in the $^3\text{He}^+$ experiment (Schuessler, Fortson, and Dehmelt, 1969) was the uncertainty in the time dilation shift, which would be reduced with cooling.

In the summer of 1975, I took a position in the Time and Frequency Division of NIST (then NBS, the National Bureau of Standards). My first task was to help make a measurement of the cesium hyperfine frequency, the frequency reference that defines the second. The apparatus, NBS-6, had been built by David Glaze of the Division. It was a traditional atomic beam apparatus but had a relatively long distance between

Ramsey zones of 3.75 m. With it, we realized a fractional accuracy of 0.9×10^{-13} (Wineland et al., 1976). At that time, the Division was more service oriented, with very little basic research. Fortunately my group leader, Helmut Hellwig, had a progressive view of the Division's future and was able to obtain NBS support to initiate laser-cooling experiments. That support, along with some seed money from the Office of Naval Research (ONR), enabled us to start a project on laser cooling in the fall of 1977. With Robert Drullinger (a local laser expert) and Fred Walls, we chose to use $^{24}Mg^+$ because of its simple electronic structure and Penning traps, because of our prior experience with them. This was a very exciting time, being able to work on a project of our choosing, and by the spring of 1978, we had obtained our first cooling results (Wineland, Drullinger, and Walls, 1978). In our experiments we observed currents in the trap electrodes induced by the ions' thermal motion and hence had a direct measurement of the ions' temperature. Meanwhile, Peter Toschek's group in Heidelberg (joined by Dehmelt, who was on sabbatical) was working toward the same goal, using Ba^+ ions confined in an rf-Paul trap. They, with colleagues Werner Neuhauser and Martin Hohenstatt, also observed the cooling at about the same time (Neuhauser et al., 1978), through the increased trapping lifetime of ions. In a near coincidence, although there was no contact between the groups, the manuscripts were received by Physical Review Letters within one day of each other (Peter Toschek's group "won" by one day!). The cooling observed in both experiments is typically called Doppler cooling, where the oscillation frequency of the ions' motion is less than the linewidth of the cooling transition. Theoretical groups were becoming interested in the cooling, and some of the earlier work is discussed in Letokhov, Minogin, and Pavlik (1977), Kazantsev (1978), and Stenholm (1986).

To us, the cooling of course provided a start toward improving clocks and in 1985, working with John Bollinger, John Prestage, and Wayne Itano, we demonstrated the first clock that utilized laser cooling (Bollinger et al., 1985). But as physicists, we were excited by just the cooling process itself. So, in addition to clock applications, it would eventually lead to reaching and controlling the lowest quantized levels of motion for a trapped particle (below).

III. CONTROLLING THE QUANTUM LEVELS OF INDIVIDUAL TRAPPED IONS

One of the obvious next steps was to isolate single ions. In addition to the aesthetic appeal of this, as for single electrons, the systematic errors in spectroscopy would be smallest in this case (Dehmelt, 1982). By observing steps in the ion laser fluorescence, the Heidelberg group was able to isolate Ba^+ single ions (Neuhauser et al., 1980). With Wayne Itano, we subsequently used this fluorescence "steps" method to observe single $^{24}Mg^+$ ions (Wineland and Itano, 1981). The Heidelberg group also made photographs of a single ion, and because of its relatively long fluorescence wavelength (493 nm), with a magnifier, a single Ba^+ ion can be observed with the human eye!

In NIST single-ion experiments we chose to focus on Hg^+ because for frequency-standard applications, $^{199}Hg^+$ has a

relatively high ground-state hyperfine clock transition frequency of 40.5 GHz (Major and Werth, 1973; Cutler, Giffard, and McGuire, 1982; Prestage, Dick, and Maleki, 1991) and also a narrow $^2S_{1/2}$-$^2D_{5/2}$ optical transition $[\tau(^2D_{5/2}) \simeq 86 \text{ ms}]$, which could potentially be used as an optical frequency standard (Bender et al., 1976). Although optical pumping of $^{199}Hg^+$ could be achieved with radiation from isotopically selected Hg^+ fluorescence lamps (Major and Werth, 1973; Cutler, Giffard, and McGuire, 1982; Prestage, Dick, and Maleki, 1991), laser excitation was made difficult because of the short (194 nm) wavelength required. Jim Bergquist in our group, with colleagues Hamid Hemmati and Wayne Itano, first developed the required source by sum-frequency mixing a doubled Ar^+ laser at 515 nm with 792 nm from a dye laser in a potassium pentaborate crystal (Hemmati, Bergquist, and Itano, 1983). We used an rf trap with a simple ring-and-end-cap structure shown in Fig. 1, similar to that used by the Heidelberg group.

By the mid-1980s ion trappers were able to directly address one of Schrödinger's questions, which formed the title for his publication "Are there quantum jumps?" (Schrödinger, 1952a, 1952b). Three similar demonstrations were made in 1986 (Bergquist et al., 1986; Nagourney, Sandberg, and Dehmelt, 1986; Sauter et al., 1986; Blatt and Zoller, 1988); for brevity, we describe the experiment of Bergquist et al. Referring to Fig. 1, a nearly harmonic binding potential called a pseudopotential (Paul, 1990) is formed by applying an rf potential between the ring electrode and the end-cap electrodes (held in common). The relevant optical energy levels of a Hg^+ ion are indicated in the upper left-hand part of the figure. The $^2S_{1/2} \rightarrow {}^2P_{1/2}$ electric-dipole transition $[\lambda = 194 \text{ nm}, \tau(^2P_{1/2}) \simeq 2.9 \text{ ns}]$ was used for Doppler laser cooling. If continuously applied, a steady fluorescence from the ion would be observed and could be used to produce images of the ion. If $^2S_{1/2} \rightarrow {}^2D_{5/2}$ resonance radiation were
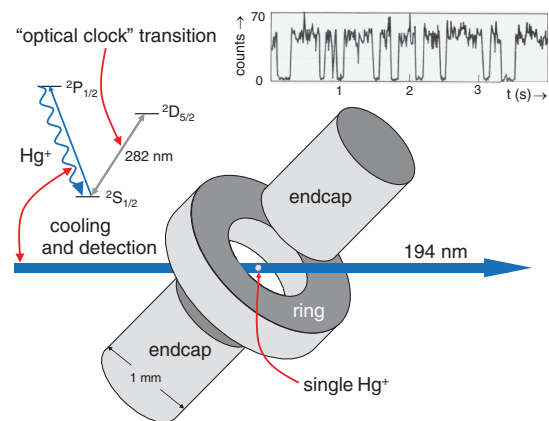

FIG. 1 (color). Schematic of the trap for single Hg^+ ion studies. An rf potential is applied between the ring electrode and end-cap electrodes (which are in common), forming an rf "pseudopotential" for the ion. The relevant Hg^+ energy levels are indicated, including the narrow $^2S_{1/2} \rightarrow {}^2D_{5/2}$ "optical clock" transition. The data in the upper right-hand part of the figure show the number of 194 nm fluorescence photons detected in 10 ms detection bins vs time when both transitions are excited simultaneously (Bergquist et al., 1986). Absence of detected counts indicates that the ion is in the $^2D_{5/2}$ state.

applied simultaneously, one would expect the 194 nm fluorescence to decrease because of excitation to the $^2D_{5/2}$ state.

A density-matrix description, valid for an ensemble of atoms, would predict a reduced but steady fluorescence rate. But what would be observed for a single ion? (Cook and Kimble, 1985; Erber and Putterman, 1985; Cohen-Tannoudji and Dalibard, 1986; Javanainen, 1986; Kimble, Cook, and Wells, 1986; Pegg, Loudon, and Knight, 1986; Schenzle, DeVoe, and Brewer, 1986). In fact the ion's fluorescence does not steadily decrease, but switches between the full value and no fluorescence, effectively indicating quantum jumps between the $^2S_{1/2}$ and $^2D_{5/2}$ states. For the data shown in the upper right-hand corner of Fig. 1, the 194 nm fluorescence photon counts registered by a photomultiplier tube were accumulated in 10 ms time bins and plotted as a function of elapsed time to show the jumps. In a more general context, a measurement of the quantum system composed of the $^2S_{1/2}$ and $^2D_{5/2}$ states can be made by applying the 194 nm "measurement" beam for 10 ms and observing the presence or absence of fluorescence. The $^2S_{1/2} \rightarrow {}^2P_{1/2}$ transition is sometimes called a "cycling transition" because when the $^2S_{1/2}$ state is excited to the $^2P_{1/2}$ state, the ion decays back to the $^2S_{1/2}$ state, emitting a photon, and the excitation/decay process is then repeated. Neglecting the occasional decays of the $^2P_{1/2}$ state to the $^2D_{3/2}$ state (Itano et al., 1987), this procedure approximates an ideal measurement in quantum mechanics because the detection of the state is nearly 100% efficient and because the state of the Hg$^+$ ion, either the $^2S_{1/2}$ or $^2D_{5/2}$ state, remains in its original condition after the measurement. Dehmelt dubbed this "electron shelving" detection (Dehmelt, 1982), where in this example the ion is shelved to the $^2D_{5/2}$ state. Such measurements are also called quantum nondemolition (QND) measurements (Braginsky and Khalili, 1996; Haroche and Raimond, 2006). The method of detection by state-dependent fluorescence has now become rather ubiquitous in atomic physics.

To perform spectroscopy on the $^2S_{1/2} \rightarrow {}^2D_{5/2}$ transition ($\lambda \simeq 282$ nm), radiation was first applied near the transition frequency in the absence of the 194 nm beam; this avoids perturbations of the energy levels from the 194 nm beam. The 282 nm beam was then switched off, followed by measurement of the ion's state with the 194 nm beam. This process was repeated many times, and by stepping the frequency of the 282 nm beam, spectra like that shown in Fig. 2 are obtained (Bergquist, Itano, and Wineland, 1987). To interpret this spectrum, we must consider the motion of the ion. Along any mode axis the motion is nearly harmonic, so in the frame of the ion, the laser beam appears to be sinusoidally frequency modulated due to the first-order Doppler shift. Thus the central feature or "carrier," which corresponds to the transition frequency, is surrounded by frequency-modulation sidebands spaced by the motional frequency of the ion (Dicke, 1953). An equivalent picture is that the ion can absorb radiation while simultaneously gaining or losing one quantum of motion, which leads to absorption features spaced by the frequency of motion around the carrier.

As in many atomic physics experiments, by using highly coherent radiation, we can initialize an ion in an eigenstate and
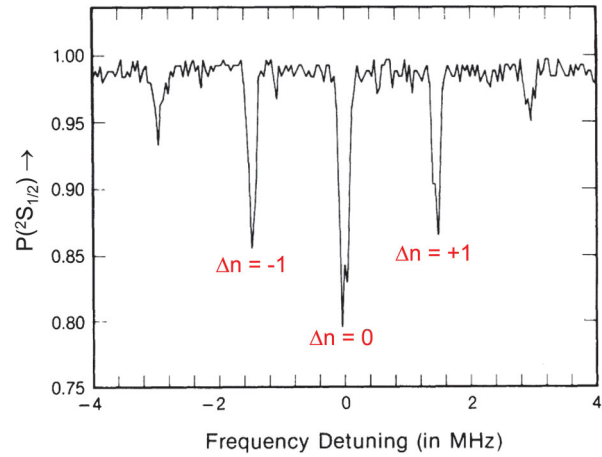

FIG. 2 (color). Spectroscopy of the $^2S_{1/2} \rightarrow {}^2D_{5/2}$ transition on a single ^{198}Hg$^+$ ion. Referring to Fig. 1, for each measurement cycle, the ion is prepared in $^2S_{1/2} \equiv |\downarrow\rangle$ state by allowing it to decay to that level. Then, application of a 282 nm "probe" laser beam is alternated with a 194 nm measurement beam. The $|\downarrow\rangle$ and $^2D_{5/2} \equiv |\uparrow\rangle$ states are detected with nearly 100% efficiency by observing the presence or absence of 194 nm scattered light. By stepping the frequency of the probe beam and averaging over many measurements, we obtain the spectrum shown where we plot the probability of the ion remaining in the $^2S_{1/2}$ state $P(^2S_{1/2})$ vs the 282 nm laser beam frequency. In a quantum picture of the motion, the central feature or "carrier" denotes transitions of the form $|\downarrow\rangle|n\rangle \rightarrow |\uparrow\rangle|n\rangle$, where n denotes the motional Fock state quantum number. "Red" and "blue" sidebands correspond to $|\downarrow\rangle|n\rangle \rightarrow |\uparrow\rangle|n + \Delta n\rangle$ transitions with $\Delta n = -1$ or $+1$, respectively. The central feature or carrier is essentially unshifted by photon recoil, since the recoil is absorbed by the entire trap apparatus as in the Mössbauer effect; see, e.g., Dicke (1953), Lipkin (1973), and Wineland et al. (1998).

deterministically prepare superpositions; e.g., $|\downarrow\rangle \rightarrow \alpha|\downarrow\rangle + \beta|\uparrow\rangle$. To extract the values of $|\alpha|$ and $|\beta|$, we detect as described above. A single measurement indicates either the $|\downarrow\rangle$ or $|\uparrow\rangle$ state with respective probabilities $P = |\alpha|^2$ and $1 - |\alpha|^2$. Quantum fluctuations or "projection noise" in the measurements are characterized with a variance $\sqrt{P(1 - P)/M}$, where M is the number of measurements on identically prepared atoms (Itano et al., 1993). Therefore, accurate measurements of P generally require many repeated experiments. Similarly, Ramsey-type experiments where the two pulses are separated in time can measure the relative phase between α and β. From these types of measurements, many ion trap groups now routinely produce and verify superposition states of single ions that have coherence times exceeding 1 s. [For ion ensembles, coherence times exceeding 10 min have been demonstrated (Bollinger et al., 1991; Fisk et al., 1995).]

The Hg$^+$ clock project at NIST, led by Jim Bergquist, has been a long but very successful story. First, an accurate clock based on the 40.5 GHz hyperfine transition of a few ^{199}Hg$^+$ ions confined in a linear Paul trap achieved systematic errors of about 4×10^{-14} (Berkeland et al., 1998). Although we felt these errors could be substantially reduced, we also realized that the future of high-performance clocks was in the optical domain, so we focused on the $^2S_{1/2} \rightarrow {}^2D_{5/2}$ optical clock transition. For many years it had been appreciated that higher frequency was advantageous in terms of measurement

precision; basically the higher oscillation frequencies allows one to divide a time interval into finer units. But two things were needed: a laser with high enough spectral purity to take advantage of narrow optical transitions, and a practical means to count cycles of the "local oscillator," in this case the laser that would excite the clock transition. In our lab, Brent Young, Bergquist, and colleagues were able to make a cavity-stabilized laser at 563 nm, which was doubled to produce the clock radiation. The 563 nm source had a line-width of less than 0.2 Hz for an averaging time of 20 s (Young *et al.*, 1999). It is now understood that the linewidth was limited by thermal fluctuations in the mirror surface, currently still the limit for the most stable lasers. The solution to the second problem is by now well known. The relatively rapid development of optical combs by Jan Hall (Hall, 2006), Ted Hänsch (Hänsch, 2006), their colleagues, and other researchers meant that it was now possible to effectively count optical cycles. Including these developments, in 2006, Bergquist and colleagues demonstrated a ^{199}Hg$^+$ optical clock with a systematic uncertainty of 7.2×10^{-17}, the first clock since the inception of atomic clocks that had smaller systematic errors than a cesium clock (Oskay *et al.*, 2006).

IV. MANIPULATING ION MOTION AT THE QUANTUM LEVEL

An interesting next step would be to control an ion's motion at the quantum level. Since a cold trapped ion's motion along any mode axis is harmonic to a very good approximation, in a quantum description (Neuhauser *et al.*, 1978; Wineland and Itano, 1979; Stenholm, 1986), we express its Hamiltonian in the usual way as $\hbar\omega_z a^\dagger a$ with ω_z the oscillation frequency (along the z axis here) and a and $a^\dagger$ the lowering and raising operators for the ion motion. The operator for the ion's position about its mean value is $z = z_0(a + a^\dagger)$, where $z_0 = \sqrt{\hbar/2m\omega_0}$ is the spread of the ground-state wave function, with m the ion's mass. In principle, we could detect the ion's motion through the current it induces in the trap electrodes, as was done for electrons. In practice, however, a far more sensitive method is to map information about the motional states onto internal states of the ion and read those out as described above. For this, we need to efficiently couple an ion's internal states to its motion. To see how this works, consider a single trapped ion that has a single-electron electric-dipole transition with resonance frequency ω_0. If this transition is excited by a laser beam of frequency ω_L propagating along the z axis, the interaction is given by

$$H_I = -e\vec{r} \cdot \hat{\epsilon} E_0 \cos(kz - \omega_L t + \phi)$$
$$= \hbar\Omega(\sigma_+ + \sigma_-)(e^{i(kz-\omega_L t+\phi)} + e^{-i(kz-\omega_L t+\phi)}), \quad (2)$$

where $\vec{r}$ is the electron coordinate relative to the ion's core, e is the electron charge, $\hat{\epsilon}$, E_0, and k are, respectively, the laser beam's electric-field polarization, amplitude, and wave vector, and ϕ is the electric-field phase at the mean position of the ion. The operators $\sigma_+ (= | \uparrow \rangle\langle \downarrow |)$ and $\sigma_- (= | \downarrow \rangle\langle \uparrow |)$ are the internal-state raising and lowering operators, and $\Omega \equiv -E_0\langle \uparrow |\vec{r} \cdot \hat{\epsilon}| \downarrow \rangle/2\hbar$, with $| \downarrow \rangle$ and $| \uparrow \rangle$ denoting the ion's ground and optically excited states as above. If we transform to an interaction picture for the ion's internal states ($\sigma_+ \rightarrow \sigma_+ e^{i\omega_0 t}$) and motion states ($a^\dagger \rightarrow a^\dagger e^{i\omega_z t}$) and assume $\omega_L \simeq \omega_0$, then

neglecting terms that oscillate near $2\omega_0$ (rotating wave approximation), Eq. (2) becomes

$$H_I \simeq \hbar\Omega\sigma_+ e^{i[kz-(\omega_L-\omega_0)t+\phi]} + \text{H.c.}$$
$$\simeq \hbar\Omega\sigma_+ e^{-i[(\omega_L-\omega_0)t-\phi]}[1 + i\eta(ae^{-i\omega_z t} + a^\dagger e^{i\omega_z t})]$$
$$+ \text{H.c.} \quad (3)$$

Here, H.c. stands for Hermitian conjugate and $\eta \equiv kz_0 = 2\pi z_0/\lambda$ is the Lamb-Dicke parameter, which we assume here to be much less than 1. For an ion of mass 40 u (e.g., ^{40}Ca$^+$) in a well with $\omega_z/2\pi = 3$ MHz and $\lambda = 729$ nm, we have $z_0 = 6.5$ nm and $\eta = 0.056$. For $\omega_L = \omega_0$ and $\eta\Omega \ll \omega_z$, to a good approximation we can neglect the nonresonant η term in Eq. (3) and obtain $H_I \simeq \hbar\Omega e^{i\phi}S_+ + \text{H.c.}$ This is the Hamiltonian for carrier transitions or, equivalently, spin-vector rotations about an axis in the x-y plane of the Bloch sphere. If we assume $\omega_L = \omega_0 - \omega_z$ (laser tuned to the "red sideband"), and absorb phase factors in the definition of Ω, the resonant term gives

$$H_I \simeq \hbar\eta(\Omega\sigma_+ a + \Omega^*\sigma_- a^\dagger). \quad (4)$$

This Hamiltonian describes the situation where a quantum of motion is exchanged with a quantum of excitation of the ion's internal state. It is most commonly known as the Jaynes-Cummings Hamiltonian from cavity QED, which expresses the exchange of energy between the internal states of an atom in a cavity and the photons confined by the cavity (Jaynes and Cummings, 1963; Haroche and Raimond, 2006). In the cavity-QED experiments of Serge Haroche, Jean-Michel Raimond, Michel Brune, and their colleagues in Paris, the atoms play much the same role as they do in the ion experiments; however, in the cavity-QED experiments, the relevant harmonic oscillator is that which describes a field mode of the cavity, whereas in the ion case, the relevant harmonic oscillator is that associated with the ion's motion (Sauter *et al.*, 1988; Blockley, Walls, and Risken, 1992). Over the years, this connection has led to some interesting and complementary experiments between the two types of experiments (Haroche and Raimond, 2006).

In the trapped-ion world, this type of exchange at the quantum level was first used in the electron g-factor experiments of Dehmelt and colleagues, where a change of the electron's cyclotron quantum number was accompanied by spin flip of the electron, which could be detected indirectly (Dehmelt, 1990). If we apply H_I of Eq. (4) to an atomic ion in the state $| \downarrow \rangle|n\rangle$, where n denotes the harmonic oscillator's quantum state (Fock state), we induce the transition $| \downarrow \rangle|n\rangle \rightarrow | \uparrow \rangle|n-1\rangle$. This corresponds to the absorption feature labeled $\Delta n = -1$ in Fig. 2, and reduces the energy of motion by $\hbar\omega_z$. When the ion decays, on average, the motion energy increases by the recoil energy $R = (\hbar k)^2/(2m)$, where $k = 2\pi/\lambda$. Typically, we can achieve the condition $R \ll \hbar\omega_z$, so that in the overall scattering process the motional energy is reduced. In Fig. 2, the carrier absorption feature is labeled $\Delta n = 0$, indicating photon absorption without changing the motional state. This is a manifestation of the "recoilless" absorption of the Mössbauer effect [see, e.g., Dicke (1953), Lipkin (1973), and Wineland *et al.* (1998)], but in the visible wavelength region.

Continuous application of the red-sideband transition provides a relatively straightforward way to laser cool the ion to near the ground state of motion. After many scattering events, the ion reaches the $| \downarrow \rangle|n = 0\rangle$ state, a "dark state" in which scattering stops, since the $| \uparrow \rangle|n = -1\rangle$ state does not exist.

The process is not perfect, since scattering in the wings of $\Delta n = 0, +1$ transitions leads to some residual recoil heating, but the condition $\langle n \rangle \ll 1$ can be achieved. This is easily verified because absorption on the $\Delta n = -1$ red sideband nearly disappears, but the $\Delta n = +1$ blue-sideband absorption remains. In 1989, with Frank Diedrich, who was a postdoc in our lab, we achieved near-ground-state laser cooling in two dimensions, in essentially the way described here (Diedrich *et al.*, 1989). Later in an experiment led by Chris Monroe, we achieved near-ground-state cooling in 3D using two-photon stimulated-Raman transitions (Monroe, Meekhof, King, Jefferts *et al.*, 1995).

In addition to suppressing Doppler shifts in spectroscopy to the highest degree possible (Wineland *et al.*, 1987), one motivation for sideband cooling was the intrinsic appeal of (actively) placing a bound particle in its ground state of motion, the lowest energy possible within the limitations imposed by quantum mechanics. Here, the ground state is a Gaussian-shaped wave packet with spread $\sqrt{\langle z^2 \rangle} = \sqrt{\hbar/2m\omega_z} \equiv z_0$ and energy $\hbar\omega_z/2$. We were also interested in generating nonclassical states of motion (Heinzen and Wineland, 1990; Cirac, Blatt *et al.*, 1993; Cirac, Parkins *et al.*, 1993; Cirac *et al.*, 1996) or entangled states of spins (Wineland *et al.*, 1992; Bollinger *et al.*, 1996). For these experiments, cooling to the ground state of motion provides a clean starting point for motional state manipulation. [In the Paris experiments, the ground state of the cavity mode can be achieved either by thermally cooling to $\langle n \rangle \ll 1$ by operating at low temperature or by extracting photons with atoms sent through the cavity in a process analogous to ion sideband cooling (Haroche and Raimond, 2006).]

The red-sideband interaction of Eq. (4) and the "blue-sideband" interaction ($H_I \simeq \hbar\eta\Omega\sigma_+ a^\dagger$ + H.c., for $\omega_L = \omega_0 + \omega_z$) that induces $|\downarrow\rangle|n\rangle \rightarrow |\uparrow\rangle|n+1\rangle$ transitions, provide simple tools for the manipulation of an ion's motional states. For example, starting from $|\downarrow\rangle|n = 0\rangle$, and applying a series of blue-sideband, red-sideband, and carrier π pulses, Fock states for a selected value of n can be deterministically prepared (Meekhof *et al.*, 1996). From $|\downarrow\rangle|n = 0\rangle$, we can also make coherent states ion motion by forcing the ion at its motion frequency with an oscillating classical uniform field (Carruthers and Nieto, 1965) or by applying an oscillating optical-dipole force (Meekhof *et al.*, 1996), which results from spatial gradients of laser-beam-induced ac Stark shifts. A coherent state of a quantum particle is very much like an oscillating classical particle but, as opposed to a classical particle that can be pointlike, the shape of the quantum particle's wave packet is the same as it is in the ground state. In a clever but straightforward scheme suggested by Chi Kwong Law and Joe Eberly (Law and Eberly, 1996) arbitrary motional state superpositions can be prepared (Ben-Kish *et al.*, 2003). As a final example, the red-sideband interaction applied for a "π-pulse" duration $t = \pi/(2\eta\Omega)$ provides internal-state to motion-state transfer

$$(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)|0\rangle \rightarrow |\downarrow\rangle(\alpha|0\rangle + \beta|1\rangle). \tag{5}$$

V. SCHRÖDINGER'S CAT

The optical-dipole force is interesting because the strength of the force can depend on the ion's internal state.

In 1996 (Monroe *et al.*, 1996), using state-dependent optical-dipole forces, we were able to produce an analog to the Schrödinger's cat state in Eq. (1), which had the form

$$\Psi = \frac{1}{\sqrt{2}}[|\uparrow\rangle|\alpha\rangle + |\downarrow\rangle|-\alpha\rangle], \tag{6}$$

where $|\alpha\rangle$ denotes a coherent state. The amplitude of the particle's oscillatory motion is equal to $2\alpha z_0$. The spatial part of the state in Eq. (6) represents two wave packets that oscillate back and forth but are 180° out of phase with each other and therefore pass through each other at the center of the trap every half cycle of oscillation. Here, the analogy to Schrödinger's cat is that the spin states of the ion are like the states of the single radioactive particle and the coherent states of the ion, which follow more macroscopic classical trajectories, are like the state of the cat; e.g., the ion at its left extremum point $\simeq$ live cat, ion at its right extremum $\simeq$ dead cat. Figure 3 describes how this state was produced.

To analyze the experiment, in Fig. 3(e), we can control the phase of the amplitude such that the coherent state is $e^{i\phi}\alpha$ rather than $-\alpha$. Near the condition $\phi = 0$, the probability $P(\downarrow)$ of the ion to be in state $|\downarrow\rangle$ oscillates as a function of ϕ due to interference of the two wave packets. This verifies the coherence between the two components of the cat superposition state. These interference oscillations are very analogous to the fringe oscillations observed in Young's-slit-type experiments performed on individual photons, electrons, neutrons, or atoms, but in those experiments the particle wave packets disperse in time, whereas the wave packets in a harmonic oscillator do not, and in principle last arbitrarily long.

In Monroe *et al.* (1996), for the condition described by Eq. (6), the maximum separation of the wave packets was
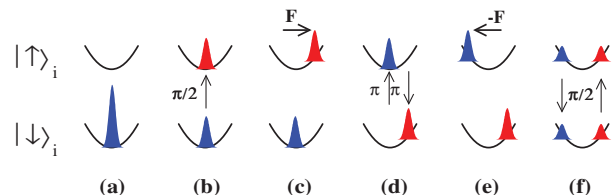

FIG. 3 (color). Depiction of the harmonic oscillator potential and the wave packets for each component of the ion's internal states, denoted $|\uparrow\rangle$ and $|\downarrow\rangle$. The images are snapshots in time; for images (c) through (f) the wave packets are shown at the extremes of their motion. The areas of the wave packets correspond to the probability of finding the atom in the given internal state. (a) The initial wave packet corresponds to the ground state of motion after laser cooling and preparation of the $|\downarrow\rangle$ internal state. (b) A $\pi/2$ carrier pulse creates the internal-state superposition $\frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle)$. (c) An oscillating optical-dipole force is applied that excites only the $|\uparrow\rangle$ component of the superposition to a coherent state of amplitude α, creating the state $\frac{1}{\sqrt{2}}(|\downarrow\rangle|n = 0\rangle + |\uparrow\rangle|\alpha\rangle)$. (d) The spin states are flipped by applying a carrier π pulse. (e) The wave packet associated with the $|\uparrow\rangle$ state is excited by the optical-dipole force to an amplitude of $-\alpha$, that is, out of phase with respect to the first excitation. This is the state of Eq. (6). (f) To analyze the state produced in step (e) and verify phase coherence between the components of the cat wave function, we apply a final $\pi/2$ carrier pulse and then measure the probability $P(\downarrow)$ of the ion to be in state $|\downarrow\rangle$ (see text). From Monroe *et al.*, 1996.

$4\alpha z_0 \simeq 83$ nm, while the size of the wave packets z_0 was 7.1 nm [see also McDonnell *et al.* (2007) and Poschinger *et al.* (2010)]. Of course, one might object to dignifying the state produced by calling it a Schrödinger cat since it is so small. In fact as we tried to make $|\alpha|$ larger, the quality of the superposition became more susceptible to decoherence caused by noisy ambient electric fields (Myatt *et al.*, 2000a, 2000b; Turchette *et al.*, 2000), limiting the size that was obtained. However, as far as we know, this is just a technical, not fundamental limitation and we should eventually be able to make a cat with $|\alpha|$ large enough that the wave packets are separated by macroscopic distances.

VI. ENTER QUANTUM INFORMATION

Following Peter Shor's development of a quantum-mechanical algorithm for efficient number factoring (Shor, 1994), there was a dramatic increase of activity in the field of quantum information science. The potential realization of general-purpose quantum information processing (QIP) is now explored in many settings, including atomic, condensed-matter, and optical systems.

At the 1994 International Conference on Atomic Physics held in Boulder, Colorado, Artur Ekert presented a lecture outlining the ideas of quantum computation (Ekert, 1995), a subject new to most of the audience. This inspired Ignacio Cirac and Peter Zoller, who attended the conference and were very familiar with the capabilities (and limitations) of trapped-ion experiments, to propose a basic layout for a quantum computer utilizing trapped ions (Cirac and Zoller, 1995). This seminal paper was the first comprehensive proposal for how a quantum information processor might be realized. In their scheme, quantum bits or "qubits" are realized with two internal states of the ion, e.g., the $|\downarrow\rangle$ and $|\uparrow\rangle$ states above. The ion qubits are held in a trap shown schematically in Fig. 4. The motion of the ions is strongly coupled by the Coulomb
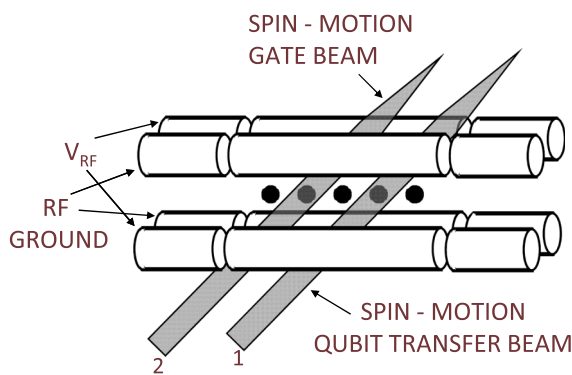

FIG. 4 (color online). Scheme for quantum computation proposed by Cirac and Zoller (Cirac and Zoller, 1995). Quadrupolar electrodes are configured to produce a linear array of trapped-ion qubits (filled black circles). Two diagonally opposite rods support an rf potential to realize a ponderomotive pseudopotential transverse to the trap's (horizontal) axis. Static potentials applied to the end segments of the electrodes confine ions along the axis. Ideally, all motional modes are laser cooled to the ground state before logic operations. The quantized modes of motion can be used as a data bus to share information between the internal-state qubits of ions that are selected by focused laser beams (see text).

interaction and is best described by the normal modes of a kind of pseudomolecule. Typically, the motion of each mode is shared among all the ions and can act as a data bus for transferring information between ions. A single-qubit gate or rotation (the relatively easy part) is implemented by applying a focused laser beam or beams onto that ion and coherently driving a carrier transition as described above. The harder part is to perform a logic gate between two selected ions. This can be accomplished by first laser cooling all modes to the ground state. The internal qubit state of one ion is then transferred onto the qubit formed from the ground and first excited state of a particular mode of motion (laser beam 1 in Fig. 4), as indicated in Eq. (5). Laser beam 2 then performs a logic gate between the (shared) motion qubit state and a second selected ion. Since the second ion is generally in a superposition state, before the gate operation is performed, the wave function for the spin and motional state of the second qubit can be written as $\alpha|\downarrow\rangle|0\rangle + \beta|\downarrow\rangle|1\rangle + \xi|\uparrow\rangle|0\rangle + \zeta|\uparrow\rangle|1\rangle$. One type of logic gate imparts a minus sign to the $|\uparrow\rangle|1\rangle$ component of the wave function by coherently driving a 2π transition $|\uparrow\rangle|1\rangle \rightarrow |aux\rangle|0\rangle \rightarrow -|\uparrow\rangle|1\rangle$, where $|aux\rangle$ is a third "auxiliary" internal state of the ion (Cirac and Zoller, 1995). Flipping the sign of the $|\uparrow\rangle|1\rangle$ component of the wave function realizes an entangling two-qubit "π-phase" gate and is universal for computation. Finally, the initial transfer step on the first ion is reversed, restoring the motion to the ground state and effectively having performed the logic gate between the internal qubit states of the two laser-beam-selected ions. At NIST, since we had recently achieved ground-state cooling with stimulated-Raman transitions on hyperfine qubit states, we were able to quickly demonstrate a universal gate between a hyperfine qubit and a motional mode qubit (Monroe, Meekhof, King, Itano, and Wineland, 1995). The complete Cirac-Zoller gate between two selected qubits was subsequently demonstrated by the Innsbruck group, led by Rainer Blatt (Schmidt-Kaler *et al.*, 2003).

More streamlined gates were subsequently devised in which multiple ions are addressed simultaneously by the same laser beams (Sørensen and Mølmer, 1999, 2000; Solano, de Matos Filho, and Zagury, 1999; Milburn, Schneider, and James, 2000; Wang, Sørensen, and Mølmer, 2001). These gates also have the advantage that it is not necessary to prepare all modes in the ground state; it is only necessary that each ion is maintained well within the Lamb-Dicke regime [$\langle z^2 \rangle \ll (\lambda/2\pi)^2$]. These "geometric" gates can be viewed as arising from quantum phases that are acquired when a mode of the ions' motion is displaced in phase space around a closed path; the phases accumulated are proportional to the enclosed area in phase space. The different gates can be viewed in a common framework, the main difference being whether or not the forces act on the spin states in the z basis (eigenstates $|\downarrow\rangle$, $|\uparrow\rangle$) or in the x, y basis [eigenstates of the form $\frac{1}{\sqrt{2}}(|\downarrow\rangle + e^{i\xi}|\uparrow\rangle)$, $\frac{1}{\sqrt{2}}(|\downarrow\rangle - e^{i\xi}|\uparrow\rangle)$] (Lee *et al.*, 2005). The forces required for the displacements are usually implemented with optical-dipole forces as in the Schrödinger cat example. Since the forces are state dependent, the differential geometric phases generate entangling gates. Two-qubit phase gates have been implemented in the z basis (Leibfried *et al.*, 2003; Home *et al.*, 2006) and in the x, y basis (Sackett *et al.*, 2000; Haljan *et al.*, 2005; Benhelm *et al.*, 2008;

Kim *et al.*, 2009). In the Innsbruck experiment of Benhelm *et al.* (2008), a Bell state with fidelity 0.993(1) was produced, setting a standard for all QIP experiments. The use of single- and multiqubit gates has enabled the demonstration of several ion-based QIP algorithms; see, for example, Blatt and Wineland (2008) and Blatt and Roos (2012). At NIST most such demonstrations were led by Didi Leibfried. Chris Monroe's group at the University of Maryland is leading efforts on an entirely different scheme for ion entanglement generation based on performing joint measurements on photons that are first entangled with ion qubits (Moehring *et al.*, 2007; Olmschenk *et al.*, 2010; Monroe *et al.*, 2012). This scheme has the advantage that the ions don't have to be in the Lamb-Dicke regime, and it also enables entanglement of widely separated qubits because of the relative ease of transferring photons over large distances.

The basic elements of the Cirac-Zoller proposal are carried forward in the different variations of trapped-ion QIP. This proposal rejuvenated the field of trapped ions and today there are over 30 groups in the world working on various aspects of quantum information processing. These include groups at the University of Aarhus; Amherst College; University of California, Berkeley; University of California, Los Angles; Duke University; ETH Zürich; University of Freiburg; Georgia Tech; Griffiths University; Imperial College; University of Innsbruck; Lincoln Laboratories; Mainz University; University of Hannover and PTB (Germany); MIT; NIST (USA); NPL (UK); Osaka University; Oxford University; Joint Quantum Institute at the University of Maryland; Université de Paris; Saarland University (Saarbrücken); Sandia National Laboratory (USA); Siegen University; Simon Fraser University; National University of Singapore; Sussex University; University of Sydney; Tsinghua University; University of Ulm; University of Washington; Wabash College; and the Weizmann Institute.

A. Quantum simulation

In the early 1980s, Richard Feynman proposed that one quantum system might be used to efficiently simulate the dynamics of other quantum systems of interest (Feynman, 1982; Lloyd, 1996). This is now a highly anticipated application of QIP and will likely occur well before useful factorization is performed. Of course, the universality of a large-scale quantum computer will allow it to simulate any quantum system of interest. However, it is also possible to use the built-in available interactions in a quantum processor to simulate certain classes of physical problems. For trapped ions, it has been possible to use the interactions employed in the various gates to simulate other systems of interest, for example, nonlinear optical systems (Leibfried *et al.*, 2002), motional quantum dynamics as in an electron's Zitterbewegung (Gerritsma *et al.*, 2010), or the properties of a "quantum walk" (Schmitz, Matjeschk *et al.*, 2009; Zähringer *et al.*, 2010). Currently, efforts are underway in several laboratories to use QIP interactions to simulate various dynamics including those of condensed-matter systems. Some of the basic ideas for how this might work with ions have been outlined in Wunderlich and Balzer (2003), Porras and Cirac (2004, 2006), Deng, Porras, and Cirac (2005), Pons *et al.*

(2007), Schätz *et al.* (2007), Chiaverini and Lybarger (2008), Taylor and Calarco (2008), Clark *et al.* (2009), Johanning, Varon, and Wunderlich (2009), Schmitz, Friedenauer *et al.* (2009), Schmied, Wesenberg, and Leibfried (2011), Blatt and Roos (2012), Britton *et al.* (2012), Korenblit *et al.* (2012), and Schneider, Porras, and Schaetz (2012). Here, logic gate interactions between ions i and j invoke a spin-spin–like interaction of the form $\sigma_{\hat{u}i}\sigma_{\hat{u}j}$, where $\hat{u} \in \{\hat{x}, \hat{y}, \hat{z}\}$. Spin rotations about a direction $\hat{u}$ act like magnetic fields along $\hat{u}$. These basic interactions have been implemented on up to 16 ions in an rf trap (Schätz *et al.*, 2007; Friedenauer *et al.*, 2008; Kim *et al.*, 2009, 2010; Edwards *et al.*, 2010; Islam *et al.*, 2012; Korenblit *et al.*, 2012). One interesting aspect of this work is the study of quantum phase transitions by varying the relative strengths of the (simulated) spin-spin and magnetic field interactions. Under appropriate conditions, the effects of spin "frustration" are now becoming apparent. The basic interactions have also been implemented on over 100 spins in a Penning trap experiment led by John Bollinger at NIST (Britton *et al.*, 2012), where the ions naturally form into a triangular array. In the Innsbruck group, simulations including engineered dissipation have also been implemented (Barreiro *et al.*, 2011; Blatt and Roos, 2012), and a striking demonstration of a digital quantum simulator has been made (Lanyon *et al.*, 2011; Blatt and Roos, 2012), in essence the first universal quantum computer.

B. Spectroscopy and quantum metrology

Some potential applications of quantum control and QIP are motivated by the idea of using entangled states to improve spectroscopic sensitivity (Wineland *et al.*, 1992, 1994; Bollinger *et al.*, 1996; Leibfried *et al.*, 2004; Roos *et al.*, 2006; Goldstein *et al.*, 2009) and demonstrations of this increased sensitivity have been made (Meyer *et al.*, 2001; Leibfried *et al.*, 2004, 2005; Roos *et al.*, 2006; Leroux, Schleier-Smith, and Vuletić, 2010; Monz *et al.*, 2011). These demonstrations were made in the limit that noise was dominated by "projection noise," the fundamental noise arising from the fluctuations in which state the system is projected into upon measurement (Wineland *et al.*, 1982; Itano *et al.*, 1993). This might be the case in a spectroscopy experiment where the interrogation time is limited by a particular experimental constraint, like the duration of flight of atoms in a cesium fountain clock or by the desire to hold the temperature of ions below a certain value if they are heated during interrogation. However, if significant phase noise is present in either the atoms themselves (Huelga *et al.*, 1997) or the interrogating radiation (Wineland *et al.*, 1998; Buzek, Derka, and Massar, 1999; André, Sørensen, and Lukin, 2004; Rosenband, 2012), the gain from entanglement can be lost. This puts a premium on finding probe oscillators that are stable enough that the projection noise dominates for the desired probe duration.

Some ions of spectroscopic interest may be difficult to detect because they either don't have a cycling transition or lack a cycling transition at a convenient wavelength. In some cases, this limitation can be overcome by simultaneously storing the ion(s) of spectroscopic interest with a "logic" ion or ions whose states can be more easily

detected. Following the Cirac and Zoller scheme, we can use the internal-to-motion-state-transfer process described above. Here, the idea is to first transfer the two states of interest in the spectroscopy ion to the ground and first excited states of a mode of the ions' coupled motion. This is then followed by mapping the motional states onto the logic ion, which is subsequently measured (Wineland *et al.*, 2002). In a project led by Till Rosenband at NIST, this technique has been used to detect optical transitions in $^{27}\text{Al}^+$ ions by transferring the relevant $^{27}\text{Al}^+$ states to a $^9\text{Be}^+$ or $^{25}\text{Mg}^+$ logic ion, which is then measured (Schmidt *et al.*, 2005). It is now used routinely in an accurate optical clock based on $^{27}\text{Al}^+$ (Rosenband *et al.*, 2008; Chou, Hume, Koelemeij *et al.*, 2010) and might also be extended to molecular ions. Currently, the $^{27}\text{Al}^+$ single-ion optical clock has the smallest systematic error of any clock at somewhat below 1 part in 10^{17} (Chou, Hume, Koelemeij *et al.*, 2010). This level of precision has enabled observations of the predictions of Einstein's general theory of relativity on a human scale, such as time dilation for bicycling speeds and the gravitational redshift for height changes of around 30 cm (Chou, Hume, Rosenband, and Wineland, 2010). Such clocks may become useful tools in geodesy.

The information transfer and readout process employed in the $^{27}\text{Al}^+/^9\text{Be}^+$ clock experiments typically had a fidelity of about 0.85, limited by errors caused by the ions' thermal motion in modes not used for information transfer [so-called "Debye-Waller" factors from Mössbauer spectroscopy (Lipkin, 1973; Wineland *et al.*, 1998)]. However, the quantum logic detection process is a QND type of measurement in that it doesn't disturb the detected populations of the $^{27}\text{Al}^+$ ion. It can therefore be repeated to gain better information on the $^{27}\text{Al}^+$ ion's (projected) state. By use of real-time Bayesian analysis on successive detection cycles, the readout fidelity was improved from 0.85 to 0.9994 (Hume, Rosenband, and Wineland, 2007). This experiment shares similarities with those of the Paris cavity-QED group, where successive probe atoms are used to perform QND measurements of the photon number in a cavity (Deléglise *et al.*, 2008). In Hume, Rosenband, and Wineland (2007), the same atom ($^9\text{Be}^+$) is reset after each detection cycle and used again. Also, because the detection was accomplished in real time, the procedure was adaptive, requiring on each run a minimum number of detection cycles to reach a certain measurement fidelity.

VII. SUMMARY

I have tried to give a brief account of some of the developments that have taken place in the area of quantum state manipulation of small numbers of trapped atomic ions. With apologies, I have omitted several aspects of this subject and for the topics discussed here, I primarily used examples from the NIST, Boulder group. Much of the other work has been discussed in various comprehensive articles and reviews; see, for example, Cirac *et al.* (1996), Wineland *et al.* (1998), Šašura and Bužek (2002), Leibfried, Blatt, Monroe, and Wineland (2003), Lee *et al.* (2005), Blatt and Wineland (2008), Duan and Monroe (2008, 2010), ; Häffner, Roos, and Blatt (2008), Kielpinski (2008), Monroe and Lukin

(2008), Blatt and Roos (2012), Korenblit *et al.* (2012), Monroe *et al.* (2012), and Schneider, Porras, and Schaetz (2012). Reviews on advanced clocks including those based on ions are contained in Gill (2005, 2011), Maleki (2008), and Margolis (2009) [see also Madej *et al.* (2012) and references therein].

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